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Optimum design of laterally-supported castellated beams using tug of war optimization algorithm

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Abstract. In this paper, the recently developed meta-heuristic algorithm called tug of war optimization is applied to optimal design of castellated beams. Two common types of laterally supported castellated beams are considered as design problems: beams with hexagonal openings and beams with circular openings. Here, castellated beams have been studied for two cases: beams without filled holes and beams with end-filled holes. Also, tug of war optimization algorithm is utilized for obtaining the solution of these design problems. For this purpose, the minimum cost is taken as the objective function, and some benchmark problems are solved from literature.

Keywords: meta-heuristic algorithm; tug of war optimization; optimal design; hexagonal opening; cellular opening

1. Introduction

Since the 1940's, the manufacturing of structural beams with higher strength and lower cost has been an asset to engineers in their efforts to design more efficient steel structures. Due to the limitations on maximum allowable deflections, using of section with heavy weight and high capacity in the design problem cannot always be utilized to the best advantage. As a result, several new methods have been created for increasing the stiffness of steel beams without increase in the weight of steel required. Castellated beam is one of them that become basic structural elements within the design of building, like a wide-flange beam (Konstantinos and D'Mello 2012).

A castellated beam is constructed by expanding a standard rolled steel section in such a way that a predetermined pattern (mostly circular or hexagonal) is cut on section webs and the rolled section is cut into two halves. The two halves are shifted and connected together by welding to form a castellated beam. In terms of structural performance, the operation of splitting and expanding the height of the rolled steel sections helps to increase the section modulus of the beams.

The main initiative for manufacturing and use of such sections is to suppress the cost of material by applying more efficient cross sectional shapes made from standard rolled beam. Web-openings have been used for many years in structural steel beams in a great variety of applications

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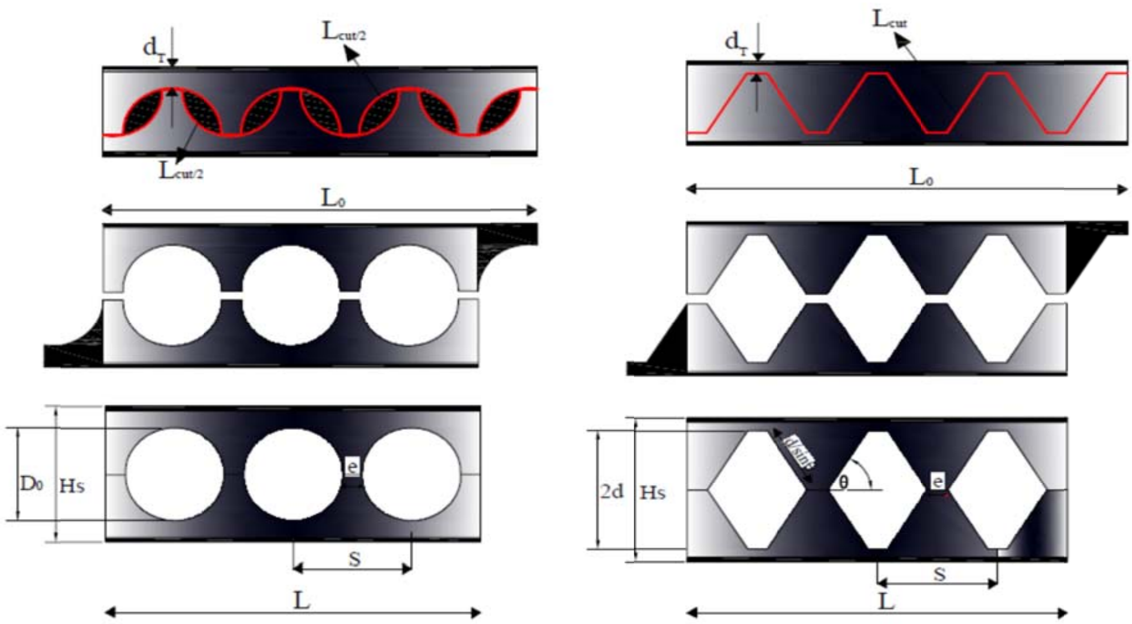


(a) (b)

Fig. 1 (a) A castellated beam with circular opening, (b) A castellated beam with hexagonal opening

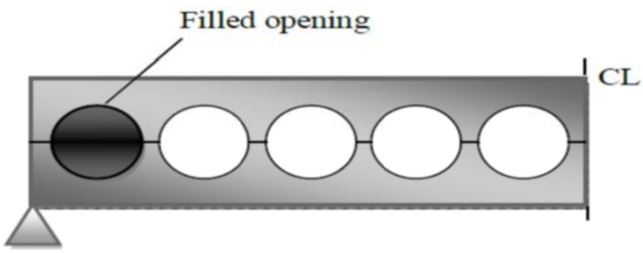


Fig. 2 Example of a beam with filled opening

because of the necessity and economic advantages. The principle advantage of the steel beam castellation process is that designer can increase the depth of a beam to raise its strength without adding steel. The resulting castellated beam is approximately 50% deeper and much stronger than the original unaltered beam (Soltani *et al*. 2012, Zaarour and Redwood 1996, Redwood and Demirdjian 1998, Sweedan 2011, Konstantinos and D'Mello 2011).

In recent years, a great deal of progress has been made in the design of steel beams with web-openings, and a cellular beam is one of them. A cellular beam is the modern form of the traditional castellated beam, but with a far wider range of applications in particular as floor beams. Cellular beams are steel sections with circular openings that are made by cutting a rolled beam web in a half circular pattern along its centerline and re-welding the two halves of hot rolled steel sections as shown in Fig. 1. An increase in beam depth provides greater flexural rigidity and strength to weight ratio.

In practice, in order to support high shear forces close to the connections, sometimes it becomes necessary to fill certain openings. In cellular beams, this is achieved by inserting discs made of steel plates and welding from both sides, Fig. 2. The openings are usually filled for one of two reasons:

i) At positions of higher shear, especially at the ends of a beam or under concentrated loads.

ii) At incoming connections of secondary beams.

It should be noted that for maximum economy infills should be avoided whenever possible, even to the extent of increasing the section mass.

In the last two decades, many metaheuristic algorithms have been developed to help solving optimization problems that were previously difficult or impossible to solve using mathematical programming algorithms. Metaheuristic algorithms provide mechanisms to escape from local optima by balancing exploration and exploitation phases, being based either on solution populations or iterated solution paths, for instance, by using neighborhoods. In general, these algorithms are simple to implement, present (near) optimal solutions in acceptable computational times even in complex search spaces. There are different meta-heuristic optimization methods; Genetic Algorithms (GA) (Goldberg and Holland 1988), Ant Colony Optimization (ACO) (Dorigo *et al.* 1996), Harmony Search algorithm (HS) (Geem *et al.* 2001), Particle Swarm Optimizer (PSO) (Eberhart and Kennedy 1996), Charged System Search method (CSS) (Kaveh and Talatahari 2010), Bat algorithm (Yang 2011), Ray optimization algorithm (RO) (Kaveh and Khayatazad 2012), Krill-herd algorithm (Gandomi and Alavi 2012), Dolphin Echolocation Optimization (DEO) (Kaveh and Farhoudi 2013), Colliding Bodies Optimization (CBO) (Kaveh and Mahdavi 2014), are some of such meta-heuristic algorithms. In this study, one of the newly developed algorithms called tug of war optimization (Kaveh and Zolghadr 2016) is used for optimal design of castellated beams. TWO is a multi-agent meta-heuristic algorithm, which considers each candidate solution $X_i=\{x_{i,j}\}$ as a team engaged in a series of tug of war competitions.

