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A new formulation of the *J* integral of bonded composite repair in aircraft structures

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Abstract. A three-dimensional finite element method is used for analysis of repairing cracks in plates with bonded composite patch in elastic and elastic plastic analysis. This study was performed in order to establish an analytical model of the *J*-integral for repair crack. This formulation of the *J*-integral to establish models of fatigue crack growth in repairing aircraft structures. The model was developed by interpolation of numerical results. The obtained results were compared with those calculated with the finite element method. It was found that our model gives a good agreement of the *J*-integral. The arrow shape reduces the *J* integral at the crack tip, which improves the repair efficiency.

Keywords: composite; Finite Element Method; modelling; fracture mechanic; elastic-plastic

1. Introduction

Bonded composite repairs of metallic structure have become a useful aircraft structural life extension solution over the last two decades. These repairs provide an efficient method for restoring the ultimate load capability of the structure (Bachir Bouiadjra *et al*. 2010). Alan Baker (1984, 1996) was the first otherwise the pioneer of these searches in the aeronautical and maritime research laboratory of the Royal Australian Air Force. One important aspect in the design of a bonded patch is an accurate tool of predicting stresses, stress intensity factors and failure strength (Cao and Liu 2012, Cetisli and Kaman 2014, Gu *et al*. 2011, Hosseini-Toudeshky *et al*. 2011). It is known that the notches are the main causes of crack initiation. This is why the use of the bonded composite repair can play a significant role in the improvement of fatigue life of the notched structures (Sapora *et al*. 2014). Several authors have computed the stress intensity factor at the crack tip of repairing cracks among them (Jones and Callinan 1979, Ouinas *et al*. 2012). These researches have shown that, after repair, the stress intensity factor exhibits an asymptotic behaviour as the crack length increases. This behaviour is due to the fact that there is a stress transfer from the cracked aluminium plate to the composite patch throughout the adhesive layer.

Recently, some studies have taken into account the effect of the material plasticity. Oudad *et al*. (2009) investigated the influence of the patch parameters on the size of the plastic zone at the tip

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the size of the plastic zone ahead of the crack. Albedah *et al*. (2013) presented the analytical formulation of the stress intensity for repairs cracks emanating from central elliptical and circular holes. Kujawski (1991) gives a good approximation of the stress intensity for repairs cracks emanating from central elliptical and circular holes. In addition, the developed model highlights clearly the difference in the value of SIF for small andlong cracks. The use of this model can be very useful for the repair designers in aircraft technology.

A three-dimensional finite element method is used for analysis of repairing cracks in plates with bonded composite patch. The comparison is done by analysing the *J* integral at the tip of repairing cracks for the two patch shapes are presented. This study was performed in order to establish an analytical model of the *J*-integral for repair crack. The model was developed by interpolation of numerical results. The obtained results were compared with those calculated with the finite element method.

2. Geometrical and materialsmodels

The basic geometry of the cracked structure considered in this study is shown in Fig. 1. Consider a plate with the following dimensions: height H_p=254 mm, width w_p=254 mm, thickness e_p=3 mm. The plate is subjected to uniaxial tensile load giving a remote stress state of σ=100 MPa for elastic analysis. A central crack of length 2*a* perpendicular to the loading axis is supposed to exist in the plate. This crack is repaired with unidirectional Boron/Epoxy composites patches. The ply orientation is parallel to the loading axis. The initial dimensions of the patch are: height H_r=80 mm, width w_r=100 mm and thickness e_r=2 mm. The adhesive are used to bond the patch on cracked plate: FM 73, Epoxy adhesive. The adhesive thickness (e_a) is taken equal to 0.2 mm.Two patch shapes were analysed in this study as shown in Fig. 2:

- Rectangular shape (shape 1): It is the most used more use shape because of its simplicity.
- Arrow patch shape (patch 2).

