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Seismic response of a rigid foundation embedded in a viscoelastic soil by taking into account the soil-foundation interaction

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Abstract. This study analyses the seismic response of a three-dimensional (3-D) rigid massless square foundation resting or embedded in a viscoelastic soil limited by rigid bedrock. The foundation is subjected to harmonic oblique seismic waves P, SV, SH and R. The key step is the characterization of the soil-foundation interaction by computing the impedance matrix and the input motion matrix. A 3-D frequency boundary element method (BEM) in conjunction with the thin layer method (TLM) is adapted for the seismic analysis of the foundation. The dynamic response of the rigid foundation is solved from the wave equations by taking into account the soil-foundation interaction. The solution is formulated using the frequency BEM with the Green's function obtained from the TLM. This approach has been applied to analyze the effect of soil-structure interaction on the seismic response of the foundation as a function of the kind of incident waves, the angles of incident waves, the wave's frequencies and the embedding of foundation. The parametric results show that the non-vertical incident waves, the embedment of foundation, and the wave's frequencies have important impact on the dynamic response of rigid foundations.

Keywords: harmonic seismic waves; foundation; soil; BEM-TLM; soil-structure interaction

1. Introduction

The analysis of the behavior of foundations under dynamic loads has grown considerably over the past four decades. Stringent security requirements imposed on design of certain types of structures have played a particularly important role in the development of analytical methods. The key step in studying the dynamic response of foundation is the determination of the relationship between forces. This relationship which results in displacement is expressed using impedance functions (dynamic stiffness) or the compliance functions (dynamic flexibility). The consideration of the soil-structure interaction in the analysis of the dynamic behaviour of the foundations allows

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to take realistically into account the influence of soil on their vibrations.

A myriad of methods has been proposed to solve the soil-structure interaction problem. To simplify the problem linear-analysis techniques have been developed. One of the most commonly used approaches is the substructuring method that allows the problem to be analyzed in two parts Kausel *et al.* (1978), Aubry and Clouteau (1992), Pecker (1984). In this approach the dynamic response of superstructure elements and substructure are examined separately. The analysis of foundation systems can be reduced to the study of the dynamic stiffness at the soil-foundation interface (known as impedance function) and driving forces from incident waves. The kinematic interaction of the foundation with incident waves is implemented in the form of a driving-force vector

Determining the foundation response thereby becomes a wave propagation problem. Due to the mixed-boundary conditions of the problem (displacement compatibility with stress distribution underneath the foundation and zero tension outside) solutions are complex. The determination of impedance functions and forces of movement related to the incident waves is a complex process. Several studies have been conducted on the dynamic response of foundation using the finite-element and boundary-element methods. Wong and Luco (1978) have shown the importance of the effect of non-verticality of SV, SH harmonics on the response of a foundation.

Apsel and Luco (1987) used an integral-equation approach based on Green's functions for multilayered soils determined to calculate the impedance functions of foundation. Using this approach, Wong and Luco (1986) studied the dynamic interaction between rigid foundations resting on a half-space. Boumekik (1985) studied the problem of 3-D foundations embedded in soil limited by a rigid substratum. The finite-element method was applied by Kausel *et al.* (1978), Kausel and Roesset (1981), Lin *et al.* (1987) to determine the behavior of rigid foundations placed on or embedded in soil layer limited by a rigid substratum. A formulation of the boundary-element method in the frequency domain has been developed to address wave-propagation problems of soil-structure interaction and structure-soil-structure which limits the discretization at the interface soil-foundation. In this approach, the field of displacement is formulated as integrals equation in terms of Green's functions Beskos (1987), Aubry and Clouteau (1992), Qian and Beskos (1996), Karabalis and Mohammadi (1991) and Mohammadi (1992). Celebi *et al.* (2006) used the boundary-element method with integral formulation (BIEM) to compute the dynamic impedance of foundations. In this context, the analytical solutions of 3-D wave equations in cylindrical coordinates in layered medium with satisfying the necessary boundary conditions are employed by Liou (1992), Liou and Chung (2009). However, Chen and Hou (2015) used a modal analysis to evaluate dynamic vertical displacements of a circular flexible foundation resting on soil media subjected to horizontal and rocking motions.

Sbartai and Boumekik (2008) used the BEM-TLM method to calculate the one hand, the dynamic impedance of rectangular foundations placed or embedded in the soil layered limited by a substratum and also the propagation of vibrations in the vicinity of a vibrating foundation. In this study, the method has been applied to analyze the effect of some parameters on the dynamic response of the foundations (depth of the substratum, embedding, masses and shape of the foundation, soil heterogeneity and frequency). However, Sbartai (2015) have studied the dynamic response of two square foundations placed or embedded in soil layered limited by a substratum.
Spryakos and Xu (2004) have developed a hybrid BEM-FEM and have conducted several studies for parametric analysis of soil-structure interaction.

McKay (2009) used the reciprocity theorem based on the boundary integral element method (BIEM) to analyze the influence of soil-structure interaction on the seismic response of foundations. However, Suarez *et al.* (2002) applied the BIEM to determine the seismic response of an L-shaped foundation.

In addition, experimental work has been carried out by researchers in Japan to determine the effect of soil-structure interaction on the response of real structure Fujimori *et al.* (1992), Akino *et al.* (1996), Mizuhata *et al.* (1988), Watakabe *et al.* (1992), Imamura *et al.* (1992).

Recently, Lee *et al.* (2012) have proposed a procedure that combines a consistent transmitting boundary with continued-fraction absorbing boundary conditions to compute the impedance functions and input motions of foundations in layered half-space. However, Kazakov (2012) proposed elastodynamic infinite elements with united shape functions (EIEUSF) to study its effectiveness in solving soil-structure interaction problems.

