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Dynamic stiffness approach and differential transformation for free vibration analysis of a moving Reddy-Bickford beam

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Abstract. In this study, the free vibration analysis of axially moving beams is investigated according to Reddy-Bickford beam theory (RBT) by using dynamic stiffness method (DSM) and differential transform method (DTM). First of all, the governing differential equations of motion in free vibration are derived by using Hamilton's principle. The nondimensionalised multiplication factors for axial speed and axial tensile force are used to investigate their effects on natural frequencies. The natural frequencies are calculated by solving differential equations using analytical method (ANM). After the ANM solution, the governing equations of motion of axially moving Reddy-Bickford beams are solved by using DTM which is based on Finite Taylor Series. Besides DTM, DSM is used to obtain natural frequencies of moving Reddy-Bickford beams. DSM solution is performed via Wittrick-Williams algorithm. For different boundary conditions, the first three natural frequencies that calculated by using DTM and DSM are tabulated in tables and are compared with the results of ANM where a very good proximity is observed. The first three mode shapes and normalised bending moment diagrams are presented in figures.

Keywords: axially moving beam; Reddy-Bickford beam theory; dynamic stiffness method; differential transform method; free vibration analysis; natural frequency

1. Introduction

Dynamic analysis of axially moving beams has became important for engineering systems due to developments in industrial technology. A large literature exists about investigation of free vibration analysis of axially moving beams using Euler-Bernoulli beam theory (EBT) and Timoshenko beam theory (TBT). Wickert and Mote (1989) derived the governing equations of motion of axially moving Euler-Bernoulli beams. In this study, the variation of the total mechanical energy between supports is investigated for simply supported and fixed supported beams. Özkaya and Öz (2002) investigated the free vibration analysis of axially moving simply supported beams according to EBT by using artificial neural networks (ANN). In this research, the axial speed is assumed as harmonically varying about a constant speed. The input data of ANN are flexural stiffness, mean axial speed and speed fluctuation frequencies. The authors stated that the results obtained from ANN are close to analytical solutions. Lee *et al.* (2004) researched the free

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vibration analysis of axially moving Timoshenko beams by using exact Dynamic Stiffness Method (DSM). In this research, the effects of axial speed and axial tensile force on vibration characteristics are investigated analytically. Banarjee and Gunawardana (2007) investigated the free vibration analysis of axially moving Euler-Bernoulli beams by using DSM. In this study, the first three natural frequencies are calculated for simply supported and fixed supported beams with different axial speed and axial tensile force values. The fundamental mode shapes are figured for both two boundary conditions. Chen *et al.* (2010) studied on dynamic stability of axially accelerating viscoelastic Timoshenko beams using with Kelvin viscoelastic model. The effects of shear deformation and rotational inertia on stability conditions are emphasized with graphs. In another study about axially accelerating viscoelastic Timoshenko beams, periodic responses of system are investigated by using Galerkin method and fourth order Runge-Kutta algorithm (Yan *et al.* 2014). Bağdatli *et al.* (2011) obtained natural frequencies of simply supported two span Euler-Bernoulli beam by using perturbation techniques. In this study, the influences of axial speed, flexural rigidity and intermediate support on dynamic behaviour of the beam are investigated. In the literature summarized above, EBT or TBT are considered for the free or forced vibration analysis. The free vibration analysis of axially moving beams according to high order shear deformation theories has not been investigated by any of the researches yet. In this study, Reddy-Bickford beam theory (RBT), also known as parabolic shear deformation theory, is used.

Even the shear deformation and rotational inertia are considered in TBT, the assumption about the cross section that remains plane after bending is still valid like EBT. The more realistic behaviour of cross sections of beams under bending is described by high order shear deformation theories. In the last years, studies about high order shear deformation theories have been noticed by many of the researchers. Levinson (1981) obtained the governing equations of motion of beams for different boundary conditions by using third order shear deformation theory which predicts that the cross section does not remain plane after bending. The displacement and moment functions are compared with TBT. Bickford (1982), Reddy (1984) independently presented a new shear deformation theory. The study of Bickford is restricted for beams. However, the research of Reddy is valid for laminated composite plates. Heyliger and Reddy (1988) investigated the linear and nonlinear transverse vibrations of rectangular beams using high order beam theory for different boundary conditions. In this study, it is stated that high order beam theories provide successful results for low modes. Reddy, Wang and Lee (1997) obtained the relationships between EBT, TBT and RBT alphanumerically. In this research, the displacement and moment functions are investigated by using the obtained relationships. Soldatos and Sophocleous (2001) studied on free vibrations of beams according to EBT, TBT and RBT. The frequency equations are obtained and first six natural frequencies are presented for different boundary conditions in this study. Eisenberger (2003a) obtained displacement and rotation functions analytically for a simply supported beam according to RBT. The results of numerical analysis that performed for Reddy-Bickford beams by using DSM are compared to EBT and TBT. In another study of Eisenberger (2003b), the normalized end moments of cantilever and simply supported beams for various loading types and length/height ratios are presented by using RBT.

DTM which is an effective mathematical technique was first presented by Zhou (1968). In the study of Chen and Ho (1986), DTM was applied to eigenvalue problem for the first time. Ho and Chen (2006) applied DTM to investigate free vibration analysis of axially loaded nonuniform spinning Timoshenko beams. In this study, the effects of axial compression load and angular speed are discussed and obtained results are compared with EBT. Arikoglu and Ozkol (2010) investigated the free vibration analysis of a three layered composite beam with an elastic core by using DTM for the first time. In this study, the results obtained from DTM were compared with the findings of previous studies and a good proximity was observed. Lal and Ahlawat (2015) applied DTM for the analysis of the axisymmetric vibrations of functionally graded circular plates subjected to uniform in-plane force using classical plate theory. The critical buckling loads for simply supported and clamped boundary conditions were calculated and two dimensional mode shapes were plotted. Yesilce (2011) obtained the governing equations of motion of multi span Reddy-Bickford beams that have mass-spring systems on spans by using Hamilton’s principle. In this study, the free vibration analysis is investigated by using DTM and the effects of spring-mass systems on the free vibration characteristics of multi span beams are discussed. Yesilce and Catal (2009) investigated free vibration analysis of axially loaded Reddy-Bickford beam on elastic foundation using DTM. The first three natural frequencies are represented for different axial compression load and modulus of subgrade reaction values. Catal and Catal (2006) applied differential transformation for buckling analysis of partially embedded pile in elastic soil. In another research of Catal (2014), differential transformation is used for buckling analysis of semi-rigid connected partially embedded beams on elastic foundation. Free vibration analysis of beams on elastic foundations is investigated using DTM by Catal (2006, 2008). Catal (2012) researched response of forced beams via differential transformation according to EBT. Wattanasakulpong and Charoensuk (2015) researched vibration characteristics of stepped beams made of functionally graded materials and natural frequencies are obtained with various boundary conditions by using DTM. Yesilce (2015) investigated the natural frequencies and mode shapes of axially loaded Timoshenko beams carrying a number of intermediate lumped masses and rotary inertias by using numerical assembly technique and DTM. In the other studies of Yesilce (2010, 2013), the free vibration analysis of moving beams according to EBT and TBT are investigated by using DTM. In the study of Ebrahimi and Salari (2015), vibrations of functionally graded size dependent nanobeams were investigated by using DTM and Navier-based analytic method. Nondimensional frequencies for different material distribution parameters are presented for various boundary conditions. Semnani *et al.* (2013) studied the free vibration analysis of thin plates with varying thickness by using two-dimensional DTM. It was stated that the results obtained from 2D-DTM were consistent with the results in the literature.

