Structural Engineering and Mechanics, *Vol. 58, No. 4 (2016) 689-705*
DOI: http://dx.doi.org/10.12989/sem.2016.58.4.689

Stress analysis of rotating annular hyperbolic discs obeying a pressure-dependent yield criterion

Woncheol Jeong[1a] and Kwansoo Chung[*2]

[1]*Department of Materials Science and Engineering, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 08826, Republic of Korea*
[2]*Department of Materials Science and Engineering, Research Institute of Advanced Materials, Engineering Research Institute, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 08826, Republic of Korea*

(Received October 30, 2015, Revised March 20, 2016, Accepted March 26, 2016)

Abstract. The Drucker-Prager yield criterion is combined with an equilibrium equation to provide the elastic-plastic stress distribution within rotating annular hyperbolic discs and the residual stress distribution when the angular speed becomes zero. It is verified that unloading is purely elastic for the range of parameters used in the present study. A numerical technique is only necessary to solve an ordinary differential equation. The primary objective of this paper is to examine the effect of the parameter that controls the deviation of the Drucker-Prager yield criterion from the von Mises yield criterion and the geometric parameter that controls the profile of hyperbolic discs on the stress distribution at loading and the residual stress distribution.

Keywords: rotating annular disc; variable thickness; plastic yielding; Drucker-Prager yield criterion

1. Introduction

Rotating discs are widely used in mechanical engineering and a great number of solutions for elastic/plastic disc are available in the literature. A review of solutions for discs of constant thickness obeying the Tresca and von Mises yield criteria has been provided in Rees (1999). More general yield criteria for discs of constant thickness have been adopted in Guo-wei *et al*. (1995), Alexandrova and Alexandrov (2004), Callioglu *et al*. (2006). However, discs of variable thickness are advantageous for many applications. There are a great number of solutions for such discs as well. However, most of analytic solutions are for the Tresca yield criterion and Hencky's deformation theory of plasticity. The latter is usually based on the von Mises yield criterion. In particular, the Tresca yield criterion has been adopted in Guven (1992, 1998), Orcan and Eraslan (2002), Eraslan and Orcan (2002a, 2002b), Eraslan (2002, 2003). These analyses have been based on the associated flow rule and linear strain hardening. Various disc profiles and boundary conditions have been assumed. Hencky's deformation theory of plasticity has been adopted in You *et al*. (2000), Eraslan (2003), Hojjati and Hassani (2008). It is worthy of note that the deformation

∗Corresponding author, Professor, E-mail: kchung@snu.ac.kr
[a]Ph.D. Student, E-mail: rippler@snu.ac.kr

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

theory of plasticity is valid only when dealing with proportional loadings. On the other hand, it is known that the strain path is not proportional in thin discs (see, for example, Pirumov *et al.* 2013).

Many metallic materials reveal pressure-dependence of plastic yielding (Spitzig *et al.* 1976, Kao *et al.* 1990, Wilson 2002, Liu 2006). It has been demonstrated in Alexandrov *et al*. (2011) and Pirumov *et al*. (2013) that this material property may have a significant effect on the distribution of stresses in thin discs. However, available solutions for rotating discs do not account for pressure-dependence of plastic yielding. The present paper provides such a solution for an annular disc assuming that the yield criterion proposed in Drucker and Prager (1952) is valid.

2. Statement of the problem

Consider a thin annual rotating disc of variable thickness. It is assumed that the outer and inner radii of the disc are stress free. These radii are denoted by a_0 and b_0, respectively (Fig. 1). It is convenient to introduce a cylindrical coordinate system (r,θ,z) whose z-axis coincides with the axis of symmetry of the disc. Let σ_r, σ_θ and σ_z be the normal stresses in this coordinate system. Symmetry of the problem dictates that these stresses are the principal stresses. Moreover, $\sigma_z=0$ under plane stress conditions. In the cylindrical coordinate system the boundary conditions are written as

$$\sigma_r = 0 \tag{1}$$

for $r=a_0$ and $r=b_0$. The only non-trivial equilibrium equation is (Timoshenko and Goodier 1970)

$$\frac{d}{dr}\left(hr\sigma_r\right) - h\sigma_\theta + h\rho\omega^2 r^2 = 0 \tag{2}$$

where ρ is the material density, and ω is the angular velocity of the disc about the z-axis. The thickness of the disc is assumed to vary according to the equation

$$h = h_0\left(\frac{r}{a_0}\right)^m \tag{3}$$

where h_0 is the thickness at the edge of the disc and m is a constant. This dependence of the thickness on the radius is of practical importance (Guven 1998, You *et al*. 2000, Hojjati and Hassani 2008). Substituting Eq. (3) into Eq. (2) yields

$$\frac{d\sigma_r}{dr} + \frac{(m+1)\sigma_r - \sigma_\theta}{r} + \rho\omega^2 r = 0\,. \tag{4}$$

Since $\sigma_z=0$, the Hooke's law in the cylindrical coordinate system reads

$$\varepsilon_r^e = \frac{\sigma_r - \nu\sigma_\theta}{E}, \quad \varepsilon_\theta^e = \frac{\sigma_\theta - \nu\sigma_r}{E}, \quad \varepsilon_z^e = -\frac{\nu\left(\sigma_r + \sigma_\theta\right)}{E}\,. \tag{5}$$

Here ν is Poisson's ratio and E is Young's modulus. The superscript e denotes the elastic part of the strain. Since the boundary value problem is statically determinate, no relation between stress

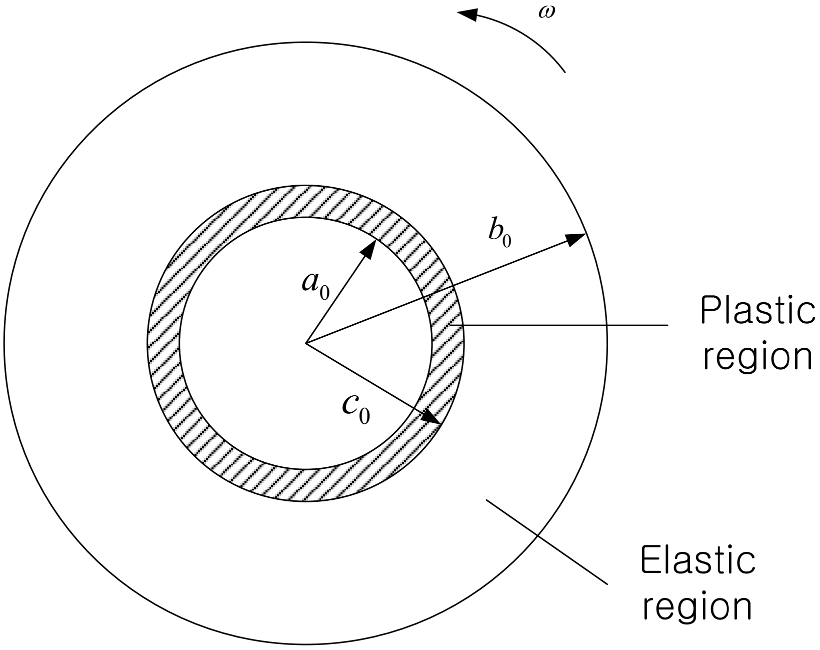

Fig. 1 Disc configuration

and plastic strain (or plastic strain rate) is required for stress analysis. Under plane stress conditions the yield criterion proposed in Drucker and Prager (1952) becomes

$$\frac{\alpha}{3}(\sigma_r+\sigma_\theta)+\sqrt{\sigma_\theta^2+\sigma_r^2-\sigma_r\sigma_\theta}=\sigma_0 \tag{6}$$

where α and σ_0 are material constants. It is worthy of note that this yield criterion adequately describes yielding of many metallic materials (Spitzig *et al*. 1976, Kao *et al*. 1990, Wilson 2002, Liu 2006). It is seen from Eq. (6) that the value of α controls the deviation of the pressure-dependent yield criterion adopted from the von Mises yield criterion and that the yield criterion (6) becomes the von Mises yield criterion at α=0. It is convenient to rewrite (6) in the form

$$\left(1-\frac{\alpha^2}{9}\right)\sigma_r^2+\left(1-\frac{\alpha^2}{9}\right)\sigma_\theta^2-\left(1+\frac{2\alpha^2}{9}\right)\sigma_r\sigma_\theta+\frac{2\alpha}{3}\sigma_0(\sigma_r+\sigma_\theta)=\sigma_0^2 \tag{7}$$

and to introduce the following dimensionless quantities

$$\Omega=\frac{\rho\omega^2b_0^2}{\sigma_0},\quad a=\frac{a_0}{b_0},\quad \gamma=\frac{r}{b_0}. \tag{8}$$