The main aim of this study is to optimize the cost of castellated beams with and without end-filled openings. For this purpose, the tug of war optimization approach is utilized for design of such beams with circular and hexagonal holes.

The present paper is organized as follows: In the next section, the design of castellated beam is introduced. In Section 3, the problem formulation including the mathematical model is presented, based on the Steel Construction Institute Publication Number 100 and Eurocode3. In Section 4, the algorithm is briefly introduced. In Section 5, numerical examples are studied, and finally the concluding remarks are provided in Section 6.

2. Design of castellated beams

The theory behind the castellated beam is to reduce the weight of the beam and to improve the stiffness by increasing the moment of inertia resulting from increased depth without usage of additional material. Due to the presence of holes in the web, the structural behavior of castellated steel beam is different from that of the standard beams. At present, there is no prescribed design method due to the complexity of the behavior of castellated beams and their associated modes of failure (Soltani *et al*. 2012). The strength of a beam with different shapes of web opening is determined by considering the interaction of the flexure and shear at the openings. There are many failure modes to be considered in the design of a beam with web opening, consisting of lateral-torsional buckling, Vierendeel mechanism, flexural mechanism, rupture of welded joints, and web post buckling. Lateral-torsional buckling may occur in an unrestrained beam. A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. In this paper it is assumed that the compression flange of the castellated beam is restrained by the floor system. Therefore, the overall buckling strength of the castellated beam is omitted from the design consideration. These modes are closely associated with beam geometry, shape parameters, type of loading, and provision of lateral supports. In the design of castellated beams, these criteria should be considered (EN 1993-1-1 2005, Ward 1990, Erdal *et al*. 2011, Saka 2009, Raftoyiannis and Ioannidis 2006, British Standards 2000, LRFD-AISC 1986).

2.1 Overall beam flexural capacity

This mode of failure can occur when a section is subjected to pure bending. In the span subjected to pure bending moment, the tee-sections above and below the openings yields in a manner similar to that of a standard webbed beam. Therefore, the maximum moment under factored dead and imposed loading, should not exceed the plastic moment capacity of the castellated beam (Soltani *et al*. 2012, Erdal *et al*. 2011).

$$M_U \leq M_P = A_{LT} P_Y H_U \tag{1}$$

where A_{LT} is the area of lower tee, P_Y is the design strength of steel, and H_U is distance between center of gravities of upper and lower tees.

2.2 Beam shear capacity

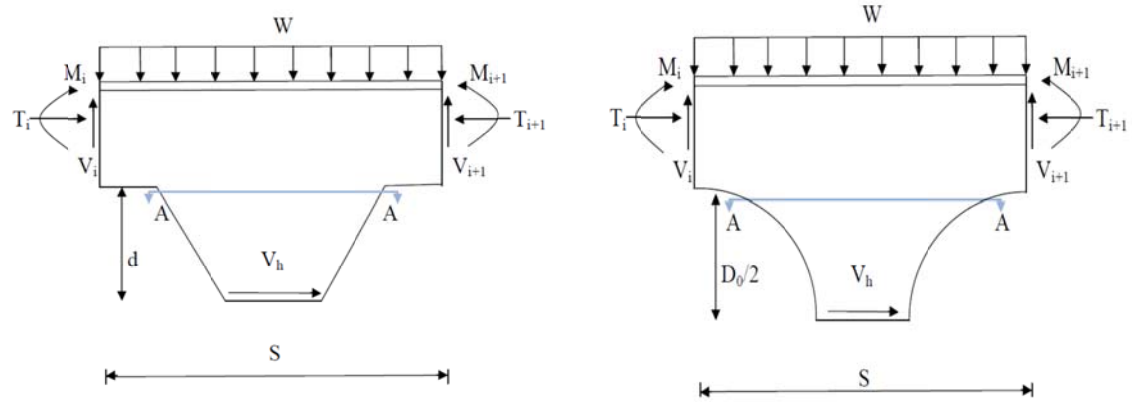
In the design of castellated beams, two modes of shear failure should be checked. The first one is the vertical shear capacity and the upper and lower tees should undergo that. The vertical shear capacity of the beam is the sum of the shear capacities of the upper and lower tees. The factored shear force in the beam should not exceed the following limits

$$\begin{aligned} P_{VY} &= 0.6P_Y\left(0.9A_{WUL}\right) && \text{circular opening} \\ P_{VY} &= \frac{\sqrt{3}}{3}P_Y\left(A_{WUL}\right) && \text{hexagonal opening} \end{aligned} \tag{2}$$

The second one is the horizontal shear capacity. It is developed in the web post due to the change in axial forces in the tee-section as shown in Fig. 3. Web post with too short mid-depth welded joints may fail prematurely when horizontal shear exceed the yield strength. The horizontal shear capacity is checked using the following equations (Soltani *et al*. 2012, Erdal *et al*. 2011)

$$\begin{aligned} P_{VH} &= 0.6P_Y\left(0.9A_{WP}\right) && \text{circular opening} \\ P_{VH} &= \frac{\sqrt{3}}{3}P_Y\left(A_{WP}\right) && \text{hexagonal opening} \end{aligned} \tag{3}$$

where $A_{WUL}$is the total area of the webs of the tees and A_{WP} is the minimum area of web post.



(a) (b)

Fig. 3 Horizontal shear in the web post of castellated beams, (a) Hexagonal opening, (b) Circular opening

2.3 Flexural and buckling strength of web post

In this study, it is assumed that the compression flange of the castellated beam is restrained by the floor system. Thus the overall buckling of the castellated beam is omitted from the design consideration. The web post flexural and buckling capacity in a castellated beam is given by (Soltani *et al*. 2012, Erdal *et al*. 2011)

$$\frac{M_{MAX}}{M_E} = [C_1 \times \alpha - C_2 \times \alpha^2 - C_3] \tag{4}$$

where M_{MAX} is the maximum allowable web post moment and M_E is the web post capacity at critical section *A-A* shown in Fig. 3. C_1, C_2 and C_3 are constants obtained by folllowing expressions

$$C_1 = 5.097 + 0.1464\beta - 0.00174\beta^2 \tag{5}$$

$$C_2 = 1.441 + 0.0625\beta - 0.000683\beta^2 \tag{6}$$

$$C_3 = 3.645 + 0.0853\beta - 0.00108\beta^2 \tag{7}$$

where $\alpha = \frac{S}{2d}$ for hexagonal openings, and $\alpha = \frac{S}{D_0}$ for circular openings, also $\beta = \frac{2d}{t_w}$ for hexagonal openings, and $\beta = \frac{D_0}{t_w}$ for circular openings, S is the spacing between the centers of holes, d is the cutting depth of hexagonal opening, D_0 is the holes diameter and t_w is the web thickness.