DSM that based on exact shape functions obtained from exact solutions, is used for solving free or forced vibration problems of structures. As the method uses the exact shape functions, the results obtained from DSM are exact natural frequencies for free vibration analysis. A nonlinear eigenvalue problem is experienced due to characteristics of the method. Thus, the Wittrick-Williams algorithm can be used as a reliable root-finding algorithm (Banerjee 1997). In the recent years, DSM was applied to various type of beams and plates. Jun *et al.* (2008) investigated the effects of axial compressive force on natural frequencies of laminated composite beams by using DSM. It is stated that DSM is effective for solving free vibration problem of laminated composite beams. Bao-hui *et al.* (2011) studied on free vibration analysis of Timoshenko element pipe conveying fluid. In this study, first three natural frequencies of a multiple-span pipe conveying fluid are obtained and some of the results are compared with Abaqus. Banarjee (2012) searched free vibrations of beams carrying spring-mass systems by using DSM. The effect of spring-mass systems on natural frequencies of beams is emphasized. Banerjee and Jackson (2013) obtained natural frequencies of rotating tapered Rayleigh beams using DSM. Percentage error in the first three natural frequencies between Bernoulli-Euler beam theory and Rayleigh beam theory is presented. In the study of Su and Banarjee (2015), DSM is used to obtain non-dimensional natural frequencies of functionally graded Timoshenko beams with various boundary conditions.

Nefovska-Danilovic and Petronijevic (2015) applied DSM to in-plane free vibration and response analysis of isotropic rectangular plates. Different boundary conditions and length/width ratios are considered in the analysis.

The free vibration analysis of axially moving beams using a high order beam theory has not been investigated yet by any of the studies in open literature. In this study, the free vibration analysis of moving beams with different boundary conditions such as fixed supported, one end fixed, the other end simply supported and simply supported are investigated according to RBT by using DTM and DSM. First of all, Hamilton's principle is applied to obtain the governing equations of motion. Secondly, the parameters for the nondimensionalized multiplication factors for axial speed (α) and axial tensile force (β) are incorporated into the equations of motion in order to investigate their effects on the natural frequencies. The natural frequencies are obtained from analytical solution of differential equations according to RBT. After the ANM solution, DTM which is an effective mathematical technique based on finite Taylor series, is used to solve the governing equations of motion. Also dynamic stiffness matrix of axially moving Reddy-Bickford beam is constructed and the natural frequencies of the beam are obtained using Wittrick-Williams algorithm. The natural frequencies obtained from DTM and DSM are presented in tables for three different boundary conditions. Moreover, the results of DTM and DSM are compared with ANM and very good proximity is observed. Finally, the mode shapes and normalised bending moment diagrams are presented in graphs for the first three modes.

2. Theory, model and formulation

The free vibration analyses of beams have been investigated using EBT and TBT in numerous studies. Even if TBT considers the shear deformation and rotational inertia, the assumption about cross section that remaining plane after bending is still valid like EBT. EBT which does not consider shear deformation is not preferred for the analysis of thick and short beams as shear deformation becomes very important for them.

According to TBT, a uniform shear deformation distribution is predicted. Thus, shear correction factor is used to consume the mistakes of this assumption. However, high order shear deformation theories do not need any shear correction factor due to assumption of variable shear strain and shear stress along the height of the cross-section. The real behaviour of cross sections after bending is defined by high order shear deformation theories. These theories are due to Bickford, Levinson, Heyliger and Reddy, Wang et al. and others all consider the warping of the cross-section. The cross-sectional displacements are presented in Fig. 1, where $w_0(x,t)$ is the lateral displacement of the beam neutral axis and z is the distance from the beam neutral axis (Reddy *et al.* 1997). In this study, RBT also known as Parabolic Shear Deformation Theory (PSDT) is used. The cross-sectional displacements of Reddy-Bickford beam are presented in Fig. 1(c).

The following assumptions about axially moving Reddy-Bickford beams are considered in this study:

1. The cross-sectional area is uniform.
2. The beam material is homogenous and isotropic.
3. The behaviour of the beam is linear and elastic.
4. The axial tensile force acting along the beam length is constant.
5. The damping is neglected.

RBT defines the displacements functions as (Yesilce 2011)

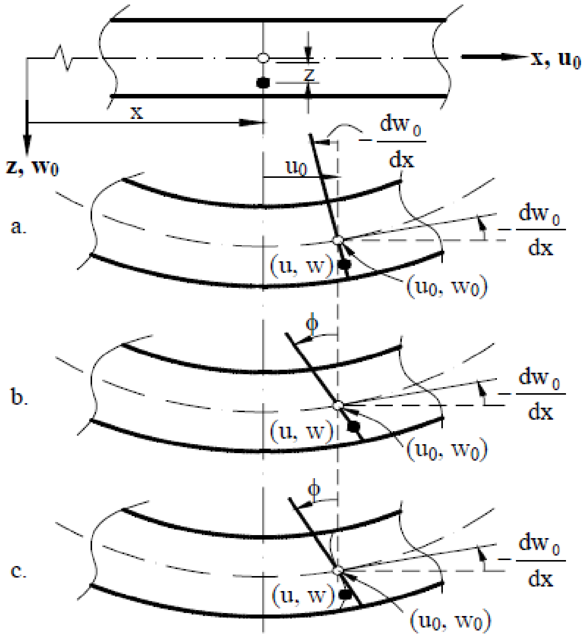

Fig. 1 Cross-section displacements in different beam theories (Yesilce 2011), (a) EBT (b) TBT (c) RBT

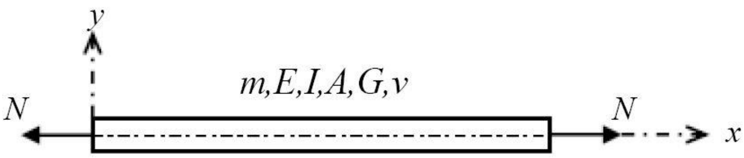

Fig. 2 Axially moving Reddy-Bickford beam with coordinate system

$$u(x,y,t) = y.\phi(x,t) - \alpha.y^3\left(\phi(x,t) + \frac{dw_0}{dx}\right) \tag{1}$$

$$w(x,y,t) = w_0(x,t) \tag{2}$$

$$\alpha = \frac{4}{\left(3h^2\right)} \tag{3}$$

Here, $u(x,y,t)$ is the axial displacement function, $w(x,y,t)$ is the lateral displacement function, $w_0(x,t)$ is the lateral displacement of the neutral beam axis, $\phi(x,t)$ is the rotation of a normal to axis of the beam, h is the height of the beam and t is the time.

An axially moving Reddy-Bickford beam model is presented with coordinate system in Fig. 2 where; N is the axial tensile force, m is the distributed mass, E is elastic modulus, I is area moment of inertia, A is cross-sectional area, G is shear modulus and v is the axial speed.

The governing equations of motion of Reddy-Bickford beams under axial tensile force are obtained by using Eqs. (1)-(2) with Hamilton's principle

$$-\frac{68}{105}\cdot EI\frac{\partial^2\phi(x,t)}{\partial x^2}+\frac{16}{105}\cdot EI\cdot\frac{\partial^3 w(x,t)}{\partial x^3}+\frac{8}{15}\cdot AG\cdot\left[\phi(x,t)+\frac{\partial w(x,t)}{\partial x}\right]=0 \tag{4}$$

$$-m\cdot\frac{\partial^2 w(x,t)}{\partial t^2}+\frac{8}{15}\cdot AG\left[\frac{\partial\phi(x,t)}{\partial x}+\frac{\partial^2 w(x,t)}{\partial x^2}\right]+\frac{16}{105}\cdot EI\cdot\frac{\partial^3\phi(x,t)}{\partial x^3}-\frac{1}{21}\cdot EI\cdot\frac{\partial^4 w(x,t)}{\partial x^4}+N\cdot\frac{\partial^2 w(x,t)}{\partial x^2}=0 \tag{5}$$

The axial speed v is incorporated into Eqs. (4)-(5) to obtain the governing equations of motion of axially moving Reddy-Bickford beams by using Eq. (6)

$$\frac{d}{dt}\equiv\frac{\partial}{\partial t}+v.\frac{\partial}{\partial x} \tag{6}$$

$$-\frac{68}{105}\cdot EI\cdot\frac{\partial^2\phi(x,t)}{\partial x^2}+\frac{16}{105}\cdot EI\cdot\frac{\partial^3 w(x,t)}{\partial x^3}+\frac{8}{15}\cdot AG\cdot\left[\phi(x,t)+\frac{\partial w(x,t)}{\partial x}\right]=0 \tag{7}$$

$$-m\left(\frac{\partial^2 w(x,t)}{\partial t^2}+2\cdot v\cdot\frac{\partial^2 w(x,t)}{\partial x.\partial t}+v^2\cdot\frac{\partial^2 w(x,t)}{\partial x^2}\right)+\frac{8}{15}\cdot AG\left[\frac{\partial\phi(x,t)}{\partial x}+\frac{\partial^2 w(x,t)}{\partial x^2}\right]$$
$$+\frac{16}{105}\cdot EI\cdot\frac{\partial^3\phi(x,t)}{\partial x^3}-\frac{1}{21}\cdot EI\cdot\frac{\partial^4 w(x,t)}{\partial x^4}+N\cdot\frac{\partial^2 w(x,t)}{\partial x^2}=0 \tag{8}$$

Eqs. (7)-(8) are obtained as the governing equations of motion of axially moving Reddy-Bickford beams where $w(x,t)$ is the lateral displacement function and $\phi(x,t)$ is the rotation of a normal to axis of the beam.