3. Purely elastic solution

The entire disc is elastic if Ω is small enough. The general solution of Eqs. (4) and (5) supplemented with the equation of strain compatibility is well known (see, for example, Timoshenko and Goodier 1970). In particular, the radial distribution of stress is given by

$$\frac{\sigma_r}{\sigma_0} = A\gamma^{n_1} + B\gamma^{n_2} + \Omega D_1\gamma^2 \ , \ \frac{\sigma_\theta}{\sigma_0} = A\left(m+1+n_1\right)\gamma^{n_1} + B\left(m+1+n_2\right)\gamma^{n_2} + \Omega D_2\gamma^2 \tag{9}$$

where $n_1 = \dfrac{-\left(m+2\right)+\sqrt{\left(m+2\right)^2-4m\left(1+\nu\right)}}{2}$, $n_2 = \dfrac{-\left(m+2\right)-\sqrt{\left(m+2\right)^2-4m\left(1+\nu\right)}}{2}$,

$$D_1 = \frac{3+\nu}{1+3\nu-\left(3+\nu\right)\left(3+m\right)} \ , \ D_2 = \frac{3\nu+1}{1+3\nu-\left(3+\nu\right)\left(3+m\right)} .$$

Here A and B are constants of integration. In the case of purely elastic discs the solution (9) should satisfy the boundary conditions (1). Therefore

$$A = A_e = \frac{a^{n_2}-a^2}{a^{n_1}-a^{n_2}}D_1\Omega, \quad B = B_e = -\frac{a^{n_1}-a^2}{a^{n_1}-a^{n_2}}D_1\Omega \ . \tag{10}$$

Substitution of Eq. (10) into Eq. (9) provides the stress distribution in the purely elastic disc in the form

$$\begin{aligned} \frac{\sigma_r}{\sigma_0} &= \left[\left(\frac{a^{n_2}-a^2}{a^{n_1}-a^{n_2}}\right)\gamma^{n_1} - \left(\frac{a^{n_1}-a^2}{a^{n_1}-a^{n_2}}\right)\gamma^{n_2} + \gamma^2\right]D_1\Omega, \\ \frac{\sigma_\theta}{\sigma_0} &= \left[D_1\left(\frac{a^{n_2}-a^2}{a^{n_1}-a^{n_2}}\right)\left(m+1+n_1\right)\gamma^{n_1} - D_1\left(\frac{a^{n_1}-a^2}{a^{n_1}-a^{n_2}}\right)\left(m+1+n_2\right)\gamma^{n_2} + D_2\gamma^2\right]\Omega. \end{aligned} \tag{11}$$

The plastic yielding is assumed to begin at $\gamma=a$ and this assumption should be verified *a posteriori*. Since $\sigma_r=0$ at $\gamma=a$, it follows from Eq. (7) that

$$\frac{\sigma_\theta}{\sigma_0} = \frac{3}{\alpha+3} \tag{12}$$

at $\gamma=a$ on the initiation of plastic yielding. Replacing γ in Eq. (11) with a and eliminating σ_θ/σ_0 with the use of Eq. (12) yield

$$\Omega_e = \frac{3}{\left(\alpha+3\right)}\left[D_1\left(\frac{a^{n_2}-a^2}{a^{n_1}-a^{n_2}}\right)\left(m+1+n_1\right)a^{n_1} - D_1\left(\frac{a^{n_1}-a^2}{a^{n_1}-a^{n_2}}\right)\left(m+1+n_2\right)a^{n_2} + D_2a^2\right]^{-1} \tag{13}$$

where Ω_e is the value of Ω corresponding to the initiation of plastic yielding.

4. Elastic/plastic solution

Plastic yielding occurs in the disc if $\Omega_e \le \Omega$. Let Ω_p be the angular velocity at which the whole disc plastic. If $\Omega_e < \Omega < \Omega_p$ then the disc has an inner plastic part, $a \le \gamma \le \gamma_c$ (or $a_0 \le r \le c_0$), and an outer elastic part, $\gamma_c \le \gamma \le 1$ (or $c_0 \le r \le b_0$). Here c_0 is the radius of the plastic/elastic boundary and $\gamma_c = c_0/b_0$ is its dimensionless representation (Fig. 1). The general solution (9) is valid in the elastic region. However, A and B are not given by Eq. (10). This solution should satisfy the boundary condition (1) at γ=1. Therefore

$$A + B + \Omega D_1 = 0 . \tag{14}$$

In the plastic region, it is necessary to solve Eqs. (4) and (7). The yield criterion (7) is satisfied by the following substitution (Alexandrov *et al*. 2011)

$$\begin{aligned}
&\frac{\sigma_r}{\sigma_0} = 3\beta_0 - \frac{\beta_1}{2}\left(1+3\sqrt{3}\beta_1\right)\sin\psi + \frac{\sqrt{3}}{2}\beta_1\left(1-\sqrt{3}\beta_1\right)\cos\psi,\\
&\frac{\sigma_\theta}{\sigma_0} = 3\beta_0 + \frac{\beta_1}{2}\left(1-3\sqrt{3}\beta_1\right)\sin\psi - \frac{\sqrt{3}}{2}\beta_1\left(1+\sqrt{3}\beta_1\right)\cos\psi,\\
&\beta_0 = \frac{2\alpha}{4\alpha^2-9}, \quad \beta_1 = \frac{\sqrt{3}}{\sqrt{9-4\alpha^2}}.
\end{aligned} \tag{15}$$

Here ψ is a new function of γ. Eliminating σ_r and σ_θ in Eq. (4) by means of Eq. (15) leads to

$$\begin{aligned}
&\beta_1\gamma\left[\left(1+3\sqrt{3}\beta_1\right)\cos\psi + \sqrt{3}\left(1-\sqrt{3}\beta_1\right)\sin\psi\right]\frac{\partial\psi}{\partial\gamma} =\\
&6\beta_0 m + \beta_1\left[-3\beta_1 m + \sqrt{3}\left(2+m\right)\right]\cos\psi - \beta_1\left(2+m+3\sqrt{3}\beta_1 m\right)\sin\psi + 2\gamma^2\Omega
\end{aligned} \tag{16}$$

This equation should be solved numerically. Let ψ_a be the value of ψ at γ=a. Then, the boundary condition for Eq. (16) is

$$\psi = \psi_a \tag{17}$$

for γ=a. The solution of Eq. (16) satisfying the boundary condition (17) is denoted as

$$\psi = \Psi\left(\gamma, \Omega\right). \tag{18}$$

The second argument of the function $\Psi(\gamma, \Omega)$ emphasizes that the solution depends on Ω. It follows from Eqs. (15) that

$$\frac{\sigma_\theta - \sigma_r}{\sigma_0} = \frac{2\sqrt{3}}{\sqrt{9-4\alpha^2}}\sin\left(\psi - \frac{\pi}{3}\right), \quad \frac{\sigma_\theta + \sigma_r}{\sigma_0} = \frac{12\alpha}{4\alpha^2-9} + \frac{18}{\left(4\alpha^2-9\right)}\sin\left(\psi + \frac{\pi}{6}\right). \tag{19}$$

It is reasonable to assume that $\sigma_\theta > \sigma_r$. Then, it is seen from Eq. (19) that

$$\frac{\pi}{3} < \psi < \frac{4\pi}{3} . \tag{20}$$

The boundary condition (1) at γ=a and Eq. (15) for σ_r combine to give

$$\psi_a = \frac{4\pi}{3} - \arcsin q, \quad q = \frac{\sqrt{3}\sqrt{9-4\alpha^2}}{2\left(3+\alpha\right)}. \tag{21}$$

The inequality (20) has been here taken into account. The disc becomes fully plastic when $\gamma_c=1$. It follows from the boundary condition (1) at $r=b_0$ (or $\gamma=1$) that $\psi_c=\psi_a$ at this instant. Therefore, Ω_p is determined by the condition

$$\frac{4\pi}{3}-\arcsin q=\Psi\left(1,\Omega_p\right). \tag{22}$$

Let ψ_c be the value of ψ at $\gamma=\gamma_c$. The radial and circumferential stresses must be continuous across the plastic/elastic boundary. Then, it follows from Eqs. (9), (14), (18) and (19) that

$$\begin{gathered}B\left[\left(m+n_2\right)\gamma_c^{n_2}-\left(m+n_1\right)\gamma_c^{n_1}\right]+\left[\left(D_2-D_1\right)\gamma_c^2-D_1\left(m+n_1\right)\gamma_c^{n_1}\right]\Omega= \\ \frac{2\sqrt{3}}{\sqrt{9-4\alpha^2}}\sin\left(\Psi\left(\gamma_c,\Omega\right)-\frac{\pi}{3}\right), \\ B\left[\left(m+2+n_2\right)\gamma_c^{n_2}-\left(m+2+n_1\right)\gamma_c^{n_1}\right]+\left[\left(D_2+D_1\right)\gamma_c^2-D_1\left(m+2+n_1\right)\gamma_c^{n_1}\right]\Omega= \\ =\frac{12\alpha}{4\alpha^2-9}+\frac{18}{4\alpha^2-9}\sin\left(\Psi\left(\gamma_c,\Omega\right)+\frac{\pi}{6}\right).\end{gathered} \tag{23}$$