2.4 Vierendeel bending of upper and lower tees

Vierendeel mechanism is always critical in steel beams with web openings, where global shear force is transferred across the opening length, and the Vierendeel moment is resisted by the local moment resistances of the tee-sections above and below the web openings. This mode of failure often occurs in web-expanded beams with long horizontal opening lengths.

Vierendeel bending results in the formation of four plastic hinges above and below the web opening. The overall Vierendeel bending resistance depends on the local bending resistance of the web-flange sections. This mode of failure is associated with high shear forces acting on the beam. The Vierendeel bending stresses in the circular opening obtained by using the Olander's approach. The interaction between Vierendeel bending moment and axial force for the critical section in the tee should be checked as follows (Erdal *et al*. 2011)

$$\frac{P_0}{P_U}+\frac{M}{M_P}\leq 1.0 \tag{8}$$

where P_0 and M are the force and the bending moment on the section, respectively. P_U is equal to the area of critical section$\times P_Y$, M_P is calculated as the plastic modulus of critical section$\times P_Y$ in plastic section or elastic section modulus of critical section$\times P_Y$ for other sections.

The plastic moment capacity of the tee-sections in castellated beams with hexagonal opening are calculated independently. The total of the plastic moment is equal to the sum of the Vierendeel resistances of the above and below tee-sections (Soltani *et al*. 2012). The interaction between Vierendeel moment and shear forces should be checked by the following expression

$$V_{OMAX}\times e-4M_{TP}\leq 0 \tag{9}$$

where V_{OMAX} and M_{TP} are the maximum shear force and the moment capacity of tee-section, respectively.

2.5 Deflection of castellated beam

Serviceability checks are of high importance in the design, especially in beams with web opening where the deflection due to shear forces is significant. The deflection of a castellated beam under applied load combinations should not exceed span/360. Methods for calculating the deflection of castellated beam with hexagonal and circular openings are shown in Ref. (Raftoyiannis and Ioannidis 2006), and Ref. (Erdal *et al*. 2011), respectively.

3. Problem formulation

In optimization problem of castellated beams, the objective is to minimize the manufacturing cost of the beam while satisfying certain constraints. In a castellated beam, there are many factors that require special considerations when estimating the cost of beam, such as man-hours of fabrication, weight, price of web cutting and welding process. In this study, it is assumed that the costs associated with man-hours of fabrication for hexagonal and circular opening are identical. Thus, the objective function comprises of three parts: the beam weight, price of the cutting, and price of the welding. The objective function can be expressed as

$$\mathrm{F}_{\mathrm{cost}}=\rho A_{\mathrm{initial}}(L_0)\times p_1+L_{cut}\times p_2+L_{weld}\times p_3 \tag{10}$$

In practice, in order to support high shear forces close to the connection or for reasons of fire safety, sometimes it becomes necessary to fill certain openings using steel plates. In this case, the price of plates is added to the total cost. Therefore, the objective function can be expressed as

$$\mathrm{F}_{\text{cost-filled}} = \rho(A_{\text{initial}}(L_0) + 2A_{hole} \times t_w) \times p_1 + L_{cut} \times p_2 + (L_{weld}) \times p_3 \tag{11}$$

where p_1, p_2 and p_3 are the price of the weight of the beam per unit weight, length of cutting and welding per unit length, L_0 is the initial length of the beam before castellation process, ρ is the density of steel, $A_{initial}$ is the area of the selected universal beam section, A_{hole} is the area of a hole, L_{cut} and L_{weld} are the cutting length and welding length, respectively. The length of cutting is different for hexagonal and circular web-openings. The dimension of the cutting length is described by following equations:

For circular opening

$$L_{\text{cut}} = \pi D_0 \times NH + 2e(NH+1) + \frac{\pi D_0}{2} + e \tag{12}$$

$$L_{\text{cut-infill}} = \pi D_0 \times NH + 2e(NH+1) + \frac{\pi D_0}{2} + e + 2 \times P_{hole} \tag{13}$$

For hexagonal opening

$$L_{cut} = 2NH(e + \frac{d}{\sin(\theta)}) + 2e + \frac{d}{\sin(\theta)} \tag{14}$$

$$L_{cut-infill} = 2NH(e + \frac{d}{\sin(\theta)}) + 2e + \frac{d}{\sin(\theta)} + 2 \times P_{hole} \tag{15}$$

where NH is the total number of holes, e is the length of horizontal cutting of web, D_0 is the diameter of holes, d is the cutting depth, θ is the cutting angle, and P_{hole} is the perimeter of hole related to filled opening.

Also, the welding length for both of circular and hexagonal openings is determined by Eqs. (16) and (17).

$$L_{weld} = e(NH+1) \tag{16}$$

$$L_{weld-infill} = e(NH+1) + 4 \times P_{hole} \tag{17}$$

3.1 Design of castellated beam with circular opening

Design process of a cellular beam consists of three phases: the selection of a rolled beam, the selection of a diameter, and the spacing between the center of holes or total number of holes in the beam as shown in Fig. 1, (Erdal *et al*. 2011, Saka 2009), Hence, the sequence number of the rolled beam section in the standard steel sections tables, the circular holes diameter and the total number of holes are taken as design variables in the optimum design problem. This problem is formulated by considering the constraints explained in the previous sections and can be expressed as the following:

Find an integer design vector $\{X\}=\{x_1,x_2,x_3\}^T$, where x_1 is the sequence number of the rolled steel profile in the standard sections list, x_2 is the sequence number for the hole diameter which contains various diameter values, and x_3 is the total number of holes for the cellular beam (Erdal *et al*. 2011). Hence the design problem can be expressed as:

Minimize Eqs. (10), (11)

