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The stability of semi-rigid skeletal structures accounting for shear deformations

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Abstract. The analysis and design of skeletal structures is greatly influenced by the behaviour of beam-to-column connections, where patented designs have led to a wide range of types with differing structural quantities. The behaviour of beam-to-column connections plays an important role in the analysis and design of framed structures. This paper presents an overview of the influence of connection behaviour on structural stability, in the in-plane (bending) mode of sway. A computer-based method is presented for geometrically nonlinear plane frames with semi-rigid connections accounting for shear deformations. The analytical procedure employs transcendental modified stability functions to model the effect of axial force on the stiffness of members. The member stiffness matrix were found. The critical load has been searched as a suitable load parameter for the loss of stability of the system. Several examples are presented to demonstrate the validity of the analysis procedure. The method is readily implemented on a computer using matrix structural analysis techniques and is applicable for the efficient nonlinear analysis of frameworks. Combined with a parametric column effective length study, connection and frame stiffness are used to propose a method for the analysis of semi-rigid frames where column effective lengths are greatly reduced and second order (deflection induced) bending moments in the column may be distributed via the connectors to the beams, leading to significant economies.

Keywords: stability, nonlinear analysis; semi-rigid connection; effective length; shear deformation

1. Introduction

The column effective length factor **k** has been widely used by practicing engineers for many years. The notion of using effective length factor **k** to asses the buckling capability of a column has found favour with designers. Simple equations and Tables for **k** have been presented in terms of column end boundary conditions and/or relative frame stiffness functions, connection stiffness factors and shear effects so that the designer may compute not only column buckling capacities but also second order deflections and ultimate second order bending moments, often termed M_{sd} in Eurocode 2 for concrete structures, EC 2 (2002) or M_{add} in the British Code for concrete structures, BS 8110 (1997). BS 8110 (1997) has adopted such an approach whereby column end conditions were equated to α, the total relative stiffness $\sum EI/L$ of the column to that of the beam(s) (or beams and slabs) framing into the ends of the column for rigid connections.

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The Turkish Code for Requirements for design and construction of reinforced concrete structures, TS 500 Building Code (2000) uses the moment magnifier design approach to account for the second-order P-δ (member instability) and P-Δ (frame instability) effects in the design of reinforced concrete columns. The validity of this approach depends to a large extent on an accurate determination of the effective length factor, often termed **k** in the TS 500 (2000), and ACI 318 Building Code (2011). The TS 500 Building Code (2000) recommends the use of simplified equations, as in the current ACI 318 Building Code (2011). The paper discusses the *k*-factor problems to highlight the basic assumptions and particular difficulties associated with reinforced concrete columns in sway frames.

The results from the connection tests by Görgün (1997) show that, although the degree of semi-rigidity (defined by K_s=joint stiffness (a semi-rigid connection of rotational stiffness J at the end of beam)/beam flexural stiffness 4EI/L) varies over a very wide range, there is clearly scope for the implementation of **k** factors that incorporate the flexural responses of the frame, the semi-rigid connections and shear effects.

This study presents the results for column effective length factors in three types of sub-frames commonly occurring in skeletal frames. Only sway frames are considered in this work. This instability was obtained using a geometric second-order two-dimensional frame computer program analysis developed by Gorgun and Yılmaz (2012). In all cases maximum column loads in each sub-frame, and hence *k*-factors are obtained for given values of α, K_s and Poisson's ratio μ_c=0.20 (defined in TS 500/2000).

It is customary in conventional analysis and design of steel and precast concrete frameworks to represent the actual joint behaviour by two extreme kinds of idealized models, i.e., the fully rigid joint model and the pinned joint model. The notions of either pinned or rigid joints are, however, simply extreme cases of true joint behaviour, and experimental investigations, many of which are referred to in (Jones *et al*. 1983), show clearly that actual joints exhibit characteristics over a wide spectrum between these extremes. The models with ideal connections simplify analysis procedure, but often cannot represent real structural behaviour. This discrepancy is reported in numerous experimental investigations of steel frames with different types of connections (Jones *et al*. 1983). The rigid connection idealization indicates that relative rotation of the connection does not exist and the end moment of the beam is entirely transferred to the columns. In contrast to the rigid connection assumption, the pinned connection idealization indicates that any restraint does exist for rotation of the connection and the connection moment is zero. Although these idealizations simplify the analysis and design process, the predicted response of the frame may be different from its real behaviour. Therefore, this idealization is not adequate as all types of connections are more or less, flexible or semi-rigid. It is proved by numerous experimental investigations that have been carried out in the past (Nethercot 1985, Davisson *et al*. 1987, Moree *et al*. 1993, Görgün 1997). The term semi-rigid is used to express the real connection behaviour. Therefore, beam-to-column connections in the analysis/design of steel and precast concrete frames should be described as semi-rigid connections.

Generally, nodal connections of plane frames are subjected to influence of bending moments, axial forces and shear forces. The effects of axial and shear forces can usually be ignored, and only the influence of bending moments is of practical interest. The constitutive moment-relative rotation relation, M-ϕ, depends on the particular type of connection. Most experiments have shown that the M-ϕ curve is nonlinear all the whole domain and for all types of connections (Görgün 1997). Therefore, modelling of the nodal connection is very important for the analysis and design of frame structure.

Based on experimental work due to static monotonic loading tests carried out for various types of beam-to column connections, many models have been suggested to approximate the connection behaviour. The simplest and the most common one is the linear model that has been broadly used for its simplicity (Aksogan and Akkaya 1991, Aksogan and Gorgun 1992, Gorgun *et al*. 2012, Gorgun and Yilmaz 2012, Gorgun 2013). This approach is based on modelling the connection as a lengthless rotational spring. This method is widely used in semi-rigid analysis of frames, and the implementation of this approach requires small modifications in the existing analysis programs. This modification does not considerably increase the computational time. Therefore, each element of the frame consists of a finite length element with a lengthless rotational spring. However, this model is good only for the low level loads, when the connection moment is quite small. In each other case, when the connection rigidity decrease compared with its initial value, a nonlinear model is necessary. Several mathematical models to describe the nonlinear behaviour of connections have been formulated and widely used in research practice (Wu and Chen 1990). Often, many authors use the so called corrective matrices to modify the conventional stiffness matrices of the beams with fully fixity at both ends. Elements of the corrective matrices are functions of the particular nondimensional parameters-fixity factors, or rigidity index.

In addition to the linear behaviour, many studies have been developed to the nonlinear analysis of the static and dynamic behaviour of frames with semi-rigid connections using different models of geometric nonlinearity of elements and nodal connections (Xu *et al*. 2005, Aristizabal-Ochoa 2007, Liu 2009, Gorgun 2013, Hadidi and Rafiee 2014, Zhang *et al*. 2014, Han *et al*. 2015). In most studies, the effect of shear deformation and axial force on elastic behaviour has been ignored as being of little consequence. However, there are steel frameworks for which shear effects may be significant (e.g., those that have deep transfer girders (Aksogan and Dincer 1991, Aristizabal-Ochoa 2012, Gorgun *et al*. 2012). Also, in the analysis of structural systems the members forming the planar frames are generally assumed to be rigidly connected among each other. However, more often than not the assumption of pin connections is also employed in such cases where the rigidity of the connection cannot be provided to a dependable degree. In fact, both of the foregoing assumptions are unrealistic when one is treating steel frames and especially, nowadays, widely used precast reinforced concrete structures. In such structures beams and columns behave as if they are semi-rigidly, or flexibly, connected among themselves, as far as the rotations of the ends are concerned. Hence, experimentally determined effective rotational spring constants for those connections should be used in the analyses of such structures. This paper presents a computer-based method for geometrically nonlinear analysis of planar frameworks with semi-rigid connections to explicitly account for the influence of axial force on elastic behaviour. Stability functions are employed to model the effect of axial force on the elastic bending stiffness of members (Chen and Lui 1991), and the influence of semi-rigid connections is taken into account. The shear-stiff stability functions presented in (Chen and Lui 1991) are modified to take shear deformability into account for comparison. The history of the stability functions for shear-flexible members is given in (Al-Sarraf 1986, Mottram 2008).