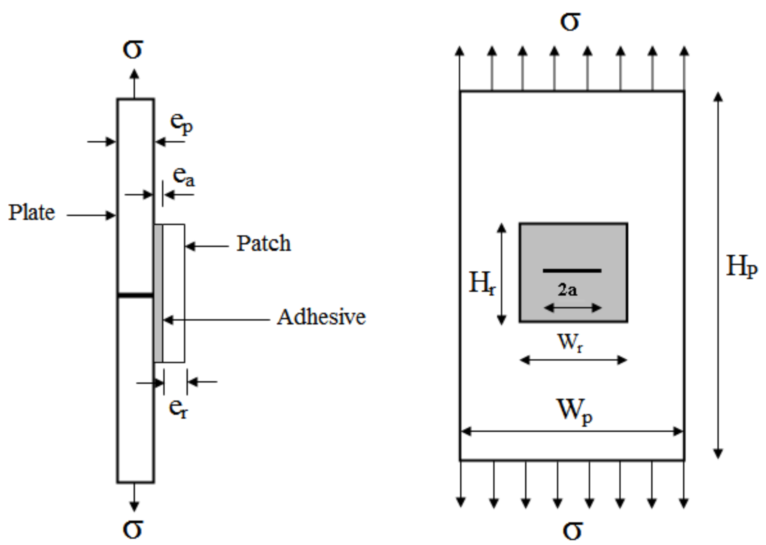

Fig. 1 Geometrical model

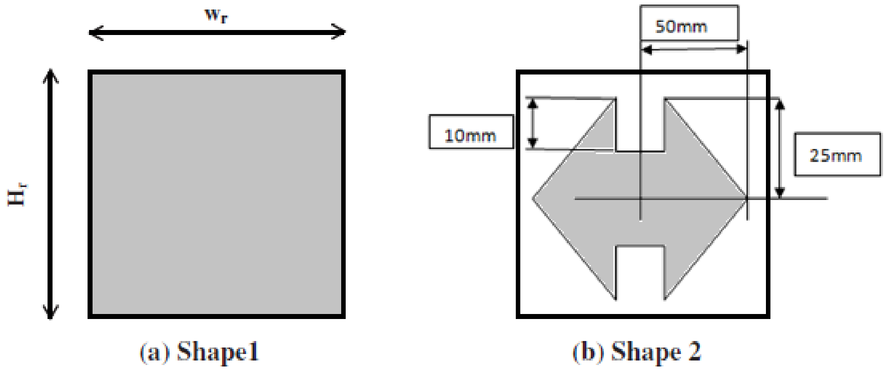

Fig. 2 Shapes of the patch

3. Material properties

The elastic properties of the plate, the patch and the adhesives are given in Table1:

Table 1 Elastic properties of different materials

	Aluminium alloy T3	Boron/epoxy	Adhésive (FM 73)
E_1 (GPa)	72	200	2.50
E_2 (GPa)		25	
E_3 (GPa)		25	
υ_{12}	0.33	0.21	0.30
υ_{13}		0.21	
υ_{23}		0.21	
G_{12} (GPa)		7.2	
G_{13} (GPa)		5.5	
G_{23} (GPa)		5.5	

For elastic-plastic analyses the Tables 1 give the mechanical properties of the material used in this study.

Table 2 Summary of tensile properties for aluminum 2024-T3 (Pyo *et al*. 1995)

E (GPa)	σ_y (MPa)	υ	α	n
72	345	0.33	0.2	8

4. Finite element modeling

The three-dimensional finite element analysis is carried out using the commercial finite element code ABAQUS (2007). The finite element model consists of three subsections tomodel the

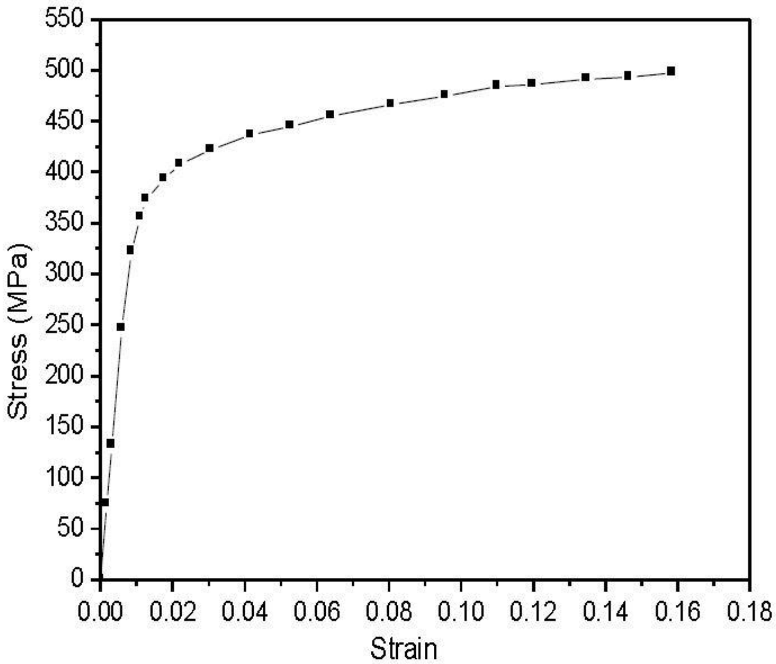

Fig. 3 Stress-strain curves for aluminum 2024-T3 (Oudad *et al*. 2009)