Eliminating B between these two equations yields

$$\begin{gathered}\left[\left(m+n_2\right)\gamma_c^{n_2}-\left(m+n_1\right)\gamma_c^{n_1}\right]\left\{\begin{matrix}\dfrac{12\alpha}{4\alpha^2-9}+\dfrac{18}{4\alpha^2-9}\sin\left(\Psi\left(\gamma_c,\Omega\right)+\dfrac{\pi}{6}\right)- \\ \left[\left(D_2+D_1\right)\gamma_c^2-D_1\left(m+2+n_1\right)\gamma_c^{n_1}\right]\Omega\end{matrix}\right\}= \\ \left[\left(m+2+n_2\right)\gamma_c^{n_2}-\left(m+2+n_1\right)\gamma_c^{n_1}\right]\left\{\begin{matrix}\dfrac{2\sqrt{3}}{\sqrt{9-4\alpha^2}}\sin\left(\Psi\left(\gamma_c,\Omega\right)-\dfrac{\pi}{3}\right)- \\ \left[\left(D_2-D_1\right)\gamma_c^2-D_1\left(m+n_1\right)\gamma_c^{n_1}\right]\Omega\end{matrix}\right\}.\end{gathered} \tag{24}$$

This equation supplies the dependence of γ_c on Ω in implicit form. The variation of ψ_c with Ω is determined by substituting this dependence into Eq. (18). Then, the dependence of B on Ω can be found from any of Eqs. (23). Finally, the variation of A with Ω is given by Eq. (14).

Let Ω_f be the maximum angular velocity. It is assumed that $\Omega_e<\Omega_f<\Omega_p$ where Ω_p is determined from Eq. (22). Having found A, B, γ_c and ψ_c as functions of Ω it is possible to calculate the values of these functions at $\Omega=\Omega_f$. Then, the distribution of σ_r and σ_θ is determined from Eq. (9) in the range $\gamma_c\le\gamma\le 1$ and from Eqs. (15) and (18) in the range $a\le\gamma\le\gamma_c$. The latter is in parametric form with ψ being the parameter.

Typical values of m are $m=-0.5$ (Guven 1992) and $m=-0.25$ (Guven 1998). Using these values of m and $m=0$ (constant thickness) calculation has been performed assuming that $\gamma_c=0.3$ and $\gamma_c=0.5$ in an $a=0.2$ disc. In order to illustrate the effect of pressure-dependency of the yield criterion, two values of α have been chosen, $\alpha=0$ (pressure-independent material) and $\alpha=0.3$ (Liu 2006). It is assumed that $\nu=0.3$. Table 1 shows the corresponding values of Ω_f. The effect of m - value on the distribution of the radial stress with γ at $\gamma_c=0.3$ is illustrated in Fig. 2 for $\alpha=0$ and in Fig. 3 for $\alpha=0.3$. It is seen from these figures that the radial stress increases as the value of m decreases. The same effect is seen for $\gamma_c=0.5$ in Fig. 4 for $\alpha=0$ and in Fig. 5 for $\alpha=0.3$. The effect of m-value on the circumferential stress is not so significant. In particular, the distribution of this stress component with γ for γ_c=0.3 is depicted in Fig. 6 for α=0 and in Fig. 7 for α=0.3. In the case of γ_c=0.5 the variation of the circumferential stress with γ is shown in Fig. 8 for α=0 and in Fig. 9 for α=0.3.

Table 1 Dependence of Ω_f on geometric and material parameters in an a=0.2 disc.

γ_c	α	m	Ω_f
0.3	0	0	1.78988
		-0.25	2.03404
		-0.5	2.31910
	0.3	0	1.57240
		-0.25	1.78647
		-0.5	2.03664
0.5	0	0	2.27276
		-0.25	2.51241
		-0.5	2.78766
	0.3	0	1.96664
		-0.25	2.17068
		-0.5	2.40528

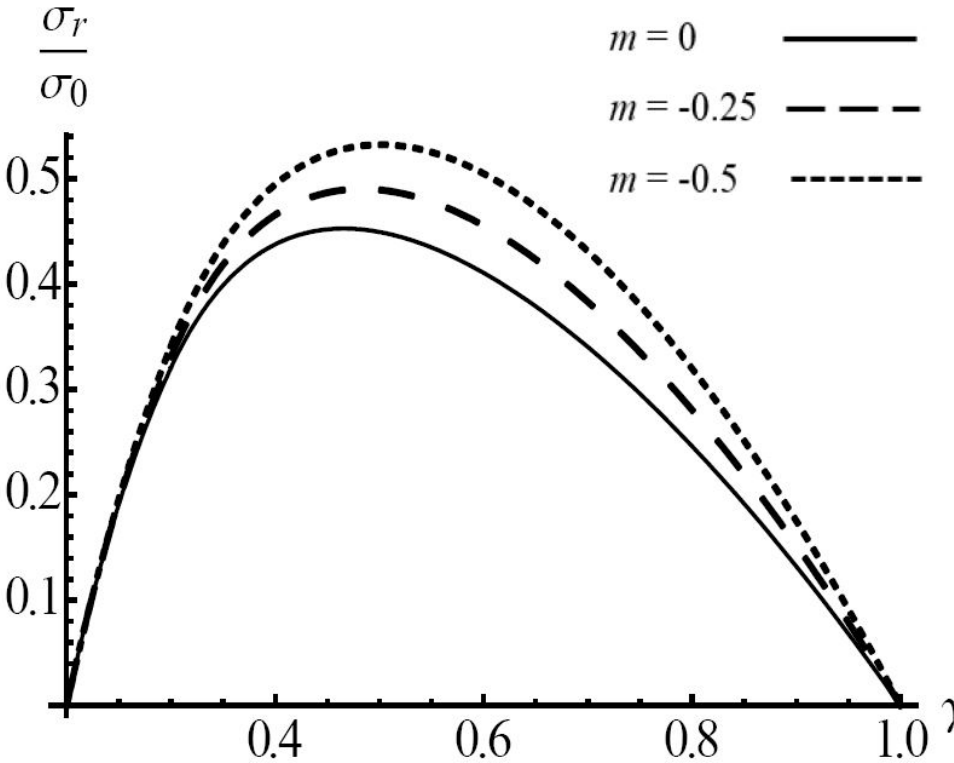

Fig. 2 Variation of the radial stress with γ at γ_c=0.3, α=0, and several m-values

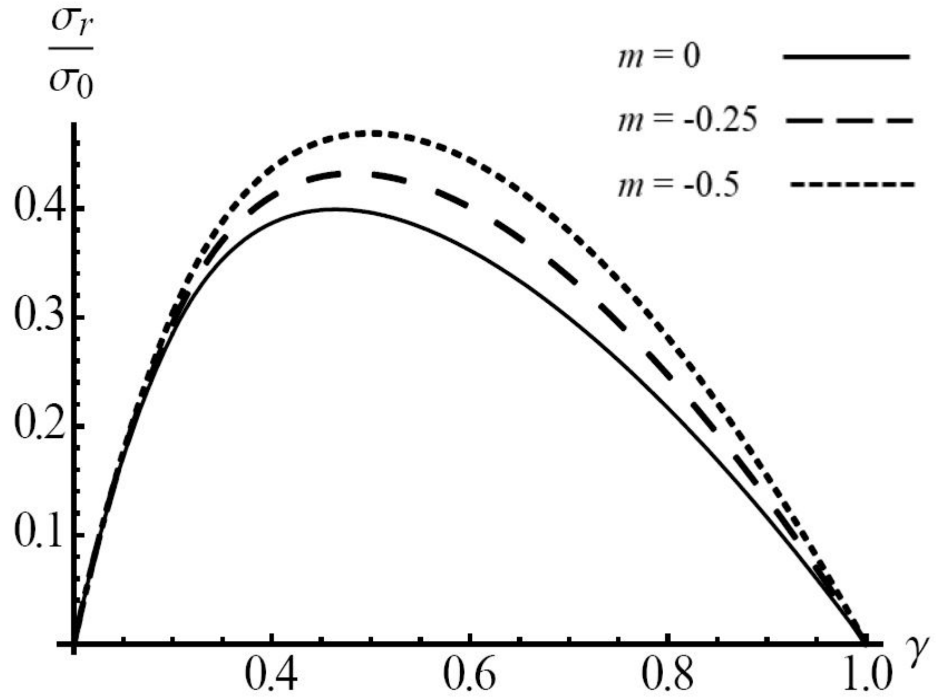

Fig. 3 Variation of the radial stress with γ at γ_c=0.3, α=0.3, and several m-values