In our study, the solution is derived from the BEM in the frequency domain with constant quadrilateral elements and the TLM is used to analyze the influence of soil-structure interaction on the response of seismic foundation. The results are presented as coefficients of movement and in terms of displacement as a function of dimensionless frequency, angle of incidence (vertically and horizontally) and embedding of foundation. This paper represents a continuation of the paper was previously published in Sbartai and Boumekik (2008) where the impedance functions have been well studied in details, but they will not be discussed in this article.

2. Harmonic seismic response of the foundations

2.1 Physical model and basic equations

Thegeometry of the calculation model is shown in Fig. 1. Considering a 3-D rigid massless square foundation resting or embedded in a viscoelastic soil limited by rigid bedrock. The soil is characterized by its density ρ, shear modulus μ, damping coefficient β and Poisson's ratio ν. The foundation is subject to harmonic oblique-incident waves that are time-dependent: *P*, *SV*, *SH* and *R*.

The movement of a not-specified-point "ζ" can be obtained from solving the wave equation

$$(C_P^2 - C_S^2)u_{j,ij} + C_S^2 u_{i,jj} - \omega^2 u_i = 0, \qquad (1)$$

where,

C_s, and C_p are the velocities of shear and compression waves and ω the angular frequency of excitation; u_i is the component of the harmonic displacement-vector in the *x*-direction; $u_{j,ij}$ is the partial derivative of the displacement field with respect to *x* and *y* and $u_{j,ij}$ is the second partial derivative of the displacement field with respect to y.

The solution of Eq. (1) may be expressed by the following integral equation

$$u_j(x,\omega) = \int_S G_{ij}(x,\xi,\omega)\ T_i(\xi,\omega)\ ds(\xi) \qquad (2)$$

where,

G_{ij} denoting the Green's functions at point *i* due to unit-harmonic load (vertical and horizontal) of at point *j* and T_i being a load (traction) distributed over an area of soil.

If the medium is continuous the Eq. (2) is very difficult to assess. However, if the soil mass is discretized appropriately, this relationship can be made algebraic and displacement can be

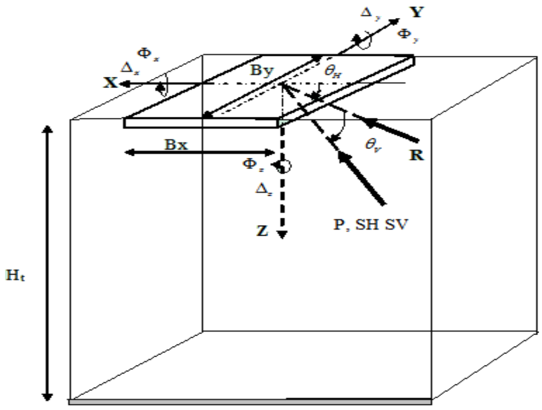

Fig. 1 Geometry of a foundation subjected to harmonic seismic

calculated. The key step of this study is to determine the impedance matrix linking the harmonic forces applied to the resulting harmonic displacement. With a continuous medium the determination of the impedance matrix is still very difficult due to the wave propagation problem and of the mixed-boundary conditions. However, if the medium is discretized vertically and horizontally then it is possible to making the problem algebraic by considering that the variation of interface displacement is a linear function.

2.2 Discretization of the model

The principle of horizontal and vertical discretization of soil mass is shown in Fig. 2. The principle of vertical discretization based on the division of every soil layer into a number of sub-layers of height h_j with similar physical characteristics. Each sublayer is assumed to be horizontal, viscoelastic, and isotropic, and characterized by constant of Lame λ_j, a shear modulus μ_i and a density ρ_j. The bedrock at depth H_t is considered infinitely rigid wherethe reflection of waves is assumed to be total and the displacements are null.

Within a given sublayer, the displacement is assumed to be a linear function. This is true when the thickness of the sublayer is small in relation to the wavelength considered (in the order of $\lambda/10$). This method is comparable to the finite element method (FEM) in the sense that the movements within each sublayer are completely defined from the displacements in the middle of the interfaces. The interaction between the elements is done only through the nodes. The degrees of freedom of the soil mass are reduced to the degrees of freedom of the nodes.

The stiffness matrix of soil mass is obtained in a similar manner to that obtained by the FEM. This technique has been developed by Lysmer and Waas (1972) and is known as the thin-layer method (TLM) and is used mainly for horizontal soil layers. This method has the advantage to making the problem algebraic and thus obtains the Green's functions by applying the BEM in the soil-foundation interface. For this reason a horizontal discretization of the interface soil-foundation is established. The horizontal discretization subdivides any horizontal interface by square elements of sections S_k. By seeking the simplicity of integration calculation and economy of computing time, the square elements are approximated by disc elements. If the units loads (along the direction x, y, z) are applied to disc j, the Green's functions at the center of disc i can be determined. By successively applying these loads on all discs, the matrix of flexibility of soil at a given frequency

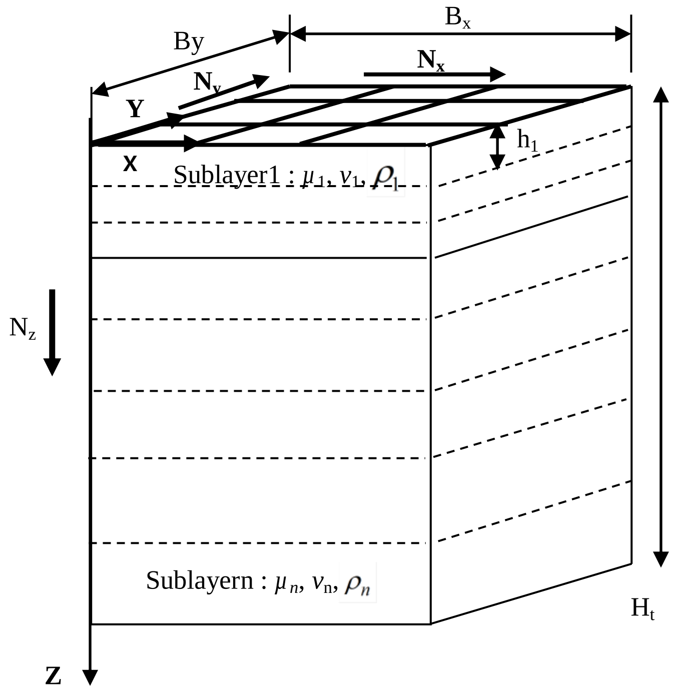

Fig. 2 The model of calculation

ω can be formed.The discretized model to calculate the impedance functions of the foundation is presented in Fig. 2. In the discrete model, Eq. (2) is expressed in algebraic form as follows