The geometrically nonlinear elastic analysis procedure is a direct extension of the conventional matrix displacement method of linear-elastic analysis. The nonlinear analysis method is verified for three example subframes from the literature (Elliott *et al*. 1996, Elliott *et al*. 1998, Görgün and Kaymak 2012).

The present study is an attempt to prepare a computer program that treats the aforementioned type of structures elegantly, taking into consideration the behaviour of the flexible connections on elastic behaviour along with the effect of geometric nonlinearity due to the axial forces in the members. As is well known, the upper limit of the load in any structure is the critical value of the load, the buckling load, which is found by taking geometric nonlinearity into consideration. Hence, the results of the present study will constitute the basis of the stability analysis of the same type of structures.

The method used in the present study is the well-known stiffness method of structural analysis. The stiffness matrix of a member elastically supported against rotation at both ends is obtained using the second order analysis, along with the use of differential equations which yielded trigonometric functions for the case of axial compressive force and hyperbolic functions for the case of axial tensile force.

The computer program that was prepared can be used to solve stability and static problems of plane frames composed of members that are semi-rigidly connected at the joints.

2. Research significance

This paper evaluates the current Turkish Code for Requirements for design and construction of reinforced concrete structures, TS 500 Building Code (2000) and BS 8110 (1997) Building Code effective length factors and develops alternative **k**-factor equations based on a parametric study for framed columns. It is found that very conservative results are obtained by the codes simplified equations when the end restraints of columns are large. The proposed **k**-factor equations are found simple to use, can be programmed on small calculators, and in good agreement with results obtained from the exact solution considering shear effect for fully rigid connections. It is concluded that the proposed equation may be suitable for adoption in practice for rigid connections, and **k**-factors may be obtained for given values of frame stiffness α and connection stiffness factor K_s from Tables given below for semi-rigid connections.

3. BS 8110 (1997) equations for k-factor of sway framed structures

The present BS 8110 (1997) recommends the use of following equations to obtain a more rigorous assessment of the effective length factor. It should be noted that the effective length factor of a column in the two plan directions may be different.

Sway columns: effective length factor for framed structures may be taken as the lesser of

$$k = 1.0 + 0.15\left(\alpha_{c,1} + \alpha_{c,2}\right) \tag{1}$$

$$k = 2.0 + 0.3\alpha_{c,\min} \tag{2}$$

where $\alpha_{c,1}$ and $\alpha_{c,2}$ are ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the lower and upper ends of a column, respectively. $\alpha_{c,\min}$ is the lesser of $\alpha_{c,1}$ and $\alpha_{c,2}$.

4. The current TS 500 (2000) simplified equations for k-factor of sway framed columns

For columns in a sway frame, TS 500 (2000) adopted the Furlong (1971) formulas with the following adjustments

For $\alpha_m < 2$

$$k = \frac{20-\alpha_m}{20}\sqrt{1+\alpha_m} \tag{3}$$

For $\alpha_m \geq 2$

$$k = 0.9\sqrt{1+\alpha_m} \tag{4}$$

For an unbraced column with a hinge at one end, the following formula is suggested

$$k = 2.0 + 0.3\alpha \tag{5}$$

where α is the value at the restrained end.

$$\alpha_{1,2} = \sum(I/\ell)_{column} \Big/ \sum(I/\ell)_{beam}\ ; \ \ \alpha_m = 0.5(\alpha_1 + \alpha_2) \tag{6}$$

The α_m factor is defined as the mean value of the α_1 and α_2 ($\alpha_1 \leq \alpha_2$) at the two ends of a compression member.

5. Parametric study

Generally reinforced concrete sway plane frames are analysed either as *fully unbraced* frames, Fig. 1(a), or as *partially braced* frames, Fig. 1(b), where shear walls (or cores) provide lateral bracing up to a certain level and the frame is unbraced above this point.

Three sub frames, labelled F1, F2 and F3 in Fig. 2, were identified for the analysis. Sub frames F1 and F2 represent the upper floor and the ground floor levels, respectively, in an unbraced frame, whilst sub frame F3 represents the upper floor in a partially braced frame immediately above the level of the bracing. It may be seen in Fig. 1(b) that the columns adjacent to the bracing walls are

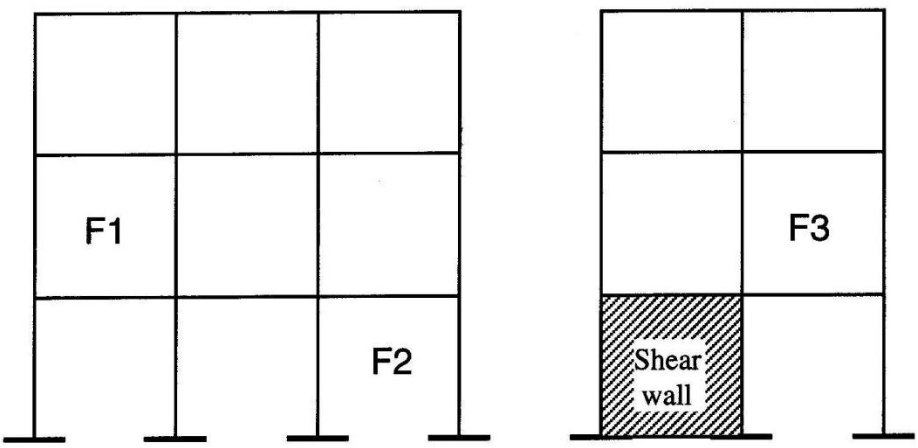

Fig. 1 Types of frames used in the analysis (a) unbraced (left) and (b) partially braced (right)

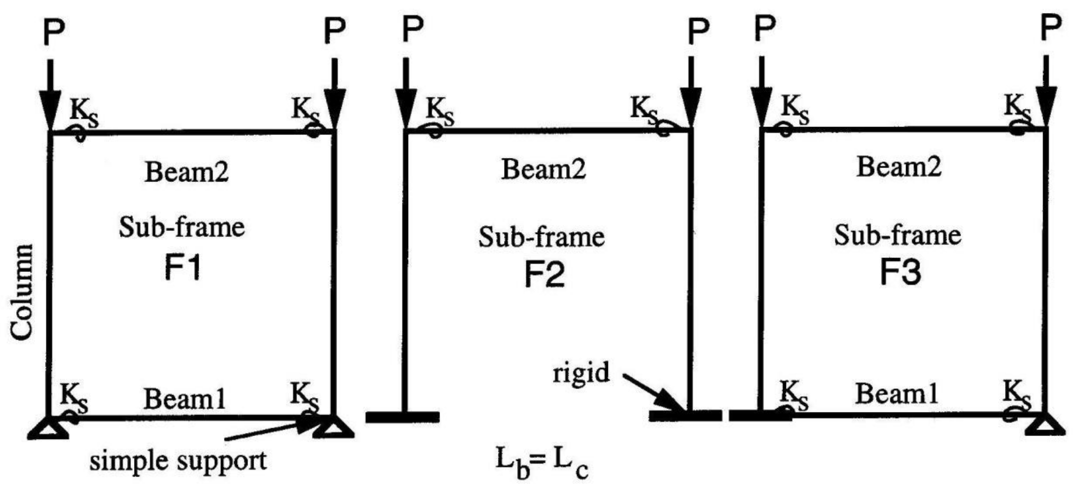

Fig. 2 Definitions of sub-frames used in effective length study

fully encastre at their upper end, and may therefore be considered fully rigid at their lower end in the sub frame F3 (left column). There is only axial deformation of columns if the frames are perfectly symmetric. To allow a large deflection problem non vertical frames are considered assuming 1 mm sway deflection at the top of the columns before applying *P*.