It is assumed that the motion is harmonic, $w(x,t)$ and $\phi(x,t)$ can be written as

$$w(x,t)=w(x).e^{i\omega t} \tag{9}$$

$$\phi(x,t)=\phi(x).e^{i\omega t} \tag{10}$$

where ω is the natural frequency and $i=\sqrt{-1}$.

Eqs. (7)-(8) are turned into ordinary differential equations by using Eqs. (9) and (10).

$$\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)\frac{d^3 w}{dz^3}+\left(\frac{8}{15}\cdot\frac{AG}{L}\right)\frac{dw}{dz}+\left(-\frac{68}{105}\cdot\frac{EI}{L^2}\right)\frac{d^2\phi}{dz^2}+\left(\frac{8}{15}\cdot AG\right)\phi(z)=0 \tag{11}$$

$$\left(-\frac{1}{21}\cdot\frac{EI}{L^4}\right)\frac{d^4 w}{dz^4}+\left(\frac{N}{L^2}+\frac{8}{15}\cdot\frac{AG}{L^2}-\frac{m.v^2}{L^2}\right)\frac{d^2 w}{dz^2}+\left(-\frac{2.m.v.i.\omega}{L}\right)\frac{dw}{dz}+\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)\frac{d^3\phi}{dz^3}$$
$$+\left(\frac{8}{15}\cdot\frac{AG}{L}\right)\frac{d\phi}{dz}+\left(m.\omega^2\right)w(z)=0 \tag{12}$$

where $z=x/L$.

Nondimensional $w(z)$ and $\phi(z)$ can be assumed as

$$w(z)=\bar{C}.e^{isz} \tag{13}$$

$$\phi(z)=\bar{D}.e^{isz} \tag{14}$$

Eqs. (13)-(14) are integrated into Eqs. (11)-(12) to construct the coefficient matrix that is used for calculating natural frequencies.

$$\left(-\frac{16}{105}\cdot\frac{EI.i.s^3}{L^3}+\frac{8}{15}\cdot\frac{AG.i.s}{L}\right)\left\{\bar{C}\right\}+\left(\frac{68}{105}\cdot\frac{EI.s^2}{L^2}+\frac{8}{15}\cdot AG\right)\left\{\bar{D}\right\}=0 \tag{15}$$

$$\left[-\frac{1}{21}\cdot\frac{EI.s^4}{L^4}-\left(\frac{N-m.v^2}{L^2}+\frac{8}{15}\cdot\frac{AG}{L^2}\right)\cdot s^2+\frac{2.m.v.\omega.s}{L}+\left(m.\omega^2\right)\right]\left\{\bar{C}\right\}$$
$$-\left[\frac{16}{105}\cdot\frac{EI.i.s^3}{L^3}+\frac{8}{15}\cdot\frac{AG.i.s}{L}\right]\left\{\bar{D}\right\}=0 \tag{16}$$

Eqs. (15)-(16) can be written in matrix form for the two unknowns $\bar{C}$ and $\bar{D}$ as

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}\begin{Bmatrix} \bar{C} \\ \bar{D} \end{Bmatrix}=0 \tag{17}$$

where

$$J_{11}=\left(-\frac{16}{105}\cdot\frac{EI.i.s^3}{L^3}+\frac{8}{15}\cdot\frac{AG.i.s}{L}\right) \tag{18}$$

$$J_{12}=\left(\frac{68}{105}\cdot\frac{EI.s^2}{L^2}+\frac{8}{15}\cdot AG\right) \tag{19}$$

$$J_{21}=\left[-\frac{1}{21}\cdot\frac{EI.s^4}{L^4}-\left(\frac{N-m.v^2}{L^2}+\frac{8}{15}\cdot\frac{AG}{L^2}\right)\cdot s^2+\frac{2.m.v.\omega.s}{L}+\left(m.\omega^2\right)\right] \tag{20}$$

$$J_{22}=-\left(\frac{16}{105}\cdot\frac{EI.i.s^3}{L^3}+\frac{8}{15}\cdot\frac{AG.i.s}{L}\right) \tag{21}$$

For non-trivial solution, determinant of the coefficient matrix must be equal to zero. Since a sixth order equation with unknowns is encountered, the general solutions that have six unknowns for both $w(z,t)$ and $\phi(z,t)$ can be written as

$$w(z,t)=\left[\bar{C}_1.e^{is_1z}+\bar{C}_2.e^{is_2z}+\bar{C}_3.e^{is_3z}+\bar{C}_4.e^{is_4z}+\bar{C}_5.e^{is_5z}+\bar{C}_6.e^{is_6z}\right].e^{i\omega t} \tag{22}$$

$$\phi(z,t)=\left[\bar{D}_1.e^{is_1z}+\bar{D}_2.e^{is_2z}+\bar{D}_3.e^{is_3z}+\bar{D}_4.e^{is_4z}+\bar{D}_5.e^{is_5z}+\bar{D}_6.e^{is_6z}\right].e^{i\omega t} \tag{23}$$

The bending moment $M(z,t)$ and shear force $Q(z,t)$ functions can be obtained by using Eqs. (22)-(23) as

$$M(z,t)=\left[\frac{16.EI}{105.L}\cdot\frac{d\phi(z)}{dz}-\frac{EI}{21.L^2}\cdot\frac{d^2w(z)}{dz^2}-N.w(z)\right].e^{i\omega t} \tag{24}$$

$$Q(z,t)=\left[\frac{EI}{21.L^3}\cdot\frac{d^3w(z)}{dz^3}-\frac{N}{L}\frac{dw(z)}{dz}-\frac{8.AG}{15}\left(\phi(z)+\frac{1}{L}\cdot\frac{dw(z)}{dz}\right)-\frac{16.EI}{105.L^2}\cdot\frac{d^2\phi(z)}{dz^2}\right].e^{i\omega t} \tag{25}$$

The high order moment function $M_h(z,t)$ can be obtained as

$$M_h(z,t)=\left[\frac{16.EI}{105.L^2}\cdot\frac{d^2w(z)}{dz^2}-\frac{68.EI}{105.L}\cdot\frac{d\phi(z)}{dz}\right].e^{i\omega t} \tag{26}$$

The slope $w'(z,t)$ of the cross-section can be defined as

$$w'(z,t)=\frac{1}{L}\cdot\frac{dw(z)}{dz}\cdot e^{i\omega t} \tag{27}$$

The six boundary conditions must be applied by using Eqs. (22)-(23) with Eqs. (24)-(27) to construct the coefficient matrix that is used for calculating the natural frequencies of the beam.

3. Differential Transform Method (DTM)

DTM is a semi-analytic transformation technique based on Taylor series expansion and is an effective tool to obtain analytical solutions of the differential equations. Certain transformation rules are applied and the governing differential equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions in DTM. The solution of these algebraic equations gives the desired solution of the problem. The difference of DTM from high-order Taylor series method is; Taylor series method requires symbolic computation of the necessary derivatives of the data functions and is expensive for large orders. DTM is an iterative procedure to obtain analytic Taylor series solutions of differential equations (Yesilce 2011).