Subjected to

$$g_1 = (1.08 \times D_0) - S \leq 0 \tag{18}$$

$$g_2 = S - (1.60 \times D_0) \leq 0 \tag{19}$$

$$g_3 = (1.25 \times D_0) - H_S \leq 0 \tag{20}$$

$$g_4 = H_S - (1.75 \times D_0) \leq 0 \tag{21}$$

$$g_5 = M_U - M_P \leq 0 \tag{22}$$

$$g_6 = V_{MAXSUP} - P_V \leq 0 \tag{23}$$

$$g_7 = V_{OMAX} - P_{VY} \leq 0 \tag{24}$$

$$g_8 = V_{HMAX} - P_{VH} \leq 0 \tag{25}$$

$$g_9 = M_{A-AMAX} - M_{WMAX} \leq 0 \tag{26}$$

$$g_{10} = V_{TEE} - (0.50 \times P_{VY}) \leq 0 \tag{27}$$

$$g_{11} = \frac{P_0}{P_U} + \frac{M}{M_P} - 1.0 \leq 0 \tag{28}$$

$$g_{12} = Y_{MAX} - L/360 \leq 0 \tag{29}$$

where t_W is the web thickness, H_S and L are the overall depth and the span of the cellular beam, and S is the distance between centers of holes. M_U is the maximum moment under the applied loads, M_P is the plastic moment capacity of the cellular beam, $V_{\mathrm{MAX}SAP}$ is the maximum shear at support, $V_{O\mathrm{MAX}}$ is the maximum shear at the opening, $V_{H\mathrm{MAX}}$ is the maximum horizontal shear, $M_{A\text{-}A\mathrm{MAX}}$ is the maximum moment at *A-A* section shown in Fig. 3. $M_{W\mathrm{MAX}}$ is the maximum allowable web post moment, V_{TEE} represent the vertical shear on top of the hole, P_0 and M are the internal forces on the web section, and Y_{MAX} denotes the maximum deflection of the cellular beam (Erdal *et al.* 2011, LRFD-AISC 1986).

3.2 Design of castellated beam with hexagonal opening

In design of castellated beams with hexagonal openings, the design vector includes four design variables: the selection of a rolled beam, the selection of a cutting depth, the spacing between the center of holes or total number of holes in the beam and the cutting angle as shown in Fig. 1. Hence the optimum design problem is formulated by the following expression:

Find an integer design vector $\{X\}=\{x_1,x_2,x_3\}^T$ where x_1 is the sequence number of the rolled steel profile in the standard sections list, x_2 is the sequence number for the cutting depth which contains various values, x_3 is the total number of holes for the castellated beam and x_4 is the cutting angle. Thus, the design problem turns out to be as follows:

Minimize Eq. (10), Eq. (11)

Subjected to

$$g_1 = d - \frac{3}{8}(H_S - 2t_f) \le 0 \tag{30}$$

$$g_2 = (H_S - 2t_f) - 10 \times (d_T - t_f) \le 0 \tag{31}$$

$$g_3 = \frac{2}{3}d \cot\theta - e \le 0 \tag{32}$$

$$g_4 = e - 2d \cot\theta \le 0 \tag{33}$$

$$g_5 = 2d \cot\theta + e - 2d \le 0 \tag{34}$$

$$g_6 = 45^\circ - \theta \le 0 \tag{35}$$

$$g_7 = \theta - 64^\circ \le 0 \tag{36}$$

$$g_8 = M_U - M_P \le 0 \tag{37}$$

$$g_9 = V_{MAXSUP} - P_V \le 0 \tag{38}$$

$$g_{10} = V_{OMAX} - P_{VY} \le 0 \tag{39}$$

$$g_{11} = V_{HMAX} - P_{VH} \le 0 \tag{40}$$

$$g_{12} = M_{A-AMAX} - M_{WMAX} \le 0 \tag{41}$$

$$g_{13} = V_{TEE} - (0.50 \times P_{VY}) \le 0 \tag{42}$$

$$g_{14} = V_{OMAX} \times e - 4M_{TP} \le 0 \tag{43}$$

$$g_{15} = Y_{MAX} - L/360 \le 0 \tag{44}$$

where t_f is the flange thickness, d_T is the depth of the tee-section, M_P is the plastic moment capacity of the castellated beam, $M_{A\text{-}A\mathrm{MAX}}$ is the maximum moment at *A-A* section shown in Fig. 3, $M_{W\mathrm{MAX}}$ is the maximum allowable web post moment, V_{TEE} is the vertical shear on the tee, M_{TP} is the moment capacity of tee-section and Y_{MAX} denotes the maximum deflection of the castellated beam with hexagonal opening (Soltani *et al*. 2012).

4. Optimization algorithm

In this section, the new meta-heuristic algorithm developed by Kaveh and Zolghadr (2016) is briefly introduced. The TWO is a population-based search method, where each agent is considered as a team engaged in a series of tug of war competitions. The weight of the teams is determined based on the quality of the corresponding solutions and the amount of pulling force that a team can exert on the rope is assumed to be proportional to its weight. Naturally, the opposing team will have to maintain at least the same amount of force in order to sustain its grip of the rope. The lighter team accelerates toward the heavier team and this forms the convergence operator of the TWO. The algorithm improves the quality of the solutions iteratively by maintaining a proper exploration/exploitation balance using the described convergence operator. A summary of this method is provided in the following steps.

Step 1: Initialization

The initial positions of teams are determined randomly in the search space

$$x_{ij}^{0} = x_{j,min} + rand(x_{j,max} - x_{j,min}) \qquad j = 1,2,...,n \tag{45}$$

where x_{ij}^{0} is the initial value of the jth variable of the ith candidate solution; $x_{j,\max}$ and $x_{j,\min}$ are the maximum and minimum permissible values for the jth variable, respectively; *rand* is a random number from a uniform distribution in the interval [0, 1]; n is the number of optimization variables.

Step 2: Evaluation of candidate designs and weight assignment

The objective function values for the candidate solutions are evaluated and sorted. The best solution so far and its objective function value are saved. Each solution is considered as a team with the following weight

$$W_i = 0.9\left(\frac{fit(i) - fit_{worst}}{fit_{best} - fit_{worst}}\right) + 0.1 \qquad i = 1,2,...,N \tag{46}$$

where $fit(i)$ is the fitness value for the ith particle; The fitness value can be considered as the penalized objective function value for constrained problems; *fitbest* and *fitworst* are the fitness values for the best and worst candidate solutions of the current iteration; According to Eq. (46) the weights of the teams range between 0.1 and 1.

Step 3: Competition and displacement

In TWO each team competes against all the others one at a time to move to its new position. The pulling force exerted by a team is assumed to be equal to its static friction force $(W\mu_s)$. Hence the pulling force between the teams i and j ($F_{p,ij}$) can be determined as $\max\{W_i\mu_s, W_j\mu_s\}$. Such a definition keeps the position of the heavier team unaltered.

The resultant force affecting team i due to its interaction with heavier team j in the kth iteration can then be calculated as follows

$$F_{r,ij}^{k} = F_{p,ij}^{k} - W_i^{k}\mu_k \tag{47}$$

where $F_{p,ij}^{k}$ is the pulling force between teams i and j in the kth iteration, and μ_k is coefficient of kinematic friction.

$$a_{ij}^k = \left(\frac{F_{r,ij}^k}{W_i^k \mu_k}\right) g_{ij}^k \tag{48}$$

in which a_{ij}^k is the acceleration of team i towards team j in the kth iteration; g_{ij}^k is the gravitational acceleration constant defined as

$$g_{ij}^k = X_j^k - X_i^k \tag{49}$$

where X_j^k and X_i^k are the position vectors for candidate solutions j and i in the kth iteration. Finally, the displacement of team i after competing with team j can be derived as

$$\Delta X_{ij}^k = \frac{1}{2} a_{ij}^k \Delta t^2 + \alpha^k (X_{max} - X_{min}) \circ (-0.5 + rand(1,n)) \tag{50}$$

The second term of Eq. (50) induces randomness into the algorithm. This term can be interpreted as the random portion of the search space traveled by team i before it stops after the applied force is removed. Here, α is a constant chosen from the interval [0,1]; $X_{\max}$ and $X_{\min}$ are the vectors containing the upper and lower bounds of the permissible ranges of the design variables, respectively; $\circ$ denotes element by element multiplication; *rand*(1,*n*) is a vector of uniformly distributed random numbers.