cracked plate, the adhesive, and the composite patch. Due to symmetry, only one quarter of the repaired plate was considered. The plate had four layers of elements in the thicknessdirection, the adhesive had only one layer of elements throughthickness and the patch had two layers of elements through thickness. To generate crack front some brick elements are replaced by "crack block". This crack-block is meshes of brick elements, which are mapped into the original elements space and merged with surrounding. The mesh was refined near the crack tip area with an element dimension of 0.067 mm using at least fifteen such fine elements in the front and back of the crack tip. The finite element mesh was generated using brick elements with 20 nodes. The number of element used in this analysis is 438216 and number of degrees of freedom DOF is: 521328 of rectangular patch andthe number of element used of arrow patch is 487359 and number of degrees of freedom DOF is: 394030. Fig. 4 shows the overall mesh of the specimen and mesh refinement in the crack tip region.

The Stress intensity factor at the crack front was computed using the virtual crack closure technique (VCCT). The VCCT is a very attractive SIF extraction technique because of its good accuracy, a relatively easy algorithm of application capability to calculate SIF for all three-fracture modes. Other authors (Leski 2007, Krueger 2002) have extended the proposed technique. Currently, the three-dimensional virtual crack closure technique (3D VCCT) is often chosen as a tool for SIF calculations

5. Results and discussion

The objective of this study is to establish an analytical model of the *J*-integral for repair crack.

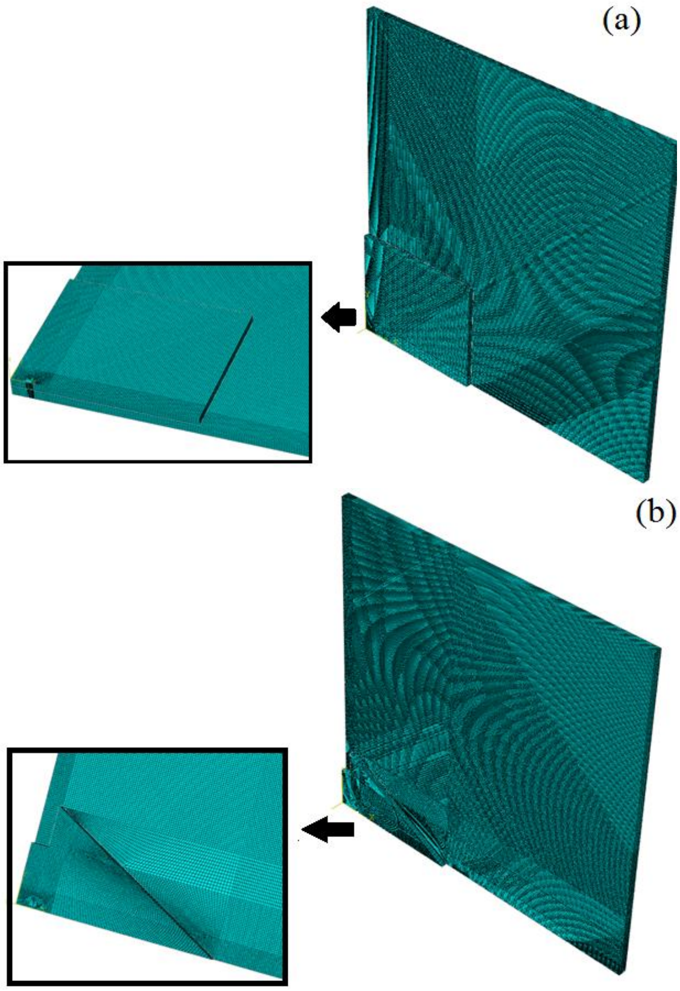

Fig. 4 Typical mesh model of the quarter of the structure (a) rectangular patch, (b) arrow patch

This formulation of the *J*-integral to establish models of fatigue crack growth in repairing aircraft structures. The model was developed by interpolation of numerical results two patch shapes were analysed in this study a rectangular shape (shape 1) and Arrow patch shape (patch 2).

5.1 Elastic analyses

Fig. 5 presents the variation of the stress intensity factor according to the crack length of the arrow patch and rectangular patch. It can be seen that there is a considerable difference in the stress intensity factor between the two shapes. Indeed the reduction of the stress intensity factor due to the use of the arrow patch is about 56% compared to the rectangular shape. This rate is constant whatever the crack length. This result confirms that the use of the arrow patch highly improve the fatigue life of the repaired structures.