$$u_j = \sum_{i=1}^{N} \int_S G_{ij} \; T_i \; ds \tag{3}$$

2.3 Determination of Green's Functions by the TLM

The Green's function for a layered stratum is obtained by an inversion of the thin-layer stiffness matrix using a spectral-decomposition procedure of Kausel and Peek (1982). The advantage of the thin-layer-stiffness matrix technique over the classical transfer-matrix technique for finite layers and the finite-layer-stiffness matrix technique of Kausel and Roesset (1981) is that the transcendental functions in the layered-stiffness matrix are linearized.

In this work, the body (B) represents a layered stratum resting on a substratum base with n horizontal layer interfaces defined by z=z_1, z_2,...,z_n and with layer j defined by z_n<z< z_{n+1}, as shown in Fig. 3. The medium of each layer n of thickness h_n is assumed to be homogeneous, isotropic, and viscoelastic. For this body, the Green's function in frequency domain is obtained with help of the TLM method.

According to the thin-layers theory of Lysmer and Waas (1972), displacements in each sub-layer vary linearly from one plane to another and still continue in the relevant direction (x, y, z). Thus, the displacements in each sublayer are obtained by linear interpolation of nodal displacements at the interface of the sublayer n as follows

$$U^{(n)}(z)=(1-\eta)U^{n}+\eta\ U^{n+1} \tag{4a}$$

$$V^{(n)}(z)=(1-\eta)\ V^{n}+\eta\ V^{n+1} \tag{4b}$$

$$W^{(n)}(z)=(1-\eta)\ W^{n}+\eta\ W^{n+1} \tag{4c}$$

where,

$\eta=\dfrac{(z-Zn)}{h_n}$ with ($0\leq\eta\leq1$), and, $U^{(n)}$, $V^{(n)}$ et $W^{(n)}$ are the displacements along the x-axis, the y-axis and the z-axis as functions of z in the layer j, and with U^n, V^n et W^n which are their nodal values at the interface layer $z=z_n$.

$$G_{ij}^{mn}=\sum_{l=1}^{2N}\frac{a_{\alpha\beta}\cdot\phi_i^{ml}\cdot\phi_j^{nl}}{k^2-k_l^2}\,, \tag{5}$$

with

$a_{\alpha\beta}=1$ if $\alpha=\beta$ and $a_{\alpha\beta}=\dfrac{k}{k_l}$ if $\alpha\neq\beta$ .i, j = x, y, z ; k, k_l: wave number; m: represents the interface where the load is applied and n: represents the interface where Green's functions are calculated.

The obtained Green's functions are complex and constitute the starting point for the determination of the flexibility matrix of an arbitrary soil volume. However, considering the geometry of the foundation, a system of cartesian coordinates was adopted. The obtained U, V, and W Green's functions are in fact the terms of the flexibility matrix of the soil. The determination of this flexibility matrix gives the impedance function of one or several foundations and the amplitudes of vibrations in the neighborhood of a foundation Sbartai and Boumekik (2008) and Sbartai (2015).

Viscoelastic soil behavior can be easily introduced in the present formulation by simply replacing the elastic constants λ and G with their complex values

$$\lambda^*(Z)=\lambda(1+2i\beta) \tag{6a}$$

$$\mu^*(Z)=\mu\ (1+2i\beta)\,, \tag{6b}$$

with β is the hysteretic damping coefficient.

3. Calculation model

The total displacement matrix of the soil is obtained by successive application of the unit loads on all elements of the discretized volume of foundation. The displacements in the soil are then expressed by

$$\{u\}=[G]\{t\} \tag{7}$$

where,

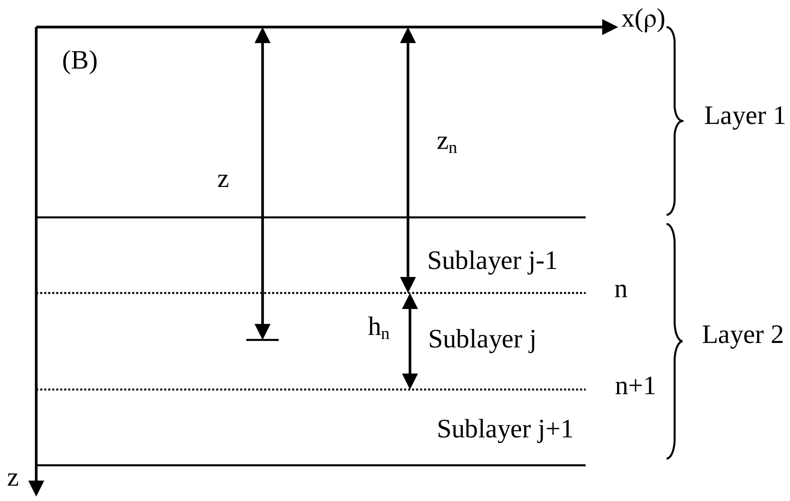

Fig. 3 Geometry of soil layers (B)

the vectors $\{u\}$ and $\{t\}$ are the nodal values of the amplitudes of displacements and tractions respectively at the interface soil-foundation; $[G]$ is the flexibility matrix of the soil.