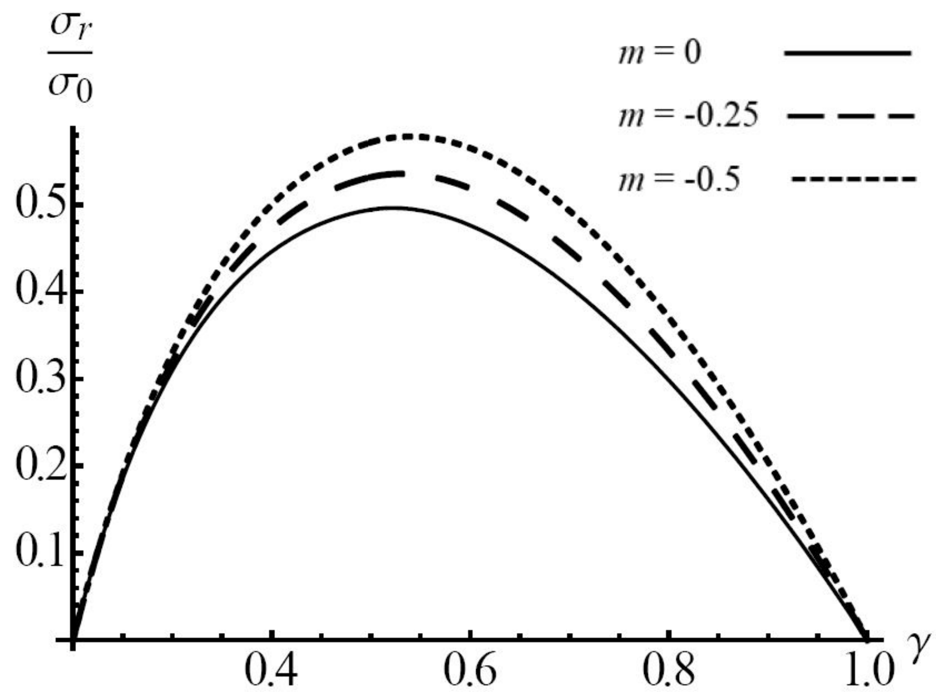

Fig. 4 Variation of the radial stress with γ at γ_c=0.5, α=0, and several m-values

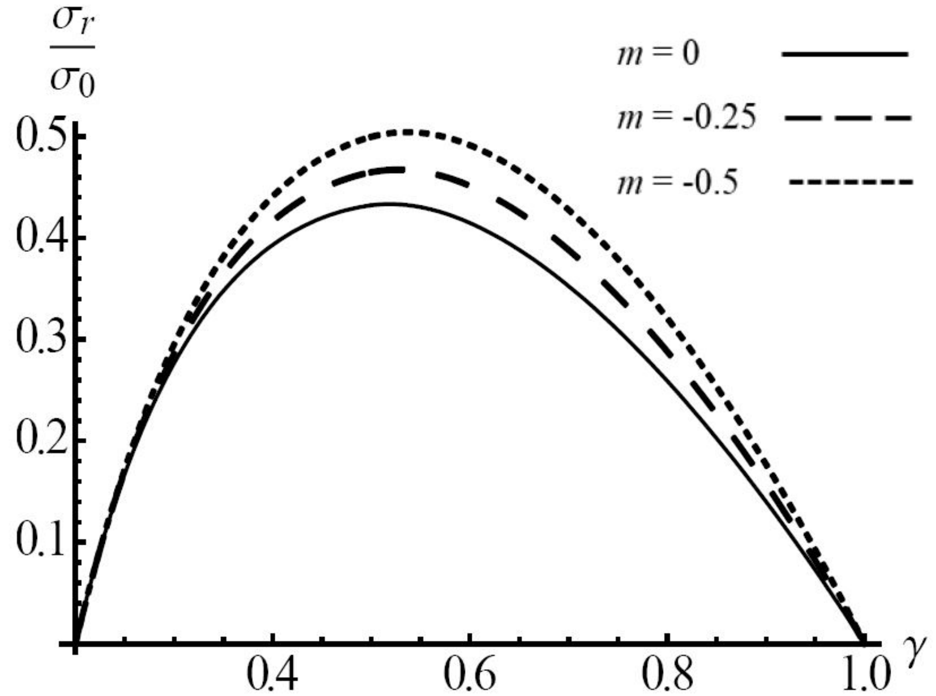

Fig. 5 Variation of the radial stress with γ at γ_c=0.5, α=0.3, and several m-values

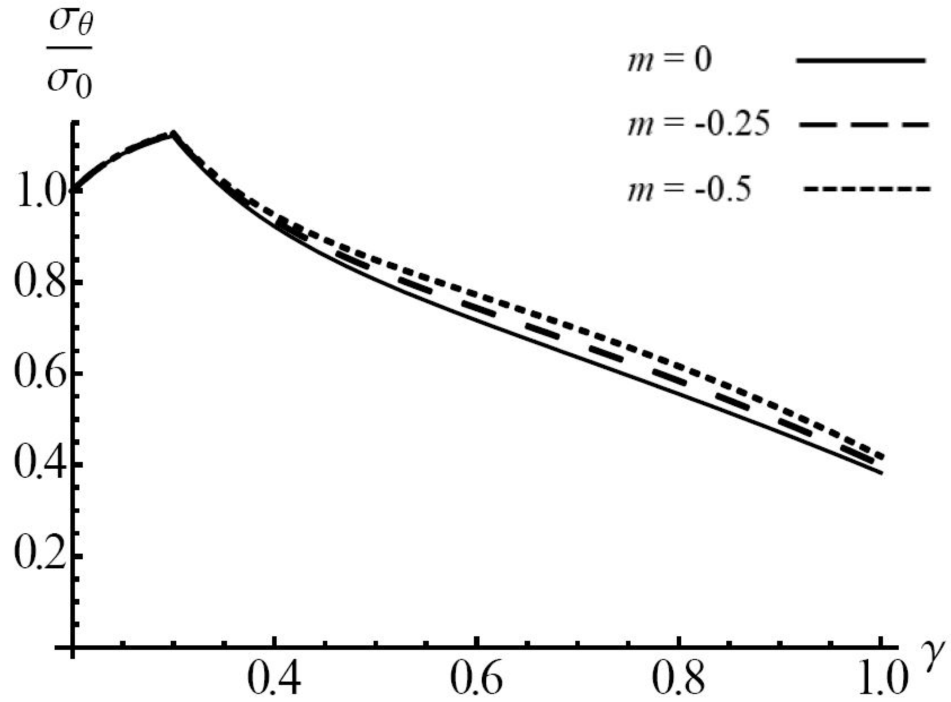

Fig. 6 Variation of the circumferential stress with γ at γ_c=0.3, α=0, and several m-values

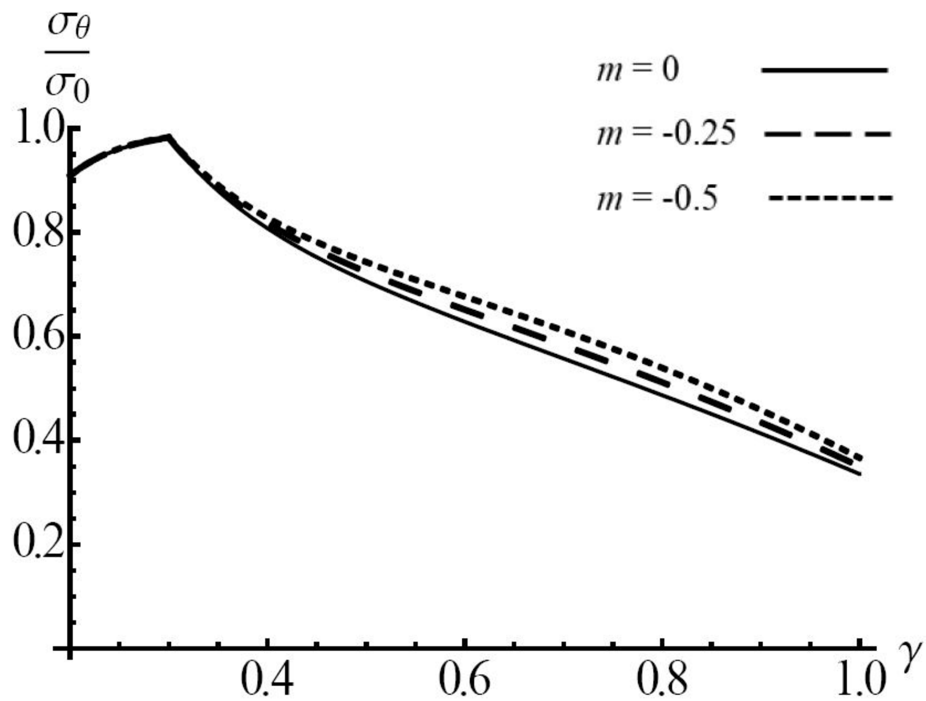

Fig. 7 Variation of the circumferential stress with γ at γ_c=0.3, α=0.3, and several *m*-values