It should be noted that when team j is lighter than team i, the corresponding displacement of team i will be equal to zero (i.e., ΔX_{ij}^k). Finally, the total displacement of team i in iteration k is equal to

$$\Delta X_i^k = \sum_{j=1}^{N} \Delta X_{ij}^k \tag{51}$$

The new position of team i at the end of kth iteration, is then calculated as:

Step 4: Handling of side constraints

It is possible for the candidate solutions to leave the search space and it is important to deal with such solutions properly. This is especially the case for the solutions corresponding to lighter teams for which the values of ΔX is usually bigger. Different strategies might be used in order to solve this problem. In this study, it is assumed that such candidate solution can be simply brought back to their previous permissible position (Flyback strategy) or they can be regenerated randomly.

Step 5: Termination

Steps 2 through 5 are repeated until a termination criterion is satisfied.

The flowchart of the TWO algorithm is shown in Fig. 4.

5. Test problems and optimization results

In this section, numerical results are presented to demonstrate the efficiency of the new meta-heuristic method (TWO) for design of castellated beams. For this purpose, three beams are

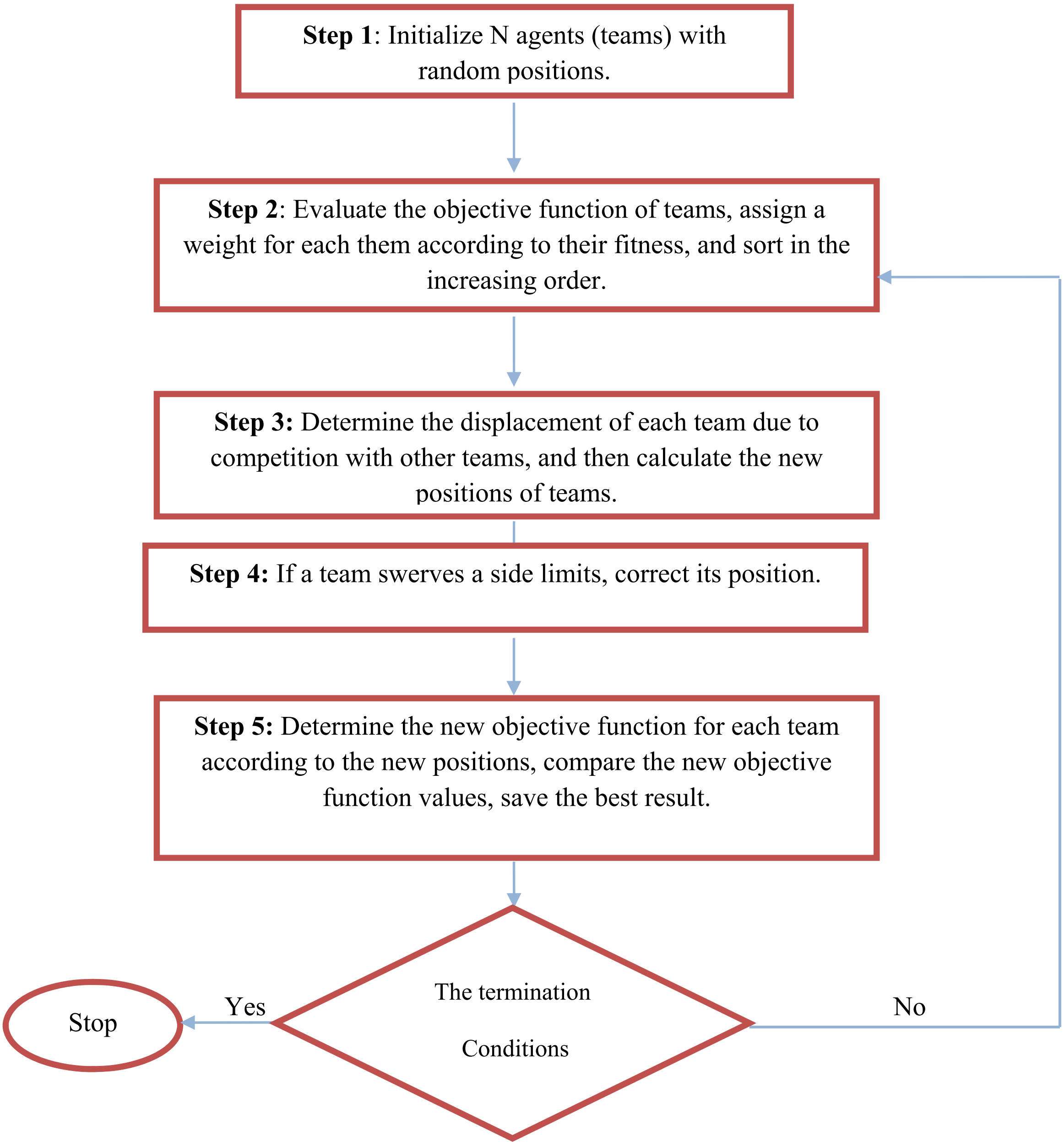


Fig. 4 Flowchart of the TWO algorithm (Kaveh and Zolghadr (2016))

selected from literature that have previously been optimized by other algorithms. Among the steel sections list of British Standards, 64 Universal Beam (UB) sections starting from 254×102×28 UB to 914×419×388 UB are chosen to constitute the discrete set of steel sections from which the design algorithm selects the sectional designations for the castellated beams. In the design pool of holes diameters 421 values are arranged which varies between 180 and 600 mm with an