When the foundation is in place, it requires different components of soil displacement consistent with rigid body motions. Compatibility of displacements at the contact area S between the soil and the rigid foundation leads to the matrix equation

$$\{u\} = [R]\{\Delta\}, \tag{8}$$

where,

$[R]$ is the transformation matrix

$$[R] = \begin{bmatrix} 1 & 0 & 0 & 0 & z & y \\ 0 & 0 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix} \tag{9}$$

and $\{\Delta\}=\{\Delta_x, \Delta_y, \Delta_z, \Phi_x, \Phi_y, \Phi_z\}$ is the displacement vector with Δ_i ($i=x,y,z$) represents translations and Φ_i ($i=x,y,z$) rotations (see Fig. 1).

If we denote $\{P\}$ the vector of load applied to the foundation, the equilibrium between the vector of applied loads and the forces (tractions) distributed over the elements discretizing the volume of the foundation is expressed by the following equation

$$\{P\} = [R]^t \{t\} \tag{10}$$

Combining Eqs. (7), (8) and (10) we obtain the following equation

$$\{P\} = \left([R]^t [G]^{-1} [R]\right)\{\Delta\} = [K(\omega)]\{\Delta\}. \tag{11}$$

Where,
ω is the circular frequency of vibration and $[K(\omega)]$ is the dynamic-stiffness matrix.

Considering the harmonic seismic waves P, SV, SH and R characterized by their incident angles (vertical and horizontal) θ_V and θ_H respectively (see in Fig. 1). The motion of the half-space due to these seismic waves can be expressed by the following equation

$$\{u^f\} = \{U^f\} e^{[-i\omega(x.\cos\theta_H + y.\sin\theta_H)/c]}, \tag{12}$$

where,
$\{U^f\}=\{U_x^f, U_y^f, U_z^f\}^t$, is known as the vector of amplitudes of the soil, that depends on the z coordinate if we want to study the embedded foundations case. However, in the case of surface foundations (z=0), it is known as the vector of amplitudes of the free field; c is the apparent velocity of the incident waves having the form $c = \dfrac{c_1}{\cos\theta_V}$ *or* $c = \dfrac{c_2}{\cos\theta_V}$ for P or S waves, respectively, and being equal to the R-wave. The explicit expressions of the vector $\{U^f\}$ of waves SH, P, SV and R may be found in Wong and Luco (1986).

The presence of a rigid foundation results a diffraction waves so that the resulting total displacement field $\{u\}$ is expressed by the following equation

$$\{u\} = \{u^f\} + \{u^s\}, \tag{13}$$

where,
$\{u^s\}$ is the scattered wave field that satisfies the equation of motion Eq. (7). Also, the total displacement field in the contact region between the foundation and the half space must be equal to the rigid body motion of the foundation.

Substituting Eq. (8) into Eq. (13), written in terms of the scattered field leads to the force-displacement relation

$$\{u^s\} = [R]\{\Delta\} - \{u^f\} \; . \tag{14}$$

Substituting Eq. (7) into Eq. (14), written in terms of the traction forces

$$\{t\} = [G]^{-1}[R]\{\Delta\} - [G]^{-1}\{u^f\} \tag{15}$$

Multiplying both sides of the Eq. (15) by the transpose of the transformation matrix

$$[R]^t\{t\} = [R]^t[G]^{-1}[R]\{\Delta\} - [R]^t[G]^{-1}\{u^f\} \tag{16}$$

The equilibrium between external forces and seismic forces and combining with Eqs. (10) and (11) yields the external forces can be as follows

$$\{P\} = [K]\{\Delta\} - [K^*]\{U^f\}, \tag{17}$$

with $[K^*]$ is the driving force matrix given by the following formula

$$[K^*] = [R]^t[G^{-1}]e^{[-i\omega(x.\cos\theta_H + y.\sin\theta_H)/c]}. \tag{18}$$

Eq. (14) can be replaced by the alternative form

$$\{\Delta\} = [C]\{P\} + [S^*]\{U^f\}, \tag{19}$$

where,
$[C]=[K]^{-1}$ is the dynamic compliance matrix and $[S^*]$ is the input motion matrix given by the following formula

$$\left[S^*\right]=\left[C\right]\left[K^*\right] \tag{20}$$

When the rigid foundation is acted upon by seismic waves only, the external forces are null ($\{P\}=0$), and the seismic response of the foundation is obtained from Eq. (17) or Eq. (19) by the following expression

$$\{\Delta\}=\left[S^*\right]\left[U^f\right], \tag{21}$$

where

$$\left[S^*\right]=\begin{bmatrix} S_{xx} & 0 & 0 \\ 0 & S_{yy} & S_{yz} \\ 0 & S_{zy} & S_{zz} \\ 0 & R_{xy} & S_{xz} \\ R_{yx} & 0 & 0 \\ S_{zx} & 0 & 0 \end{bmatrix} \tag{22}$$

When the mass of the foundation is not null, one replace simply $[K]$ by $[K]-\omega^2[M]$ in the above equations, where $[M]$ is the mass matrix of the foundation.

4. Results

4.1 Validation of the method

The accuracy of the method BEM-TLM used to study the 3D-response of foundations subject to plane-harmonic waves with variable angles of incidence and vibration frequency a_o in this section is validated through comparisons with results obtained by Luco and Wong (1977), Qian and Beskos (1996), for a semi-infinite ground. A parametric study was conducted to define the parameters of the calculation model. The influence of the discretization of the soil-foundation interface was studied. The thickness of a sublayer h must be small enough that the discrete model can transmit waves in an appropriate manner and without numerical distortion. This size depends on the frequencies involved and the velocity of wave propagation. The frequency of loading and velocity of wave propagation affect the precision of the numerical solution. Kausel and Peek (1982) showed that the thickness of sub-layer must be smaller than a quarter of the wave length λ. Consequently, the maximum dimensionless frequency must not exceed the number of sub-layer N divided by four.