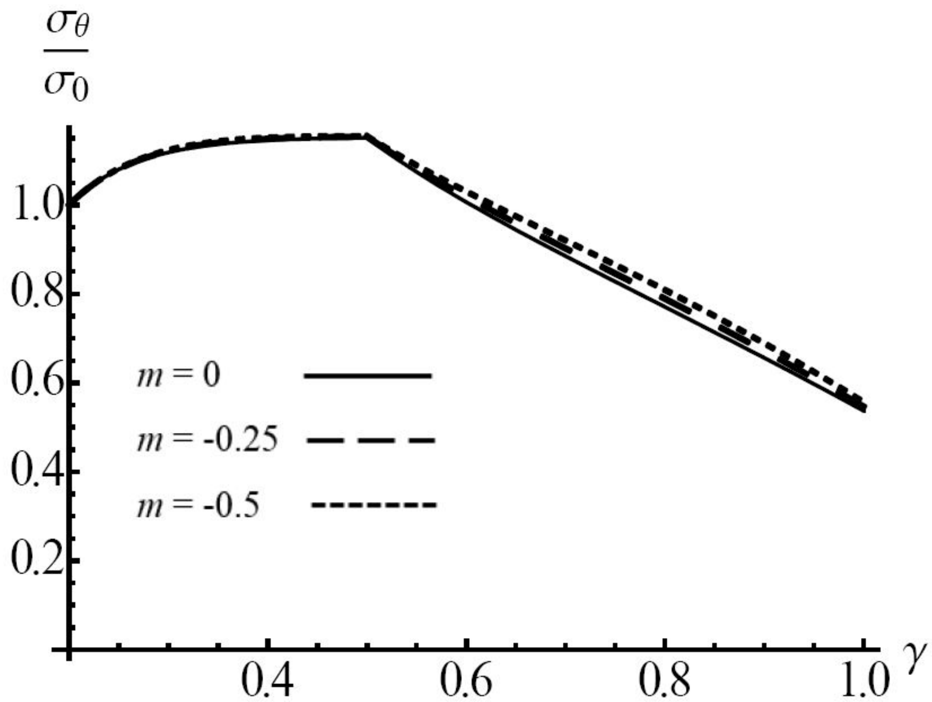

Fig. 8 Variation of the circumferential stress with γ at γ_c=0.5, α=0, and several *m*-values

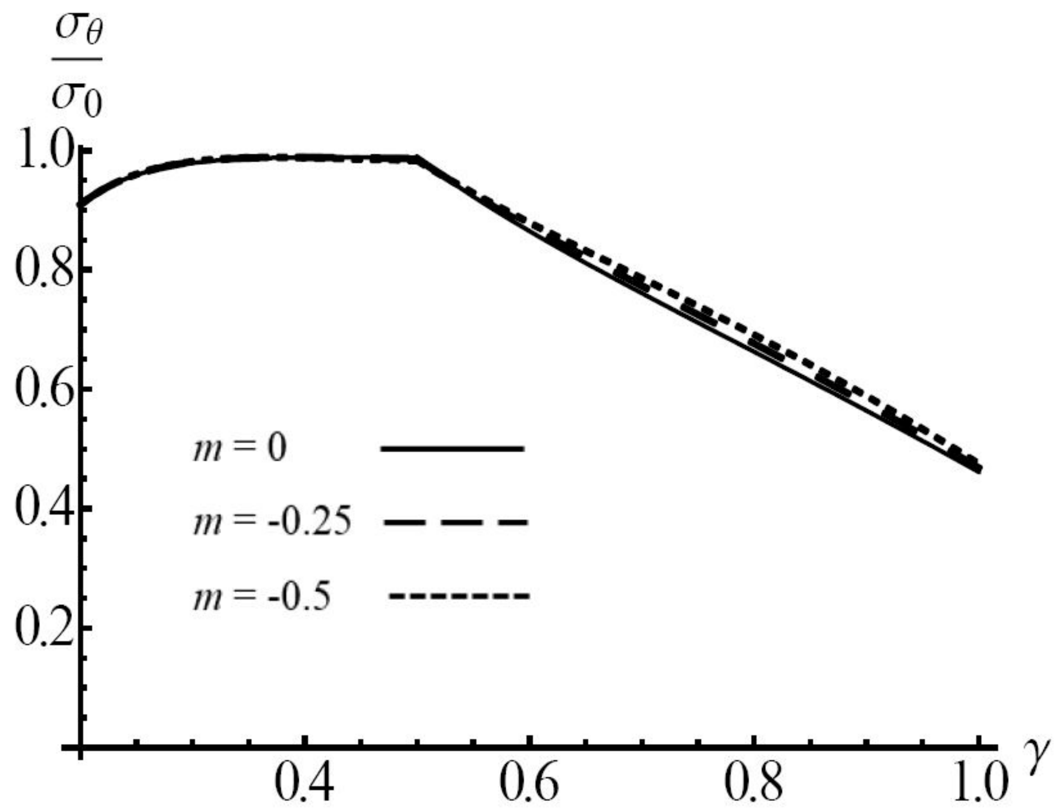

Fig. 9 Variation of the circumferential stress with γ at γ_c=0.5, α=0.3, and several m-values

5. Residual stresses

It is assumed that unloading is purely elastic. This assumption should be verified a posteriori. The stress increments, $\Delta\sigma_r$ and $\Delta\sigma_\theta$, are calculated by Eqs. (9) and (10) where Ω should be replaced with $-\Omega_f$, A with ΔA, and B with ΔB when the angular velocity decreases from Ω_f to zero. As a result

$$\begin{aligned}
\Delta A &= -\left(\frac{a^{n_2}-a^2}{a^{n_1}-a^{n_2}}\right)\frac{\Omega_f\left(3+\nu\right)}{\left[1+3\nu-\left(3+\nu\right)\left(3+m\right)\right]}, \\
\Delta B &= \left(\frac{a^{n_1}-a^2}{a^{n_1}-a^{n_2}}\right)\frac{\Omega_f\left(3+\nu\right)}{\left[1+3\nu-\left(3+\nu\right)\left(3+m\right)\right]}, \\
\frac{\Delta\sigma_r}{\sigma_0} &= \Delta A\gamma^{n_1}+\Delta B\gamma^{n_2}-\frac{\Omega_f\left(3+\nu\right)}{\left[1+3\nu-\left(3+\nu\right)\left(3+m\right)\right]}\gamma^2, \\
\frac{\Delta\sigma_\theta}{\sigma_0} &= \Delta A\left(m+1+n_1\right)\gamma^{n_1}+\Delta B\left(m+1+n_2\right)\gamma^{n_2}-\frac{\Omega_f\left(3\nu+1\right)}{\left[1+3\nu-\left(3+\nu\right)\left(3+m\right)\right]}\gamma^2 .
\end{aligned} \tag{25}$$

Then, the residual stresses are

$$\frac{\sigma_r^{res}}{\sigma_0} = \frac{\sigma_r}{\sigma_0} + \frac{\Delta\sigma_r}{\sigma_0}, \quad \frac{\sigma_\theta^{res}}{\sigma_0} = \frac{\sigma_\theta}{\sigma_0} + \frac{\Delta\sigma_\theta}{\sigma_0}. \tag{26}$$

Here σ_r and σ_θ are found from the solution given in Section 4 at $\Omega=\Omega_f$ (Figs. 2-9). Using Eq. (7) the condition of the validity of the purely elastic solution at unloading can be written as

$$\left(1-\frac{\alpha^2}{9}\right)\left(\frac{\sigma_r^{res}}{\sigma_0}\right)^2 + \left(1-\frac{\alpha^2}{9}\right)\left(\frac{\sigma_\theta^{res}}{\sigma_0}\right)^2 - \\ -\left(1+\frac{2\alpha^2}{9}\right)\frac{\sigma_r^{res}}{\sigma_0}\frac{\sigma_\theta^{res}}{\sigma_0} + \frac{2\alpha}{3}\left(\frac{\sigma_r^{res}}{\sigma_0}+\frac{\sigma_\theta^{res}}{\sigma_0}\right) - 1 \le 0. \tag{27}$$