Consider a function $y(z)$, which is analytic in a domain D, can be represented by a power series with a center at $z=z_0$, any point in D. The differential transform of the function $y(z)$ is given by Eq. (28)

$$Y(k)=\frac{1}{k!}.\left(\frac{d^k y(z)}{dz^k}\right)_{z=z_0} \tag{28}$$

where $y(z)$ is the original function and $Y(k)$ is the transformed function. The inverse transformation is defined as

$$y(z)=\sum_{k=0}^{\infty}(z-z_0)^k\cdot Y(k) \tag{29}$$

Using Eqs. (28) and (29), Eq. (30) is obtained as

$$y(z)=\sum_{k=0}^{\infty}\frac{(z-z_0)^k}{k!}\cdot\left(\frac{d^k y(z)}{dz^k}\right)_{z=z_0} \tag{30}$$

Eq. (30) implies that the concept of the differential transformation is derived from Taylor's series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by iterative procedure that are described by the transformed equations of the original functions. In real applications, the function $y(z)$ in Eq. (29) is expressed by a finite series and can be written as

Table 1 DTM theorems used for equations of motion

Original Function	Transformed Function
$y(z)=u(z)\pm v(z)$	$Y(k)=U(k)\pm V(k)$
$y(z)=a.u(z)$	$Y(k)=a.U(k)$
$y(z)=\frac{d^m u(z)}{dz^m}$	$Y(k)=\frac{(k+m)!}{k!}\cdot U(k+m)$
$y(z)=u(z).v(z)$	$Y(k)=\sum_{r=0}^{k} U(r).V(k-r)$
$y(z)=z^m$	$Y(k)=\partial(k-m)=\begin{cases}0 & if \quad k\neq m\\ 1 & if \quad k=m\end{cases}$

Table 2 DTM theorems used for boundary conditions

Original Boundary Conditions	Transformed Boundary Conditions	Original Boundary Conditions	Transformed Boundary Conditions
$y(0)=0$	$Y(0)=0$	$y(1)=0$	$\sum_{k=0}^{\infty} Y(k)=0$
$\frac{dy}{dz}(0)=0$	$Y(1)=0$	$\frac{dy}{dz}(1)=0$	$\sum_{k=0}^{\infty} k\cdot Y(k)=0$
$\frac{d^2y}{dz^2}(0)=0$	$Y(2)=0$	$\frac{d^2y}{dz^2}(1)=0$	$\sum_{k=0}^{\infty} k\cdot(k-1)\cdot Y(k)=0$
$\frac{d^3y}{dz^3}(0)=0$	$Y(3)=0$	$\frac{d^3y}{dz^3}(1)=0$	$\sum_{k=0}^{\infty} k\cdot(k-1)\cdot(k-2)\cdot Y(k)=0$

$$y(z)=\sum_{k=0}^{N^*}(z-z_0)^k.Y(k) \tag{31}$$

Eq. (31) implies that $\sum_{k=N^*+1}^{\infty}(z-z_0)^k Y(k)$ is negligibly small. Where N^* is series size and the value of N^* depends on the convergence of the eigenvalues.

DTM theorems that are frequently used in differential transformation of the differential equations and the boundary conditions are introduced in Table 1 and Table 2, respectively.

3.1 Application of DTM to solve equations of motion

The procedure is started with using Eqs. (11) and (12) in the form as follows

$$\frac{d^3w(z)}{dz^3}=\frac{\left(\frac{8}{15}\cdot\frac{AG}{L}\right)}{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}\cdot\frac{dw(z)}{dz}+\frac{\left(-\frac{68}{105}\cdot\frac{EI}{L^2}\right)}{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}\cdot\frac{d^2\phi(z)}{dz^2}+\frac{\left(\frac{8}{15}\cdot AG\right)}{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}\cdot\phi(z) \tag{32}$$

$$\frac{d^4w(z)}{dz^4}=\frac{\left(\frac{N}{L^2}+\frac{8}{15}\cdot\frac{AG}{L^2}-\frac{m.v^2}{L^2}\right)}{\left(-\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{d^2w(z)}{dz^2}+\frac{\left(\frac{2.m.v.i.\omega}{L}\right)}{\left(\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{dw(z)}{dz}+\frac{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}{\left(-\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{d^3\phi(z)}{dz^3}$$
$$+\frac{\left(\frac{8}{15}\cdot\frac{AG}{L}\right)}{\left(-\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{d\phi(z)}{dz}+\frac{\left(m.\omega^2\right)}{\left(-\frac{1}{21}\cdot\frac{EI}{L^4}\right)}.w(z) \tag{33}$$

DTM is applied to Eqs. (32) and (33) by using the theorems presented in Table 1 and the following expressions are obtained

$$W(k+3)=-\frac{\left(\frac{8}{15}\cdot\frac{AG}{L}\right)}{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}\frac{W(k+1)}{(k+2)(k+3)}+\frac{\left(\frac{68}{105}\cdot\frac{EI}{L^2}\right)}{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}\cdot\frac{\phi(k+2)}{(k+3)}-\frac{\left(\frac{8}{15}\cdot AG\right)}{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}\cdot\frac{\phi(k)}{(k+1)(k+2)(k+3)} \tag{34}$$

$$W(k+4)=-\frac{\left(\frac{N}{L^2}+\frac{8}{15}\cdot\frac{AG}{L^2}-\frac{m.v^2}{L^2}\right)}{\left(\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{W(k+2)}{(k+3)(k+4)}+\frac{\left(\frac{2.m.v.i.\omega}{L}\right)}{\left(\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{W(k+1)}{(k+2)(k+3)(k+4)}$$
$$+\frac{\left(\frac{16}{105}\cdot\frac{EI}{L^3}\right)}{\left(\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{\Phi(k+3)}{(k+4)}+\frac{\left(\frac{8}{15}\cdot\frac{AG}{L}\right)}{\left(\frac{1}{21}\cdot\frac{EI}{L^4}\right)}\cdot\frac{\Phi(k+1)}{(k+2)(k+3)(k+4)}+\frac{\left(m.\omega^2\right)}{\left(\frac{1}{21}\cdot\frac{EI}{L^4}\right)}W(k) \tag{35}$$

where $W(k)$ is the transformed function of $w(z)$ and $\Phi(k)$ is the transformed function of $\phi(z)$.

The boundary conditions of an axially moving Reddy-Bickford beam are tabulated in Table 3:

Table 3 The boundary conditions of axially moving Reddy-Bickford beam

Fixed - Fixed	Fixed - Simple	Simple - Simple
$w(z=0)=0$	$w(z=0)=0$	$w(z=0)=0$
$\frac{dw(z=0)}{dz}=0$	$\frac{dw(z=0)}{dz}=0$	$M(z=0)=0$
$\varphi(z=0)=0$	$\varphi(z=0)=0$	$M_h(z=0)=0$
$w(z=1)=0$	$w(z=1)=0$	$w(z=1)=0$
$\frac{dw(z=1)}{dz}=0$	$M(z=1)=0$	$M(z=1)=0$
$\varphi(z=1)=0$	$M_h(z=1)=0$	$M_h(z=1)=0$

Table 4 Transformed boundary conditions of axially moving Reddy-Bickford beam

Boundary Condition	z=0	z=1
Fixed - Fixed	$W(0)=\Phi(0)=\frac{dW(z=0)}{dz}=0$	$\sum_{k=0}^{N^*}W(k)=\sum_{k=0}^{N^*}\Phi(k)=\sum_{k=0}^{N^*}\frac{dW(k)}{dz}=0$
Fixed - Simple	$W(0)=\Phi(0)=\frac{dW(z=0)}{dz}=0$	$\sum_{k=0}^{N^*}W(k)=\sum_{k=0}^{N^*}\bar{M}(k)=\sum_{k=0}^{N^*}\bar{M}_h(k)=0$
Simple - Simple	$W(0)=W(2)=\Phi(1)=0$	$\sum_{k=0}^{N^*}W(k)=\sum_{k=0}^{N^*}\bar{M}(k)=\sum_{k=0}^{N^*}\bar{M}_h(k)=0$

The transformed boundary conditions of an axially moving Reddy-Bickford beam are obtained as shown in Table 4 by applying DTM to boundary conditions presented in Table 3:
where $\bar{M}(k)$ and $\bar{M}_h(k)$ are transformed functions of $M(z)$ and $M_h(z)$, respectively.