Considering a rigid, massless and square foundation ($B_x=B_y=2a$) placed to the surface of a half-space with a Poisson's ratio $\nu=1/3$ and subjected to harmonic waves *P*, *SV*, and *SH* ($\theta_H=90°$ and $\theta_V=45°$). Fig. 4 shows the variation of the real and the imaginary part of coefficient S_{xx} movement based versus the dimensionless frequency $a_o = \omega.a/C_s$. The results obtained by the proposed

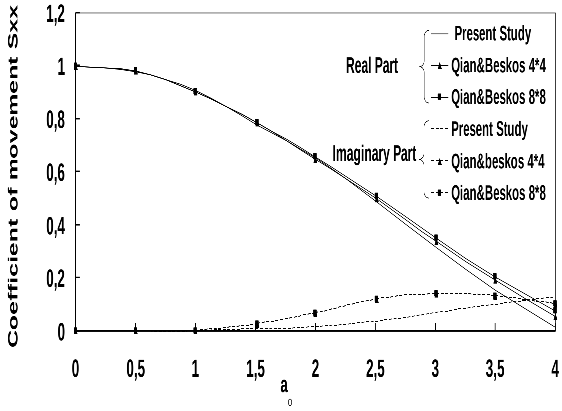

Fig. 4 The coefficient of movement S_{xx} a square foundation (θ_H=0°, θ_V=45°and c_s/c=0.70711)

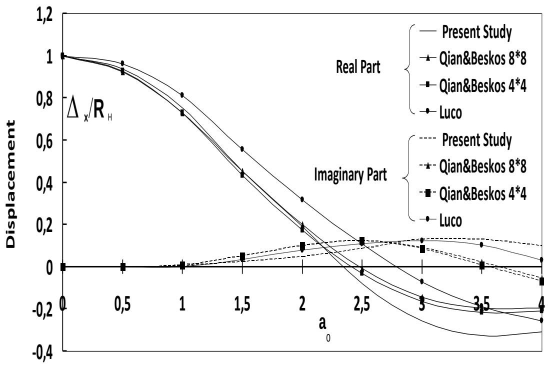

Fig. 5 The response of a square foundation under the Rayleigh wave c_R/c=0.9325

method are in good agreement with those obtained by the method used by Qian and Beskos (1996).

Considering the same foundation subjected to a Rayleigh wave where the angle of incidence is horizontal (θ_H=0) and the corresponding velocity taken is equal to c_R= 0.9325c for a Poisson's ratio v=1/3. Fig. 5 shows the real and the imaginary part of dimensionless displacement $\Delta x/H_R$ versus dimensionless frequency a_o.

The results of this study were compared with those of Qian and Beskos (1996) and Luco and Wong (1977). The results obtained are in good agreement with those of Qian and Beskos (1996) and Luco and Wong (1977). However, a difference is present only for dimensionless frequencies a_o higher than 2.5.

4.2 Parametric study and discussion

4.2.1 Surface foundation on homogeneous soil

In this section, a parametric analysis was performed by studying a square foundation of the side (B_X=2a) subjected to plane-harmonic waves with variable angles of incidence and vibration frequency a_o is presented (see in Fig. 1). The results are presented in terms of coefficients of motion as functions of the dimensionless frequency a_o. The soil is characterized by the height H_t=10 m of the bedrock to simulate a semi-infinite (To simulate a semi-infinite medium Ht/r must be superior or equal to 20), its Poisson's ratio v=1/3, its coefficient of the hysteretic damping β=0.05, its shear modulus μ=1, and its density ρ=1.The terms of coefficient of motion are presented in the Figs. 6-10 for the shear S-wave , with an horizontal angle of incidence θ_H=90° and vertical angle of incidence θ_V=0°, 30°, 45°, 60° and 90°.

Coefficients of movements in translation

For an angle of incidence θ_v=90°, the coefficients of translational movement S_{xx}, S_{yy}, S_{zz}, are equal to unity for all frequencies. Typically this value is adopted in the study of a structure subjected to seismic loading. This value induces oversized foundations. Figs. 6 and 7 show that for other angles these coefficients vary with the dimensionless frequency. The amplitude of the

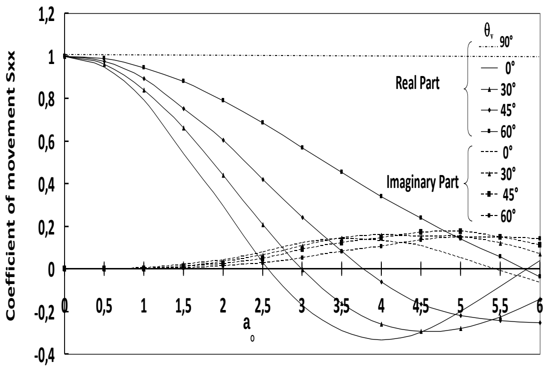

Fig. 6 The coefficient of movement S_{xx} (θ_H=90°)

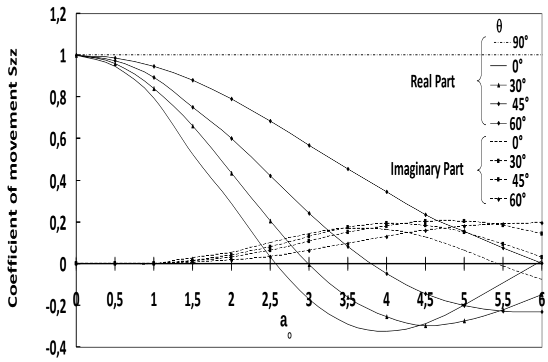

Fig. 7 The coefficient of movement S_{zz} (θ_H=90°)