For the analysis, the range of values for α was obtained from realistic joint values used in typical concrete frames, i.e., α=0.5 to 10. In fact because the computer program requires a value for α greater than 0, α=0.001 was used to simulate α=0. For simplicity and reliability in the analysis, the length of the beams and columns in the sub-frames were made equal (L=4000 mm), and in general the cross sectional properties of the column members were varied in order to necessitate a change in α, although this is not important once the results are normalised with respect to α and **k**. The maximum critical load for the column converged to within an inaccuracy of less than 0.1 per cent of the ultimate squash load for the column, so that the error in k is approximately 0.1 per cent.

6. Results

6.1 Variations in column effective length factors with rigid connections ignoring shear effects

Comparing the results obtained from this work and those calculated using BS 8110, and TS 500 equations, Figs. 3 through 6 show the results for the variation in k with α assuming fully rigid connections. Note that in the case of sub-frame F1, $\alpha_1=\alpha_2$, where α_1 and α_2 are the relative stiffnesses of the column to the lower and upper beams, respectively. In sub-frame F2, α_1=0 because the foundation is rigid. There is no equation in BS 8110, and TS 500 to deal with sub-frame F3 (Görgün and Kaymak 2012).

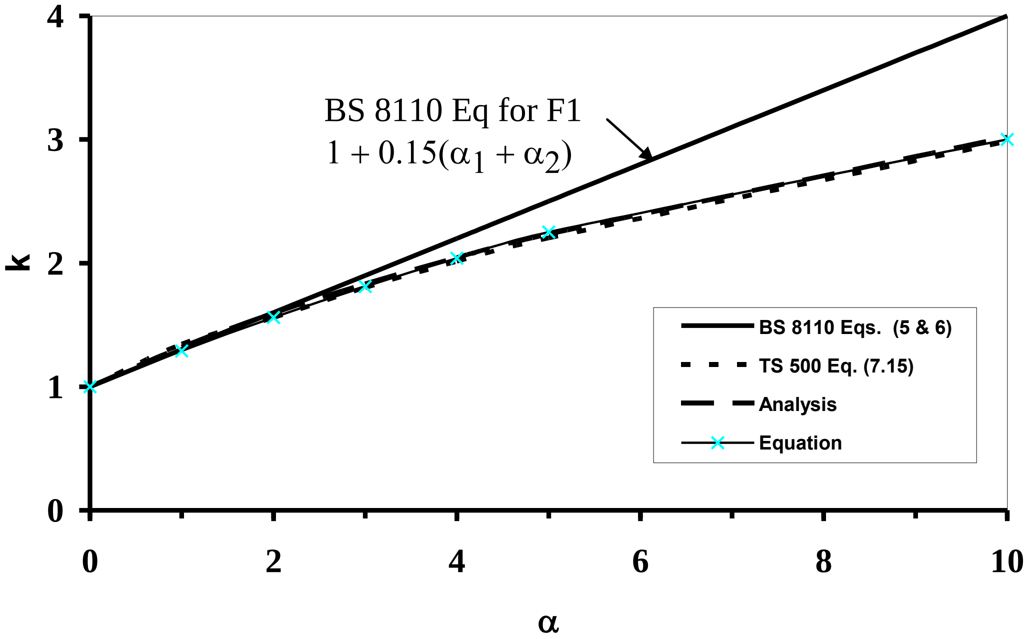

Fig. 3 Variation in column effective length factor **k** with frame stiffness α (α_1=α_2) for sub-frame F1

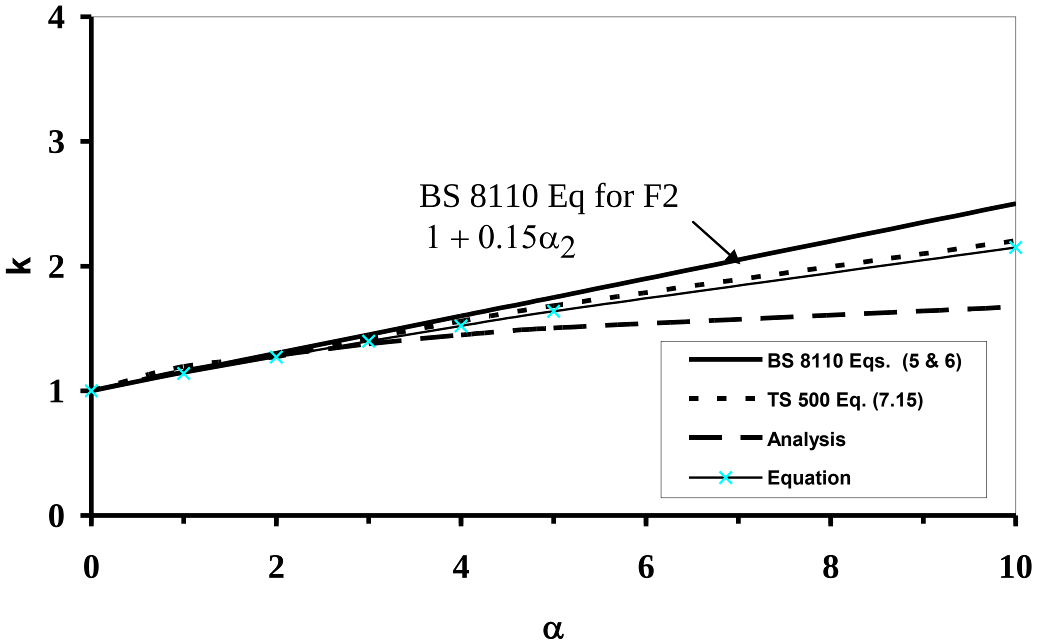

Fig. 4 Variation in column effective length factor **k** with frame stiffness α (α_1=0) for sub-frame F2

The results in Figs. 3 through 5 show that the codes equations are in good agreement with analytical results for $0<\alpha<2$, and conservative thereafter. It is postulated that an equation for sub-