For fixed supported beam, taking $W(2)=n_1$, $\Phi(1)=n_2$, $\Phi(2)=n_3$; for one end (z=0) fixed, the other end (z=1) simply supported beam, taking $W(2)=n_1$, $\Phi(1)=n_2$, $\Phi(2)=n_3$; for simply supported beam, taking $W(0)=n_1$, $W(2)=n_2$, $\Phi(1)=n_3$ and substituting the transformed boundary conditions given in Table 4 into Eqs. (34) and (35) for each boundary condition, the matrix given below is obtained.

$$\begin{bmatrix} A_{11}^{(N^*)}(\omega) & A_{12}^{(N^*)}(\omega) & A_{13}^{(N^*)}(\omega) \\ A_{21}^{(N^*)}(\omega) & A_{22}^{(N^*)}(\omega) & A_{23}^{(N^*)}(\omega) \\ A_{31}^{(N^*)}(\omega) & A_{32}^{(N^*)}(\omega) & A_{33}^{(N^*)}(\omega) \end{bmatrix} \cdot \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{36}$$

where n_1, n_2, n_3 are constants and $\bar{A}_{j1}^{(N^*)}(\omega)$, $\bar{A}_{j2}^{(N^*)}(\omega)$, $\bar{A}_{j3}^{(N^*)}(\omega)$ (j=1, 2, 3) are polynomials of ω corresponding N^*.

Finally, the natural frequencies are obtained by non-trivial solution for which determinant of coefficient matrix equal to zero.

$$\begin{vmatrix} A_{11}^{(N^*)}(\omega) & A_{12}^{(N^*)}(\omega) & A_{13}^{(N^*)}(\omega) \\ A_{21}^{(N^*)}(\omega) & A_{22}^{(N^*)}(\omega) & A_{23}^{(N^*)}(\omega) \\ A_{31}^{(N^*)}(\omega) & A_{32}^{(N^*)}(\omega) & A_{33}^{(N^*)}(\omega) \end{vmatrix} = 0 \tag{37}$$

The j^{th} estimated eigenvalue, $\omega_j^{(N^*)}$ corresponds to $N^*$and the value of $N^*$is determined as

$$\left|\omega_j^{(N^*)} - \omega_j^{(N^*-1)}\right| \le \varepsilon \tag{38}$$

where $\omega_j^{(N^*-1)}$ is the j^{th} estimated eigenvalue corresponding to (N^*-1) and ε is the small tolerance parameter. If Eq. (38) is satisfied, the j^{th} estimated eigenvalue, $\omega_{\text{j}}^{(N^*)}$ is obtained.

The normalised bending moment diagrams and mode shapes can be plotted by using transformed functions. As n_2 and n_3 can be written in terms of n_1 by using Eq. (39), the transformed functions $W(k)$, $\Phi(k)$ and $\bar{M}(k)$ can be expressed in terms of ω and n_1 as follows

$$A_{11}(\omega)\cdot n_1 + A_{12}(\omega)\cdot n_2 + A_{13}(\omega)\cdot n_3 = 0 \tag{39}$$

$$W\left(k\right)=W\left(\omega,n_1\right) \tag{40}$$

$$\Phi\left(k\right)=\Phi\left(\omega,n_1\right) \tag{41}$$

$$\bar{M}\left(k\right)=\bar{M}\left(\omega,n_1\right) \tag{42}$$

The mode shapes and normalised bending moment diagrams can be plotted for different values of ω by using Eq. (40) and (42), respectively.

4. Dynamic stiffness approach

The dynamic stiffness matrix of an axially moving Reddy-Bickford beam is constructed using displacements and forces at the ends of the beam according to boundary conditions. The vector of end displacements of beam and the vector of coefficients are defined in Eq. (43) and (44), respectively.

$$\delta=[w_0 \quad w'_0 \quad \phi_0 \quad w_1 \quad w'_1 \quad \phi_1]^T \tag{43}$$

$$C=[C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5 \quad C_6]^T \tag{44}$$

where

$$w_0=w(z=0), \quad w'_0=w'_0(z=0), \quad \phi_0=\phi(z=0), \quad w_1=w(z=1), \quad w'_1=w'(z=1) \quad \phi_1=\phi(z=1) \tag{45}$$

Using Eqs. (22)-(23) and Eq. (27), the matrix form of Eq. (45) can be written as

$$\begin{bmatrix} w_0 \\ w'_0 \\ \phi_0 \\ w_1 \\ w'_1 \\ \phi_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ is_1 & is_2 & is_3 & is_4 & is_5 & is_6 \\ K_1 & K_2 & K_3 & K_4 & K_5 & K_6 \\ e^{is_1} & e^{is_2} & e^{is_3} & e^{is_4} & e^{is_5} & e^{is_6} \\ is_1e^{is_1} & is_2e^{is_2} & is_3e^{is_3} & is_4e^{is_4} & is_5e^{is_5} & is_6e^{is_6} \\ K_1e^{is_1} & K_2e^{is_2} & K_3e^{is_3} & K_4e^{is_4} & K_5e^{is_5} & K_6e^{is_6} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} \tag{46}$$

where

$K_j = D_j / C_j$, j=1, 2, 3, 4, 5, 6

The closed form of Eq. (46) is given in Eq. (47)

$$\delta = \Delta C \tag{47}$$

where

$$\Delta=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ is_1 & is_2 & is_3 & is_4 & is_5 & is_6 \\ K_1 & K_2 & K_3 & K_4 & K_5 & K_6 \\ e^{is_1} & e^{is_2} & e^{is_3} & e^{is_4} & e^{is_5} & e^{is_6} \\ is_1e^{is_1} & is_2e^{is_2} & is_3e^{is_3} & is_4e^{is_4} & is_5e^{is_5} & is_6e^{is_6} \\ K_1e^{is_1} & K_2e^{is_2} & K_3e^{is_3} & K_4e^{is_4} & K_5e^{is_5} & K_6e^{is_6} \end{bmatrix}$$

The vector of end forces of the beam is written in Eq. (48)

$$F = [Q_0 \quad M_{h_0} \quad M_0 \quad Q_1 \quad M_{h_1} \quad M_1]^T \tag{48}$$

where

$$\begin{aligned} &Q_0 = Q(z=0), \quad M_{h_0} = M_h(z=0), \quad M_0 = M(z=0), \\ &Q_1 = Q(z=1), \quad M_{h_1} = M_h(z=1), \quad M_1 = M(z=1) \end{aligned} \tag{49}$$

It should be noted that according to sign convention, the following relations are valid.

$$Q_0 = -Q_1, \quad M_{h_0} = -M_{h_1}, \quad M_0 = -M_1, \tag{50}$$

The matrix form of force functions given in Eq. (49) is written below by using Eqs. (24)-(26)

$$\begin{bmatrix} Q_0 \\ M_{h_0} \\ M_0 \\ Q_1 \\ M_{h_1} \\ M_1 \end{bmatrix} = \begin{bmatrix} \psi_1 & \psi_2 & \psi_3 & \psi_4 & \psi_5 & \psi_6 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \\ \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 & \Omega_6 \\ -\eta_1 & -\eta_2 & -\eta_3 & -\eta_4 & -\eta_5 & -\eta_6 \\ -\gamma_1 & -\gamma_2 & -\gamma_3 & -\gamma_4 & -\gamma_5 & -\gamma_6 \\ -\Theta_1 & -\Theta_2 & -\Theta_3 & -\Theta_4 & -\Theta_5 & -\Theta_6 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} \tag{51}$$

where

$$\begin{aligned} &\psi_j = R_3 i s_j K_j - R_4 i s_j^3 - R_5 s_j^2 K_j + R_6 i s_j + R_7 K_j, \; j = 1,2,3,4,5,6 \\ &\lambda_j = -R_1 s_j^2 + R_2 i s_j K_j, \; j = 1,2,3,4,5,6 \\ &\Omega_j = R_1 K_j i s_j - R_8 s_j^2 + N, \; j = 1,2,3,4,5,6 \\ &\eta_j = e^{i s_j}(R_3 i s_j K_j - R_4 i s_j^3 - R_5 s_j^2 K_j + R_6 i s_j + R_7 K_j), \; j = 1,2,3,4,5,6 \\ &\gamma_j = e^{i s_j}(-R_1 s_j^2 + R_2 i s_j K_j), \; j = 1,2,3,4,5,6 \\ &\Theta_j = e^{i s_j}(R_1 K_j i s_j - R_8 s_j^2 + N), \; j = 1,2,3,4,5,6 \end{aligned} \tag{52}$$