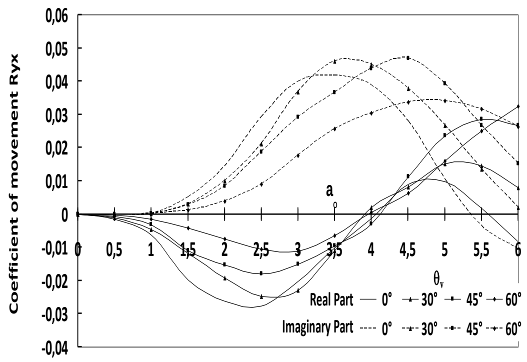

Fig. 8 The coefficient of movement R_{yx} (θ_H=90°)

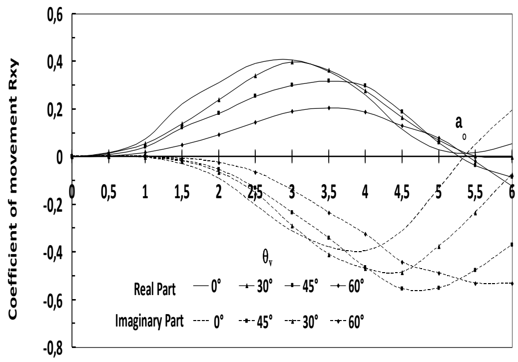

Fig. 9 The coefficient of motion R_{xy} (θ_H=90°)

response also depends on the vertical angle of incidence.

The results show that the real parts of S_{xx} and S_{zz} have a higher magnitude than the value of the imaginary parts. With low frequencies, the response is in phase with the free-field motion. These coefficients filter low frequencies and therefore behave as low-pass filters.

Coefficients of movements in rotation and torsion

Figs. 8 and 9 present the relative coefficients of the rotational movement R_{xy} and R_{yx} to the *x*-axis and *y*-axis respectively. For an angle of incidence θ_v=90°, the coefficients of rotational movement (R_{xy}, and R_{yx}) are zero and maximum for θ_v=0°. Fig. 10 shows the relative coefficient of torsion S_{zx} to the axis of *z* as a function of dimensionless frequency and the vertical angle of incidence θ_v. The results obtained show that the value of the imaginary part of S_{zx} is dominant, which shows a large damping. Thus, the answer is out of step with the free-field motion in the center of the foundation.

Figs. 8, 9, and 10 present these coefficients for other angles of incidence (0°, 30°, 45°, and 60°) and show that they vary depending on the dimensionless frequency. The amplitude of the response also depends on the vertical angle of incidence. The coefficient of movement of rotation and torsion filters high frequencies and therefore behaves as high-pass filters.

The matrix coefficients of movement shown in Figs. 7-10 are valid only for *S* waves with a

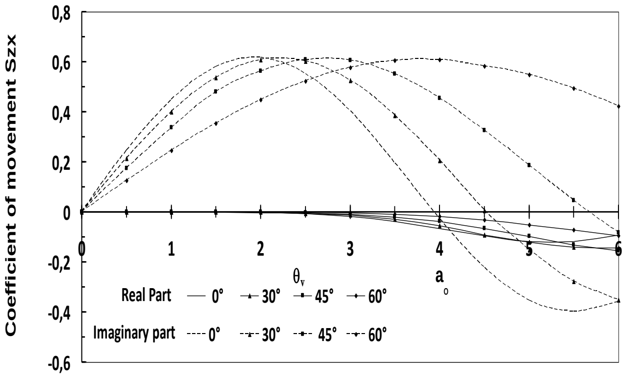

Fig. 10 Coefficient of movement S_{zx} (θ_H=90°)

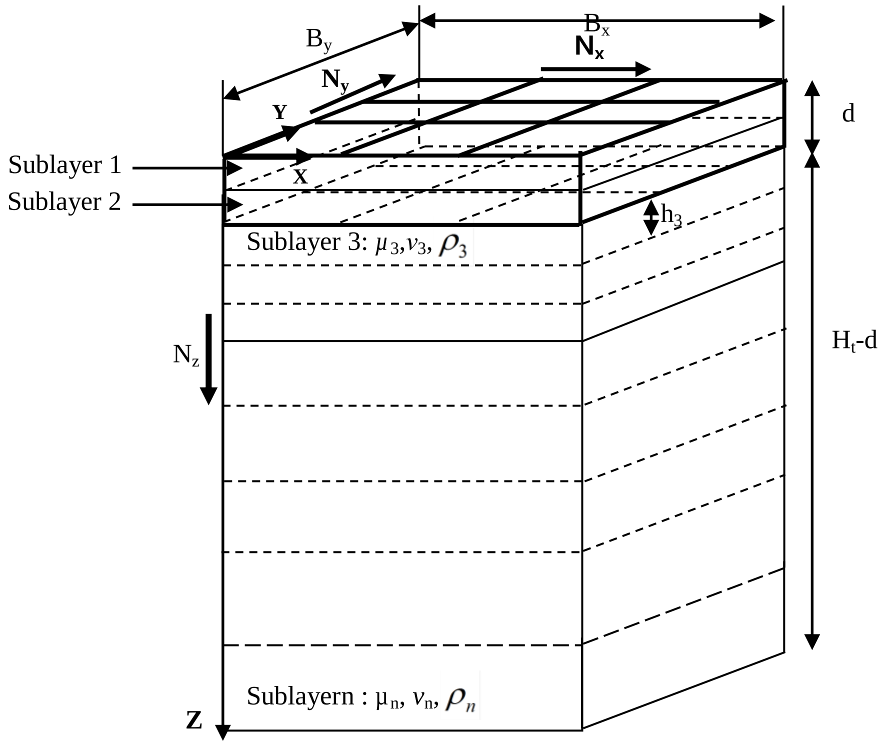

Fig. 11 Model of calculation of an embedded foundation subjected to harmonic seismic waves

horizontal angle of incidence θ_H =90°. These coefficients can be determined for other incidence angles and other types of waves.

