Structural Engineering and Mechanics, *Vol. 57, No. 4 (2016) 703-716*
DOI: http://dx.doi.org/10.12989/sem.2016.57.4.703

Influence of microstructure, heterogeneity and internal friction on SH waves propagation in a viscoelastic layer overlying a couple stress substrate

Vikas Sharma[*1] and Satish Kumar[2a]

[1]*Department of Mathematics, Lovely Professional University, Phagwara, Punjab, 144411, India*
[2]*School of Mathematics, Thapar University, Patiala, Punjab, 147004, India*

(Received March 25, 2015, Revised January 8, 2016, Accepted January 15, 2016)

Abstract. In this paper, we have investigated shear horizontal wave propagation in a layered structure, consisting of granular macromorphic rock (Dionysos Marble) substrate underlying a viscoelastic layer of finite thickness. SH waves characteristics are affected by the material properties of both substrate and the coating. The effects of microstructural parameter "characteristic length" of the substrate, along with heterogeneity, internal friction and thickness of viscoelastic layer are studied on the dispersion curves. Dispersion equation for SH wave is derived. Real and damping phase velocities of SH waves are studied against dimensionless wave number, for different combinations of various parameters involved in the problem.

Keywords: SH waves; viscoelastic; couple stress; heterogeneity; characteristic length

1. Introduction

In the traditional approach, theory of seismic wave propagation was developed within the frame work of linear elasticity, but later developments showed that earth should be more correctly regarded as a dissipative medium. To encounter dissipation of energy considerations and to overcome the shortcomings of linear elasticity the near sub surface of earth is modelled as linearly viscoelastic material. The geological evidence of heterogeneity within the earth are provided by the wide variations of rocks erupted from volcanoes. The scattering of high frequency seismic waves also support the existence of small scale heterogeneity in the earth lithosphere. Hence, for characterising the internal microstructure of solid earth, heterogeneity and viscoelasticity of the material composition of the earth subsurface has to be taken into account.

The study of guided SH waves has received much attention in the areas of non-destructive testing, exploration geophysics and in the field of seismology for estimating the damage capabilities of seismic waves. SH waves are horizontally polarised shear waves, where a single displacement component is involved. The particle motions are polarised in a single direction. SH waves are studied by many researchers under different conditions. Bhattacharya (1970) pointed out

∗Corresponding author, Assistant Professor, E-mail: vikas.sharma@lpu.co.in, vikassharma10a@yahoo.co.in
[a]Assistant Professor, E-mail: satishk.sharma@thapar.edu

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

some possible exact solution of SH-wave equation for inhomogeneous media. Schoenberg (1971) studied the transmission and reflection of plane waves at an elastic-viscoelastic interface. He concluded that some properties of these waves depend on the frequency of incident wave and the angle of incidence of impinging wave. Kaushik and Chopra (1984) studied the transmission and reflection of inhomogeneous plane SH waves at an interface between two horizontally and vertically heterogeneous viscoelastic solids. Chakraborty (1985) studied reflection and transmission of SH waves from an inhomogeneous half space. He showed effects of inhomogeneity on the reflection and transmission coefficients, graphically. Yang *et al.* (2008) investigated the transient response of SH waves in a layered half space with sub-surface and interface cracks. Borcherdt (2009) has studied the propagation of SH waves in viscoelastic media.

Chattopadhyay *et al.* (2010) studied the propagation of SH waves in an irregular non homogeneous monoclinic crustal layer over a semi-infinite monoclinic medium. They studied the influence of depth of irregularity and non-homogeneity parameters on phase velocity. Chaudhary *et al.* (2010) studied the transmission of plane SH waves through a monoclinic layer embedded between two different self-reinforced elastic solid half spaces. Sahu *et al.* (2014) studied SH waves in viscoelastic heterogeneous layer over half space with self-weight. They studied the effect of gravity, heterogeneity and internal friction on propagation of SH waves in viscoelastic layer over a half space. They observed that heterogeneity of the medium affects the velocity profile of SH wave significantly. Kakar (2015) studied propagation of SH waves in heterogeneous layer laying over an inhomogeneous isotropic elastic half-space and observed that SH wave velocity increases with increases of inhomogeneity parameter.