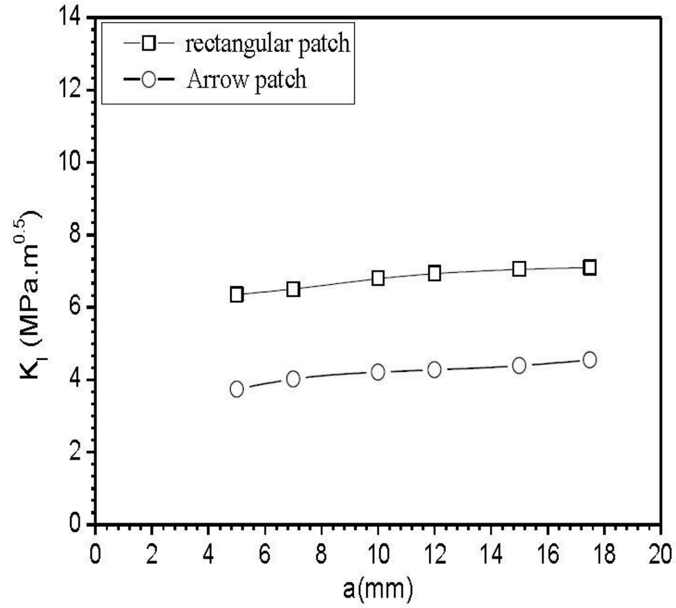

Fig. 5 Variations of the stress intensity factor, according to the crack length of the arrow patch and rectangular patch

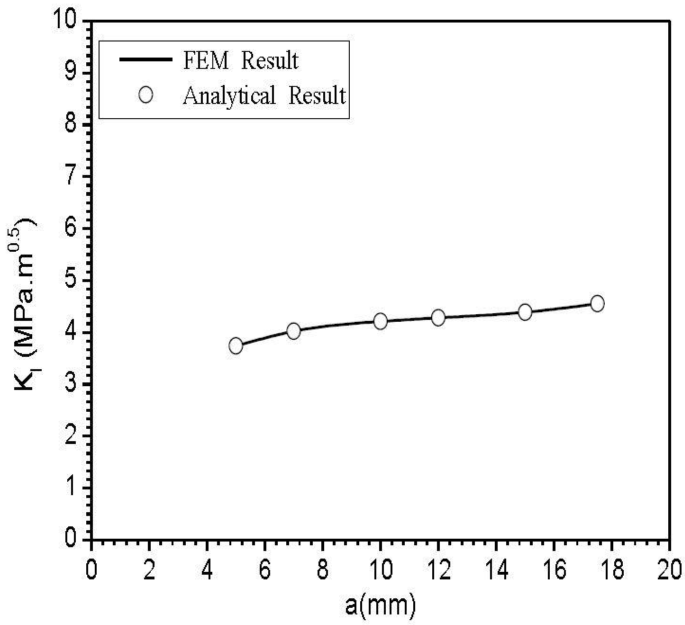

Fig. 6 Variation of the stress intensity factor, according to the crack length of the arrow patch of the analytical model compared to the FE results

In comparing the results of the three approaches, one can see that for short cracks (a<5 mm), the values of the SIF from the two analytical models (Jones *et al*. 2004, Modified Kujawski 1991) are slightly different from those obtained by the finite element method which is taken as reference. The Jones *et al*. (2004) model underestimates the stress intensity factor for repairs crack, while the modified Kujawski (1991) model over estimates it. For longer cracks (*a*>5 mm) the two analytical models give good approximation of the SIF compared to the FE analysis, but it is clear that the modified Kujawski (1991) approach is more accurate.

Fig. 6 presents the variation of the stress intensity factor according to the crack length of the arrow patch of the analytical model compared to the FE analysis. The model was developed by interpolation of numerical results.The analytical model gives a good agreement of the stress intensity factor compared to the FE analysis.A shape factor function F_t is obtained by interpolation of the numerical results. The fitted solution of the *J*-integral of repairing cracks in plates with bonded composite patch is:

The elastic of the *J*-integral can be found

$$J_e = \frac{K_I^2}{E} \tag{1}$$

Where J_e is the energy release rate for mode *I*

$$K_I = \sigma . F_t \sqrt{\pi a} \tag{2}$$

$$F_t = g . \chi . \xi . \psi\left(\frac{a}{t}\right) \tag{3}$$

$$g = \left(\frac{\nu_p}{\nu_a}\right)\left(\frac{E_a}{E_p}\right) \tag{4}$$

$$\chi = \left(\frac{E_{r1}}{G_{r12}}\right)\left(\frac{G_{r23}}{G_{r13}}\right)\left(\frac{\nu_{r23}}{\nu_{r12}}\right)\left(\frac{E_{r2}}{E_{r3}}\right)\nu_{r13} \tag{5}$$

$$\xi = \left(\frac{e_r}{e_a}\right) \tag{6}$$

$$\psi\left(\frac{a}{t}\right) = 2{,}06251\left(\frac{a}{t}\right)^2 - 26{,}71294\left(\frac{a}{t}\right) + 172{,}76153 \tag{7}$$

5.2 Elastic-plastic analyses

For better understanding of this behaviour, the J-integral variation as a function of the crack length in the case of single sided composite repair for the two forms of the patch (arrow and rectangular) is presented in Fig. 7. It can be also noted, that the difference in the *J*-integral between the two patch shapes is significant when the crack length varies between 10 and 17.5 mm. The maximal difference is recorded for *a*=17.5 mm. The reduction of the *J*-integral by the use of the rectangular patch is less significant compared with the arrow patch. The arrow shape reduces the *J* integral at the crack tip, which improves the repair efficiency.

Fig. 8 presents the distribution of the stresses along the cracked plate for the two patch shapes (arrow and rectangular) in the case of single sided composite repair. It can be seen that the

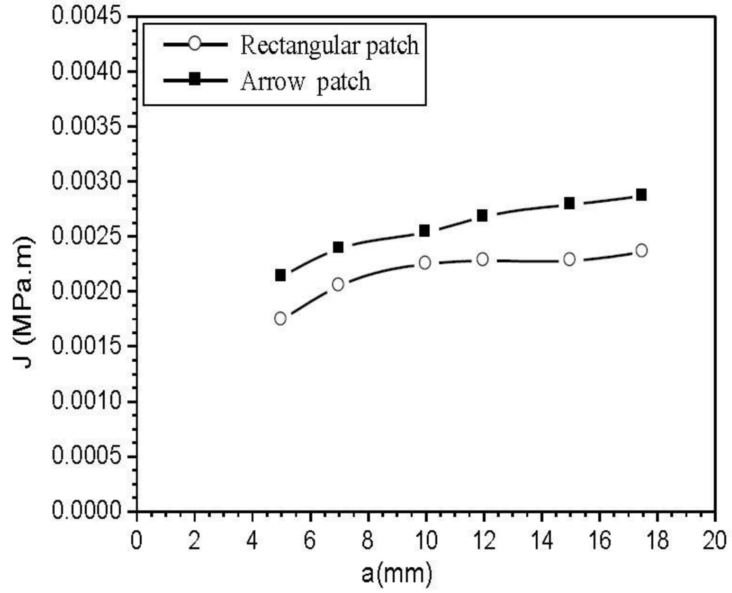

Fig. 7 Variation of the *J* integral according to the crack length of the arrow patch and rectangular patch

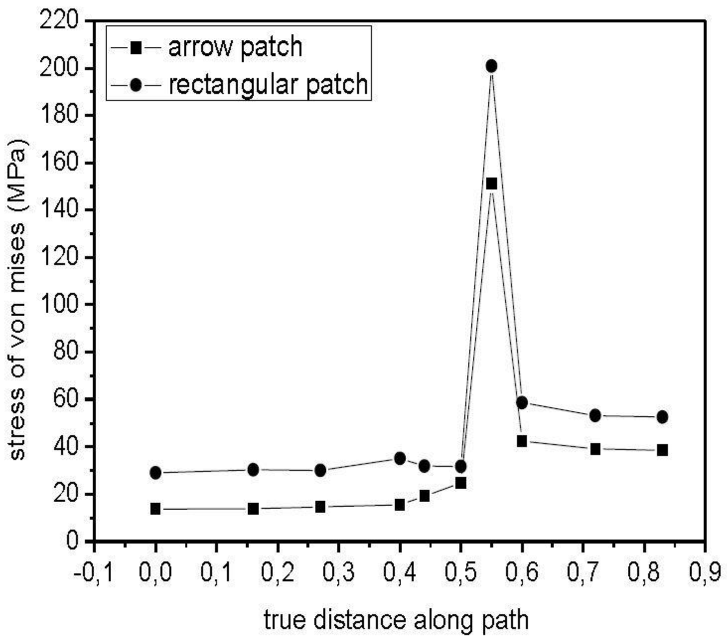

Fig. 8 Presents the distribution of the stresses along the cracked plate for the two patch shapes(arrow and rectangular)

maximum stress in the plate is recorded at the level at the crack tip. This is due to the fact that the stress on the repaired structures is concentrated near the crack tip. Beyond this region, the stresses are almost weak and the increase of the crack length increases the stress at the crack tip, the arrow