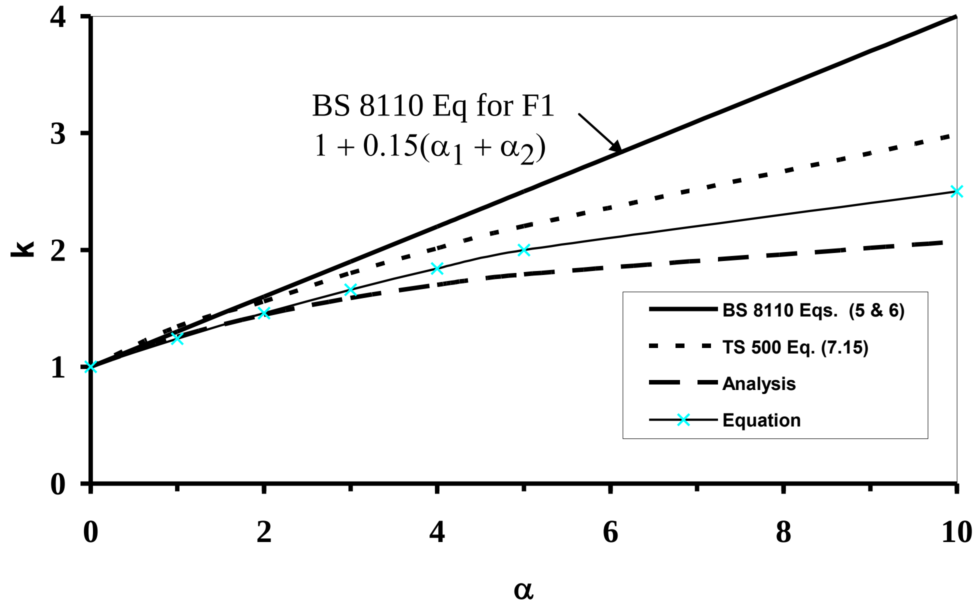

Fig. 5 Variation in column effective length factor **k** with frame stiffness α (α_1=α_2) for sub-frame F3

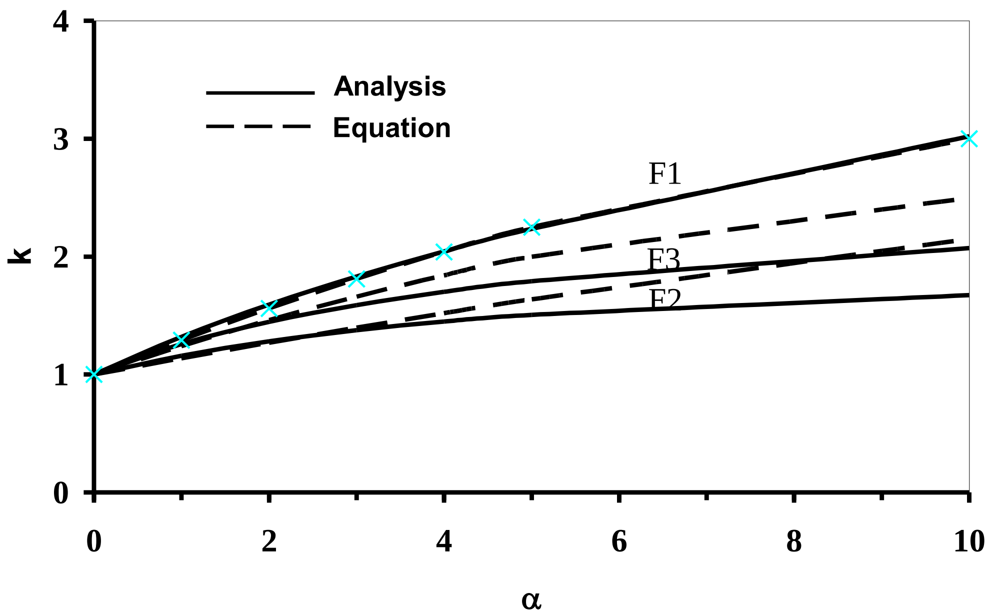

Fig. 6 Variation in column effective length factor **k** with frame stiffness for sub-frames used in the analysis

frame F3 may be taken as the mean of the equations for F1 and F2. The results suggest that the TS 500 code equations might be modified for values of α>3.

Fig. 6 shows the results for the variations in k for selected values of α in the sub-frames F1, F2, and F3. The dashed lines show the plots of the proposed parametric equations given in Section 6.2.

6.2 Parametric equations for rigid connections

From Fig. 3, the data for the upper storey sub-frame F1 may be approximated by using the following empirical relationship (Görgün and Kaymak 2012)

$$k = 1.0 + 0.30\alpha_m - 0.01\alpha_m^2 \tag{7}$$

Referring to Fig. 4, the data for the ground floor sub-frame F2 may be given as

$$k = 1.0 + 0.28\alpha_m - 0.01\alpha_m^2 \tag{8}$$

Referring to Fig. 5, the data for the upper storey sub-frame F3 may be given as

$$k = 1.0 + 0.25\alpha_m - 0.01\alpha_m^2 \tag{9}$$

6.3 Variations in column effective length factors with semi-rigid connections ignoring shear effects

Comparing the results obtained from this work with semi-rigid connections ignoring shear effects (μ_c=0.00) and those calculated using TS 500 equations with fully rigid connections ignoring the shear effects, Tables 1 through 6 show the results for the variation in critical load P_{cr} (kN) and column effective length factor k with frame stiffness α and the degree of semi-rigidity K_s.

Table 1 Variation in critical load P_{cr} with frame stiffness α and connection stiffness factor K_s for frame F1 without shear effect (μ_c=0.00)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	P_E (kN)
	Connection stiffness factor, K_s									$\frac{\pi^2 EI}{L^2}$
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	Unstable	13.637	13.703	13.746	13.760	13.767	13.770	13.772	13.774	13.805
0.010	Unstable	125	131	135	136	136.23	136.76	136.77	136.93	138.05
0.050	Unstable	436	531	606	635	651	659	663	667	690
0.200	Unstable	789	1205	1741	2031	2211	2311	2375	2422	2761
0.500	Unstable	928	1582	2721	3558	4190	4590	4864	5078	6903
1.000	Unstable	985	1757	3313	4685	5893	6753	7395	7923	13805
2.000	Unstable	1015	1858	3701	5525	7323	8738	9878	10874	27610
3.000	Unstable	1025	1893	3848	5866	7945	9652	11076	12355	41415
4.000	Unstable	1031	1912	3925	6049	8291	10174	11776	13235	55220
5.000	Unstable	1034	1923	3973	6164	8511	10511	12234	13822	69026
10.000	Unstable	1040	1945	4071	6405	8982	11245	13247	15132	138051
20.000	Unstable	1044	1956	4121	6532	9234	11644	13807	15868	276102
30.000	Unstable	1045	1960	4138	6875	9321	11782	14002	16126	414153

Table 2 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F1 without shear effect (μ_c=0.00)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	TS 500 (2000)
	Connection stiffness factor, K_s									
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	Unstable	1,006	1,004	1,002	1,002	1,001	1,001	1,001	1,001	1,000
0.010	Unstable	1,051	1,027	1,011	1,008	1,007	1,005	1,005	1,004	1,004
0.050	Unstable	1,258	1,140	1,067	1,042	1,030	1,023	1,020	1,017	1,022
0.200	Unstable	1,871	1,514	1,259	1,166	1,117	1,093	1,078	1,068	1,084
0.500	Unstable	2,727	2,089	1,593	1,393	1,284	1,226	1,191	1,166	1,194
1.000	Unstable	3,744	2,803	2,041	1,727	1,531	1,430	1,366	1,320	1,343
2.000	Unstable	5,216	3,855	2,731	2,235	1,942	1,778	1,672	1,593	1,559
3.000	Unstable	6,356	4,677	3,281	2,657	2,283	2,071	1,934	1,831	1,800
4.000	Unstable	7,318	5,374	3,751	3,021	2,581	2,330	2,165	2,043	2,012
5.000	Unstable	8,170	5,991	4,168	3,346	2,848	2,563	2,375	2,235	2,205
10.000	Unstable	11,521	8,425	5,823	4,643	3,920	3,504	3,228	3,020	2,985
20.000	Unstable	16,262	11,881	8,185	6,501	5,468	4,869	4,472	4,171	4,124
30.000	Unstable	19,908	14,536	10,004	7,761	6,666	5,929	5,439	5,068	5,011