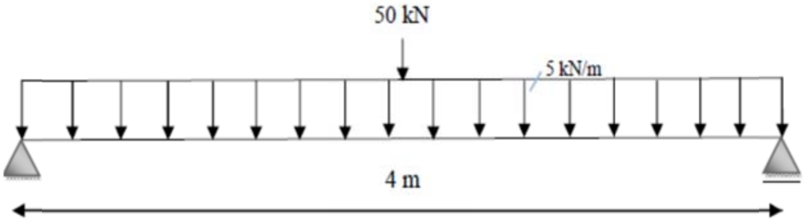


Fig. 5 Simply supported beam with a span of 4 m

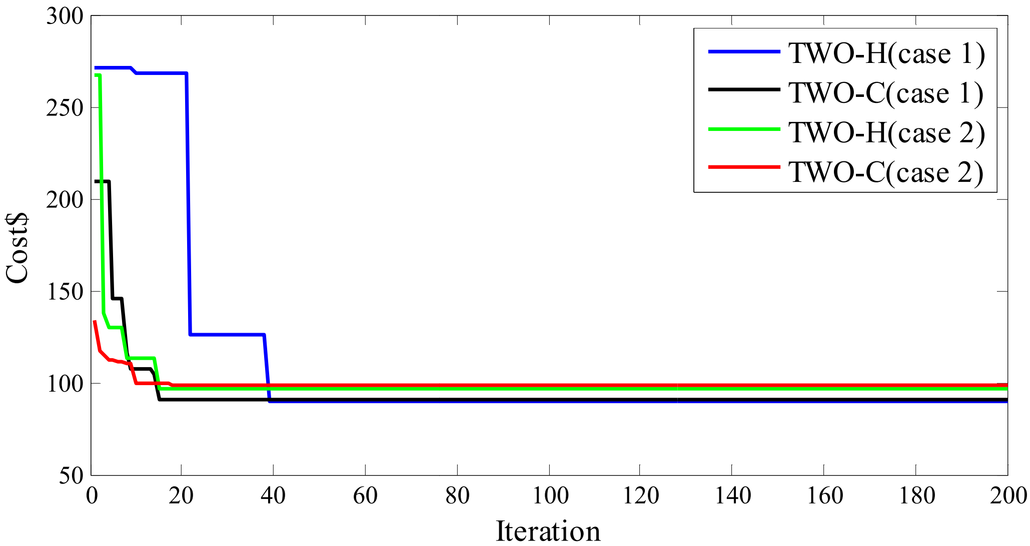


Fig. 6 Convergence curves recorded in the 4m span beam problem for the TWO best optimization runs

increment of 1 mm. Also, for cutting depth of hexagonal opening, 351 values are considered which varies between 50 and 400 mm with increment of 1 mm and cutting angle changes from 45 to 64. Another discrete set is arranged for the number of holes. Likewise, in all the design problems, the modulus of elasticity is equal to 205 GPa and Grade 50 is selected for the steel of the beam which has the design strength of 355 MPa. The coefficients P_1, P_2 and P_3 in the objective function are considered as 0.85, 0.30 and 1.00, respectively (Kaveh and Shokohi 2014, Kaveh and Shokohi 2015a, Kaveh and Shokohi 2015b, Kaveh and Shokohi 2015c) A maximum number of iterations of 200 are used as the termination criterion in all the examples, and α is taken as 0.1 for all design problems. Also, all design problems have been solved in two cases, with and without filled holes.

5.1 Castellated beam with 4m span

A simply supported beam with a span of 4m is considered as the first test problem, shown in Fig. 5. The beam is subjected to 5 kN/m dead load including its own weight. A concentrated live load of 50 kN also acts at mid-span of the beam and the allowable displacement of the beam is limited to 12 mm. For this problem the number of agents (teams) is taken as 20.

Table 1 Optimum designs of the castellated beams with 4m span

	Algorithm	Optimum UB section	Hole diameter - cutting depth(mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
Case 1	ECSS (Kaveh and Shokohi 2014)	UB 305×102×25	125	14	57°	89.78	Hexagonal
	CBO (Kaveh and Shokohi 2015a)	UB 305×102×25	125	14	57°	89.78	
	CBO-PSO (Kaveh and Shokohi 2015c)	UB 305×102×25	125	14	57°	89.78	
	Present work (TWO)	UB 305×102×25	126	13	61°	89.73	
	ECSS (Kaveh and Shokohi 2014)	UB 305×102×25	248	14	_	96.32	Circular
	CBO (Kaveh and Shokohi 2015a)	UB 305×102×25	244	14	_	91.14	
	CBO-PSO (Kaveh and Shokohi 2015c)	UB 305×102×25	243	14	_	91.08	
	Present work (TWO)	UB 305×102×25	249	14	_	91.15	
Case 2	ECSS (Kaveh and Shokohi 2015b)	UB 305×102×25	125	14	60°	96.45	Hexagonal
	CBO (Kaveh and Shokohi 2015b)	UB 305×102×25	125	14	64°	96.61	
	CBO-PSO (Kaveh and Shokohi 2015b)	UB 305×102×25	125	14	56°	96.04	
	Present work (TWO)	UB 305×102×25	125	14	56°	96.33	
	ECSS (Kaveh and Shokohi 2015b)	UB 305×102×25	244	14	_	98.62	Circular
	CBO (Kaveh and Shokohi 2015b)	UB 305×102×25	243	14	_	98.70	
	CBO-PSO (Kaveh and Shokohi 2015b)	UB 305×102×25	243	14	_	98.58	
	Present work (TWO)	UB 305×102×25	244	14	_	98.62	

Castellated beams with hexagonal and circular openings are separately designed with TWO. These beams are designed for two cases. In case 1, it is assumed that the end of the beams is not filled. Thus the objective function for this case is obtained from Eq. (10). In the second case, it is assumed that the holes in the end of the beam are filled with steel plate, and Eq. (11) is utilized for the objective function. The optimum results obtained by TWO are given in Table 1. It is apparent from the same table that the optimum cost for castellated beam with hexagonal hole is equal to 89.73$ which is obtained by TWO. Also, according to the results, the tug of war optimization algorithm has good performance in design of cellular beam. These results indicate that the castellated beam with hexagonal opening have less cost in comparison to the cellular beam. The same conclusion can be drawn for the filled opening configuration from the results listed in Table 1.

Fig. 6 shows the convergence curves of the TWO algorithm for design of castellated beams with different shapes for the openings.

5.2 Castellated beam with 8m span

In the second problem, the tug of war optimization algorithm is used to design a simply supported castellated beam with a span of 8m. Similar to the first example, this beam is also designed for two different cases. The beam carries a uniform dead load 0.40 kN/m, which includes