Using Eqs. (25) and (26) the distribution of the residual stress has been calculated. The effect of m-value on the distribution of the radial residual stress with γ at γ_c=0.3 is illustrated in Fig. 10 for α=0 and in Fig. 11 for α=0.3. It is seen from these figures that the effect is negligible at this value of γ_c. A larger effect is revealed at γ_c=0.5. It is seen from Fig. 12 for α=0 and in Fig. 13 for α=0.3. In both cases $\left|\sigma_r^{res}\right|$ increases as the value of m increases. The effect of m-value on the circumferential residual stress is also insignificant for γ_c=0.3. It is seen in Fig.14 for α=0 and in Fig. 15 for α=0.3. In the case of γ_c=0.5 the variation of the circumferential residual stress with γ is shown in Fig. 16 for α=0 and in Fig. 17 for α=0.3. It is seen from these figures that the effect of m-

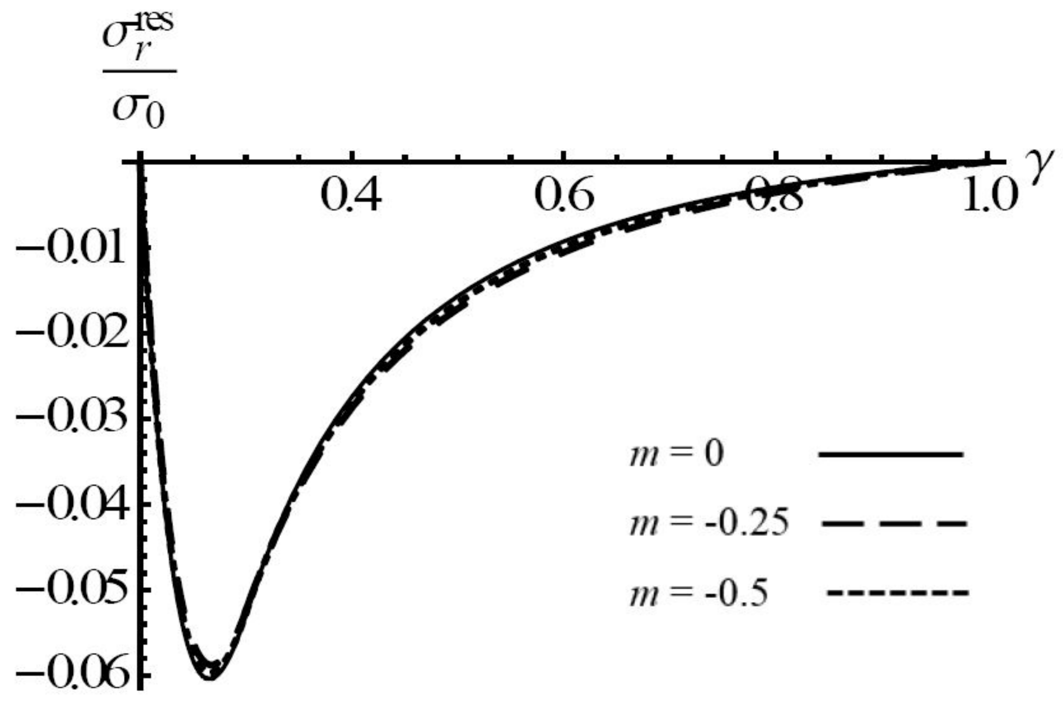

Fig. 10 Variation of the residual radial stress with γ at γ_c=0.3, α=0, and several m-values

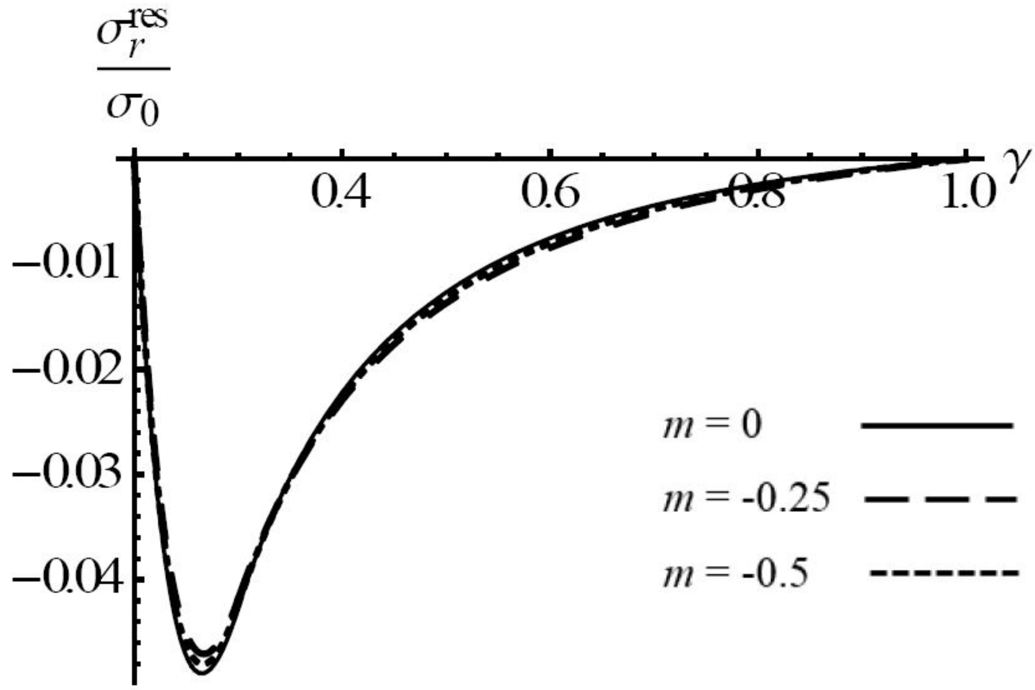

Fig. 11 Variation of the residual radial stress with γ at γ_c=0.3, α=0.3, and several m-values

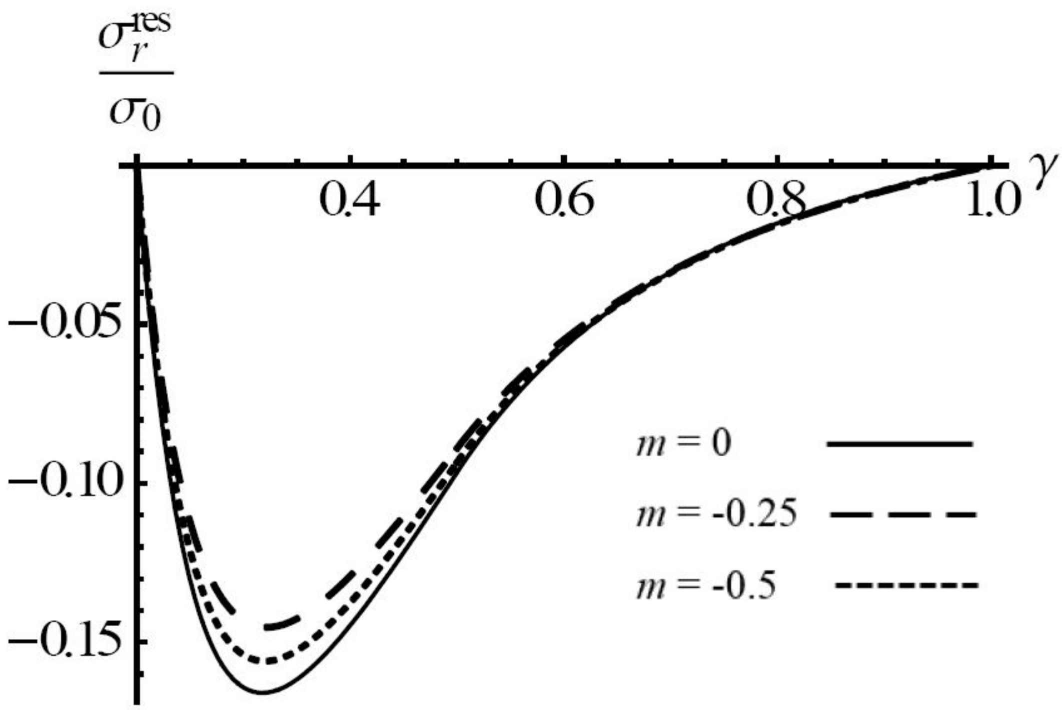

Fig. 12 Variation of the residual radial stress with γ at γ_c=0.5, α=0, and several m-values