Table 3 Variation in critical load P_{cr} with frame stiffness α and connection stiffness factor K_s for frame F2 without shear effect (μ_c=0.00)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	P_E (kN)
	Connection stiffness factor, K_s									$\frac{\pi^2 EI}{L^2}$
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.200	690	1429	1773	2164	2351	2459	2516	2552	2580	2761
0.500	1726	2625	3227	4198	4844	5293	5560	5737	5882	6903
1.000	3452	4414	5153	6580	7764	8743	9404	9876	10285	13805
2.000	6903	7893	8713	10469	12152	13752	14967	15917	16795	27610
5.000	17257	18245	19110	21097	23197	25413	27273	28852	30463	69026
10.000	34513	35467	36335	38387	40636	43103	45254	47144	49243	138051
20.000	69026	69899	70746	72791	75087	77663	79957	82010	84664	276102
30.000	103539	104328	105147	107156	109438	112025	114349	116443	119486	414153

The results in Tables 2,4,6 show that the code equations are in good agreement with analytical results for $0<\alpha<2$, and conservative thereafter for rigid connections. It is postulated that an equation for sub-frame F3 may be taken as the mean of the equations for F1 and F2 for fully-rigid connections. The results suggest that the TS 500 code equations might be modified for values of $\alpha>3$ and incorporates semi-rigid connections.

Table 4 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F2 without shear effect (μ_c=0.00)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	TS 500 (2000)
	Connection stiffness factor, K_s									
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.200	2,000	1,390	1,248	1,130	1,084	1,060	1,048	1,040	1,034	1,044
0.500	2,000	1,622	1,463	1,282	1,194	1,142	1,114	1,097	1,083	1,104
1.000	2,000	1,768	1,637	1,448	1,333	1,257	1,212	1,182	1,159	1,194
2.000	2,000	1,870	1,780	1,624	1,507	1,417	1,358	1,317	1,282	1,344
5.000	2,000	1,945	1,901	1,809	1,725	1,648	1,591	1,547	1,505	1,684
10.000	2,000	1,973	1,949	1,896	1,843	1,790	1,747	1,711	1,674	2,205
20.000	2,000	1,987	1,976	1,948	1,918	1,886	1,858	1,835	1,806	2,985
30.000	2,000	1,992	1,985	1,966	1,945	1,923	1,903	1,886	1,862	3,600

Table 5 Variation in critical load P_{cr} with frame stiffness α and connection stiffness factor K_s for frame F3 without shear effect (μ_c=0.00)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	P_E (kN)
	Connection stiffness factor, K_s									$\frac{\pi^2 EI}{L^2}$
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	1,901	13,674	13,724	13,754	13,764	13,770	13,772	13,774	13,775	13,805
0.200	380	1117	1481	1925	2155	2296	2375	2426	2470	2761
0.500	950	1811	2401	3381	4070	4579	4899	5120	5333	6903
1.000	1900	2762	3459	4810	5955	6937	7630	8145	8735	13805
2.000	3796	4496	5240	6816	8321	9771	10896	11798	13211	27610
5.000	9473	9680	9837	11394	13076	14845	16333	17606	21490	69026
10.000	18890	19117	19255	19610	19675	20497	21982	23288	32130	138051

Table 6 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F3 without shear effect (μ_c=0.00)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	TS 500 (2000)
	Connection stiffness factor, K_s									
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	2,695	1,005	1,003	1,002	1,001	1,001	1,001	1,001	1,001	NA
0.200	2,696	1,572	1,365	1,198	1,132	1,097	1,078	1,067	1,057	NA
0.500	2,696	1,952	1,696	1,429	1,302	1,228	1,187	1,161	1,138	NA
1.000	2,696	2,236	1,998	1,694	1,523	1,411	1,345	1,302	1,257	NA
2.000	2,697	2,478	2,295	2,013	1,822	1,681	1,592	1,530	1,446	NA
5.000	2,699	2,670	2,649	2,461	2,298	2,156	2,056	1,980	1,792	NA
10.000	2,703	2,687	2,678	2,653	2,649	2,595	2,506	2,435	2,073	NA

Table 7 Variation in critical load P_{cr} with frame stiffness α and connection stiffness factor K_s for frame F1 with shear effect (μ_c=0.20)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	P_E (kN)
	Connection stiffness factor, K_s									$\frac{\pi^2 EI}{L^2}$
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	Unstable	13.637	13.727	13.746	13.760	13.767	13.777	13.780	13.796	13.805
0.010	Unstable	125	131	135	136	136.23	136.76	136.77	136.93	138.05
0.050	Unstable	436	530	606	635	650	658	663	666	690
0.200	Unstable	787	1202	1735	2024	2202	2301	2364	2411	2761
0.500	Unstable	927	1577	2706	3534	4156	4550	4820	5030	6903
1.000	Unstable	983	1751	3291	4642	5826	6666	7291	7805	13805
2.000	Unstable	1013	1851	3674	5466	7219	8591	9690	10662	27610
3.000	Unstable	1023	1886	3819	5798	7822	9472	10839	12067	41415
4.000	Unstable	1029	1904	3895	5978	8157	9973	11508	12906	55220
5.000	Unstable	1032	1915	3942	6090	8370	10297	11945	13474	69026
10.000	Unstable	1038	1938	4038	6325	8825	11000	12908	14699	138051
20.000	Unstable	1042	1949	4088	6448	9068	11381	13439	15408	276102
30.000	Unstable	1043	1953	4105	6490	9152	11514	13624	15651	414153

Table 8 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F1 with shear effect (μ_c=0.20)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	TS 500 (2000)
	Connection stiffness factor, K_s									
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	Unstable	1.006	1.003	1.002	1.002	1.001	1.001	1.001	1.000	1.000
0.010	Unstable	1.051	1.027	1.011	1.008	1.007	1.005	1.005	1.004	1.004
0.050	Unstable	1.258	1.141	1.067	1.042	1.030	1.024	1.020	1.018	1.022
0.200	Unstable	1.873	1.516	1.261	1.168	1.120	1.095	1.081	1.070	1.084
0.500	Unstable	2.729	2.092	1.597	1.398	1.289	1.232	1.197	1.171	1.194
1.000	Unstable	3.747	2.808	2.048	1.725	1.539	1.439	1.376	1.330	1.344
2.000	Unstable	5.221	3.862	2.741	2.247	1.956	1.793	1.688	1.609	1.559
3.000	Unstable	6.363	4.686	3.293	2.673	2.301	2.091	1.955	1.853	1.800
4.000	Unstable	7.326	5.385	3.765	3.039	2.602	2.353	2.191	2.068	2.012
5.000	Unstable	8.178	6.004	4.185	3.367	2.872	2.589	2.404	2.263	2.205
10.000	Unstable	11.532	8.440	5.847	4.672	3.955	3.543	3.270	3.065	2.985
20.000	Unstable	16.278	11.902	8.218	6.544	5.518	4.925	4.533	4.233	4.124
30.000	Unstable	19.927	14.562	10.044	7.988	6.727	5.997	5.514	5.144	5.011