The constants R_1-R_8 are used to simplificate the solution process and are listed below

$$\begin{aligned} &R_1 = (16EI)/(105L^2), R_2 = (-68EI)/(105L) \\ &R_3 = (-8AG)/(15L), R_4 = (EI)/(21L^3) \\ &R_5 = (-16EI)/(105L^2), R_6 = (m\omega^2 I)/(21A) \\ &R_7 = (16mI\omega^2)/(105A), R_8 = (-EI)/(21L^2) \end{aligned} \tag{52}$$

The closed form of Eq. (51) is presented in Eq. (53)

$$F = \kappa C \tag{53}$$

where

$$\kappa=\begin{bmatrix} \psi_1 & \psi_2 & \psi_3 & \psi_4 & \psi_5 & \psi_6 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \\ \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 & \Omega_6 \\ -\eta_1 & -\eta_2 & -\eta_3 & -\eta_4 & -\eta_5 & -\eta_6 \\ -\gamma_1 & -\gamma_2 & -\gamma_3 & -\gamma_4 & -\gamma_5 & -\gamma_6 \\ -\Theta_1 & -\Theta_2 & -\Theta_3 & -\Theta_4 & -\Theta_5 & -\Theta_6 \end{bmatrix}$$

Finally, the relations between Eqs. (47) and (53) are used to derive the dynamic stiffness matrix.

$$F = \kappa\Delta^{-1}\delta \tag{54}$$

$$K^* = \kappa\Delta^{-1} \tag{55}$$

K^* represents the dynamic stiffness matrix of an axially moving Reddy-Bickford beam. Eq. (56) is used to obtain natural frequencies of the beam.

$$\left|K^*\right| = 0 \tag{56}$$

5. Numerical analysis and discussions

For numerical analysis, axially moving Reddy-Bickford beams with different boundary conditions such as fixed supported, one end fixed, the other end simply supported and simply supported are considered in this study. The first three natural frequencies, ω_i (i =1,2,3) of moving Reddy-Bickford beams are calculated by using computer programs prepared in Matlab by the authors. ANM and DTM solutions are based on determining values for which the determinant of the coefficient matrix is equal to zero. The change of sign method is applied in programs which is based on iterations. If there is a sign changing between two trial of ω_i (i=1,2,3), there must be a root in this interval. For DSM solution, the values that equals the determinant of dynamic stiffness matrix to zero are obtained using Wittrick-Williams algorithm. The numerical analysis is performed based on uniform, rectangular Reddy-Bickford beams with the following data:

L=5.0 m; m=0.89195 kN.sec^2/m^2; EI=6.4×10^5 kN.m^2 ; AG=1.536×10^{5}kN; β=0.20, 0.40 and 0.60; α=0.20, 0.40 and 0.60 where $\beta=N.L^2/EI$ and $\alpha = v\sqrt{m/N}$

The calculated first three natural frequency values by using ANM, DTM and DSM solutions with different nondimensionalised factors for axial tensile force and axial speed are presented in Tables 5-7 for fixed-fixed boundary conditions, in Tables 8-10 for fixed-simple boundary conditions, in Table 11-13 for simple-simple boundary conditions.

For β=0.20 and α=0.60; the first three mode shapes of axially moving fixed supported Reddy-Bickford beam are shown in Fig. 3, the first three mode shapes of axially moving Reddy-Bickford beam with fixed-simple boundary conditions are shown in Fig. 4 and the first three mode shapes of axially moving simply supported Reddy-Bickford beam are presented in Fig. 5.

For β=0.20 and α=0.60; the normalised bending moment diagrams for the first three modes of axially moving fixed supported Reddy-Bickford beam are shown in Fig. 6, the normalised bending moment diagrams for the first three modes of axially moving Reddy-Bickford beam with fixed-

Table 5 Natural frequencies of axially moving Reddy-Bickford beams with β=0.20 under fixed-fixed supported boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	239.5346	239.5346	239.5346 (N*=62)	238.2855	238.2855	238.2855 (N*=62)	236.2039	236.2039	236.2039 (N*=62)
2nd mode(rad/sec)	501.5633	501.5633	501.5633 (N*=66)	499.7285	499.7285	499.7285 (N*=66)	496.6692	496.6692	496.6692 (N*=66)
3rd mode (rad/sec)	810.3140	810.3140	810.3140 (N*=70)	808.0597	808.0597	808.0597 (N*=70)	804.3061	804.3061	804.3061 (N*=70)

Table 6 Natural frequencies of axially moving Reddy-Bickford beams with β=0.40 under fixed-fixed supported boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	244.3027	244.3027	244.3027 (N*=64)	241.8587	241.8587	241.8587 (N*=64)	237.7868	237.7868	237.7868 (N*=62)
2nd mode(rad/sec)	510.7113	510.7113	510.7113 (N*=68)	507.0788	507.0788	507.0788 (N*=66)	501.0199	501.0199	501.0199 (N*=66)
3rd mode (rad/sec)	823.0298	823.0298	823.0298 (N*=70)	818.5509	818.5509	818.5509 (N*=72)	811.0985	811.0985	811.0985 (N*=70)

Table 7 Natural frequencies of axially moving Reddy-Bickford beams with β=0.60 under fixed-fixed supported boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	248.9701	248.9701	248.9701 (N*=64)	245.3805	245.3805	245.3805 (N*=64)	239.4008	239.4008	239.4008 (N*=64)
2nd mode(rad/sec)	519.6764	519.6764	519.6764 (N*=70)	514.2826	514.2826	514.2826 (N*=66)	505.2834	505.2834	505.2834 (N*=68)
3rd mode (rad/sec)	835.5267	835.5267	835.5267 (N*=70)	828.8524	828.8524	828.8524 (N*=72)	817.7548	817.7548	817.7548 (N*=72)

Table 8 Natural frequencies of axially moving Reddy-Bickford beams with β=0.20 under fixed-simple boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	211.5277	211.5277	211.5277 (N*=64)	210.3156	210.3156	210.3156 (N*=64)	208.2955	208.2955	208.2955 (N*=62)
2nd mode(rad/sec)	471.6519	471.6519	471.6519 (N*=66)	469.7452	469.7452	469.7452 (N*=66)	466.5670	466.5670	466.5670 (N*=64)
3rd mode (rad/sec)	764.6574	764.6574	764.6574 (N*=72)	762.3954	762.3954	762.3954 (N*=70)	758.6260	758.6260	758.6260 (N*=70)

Table 9 Natural frequencies of axially moving Reddy-Bickford beams with β=0.40 under fixed-simple boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	216.7110	216.7110	216.7110 (N*=66)	214.3393	214.3393	214.3393 (N*=66)	210.3860	210.3860	210.3860 (N*=66)
2nd mode(rad/sec)	480.9647	480.9647	480.9647 (N*=68)	477.1997	477.1997	477.1997 (N*=66)	470.9231	470.9231	470.9231 (N*=68)
3rd mode (rad/sec)	777.6534	777.6534	777.6534 (N*=70)	773.1600	773.1600	773.1600 (N*=70)	765.6735	765.6735	765.6735 (N*=72)

Table 10 Natural frequencies of axially moving Reddy-Bickford beams with β=0.60 under fixed-simple boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	221.7643	221.7643	221.7643 (N*=66)	218.2803	218.2803	218.2803 (N*=66)	212.4728	212.4728	212.4728 (N*=64)
2nd mode(rad/sec)	490.0852	490.0852	490.0852 (N*=68)	484.5080	484.5080	484.5080 (N*=66)	475.2094	475.2094	475.2094 (N*=66)
3rd mode (rad/sec)	790.4144	790.4144	790.4144 (N*=74)	783.7202	783.7202	783.7202 (N*=74)	772.5684	772.5684	772.5684 (N*=72)