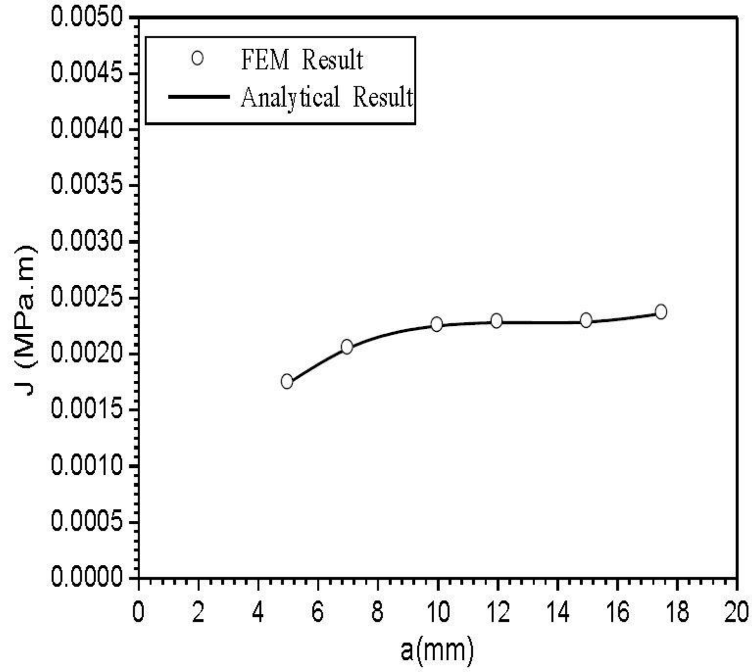

Fig. 9 variation of the *J*-integral according to the crack length of the arrow patch for the analytical model compared to the FE results

patch reduces the stresses compared with rectangular patch, the arrow shapes gives the good performances and improve the repair durability.

Fig. 9 presents the variation of the *J*-integral according to the crack length for the arrow patch for the analytical model compared to the FE analysis.The analytical model gives good agreement of the *J* integral compared to the FE analysis. A shape factor function *H* is obtained by interpolation of the numerical results. The fitted solution of the *J* integral of repairing cracks in plates with bonded composite patch is

$$J = \sigma_y . a . \alpha \, n . H \tag{8}$$

$$H = g \, .\chi \, .\xi \, .\psi\left(\frac{a}{t}\right) \tag{9}$$

$$g = \left(\frac{\nu_p}{\nu_a}\right)\left(\frac{E_a}{E_p}\right) \tag{10}$$

$$\chi = \left(\frac{E_{r1}}{G_{r12}}\right)\left(\frac{G_{r23}}{G_{r13}}\right)\left(\frac{\nu_{r23}}{\nu_{r12}}\right)\left(\frac{E_{r2}}{E_{r3}}\right)\nu_{r13} \tag{11}$$

$$\xi = \left(\frac{e_r}{e_a}\right) \tag{12}$$

$$\psi\left(\frac{a}{t}\right)=7{,}36541.10^{-7}\left(\frac{a}{t}\right)^{2}-9{,}68342.10^{-6}\left(\frac{a}{t}\right)+4{,}2389.10^{-5} \tag{13}$$

5. Conclusions

This paper presents a three-dimensional finite element method used for analysis of repairing cracks in plates with bonded composite patch in elastic and elastic plastic. This study was performed in order to establish an analytical model of the *J*-integral for repair crack. This formulation of the *J*-integral to establish models of fatigue crack growth in repairing aircraft structures. The model was developed by interpolation of numerical results. The obtained results were compared with those calculated with the finite element method. The obtained results also show that the arrow shape gives higher repair performances. These performances can be detailed as follows:

- The arrow shape reduces the SIF at the crack tip, which improves the repair efficiency.
- The arrow patch reduces the stresses, which improve the repair durability.
- It was found that our model gives a good agreement of the *J*-integral. The arrow shape reduces the J integral at the crack tip, which improves the repair efficiency.

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