Table 9 Variation in critical load P_{cr} with frame stiffness α and connection stiffness factor K_s for frame F2 with shear effect (μ_c=0.20)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	P_E (kN)
	Connection stiffness factor, K_s									$\frac{\pi^2 EI}{L^2}$
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.200	690	1427	1769	2158	2344	2450	2508	2543	2576	2761
0.500	1723	2618	3224	4191	4833	5278	5543	5717	5841	6903
1.000	3440	4394	5124	6528	7687	8643	9286	9745	10140	13805
2.000	6856	7831	8635	10349	11981	13523	14688	15595	16430	27610
5.000	16967	17920	18752	20650	22642	24725	26460	27924	29479	69026
10.000	33373	34262	35070	36965	39026	41266	43201	44889	46872	138051
20.000	64612	65371	66108	67877	69847	72036	73967	75681	77885	276102
30.000	93915	94554	95221	96849	98686	100748	102583	104224	106626	414153

Table 10 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F2 with shear effect (μ_c=0.20)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	TS 500 (2000)
	Connection stiffness factor, K_s									
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.200	2.000	1.391	1.249	1.131	1.085	1.062	1.049	1.042	1.035	1.044
0.500	2.002	1.624	1.463	1.283	1.195	1.144	1.116	1.099	1.087	1.104
1.000	2.003	1.773	1.641	1.454	1.340	1.264	1.219	1.190	1.167	1.194
2.000	2.007	1.878	1.788	1.633	1.518	1.429	1.371	1.331	1.296	1.273
5.000	2.017	1.963	1.919	1.828	1.746	1.671	1.615	1.572	1.530	1.684
10.000	2.034	2.007	1.984	1.933	1.881	1.829	1.788	1.754	1.716	2.205
20.000	2.067	2.055	2.044	2.017	1.988	1.958	1.932	1.910	1.883	2.985
30.000	2.100	2.093	2.086	2.068	2.049	2.028	2.009	1.993	1.971	3.600

6.4 Variations in column effective length factors with semi-rigid connections incorporating shear effects

Comparing the results obtained from this work with semi-rigid connections incorporating shear effects and those calculated using TS 500 equations with fully rigid connections ignoring the shear effects, Tables 7 through 12 and Figs. 7-12 show the results for the variation in critical load P_{cr} (kN) and **k** with α and K_s.

The results in Tables 8-10-12 and Figs. 7-12 show that the codes equations are in good agreement with analytical results for $0<\alpha<2$, and conservative thereafter. It is postulated that an equation for sub-frame F3 may be taken as the mean of the equations for F1 and F2 for fully-rigid connections and shear effects. The results suggest that the TS 500 code equations might be modified for values of $\alpha>3$ and incorporates semi-rigid connections and shear effects.

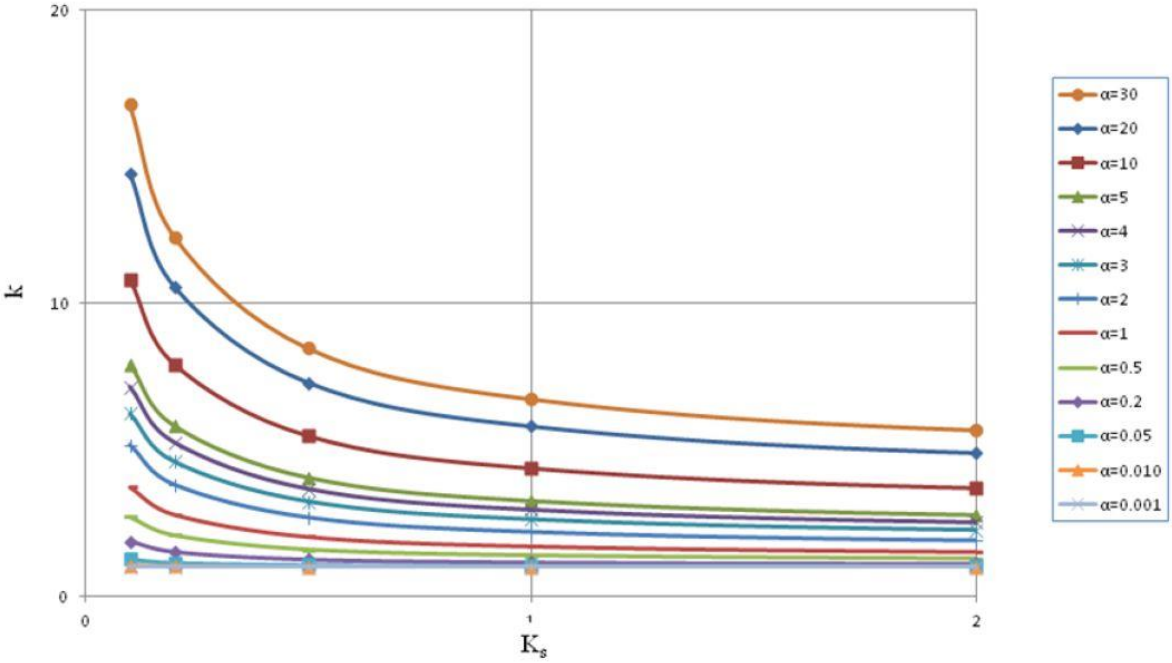

Fig. 7 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F1 with shear effect (μ_c=0.20)

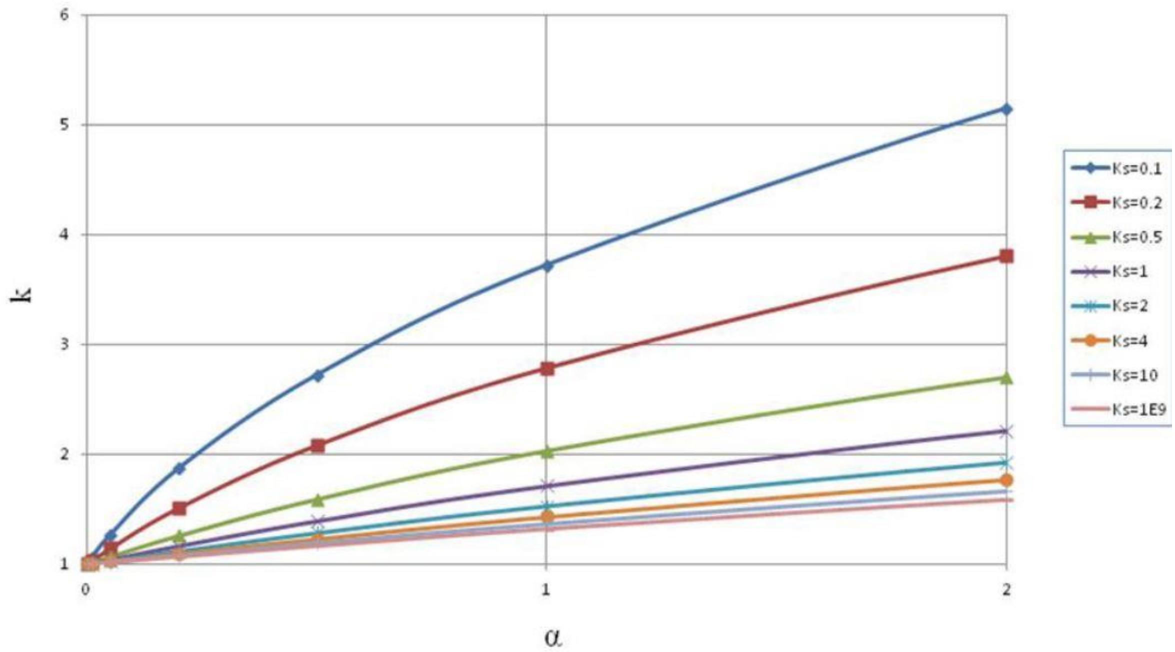

Fig. 8 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F1 with shear effect (μ_c=0.20)