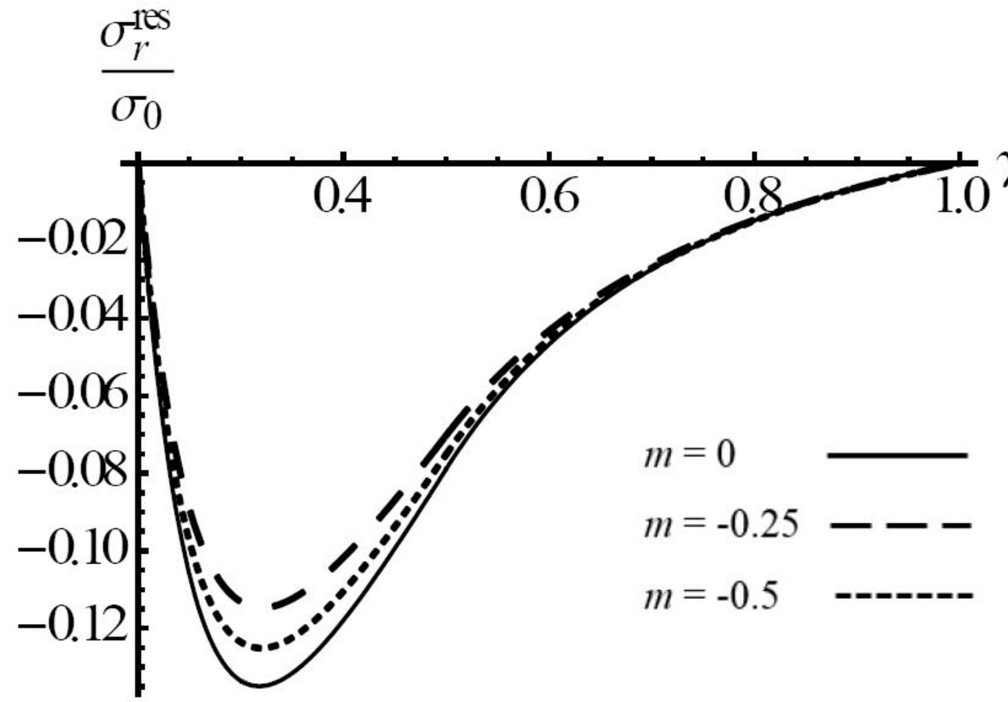

Fig. 13 Variation of the residual radial stress with γ at γ_c=0.5, α=0.3, and several m-values

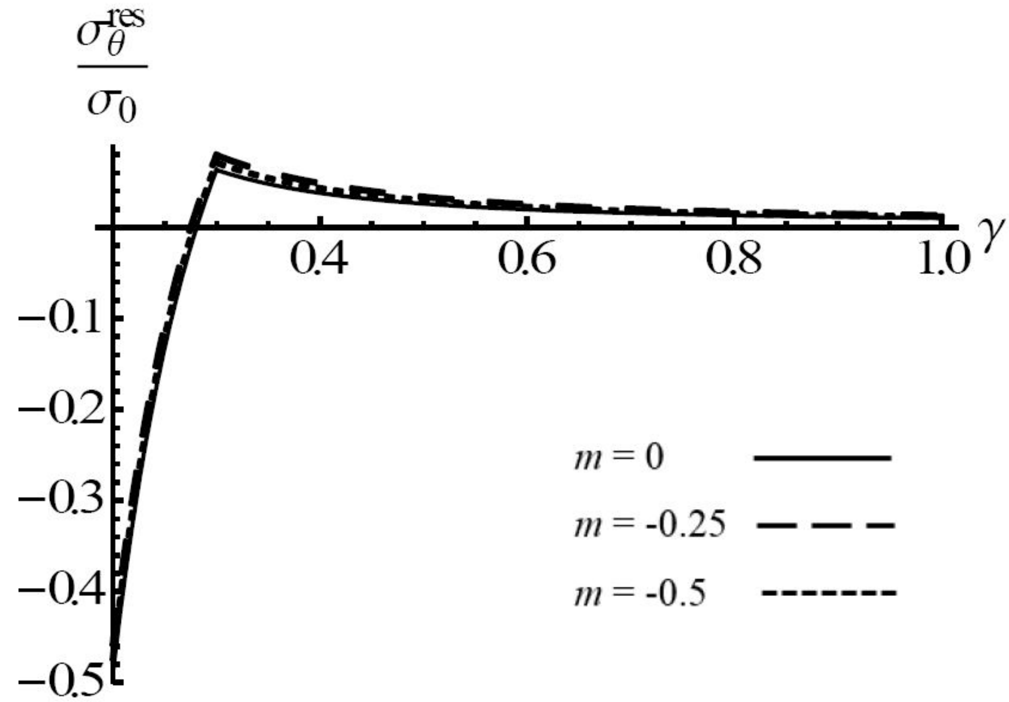

Fig. 14 Variation of the residual circumferential stress with γ at γ_c=0.3, α=0, and several m-values

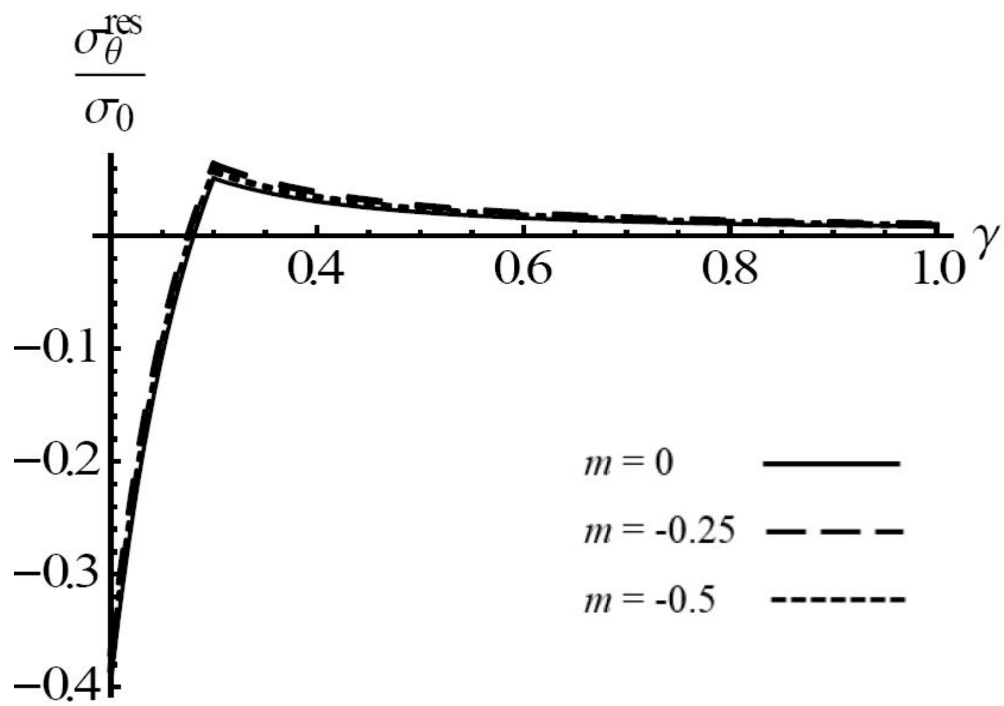

Fig. 15 Variation of the residual circumferential stress with γ at γ_c=0.3, α=0.3, and several m-values

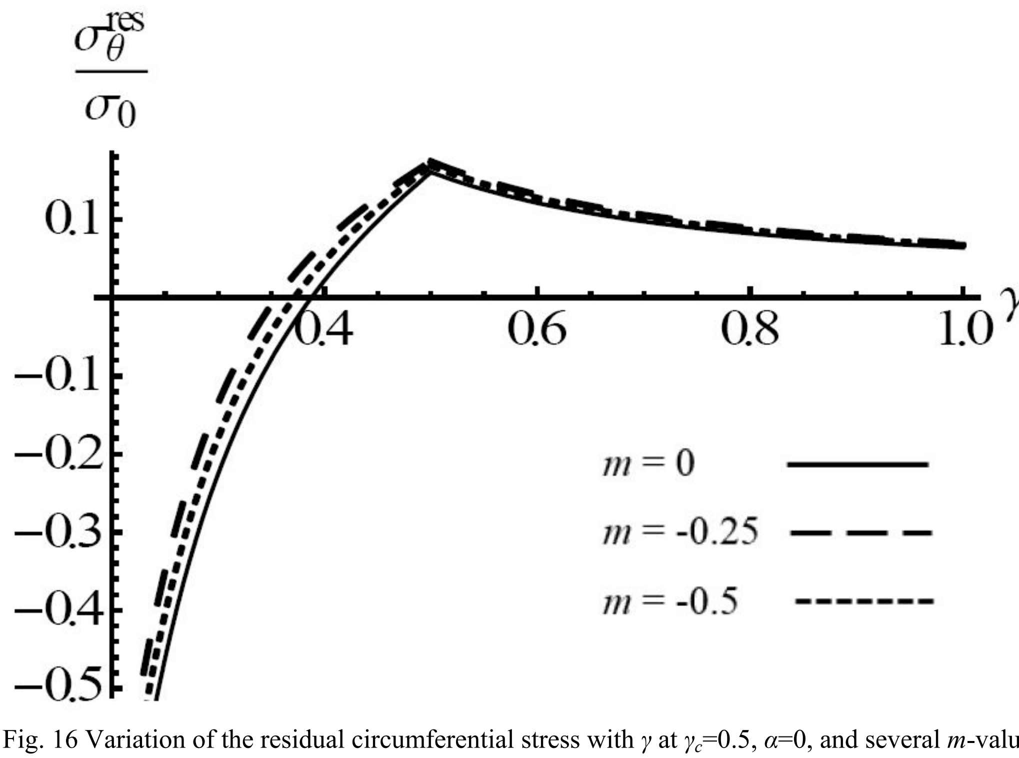

Fig. 16 Variation of the residual circumferential stress with γ at γ_c=0.5, α=0, and several m-values