Table 11 Natural frequencies of axially moving Reddy-Bickford beams with β=0.20 under simple-simple boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	189.6443	189.6443	189.6443 (N*=58)	188.4528	188.4528	188.4528 (N*=62)	186.4666	186.4666	186.4666 (N*=60)
2nd mode(rad/sec)	443.3437	443.3437	443.3437 (N*=62)	441.4135	441.4135	441.4135 (N*=64)	438.1961	438.1961	438.1961 (N*=64)
3rd mode (rad/sec)	722.7985	722.7985	722.7985 (N*=68)	720.5043	720.5043	720.5043 (N*=68)	716.6802	716.6802	716.6802 (N*=68)

Table 12 Natural frequencies of axially moving Reddy-Bickford beams with β=0.40 under simple-simple boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	195.1364	195.1364	195.1364 (N*=60)	192.8062	192.8062	192.8062 (N*=62)	188.9207	188.9207	188.9207 (N*=62)
2nd mode(rad/sec)	452.8119	452.8119	452.8119 (N*=64)	449.0056	449.0056	449.0056 (N*=68)	442.6604	442.6604	442.6604 (N*=64)
3rd mode (rad/sec)	736.0047	736.0047	736.0047 (N*=68)	731.4516	731.4516	731.4516 (N*=70)	723.8620	723.8620	723.8620 (N*=68)

Table 13 Natural frequencies of axially moving Reddy-Bickford beams with β=0.60 under simple-simple boundary conditions

Natural Frequency	α=0.20			α=0.40			α=0.60		
	Method			Method			Method		
	ANM	DSM	DTM	ANM	DSM	DTM	ANM	DSM	DTM
1st mode (rad/sec)	200.4734	200.4734	200.4734 (*N**=60)	197.0518	197.0518	197.0518 (*N**=62)	191.3456	191.3456	191.3456 (*N**=62)
2nd mode(rad/sec)	462.0775	462.0775	462.0775 (*N**=66)	456.4458	456.4458	456.4458 (*N**=66)	447.0573	447.0573	447.0573 (*N**=64)
3rd mode (rad/sec)	748.9631	748.9631	748.9631 (*N**=70)	742.1859	742.1859	742.1859 (*N**=70)	730.8885	730.8885	730.8885 (*N**=68)

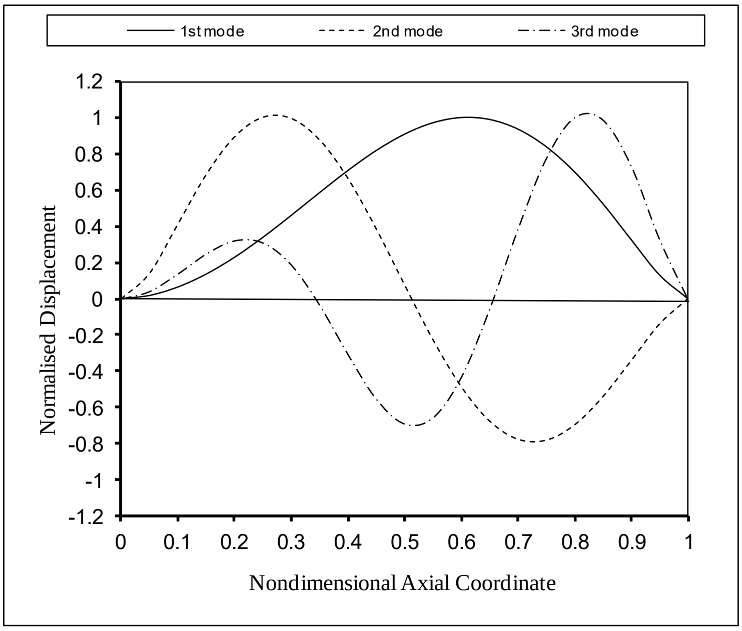

Fig. 3 The first three mode shapes of axially moving Reddy-Bickford beam under fixed-fixed boundary condition, β=0.20 and α=0.60

simple boundary conditions are shown in Fig. 7 and the normalised bending moment diagrams for the first three modes of axially moving simply supported Reddy-Bickford beam are presented in Fig. 8.

For all of the boundary conditions, the results reveal that increasing axial speed decreases the natural frequency values with constant axial tensile force. Contrary, an augmentation in axial tensile force with a constant axial speed causes higher natural frequency values for all beams. As expected, for the same axial speed and axial tensile force, the maximum frequency values are obtained in the case of fixed supported Reddy-Bickford beam and the minimum values can be seen from simply supported Reddy-Bickford beam.

The convergences of natural frequency values are very important in the application of DTM. As can be seen from Tables 5-13, the natural frequency values of the third modes are obtained for fixed supported beams when the series size is taken 72, for one end fixed, the other end simple supported beams when the series size is taken 74, for simply supported beams when series size is taken as 70. It is observed that, higher modes appear when more terms are taken into account in

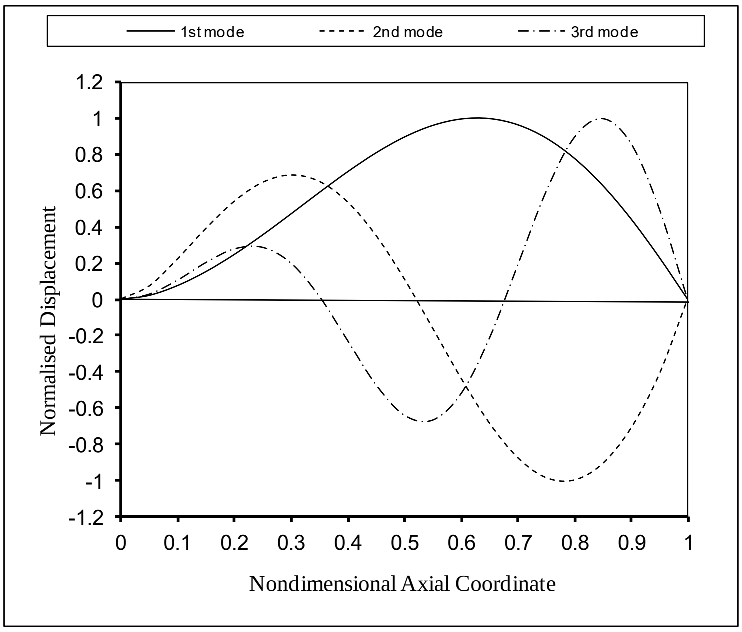

Fig. 4 The first three mode shapes of axially moving Reddy-Bickford beam under fixed-simple boundary condition, β=0.20 and α=0.60

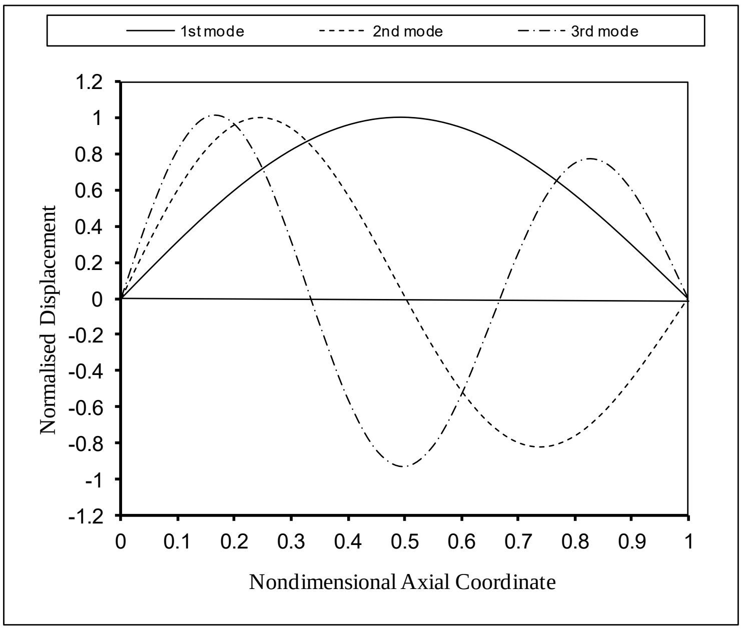

Fig. 5 The first three mode shapes of axially moving Reddy-Bickford beam under simple-simple boundary condition, β=0.20 and α=0.60