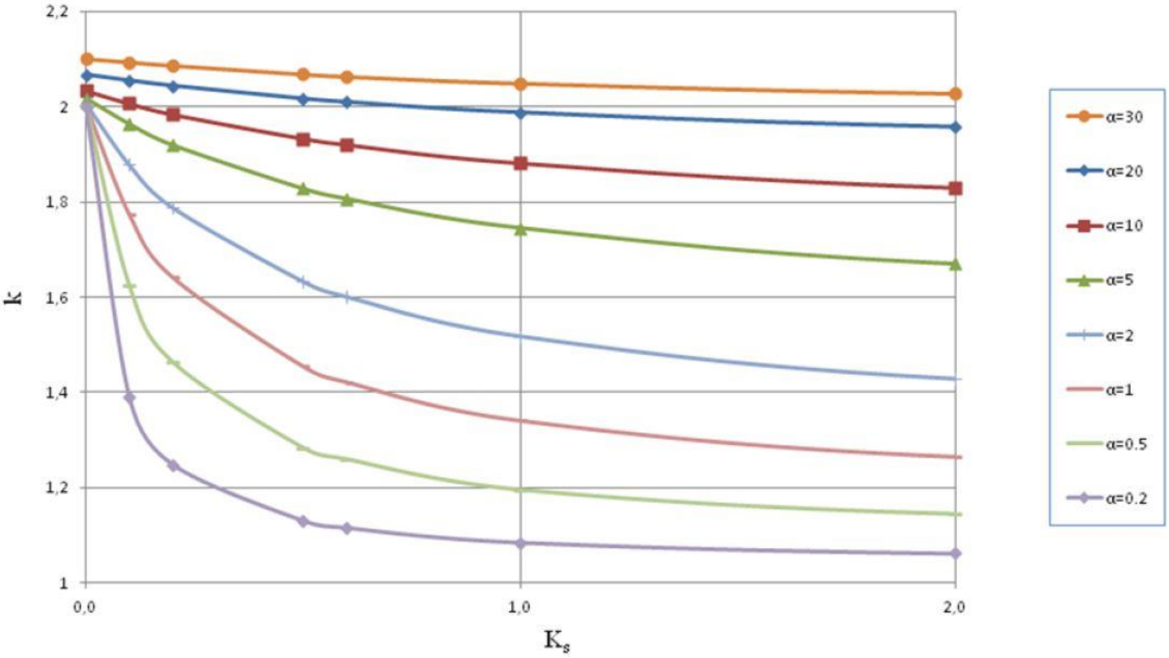

Fig. 9 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F2 with shear effect (μ_c=0.20)

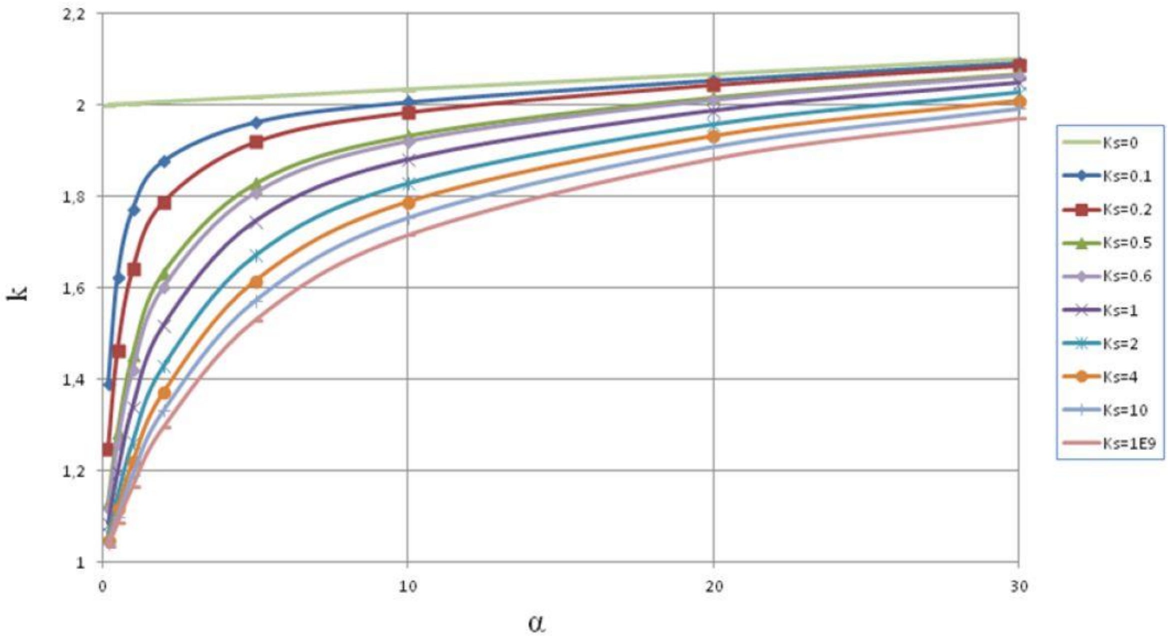

Fig. 10 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F2 with shear effect (μ_c=0.20)

Table 11 Variation in critical load P_{cr} with frame stiffness α and connection stiffness factor K_s for frame F3 with shear effect (μ_c=0.20)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	P_E (kN)
	Connection stiffness factor, K_s									$\frac{\pi^2 EI}{L^2}$
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	1.901	13.676	13.727	13.760	13.770	13.775	13.778	13.780	13.782	13.805
0.200	380	1122	1485	1927	2156	2297	2375	2426	2460	2761
0.500	949	1834	2428	3410	4095	4598	4913	5129	5287	6903
1.000	1893	2836	3548	4918	6068	7046	7727	8232	8620	13805
2.000	3770	4739	5520	7162	8713	10188	11321	12215	12946	27610
5.000	9308	10066	10701	12922	14797	16738	18348	19704	20877	69026
10.000	18241	19000	19628	21084	22635	24295	25704	29333	30736	138051

Table 12 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F3 with shear effect (μ_c=0.20)

Frame stiffness α	Pinned connections	Semi-rigid connections							Rigid connections	TS 500 (2000)
	Connection stiffness factor, K_s									
	0.0	0.1	0.2	0.5	1.0	2.0	4.0	10	∞	
0.001	2.695	1.005	1.003	1.002	1.001	1.001	1.001	1.001	1.001	NA
0.200	2.696	1.569	1.364	1.197	1.132	1.096	1.078	1.067	1.059	NA
0.500	2.697	1.940	1.686	1.423	1.298	1.225	1.185	1.160	1.143	NA
1.000	2.700	2.206	1.973	1.675	1.508	1.400	1.337	1.295	1.266	NA
2.000	2.706	2.414	2.236	1.963	1.780	1.646	1.562	1.503	1.460	NA
5.000	2.723	2.619	2.540	2.311	2.160	2.031	1.940	1.872	1.818	NA
10.000	2.751	2.696	2.652	2.559	2.470	2.384	2.317	2.169	2.119	NA

7. Discussion

It has been found that where column effective length factors **k** are determined within a structural framework, the nature of that framework, its boundary conditions, the effect of shear deformation and the degree of semi-rigidity of the connections will influence the results. All the results show an increase in **k** with:

- an increasing number of degrees of freedom
- an increase in α
- an increase in Poisson's ratio
- a decrease in K_s

The results obtained for the upper storey in the partially braced sub-frame F3 are of particular interest to designers because the boundary conditions for the column which is not adjacent to a shear wall is unspecified in codes of practice. Treating the column alone would lead to very high **k** factors.