4.2.2 Embedded foundation on homogeneous soil

To present results that are easier to understand visually, the driving-force vectors are converted to input-motion vectors by multiplying by the inverse of the impedance matrix. Three different sets of results are given. The response of the massless foundation to incident body waves of type *SH*, *P* and *SV* are considered for a square foundation dimension B_x=2*a* embedded in a homogeneous viscoelastic soil to a depth *d* (see in Fig. 11). To simplify the presentation of this paper, only one direction of wave propagation is considered with the vertical incident angle θ_v=45.

In this section, the influence of the embedding ratio (*t*=*d/a*=0, 0.3, 0.6) on the seismic response of the foundation is studied. The results are presented in terms of displacements, rotations, and torsion.

Compression wave P

Figs. 12-14 show the response of a massless foundation to an incident *P*-wave. The wave travels in the *x*-direction with its particle motion in the *z* and *x*-directions. One angle of incidence is considered θ_v=45°, where is measured with respect to the *x*-axis. The wavelength of the incident *P*-wave is twice as long as that of the incident *S*-waves; therefore, the kinematic interaction is less prominent. In general for an isotropic and homogenous medium, the *P*-wave induces displacement along the *x* and *z*-axes and rotation around the *y*-axis. Figs. 12-14 show the variation of displacement and rotation as a function of dimensionless frequency and the influence of embedding coefficient *t* on the motion of foundation.

The displacements (Δ_x, Δ_z) and rotation (ϕ_y) are strongly attenuated due to the increase of embedded foundation. Another, the horizontal displacement Δ_x is more affected by the presence of embedding than the vertical displacement Δ_z and rotation ϕ_y. The imaginary part of the two modes of translation (vertical and horizontal) is not affected by the increase of embedding. In contrast, the imaginary part of the rotation is strongly affected by the presence of embedding.

Shear wave SV

Figs. 15-17 show the response of a massless foundation subjected to an incident *SV*-wave. The wave travels in the *x*-direction with its particle motion in the *z* and x-directions. For a vertical incident angle θ_v=45°, the free-field motion for the SV-wave in the direction of propagation is zero. Similar to the P-wave case, only the horizontal, vertical, and rocking components are excited. The variation of displacement and rotation as a function of frequency, and show the influence of embedding on the motion of the foundation is presented. Fig. 15 shows that the displacement Δ_x

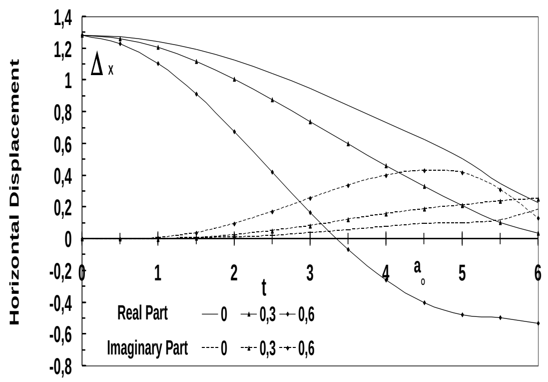

Fig. 12 Horizontal input motion Δ_x due to incident *P*-waves (θ_v=45°, θ_H=0°).

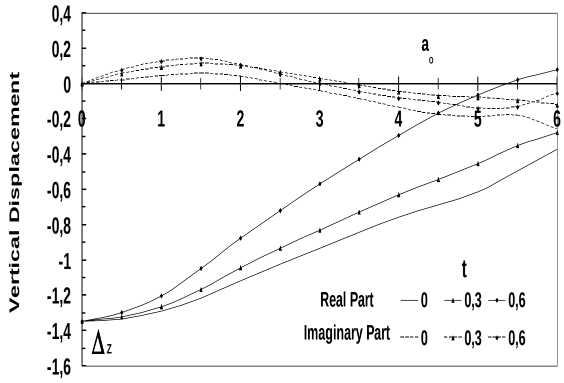

Fig. 13 Vertical input motion Δ_z due to incident *P*-waves (θ_v=45°, θ_H=0°)

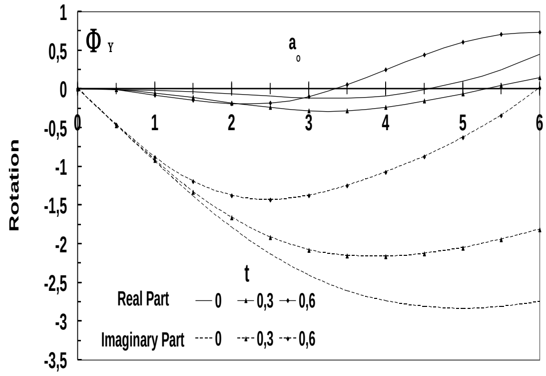

Fig. 14 Rocking input motion ϕ_y due to incident *P*-waves (θ_v=45°, θ_H=0°)

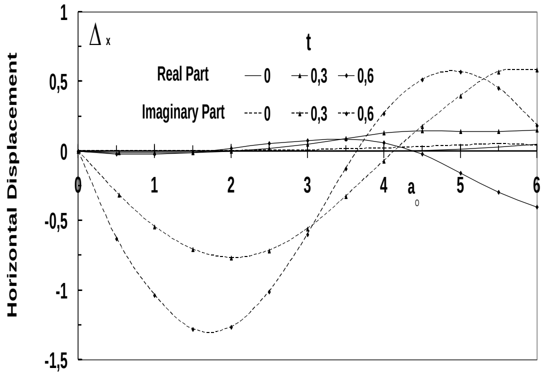

Fig. 15 Horizontal input motion Δ_x due to incident *SV*-waves (θ_v=45°, θ_H=0°)