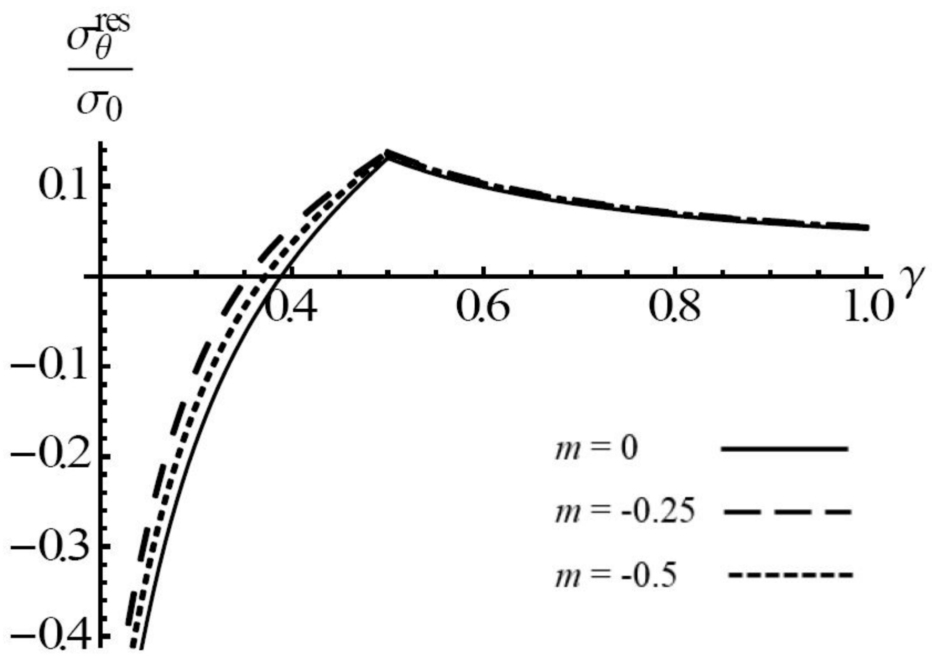

Fig. 17 Variation of the residual circumferential stress with γ at γ_c=0.5, α=0.3, and several m-values

value of the circumferential residual stress is more pronounced in the plastic region. It is also seen that the dependence of σ_θ^{res} on m at a given value of γ is not monotonic.

The distributions of the residual stresses shown in Figs. 10 to 17 have been substituted into Eq. (27) to verify that the yield criterion is not violated in the elastic region.

6. Conclusions

A new semi-analytic solution for a thin rotating annular disc has been found. A numerical technique is only necessary to solve the ordinary differential Eq. (16). The primary objective of the present paper is to reveal the effect of α involved in the yield criterion (6) and m involved in Eq. (3) on the distribution of stress at loading and on the distribution of residual stresses. Note that α=0 corresponds to the von Mises yield criterion and m=0 corresponds to the disc of constant thickness. Therefore, the value of α is a measure of the deviation of the Drucker-Prager yield criterion from the von Mises yield criterion.

Based on numerical results obtained the following conclusions can be drawn.

- The radial stress increases as m increases (Figs. 2 to 5).
- The effect of m on the circumferential stress is not so significant as on the radial stress (Figs. 6 to 9).
- The effect of m on both the radial and circumferential residual stresses is negligible at γ_c=0.3 (Figs. 10, 11, 14, and 15) and more pronounced at γ_c= 0.5(Figs. 12, 13, 16, and 17).
- The dependence of the circumferential residual stress on m at a given value of γ is not monotonic (Figs.16 and 17).

Acknowledgements

This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2010-220-D00037) and by the R&D programs on Co-operative Research Project (No. B551179-11-02-00).

Reference

Alexandrova, N. and Alexandrov, S. (2004), "Elastic-plastic stress distribution in a plastically anisotropic rotating disk", *Tran. ASME J. Appl. Mech*., **71**(3), 427-429.

Alexandrov, S., Jeng, Y. R. and Lomakin, E. (2011), "Effect of pressure-dependency of the yield criterion on the development of plastic zones and the distribution of residual stresses in thin annular disks", *J. Appl. Mech.-T*, ASME, **78**(3), 031012.

Callioglu, H., Topcu, M. and Tarakcilar, A.R. (2006), "Elastic-plastic stress analysis of an orthotropic rotating disc", *Int. J. Mech. Sci*., **48**, 985-990.

Drucker, D.C. and Prager, W. (1952), "Soil mechanics and plastic analysis for limit design", *Q. Appl. Math.*, **10**, 157-165.

Eraslan, A.N. (2002), "Inelastic deformations of rotating variable thickness solid disks by Tresca and von Mises criteria", *Int. J. Comp. Eng. Sci.*, **3**(1), 89-101.

Eraslan, A.N. (2003), "Elastoplastic deformations of rotating parabolic solid disks using Tresca's yield criterion", *Eup. J. Mech. A/Solid*., **22**, 861-874.

Eraslan, A.N. and Orcan, Y. (2002a), "Elastic-plastic deformation of a rotating solid disk of exponentially varying thickness", *Mech. Mater*., **34**, 423-432.

Eraslan, A.N. and Orcan, Y. (2002b), "On the rotating elastic-plastic solid disks of variable thickness having concave profiles", *Int. J. Mech. Sci.*, **44**, 1445-1466.

Guven, U. (1992), "Elastic-plastic stresses in a rotating annular disk of variable thickness and variable density", *Int. J. Mech. Sci*., **34**(2), 133-138.

Guven, U. (1998), "Elastic-plastic stress distribution in a rotating hyperbolic disk with rigid inclusion", *Int. J. Mech. Sci.*, **40**, 97-109.

Hojjati, M.H. and Hassani, A. (2008), "Theoretical and numerical analyses of rotating discs of non-uniform thickness and density", *Int. J. Pres. Ves. Pip.*, **85**, 694-700.

Kao, A.S., Kuhn, H.A., Spitzig, W.A. and Richmond, O. (1990), "Influence of superimposed hydrostatic pressure on bending fracture and formability of a low carbon steel containing globular sulfides", *Tran. ASME J. Eng. Mater. Technol.*, **112**, 26-30.

Liu, P.S. (2006), "Mechanical behaviors of porous metals under biaxial tensile loads", *Mater. Sci. Eng.,* **A422**, 176-183.

Orcan, Y. and Eraslan, A.N. (2002), "Elastic-plastic stresses in linearly hardening rotating solid disks of variable thickness", *Mech. Res. Commun*., **29**, 269-281.

Pirumov, A., Alexandrov, S. and Jeng, Y.R. (2013), "Enlargement of a circular hole in a disc of plastically compressible material", *Acta Mech.*, **224**(12), 2965-2976.

Rees, D.W.A. (1999), "Elastic-plastic stresses in rotating discs by von Mises and Tresca", *ZAMM*, **19**, 281-288.

Spitzig, W.A., Sober, R.J. and Richmond, O. (1976), "The effect of hydrostatic pressure on the deformation behavior of maraging and HY-80 steels and its implications for plasticity theory", *Metallurg. Tran.*, **7A**, 1703-1710.

Timoshenko, S. and Goodier, J.N. (1970), *Theory of Elasticity*, 3rd Edition, McGraw-Hill, New-York, USA.

Wilson, C.D. (2002), "A critical reexamination of classical metal plasticity", *Tran. ASME J. Appl. Mech.,* **69**, 63-68.

You, L.H., Tang, Y.Y., Zhang, J.J. and Zheng, C.Y. (2000), "Numerical analysis of elastic-plastic rotating disks with arbitrary variable thickness and density", *Int. J. Solid Struct*., **37**, 7809-7820.