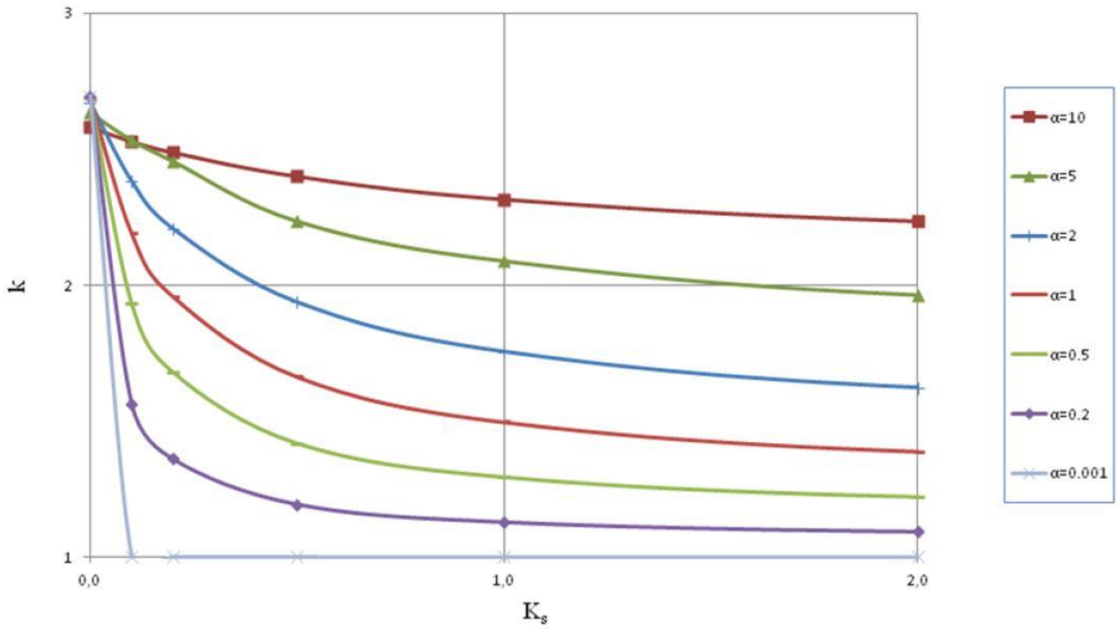

Fig. 11 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F3 with shear effect (μ_c=0.20)

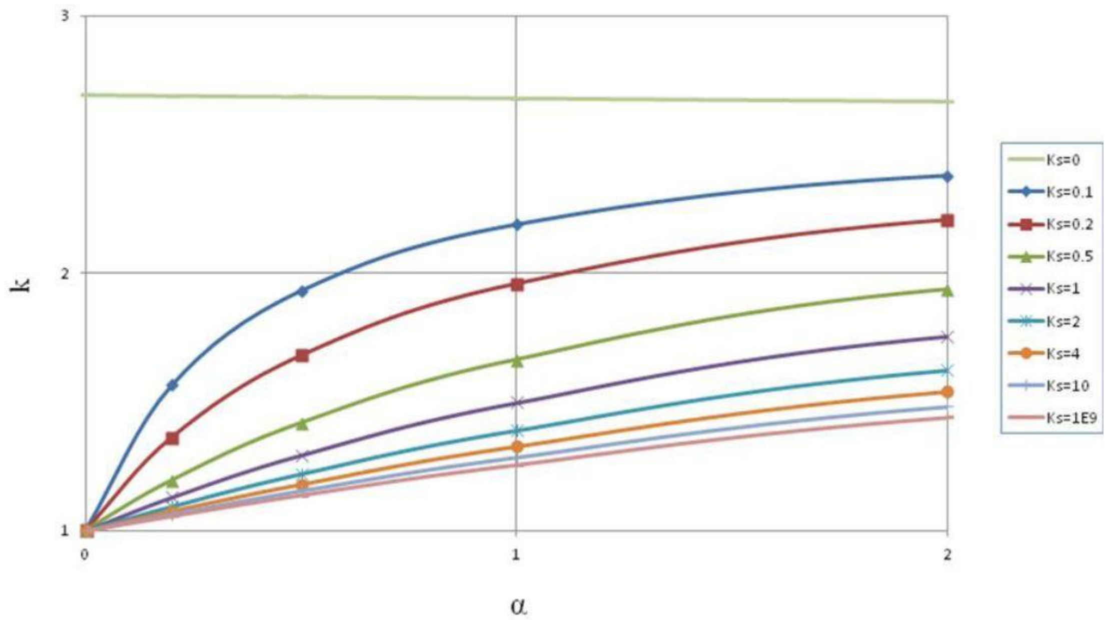

Fig. 12 Variation in column effective length factor **k** with frame stiffness α and connection stiffness factor K_s for frame F3 with shear effect (μ_c=0.20)

Since the effective length factor of sway columns will approach infinity as the α_1 and α_2 values at both column ends approach infinity, the parametric equations are not suitable for this case. Since real fixed or hinged supports for a column seldom exist in actual structure, α=1,0 and α=10 are recommended for fixed and hinged supports, respectively. Eqs. (7) through (9) should therefore be limited to a range, for example α values not greater than 10.

From the stability analysis, we arrive at following boundary conditions for rigid and pinned connections without shear effect (remember: $\alpha_1 \leq \alpha_2$, α is the value at the restrained end for an unbraced column with a hinge at one end):

1. For column fixed at bottom end and hinged at top end, Eq. (5).

$$\alpha_1 = 0,\ \alpha_2 = \infty,\ \alpha = 0\ \rightarrow k = 2.0 \quad (10)$$

2. For column hinged at bottom end and fixed at top end, Eq. (5).

$$\alpha_1 = 0,\ \alpha_2 = \infty,\ \alpha = 0\ \rightarrow k = 2.0 \quad (11)$$

3. For column fixed at both ends

$$\alpha_1 = 0,\ \alpha_2 = \infty \rightarrow k = 2.0 \quad (12)$$

4. For column with equal end restraints

$$\alpha_1 = 3,\ \alpha_2 = 3 \rightarrow k = 1.83 \quad (13)$$

5. For column fixed at bottom end and α_2=10 at top end

$$\alpha_1 = 0,\ \alpha_2 = 10 \rightarrow k = 1.67 \quad (14)$$

8. Conclusions

In this present study frame stability analyses on three types of single-storey *x* single-bay sub-frames in *unbraced* and *partially braced* skeletal frames have shown that column effective length factors *k* increase due to:

- an increasing number of total degrees of freedom at the joints in the ends of the beams and columns.
- an increase in α, the relative stiffness of the columns to the beam members;
- an increase in Poisson's ratio;
- a decrease in K_s

Parametric design equations for column effective length factors have been presented for the variations in **k** with α for rigid connections and **k** values are given in the Tables for semi-rigid connections with/without shear effects. The results enable designers to determine **k** factors for situations currently not catered for in codes of practice, in particular the upper storey in a partially braced frame and shear effects.

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