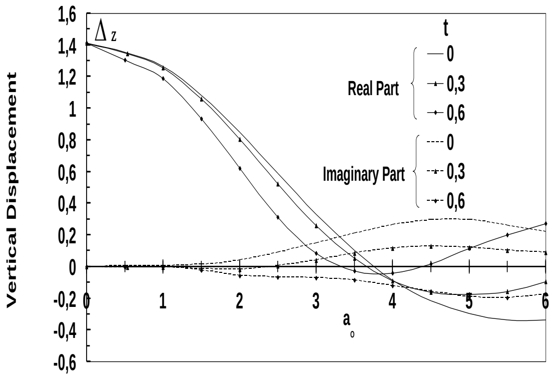

Fig. 16 Vertical input motion Δ_x due to incident *SV*-waves (θ_v=45°, θ_H =0°)

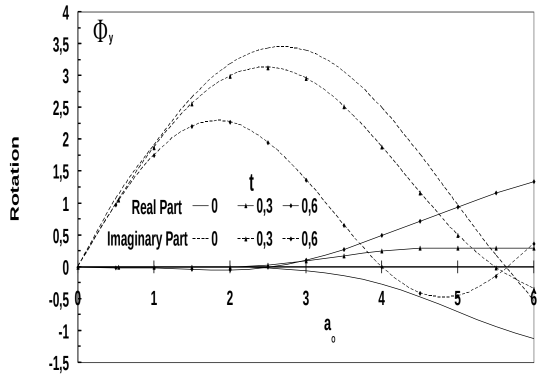

Fig. 17 Rocking input motion ϕ_y due to incident *SV*-waves (θ_v=45°, θ_H=0°)

is zero for a surface foundation (*t*=0) and becomes non-zero for relative embedding (*t*=0.3 and 0.6) with an imaginary part that is strongly affected, indicating that the foundation does not follow the free-field motion. Moreover the horizontal displacement Δ_x is more affected by the presence of embedding than the vertical displacement Δ_z and the rotation ϕ_y especially for low frequencies. The presence of the embedded foundation changes the sign of vertical displacement and rotation after the frequency a_o=4.

Shear wave SH

The response of the square, massless foundation to a *SH*-wave is presented in Figs. 18 and 19, with a horizontal angle of incidence θ_H=90°.

The incident wave travels in the *y*-direction, therefore the particle motion of the wave is in the *x*-direction. The shear wave causes displacement and torsion. Figs. 18 and 19 show the embedment influence on displacement and torsion.

For dimensionless frequencies lower than 3, the horizontal displacement Δ_x is not affected by increasing the relative embedding. This is not the case for torsion ϕ_z, which is strongly affected by an increase in the embedding. In contrast, for frequencies superior to 3, the presence of the

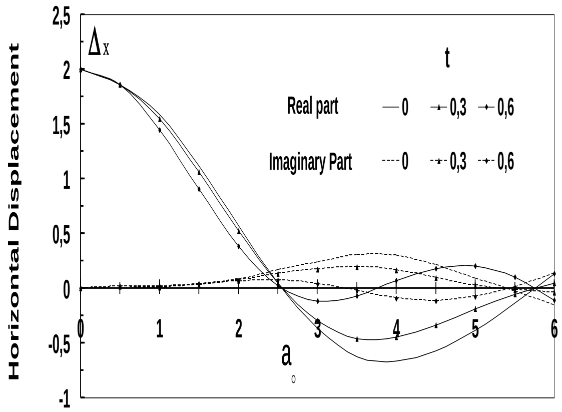

Fig. 18 Horizontal input motion Δ_x due to incident *SH*-waves (θ_v=45°, θ_H=90°)

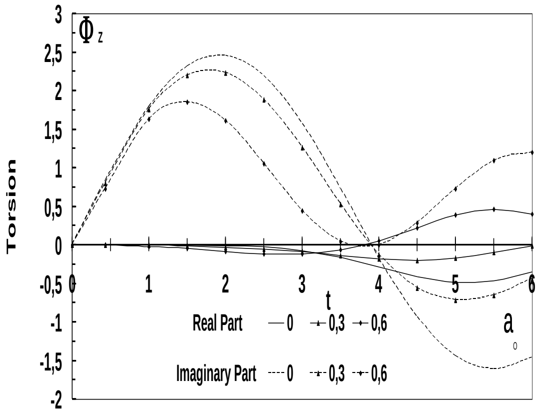

Fig. 19 Torsion-input motion ϕ_z due to incident *SH*-waves (θ_v=45°, θ_H=90°)

foundation embedding causes a change signs for Δ_x and ϕ_z.

In conclusion, the Figs. 12-19 show that the horizontal displacement caused by *P*-wave is more attenuated than the displacement caused by *SH* and *SV*-waves. However, the rotation caused by *SV*-wave is more attenuated than the rotation caused by *P*-wave.

5. Conclusions

The interaction of a seismic square-rigid foundation placed and embedded in a homogeneous viscoelastic soil and subjected to obliquely incident harmonic *P*, *SV*, and *SH*-waves was implemented. A simplified BEM-TLM was developed and used to calculate the foundation-input motion under different travelling seismic waves. The solution was formulated by the boundary-element method in the frequency domain using the formalism of Green's functions. Constant quadrilateral elements were used to study the seismic response of a foundation. The efficiency of this technique was confirmed by comparison with previous studies. This remarkably simple technique was concluded to be both highly effective and economical to determine input motions for rigid foundations of arbitrary geometry. The originality of the method lies first in the insignificance of the number of elements used in the discretization of the model, and second, in the ability to simulate an embedding foundation.

This study shows the importance of the inclination of incident waves on the behavior of a foundation. The results indicate that:

- The response of a foundation subject to non-vertical incident waves is different from that of a foundation subject to vertical-incident waves.
- Non-vertical incident waves generate the torsion, translation, and rotation. Vertical incident waves cause the translation.
- A vertical angle of incidence equal to θ_V=0° leads to an oversizing of the foundations.
- Coefficients of translational movement filter low frequencies while coefficients of rotation filter at high frequencies.
- Embedment of foundation affects displacement, rotation, and torsion in more specific ways

and acts as a favorable factor in the seismic response of foundations. The movement of the foundation is strongly attenuated for relatively deep embedded, especially at low frequencies.

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