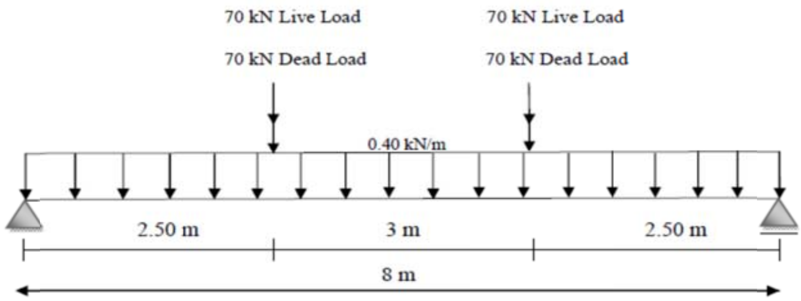


Fig. 7 Simply supported beam with a span of 8 m

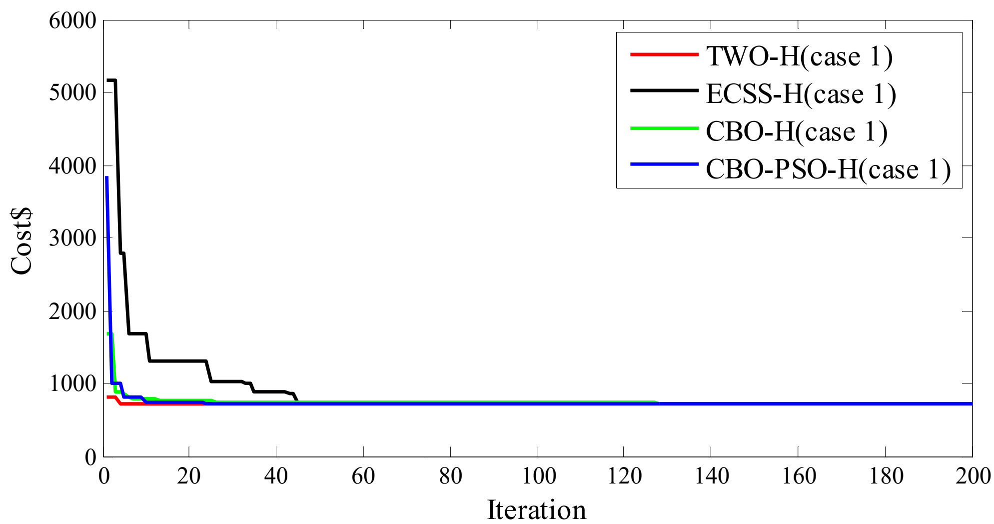


Fig. 8 Comparison of best run convergence curves recorded in the 8m span beam problem (unfilled hexagonal holes) for different metaheuristic algorithms

Table 2 Optimum designs of the castellated beams with 8m span

	Algorithm	Optimum UB section	Hole diameter - cutting depth(mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
Case 1	ECSS (Kaveh and Shokohi 2014)	UB 610×229×101	246	14	59°	719.47	Hexagonal
	CBO (Kaveh and Shokohi 2015a)	UB 610×229×101	243	14	59°	718.93	
	CBO-PSO (Kaveh and Shokohi 2015c)	UB 610×229×101	244	14	55°	718.33	
	Present work (TWO)	UB 610×229×101	243	14	56°	718.20	
	ECSS (Kaveh and Shokohi 2015a)	UB 610×229×101	487	14	–	721.55	Circular
	CBO (Kaveh and Shokohi 2015a)	UB 610×229×101	487	14	–	721.55	
	CBO-PSO (Kaveh and Shokohi 2015c)	UB 610×229×101	487	14	–	721.55	
	Present work (TWO)	UB 610×229×101	487	14	–	721.55	
Case 2	ECSS (Kaveh and Shokohi 2015b)	UB 610×229×101	246	14	56°	744.65	Hexagonal
	CBO (Kaveh and Shokohi 2015b)	UB 610×229×101	246	14	58°	745.48	
	CBO-PSO (Kaveh and Shokohi 2015b)	UB 610×229×101	246	14	55°	744.42	
	Present work (TWO)	UB 610×229×101	246	14	55°	744.42	
	ECSS (Kaveh and Shokohi 2015b)	UB 610×229×101	478	14	–	753.74	Circular
	CBO (Kaveh and Shokohi 2015b)	UB 610×229×101	479	14	–	754.02	
	CBO-PSO (Kaveh and Shokohi 2015b)	UB 610×229×101	478	14	–	753.74	
	Present work (TWO)	UB 610×229×101	478	14	–	753.74	

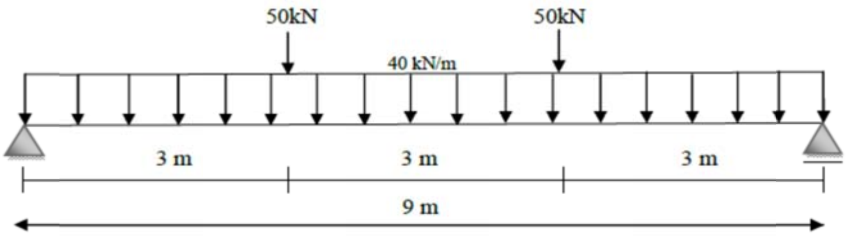


Fig. 9 Simply supported beam with 9 m span

its own weight. In addition, it is subjected to two concentrated loads as shown in Fig. 7. The allowable displacement of the beam is limited to 23 mm, and the number of agents is taken as 20.

This beam is designed by TWO and the results are compared to those of the other optimization algorithms as shown in Table 2. In design of the beam with hexagonal hole, the corresponding cost obtained by the TWO is equal to 718.2$ which is the lowest value among all the methods. Therefore, the performance of the tug of war optimization is better than other approaches (Kaveh and Shokohi 2014, Kaveh and Shokohi 2015a, Kaveh and Shokohi 2015b, Kaveh and Shokohi 2015c) for this design example. According to the obtained results, the designed beam with hexagonal opening has less cost in comparison with the cellular beam, and it is a better option in this case. In design of end-filled case, it is obvious that the presented method has the same performance. Furthermore, the maximum value of the strength ratio is equal to 0.99 for both hexagonal and circular beams, and it is shown that these constraints are dominant in the design process.

Fig. 8 shows the convergence history for optimum design of hexagonal beam which is obtained by different meta-heuristic algorithms.

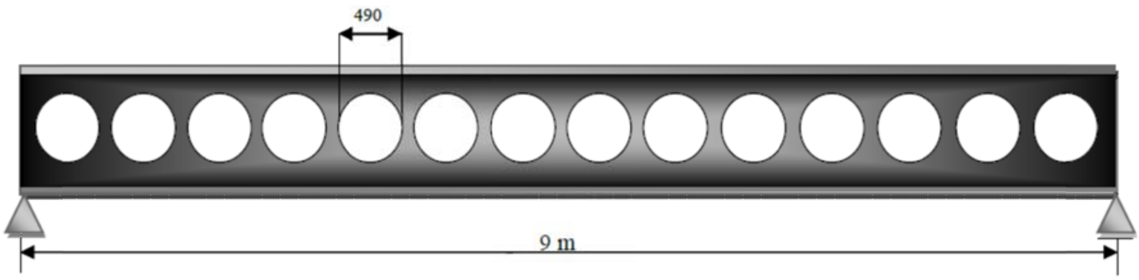
5.3 Castellated beam with 9m span

The beam with 9m span is considered as the last example of this study in order to compare the minimum cost of the castellated beams. The beam carries a uniform load of 40 kN/m including its own weight and two concentrated loads of 50 kN as shown in Fig. 9. The allowable displacement of the beam is limited to 25 mm, and the number of agent is taken as 20.

Table 3 compares the results obtained by the TWO with those of the other algorithms. In the optimum design of castellated beam with hexagonal hole, TWO algorithm selects 684×254×125 UB profile, 16 holes, and 231 mm for the cutting depth and 57° for the cutting angle. The minimum cost of the design beam is equal to 991.04$. Also, in the optimum design of cellular beam, the TWO algorithm selects 610×229×125 UB profile, 14 holes of diameter 490 mm. It can be observed from Table 3 that the optimal design has the minimum cost of 990.33$ for beam with hexagonal holes which is obtained by the CBO-PSO algorithm, however, the TWO results in better design for cellular beam. In the design of beam with filled holes, the obtained results using the tug of war optimization algorithm is slightly different from each other. This shows that in the case of holes filled with steel plates, where the beam span is large, using cellular beams can be a good design strategy. Similar to the previous example, the strength criteria is dominant in the design of this beam and it is related to the Vierendeel mechanism. The maximum ratio of these criteria is equal to 0.99 for both hexagonal and cellular cases.