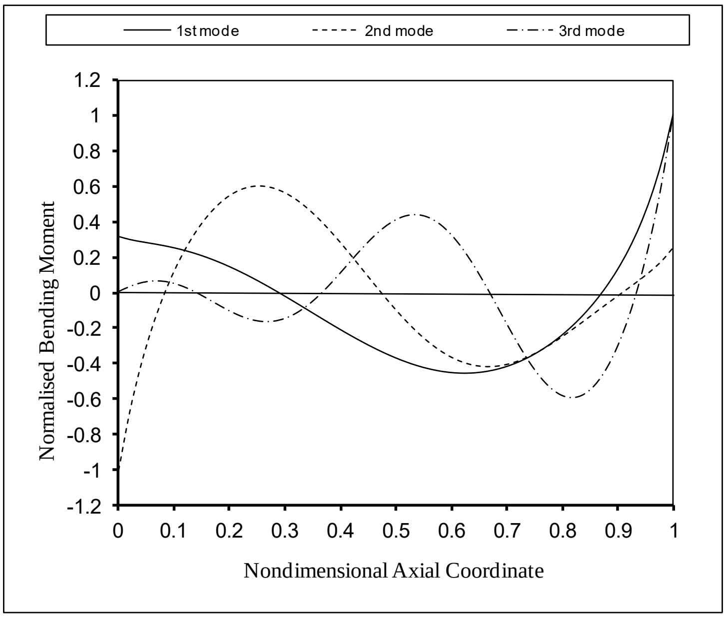

Fig. 6 The normalised bending moment diagrams for the first three modes of axially moving Reddy-Bickford beam under fixed-fixed boundary condition, β=0.20 and α=0.60

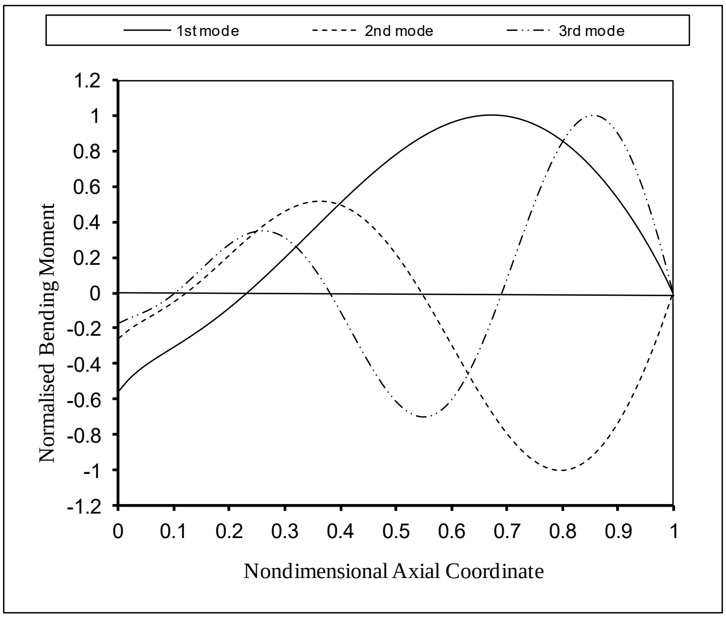

Fig. 7 The normalised bending moment diagrams for the first three modes of axially moving Reddy-Bickford beam under fixed-simple boundary condition, β=0.20 and α=0.60

DTM applications. The convergences of fundamental frequencies in DTM are presented in Fig. 9 when β=0.20 and α=0.60. For the first three modes, the mode shapes are presented in Figs. 3-5 and the normalised bending moment diagrams are presented in Figs. 6-8 for different boundary

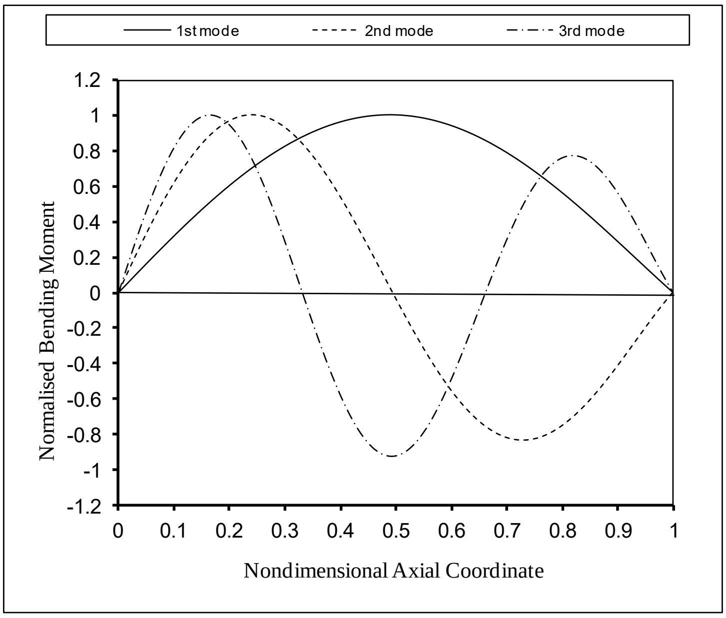

Fig. 8 The normalised bending moment diagrams for the first three modes of axially moving Reddy-Bickford beam under simple-simple boundary condition, β=0.20 and α=0.60

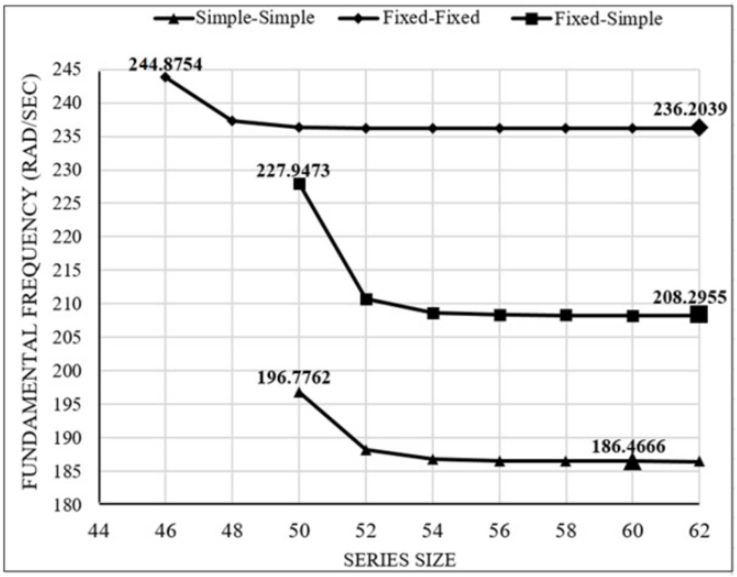

Fig. 9 Natural frequency convergences of DTM solutions for axially moving Reddy-Bickford beams, β=0.20 and α=0.60

conditions. It is seen that from Figs. 6-8; for the fundamental mode, taking β=0.20, α=0.60, the maximum bending moments occur at the end (z=1) of fixed supported beam, near z=0.70 of one end fixed, the other end simple supported beam and at the middle (z=0.5) of simply supported beam.

5. Conclusions

In this study, the free vibration analysis of axially moving beams is investigated according to a high order shear deformation theory for the first time. The fixed supported, one end fixed, the other end simple supported and simply supported beams are considered for the analysis. Different values of nondimensionalised factors for axial speed and axial tensile force are studied to have information about their effects on the free vibration of moving beams. DTM algorithms are developed and computer programs that based on iterations are prepared by using Matlab to calculate the natural frequencies. In addition to this, dynamic stiffness formulation is performed and natural frequencies are obtained from DSM solution. The obtained natural frequencies from DTM and DSM solutions are compared with ANM solutions and very good agreement is observed. The effectiveness of DTM and DSM for solving free vibration problems of moving beams are experienced. It is seen that the computer programs prepared for DTM are working significantly fast in comparison with ANM and DSM.

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