Table 3 Optimum designs of the castellated beams with 9 m span

	Algorithm	Optimum UB section	Hole diameter - cutting depth(mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
Case 1	ECSS (Kaveh and Shokohi 2014)	UB 684×254×125	277	13	56°	995.97	Hexagonal
Case 1	CBO (Kaveh and Shokohi 2015a)	UB 684×254×125	233	15	64°	993.79	Hexagonal
Case 1	CBO-PSO (Kaveh and Shokohi 2015c)	UB 684×254×125	230	16	56°	990.33	Hexagonal
Case 1	Present work (TWO)	UB 684×254×125	231	16	57°	991.04	Hexagonal
Case 1	ECSS (Kaveh and Shokohi 2014)	UB 684×254×125	539	14	_	998.94	Circular
Case 1	CBO (Kaveh and Shokohi 2015a)	UB 684×254×125	538	14	_	997.57	Circular
Case 1	CBO-PSO (Kaveh and Shokohi 2015c)	UB 684×254×125	538	14	_	998.58	Circular
Case 1	Present work (TWO)	UB 610×229×125	490	14	_	995.89	Circular
Case 2	ECSS (Kaveh and Shokohi 2015b)	UB 684×254×125	277	14	61°	1033.32	Hexagonal
Case 2	CBO (Kaveh and Shokohi 2015b)	UB 684×254×125	277	14	60°	1034.07	Hexagonal
Case 2	CBO-PSO (Kaveh and Shokohi 2015b)	UB 684×254×125	276	14	58°	1031.92	Hexagonal
Case 2	Present work (TWO)	UB 684×254×125	277	14	57°	1031.98	Hexagonal
Case 2	ECSS (Kaveh and Shokohi 2015b)	UB 684×254×125	539	14	_	1041.71	Circular
Case 2	CBO (Kaveh and Shokohi 2015b)	UB 684×254×125	539	14	_	1041.79	Circular
Case 2	CBO-PSO (Kaveh and Shokohi 2015b)	UB 684×254×125	539	14	_	1041.68	Circular
Case 2	Present work (TWO)	UB 610×229×125	489	15	_	1033.34	Circular



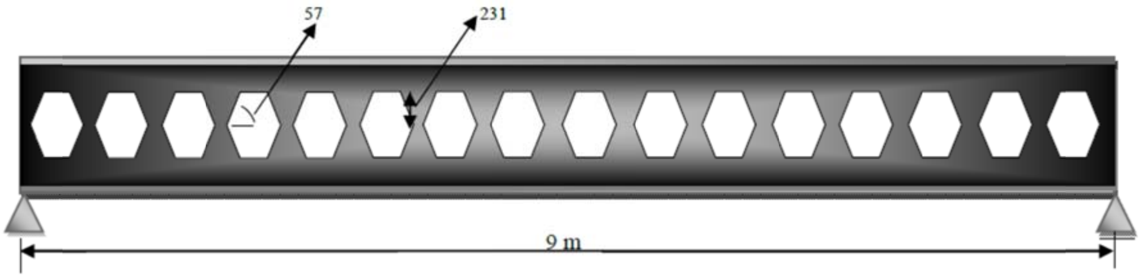


Fig. 10 Optimum profiles of the castellated beams with unfilled cellular and hexagonal openings for beam with 9 m span

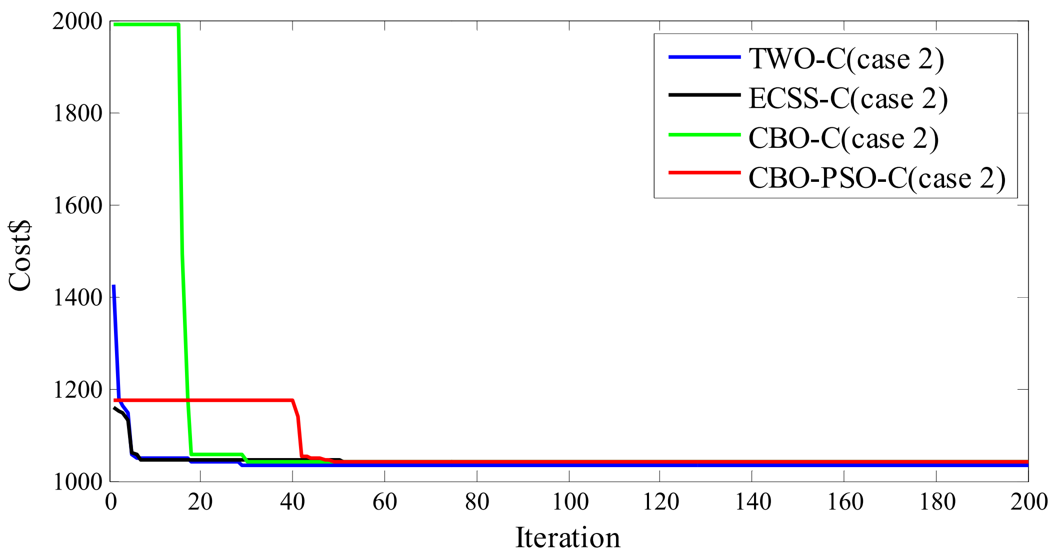


Fig. 11 Comparison of best run convergence curves recorded in the 9 m span beam problem (filled circular holes) for different metaheuristic algorithms

The optimum shapes of the hexagonal and circular openings with unfilled holes are separately shown in Fig. 10. Also, the convergence histories of metaheuristics are shown in Fig. 11 for design of cellular beam with filled openings. It is apparent from the figure that TWO has good convergence rate in design of this problem and finds better solution for cellular beam.

6. Conclusions

In this paper, a newly developed meta-heuristic algorithm called tug of war optimization is utilized for optimum design of castellated beams. Three benchmark problems are solved in order to assess the robustness and efficiency of the TWO. These beams are designed in two cases with filled openings and unfilled openings, where the hexagonal and circular holes are considered as the types of the web openings. Comparing the results obtained by TWO with those of other optimization methods demonstrates that TWO has a good performance compared to the other methods in the ability of finding the optimum solution. Also, the convergence rate of this algorithm to the optimal solution is quite good for most of problems and it requires a less number of analyses to find better solution making TWO computationally more efficient. From the results obtained in this paper, it can be concluded that the use of the beam with hexagonal openings can lead to the use of less steel material and it is better choice than cellular beam in unfilled cases. For design of castellated beam with large spans, especially in filled cases, it is observed that the cellular beam has a good performance and it can be used as an alternative to castellated beam with hexagonal opening.

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