Inner microstructure of the material plays a dominant role in deciding the properties of the materials, but these microstructures were neglected in classical theory of elasticity. Classical theory works on the assumption that matter is continuously distributed without any defects. Experimental results have shown that the materials having inner atomic structure or microstructures, behave differently at micro level as compared to macroscale. So, there was a need for modifications in classical theory of elasticity or to come up with an alternate size dependent continuum mechanics, which accounts for the microstructure of the material and can reduce to classical theory for macroscale problems.

Voigt (1887) was the first who generated the idea of couple stresses in the materials by assuming that infinitesimal surface element transmit both Cauchy stresses and couple stresses. Cosserat and Cosserat (1909) gave the mathematical model to analyse materials with couple stresses, by considering that the deformation of the medium is described by displacement vector and an independent rotation vector but the theory was not recognised at that time and later on many researchers like Toupin (1962), Mindlin and Tiersten (1962), Koiter (1964), Eringen (1968), Nowacki (1974) worked on this idea to explore microstructural effects in the material. Many problems of wave propagation in an elastic medium with microstructures under different conditions have been studied by applying couple stress theory. Sengupta and Ghosh (1974) studied the effects of couple stresses on wave propagation in elastic layer and they observed that couple stresses affect the velocity of propagation of waves in an elastic layer. Das *et al*. (1991) studied thermo viscoelastic Rayleigh waves under the influence of couple stress and gravity. They derived general equations of phase velocity for these waves and shown that it reduces to classical elastic Rayleigh waves in the absence of couple stresses, viscosity and gravity. Ottosen *et al*. (2000) studied Rayleigh waves by applying the indeterminate couple stress theory. Georgiadis and Velgaki (2003) showed the dispersive nature of Rayleigh waves propagating along the surface of a half space at high frequencies using couple stress theory and also tried to estimate the values of microstructural parameters in couple stress theory. Akgoz and Civalek (2013) did the modelling and analysis of micro-sized plates resting on elastic medium using the modified couple stress theory. Chen and Li (2014) proposed a new modified couple stress theory for anisotropic elasticity containing three length scale parameters and developed composite laminated Kirchhoff plate models under this theory. Among various studies of SH waves, one of the prominent study is to study SH waves in the context of size dependent microcontinuum theory. One such attempt was made by Vardoulakis and Georgiadis (1997), they studied SH surface waves in a homogeneous gradient-elastic half space with surface energy. They showed the existence of SH waves in a homogeneous gradient-elastic half space. They concluded that SH surface waves may exist in a homogeneous half space if the problem is analysed by a continuum theory with appropriate microstructure.

Hadjesfandiari and Dargush (2011), developed a size dependent couple stress theory for isotropic material involving two Lamé parameters (λ and μ) and one length scale parameter (η) called couple stress coefficient, which accounts for couple stress effects in the material. These three parameters can completely characterise the behaviour of an isotropic material. Here, in this theory the ratio $\frac{\eta}{\mu}$ defines square of characteristic length l. One of the major problems in these size dependent elastic theories (Toupin 1962, Mindlin and Tiersten 1962, Koiter 1964, Hadjesfandiari and Dargush 2011) is the determination of these length scale parameters. Lakes (1991) observed that characteristic length would be undetectable for macroscopic mechanical experiment, but is relevant for studies involving composite and cellular solids. It may be comparable to the average cell size of the material in cellular solids or is of the order of spacing between fibres for the fibrous composites.

Dionysos Marble is a white fine-grained metamorphic marble with a saccharoidal microstructure. Here, from the application point of view of this model, SH waves propagation is studied in the heterogeneous viscoelastic layer overlying a granular macromorphic rock (Dionysos Marble). The study is carried out by applying couple stress theory proposed by Hadjesfandiari and Dargush (2011), for observing the effects of microstructures of the substrate in terms of characteristic length along with other parameters of viscoelastic layer on the propagation of SH waves.

2. Formulation and solution of the problem

Consider a layer of viscoelastic medium of thickness H, lying over a couple stress half space with microstructures, characterised by an additional material parameter l, called characteristic length. The origin of the coordinate system O (x,y,z) lies on the interfacial surface joining half space and layer of viscoelastic medium. Here, z axis is pointing vertically downwards into the half space, the interface between layer and half space is given by z=0 and the free surface of layer is $z=-H$. For SH waves, displacement components and body forces are independent of y co-ordinate, so if (u,v,w) are the displacement co-ordinates of a point, then u=w=0 and v is function of parameters x, z and t.

2.1 Couple stress half space

The basic governing equation of motion and constitutive relations of couple stress theory for

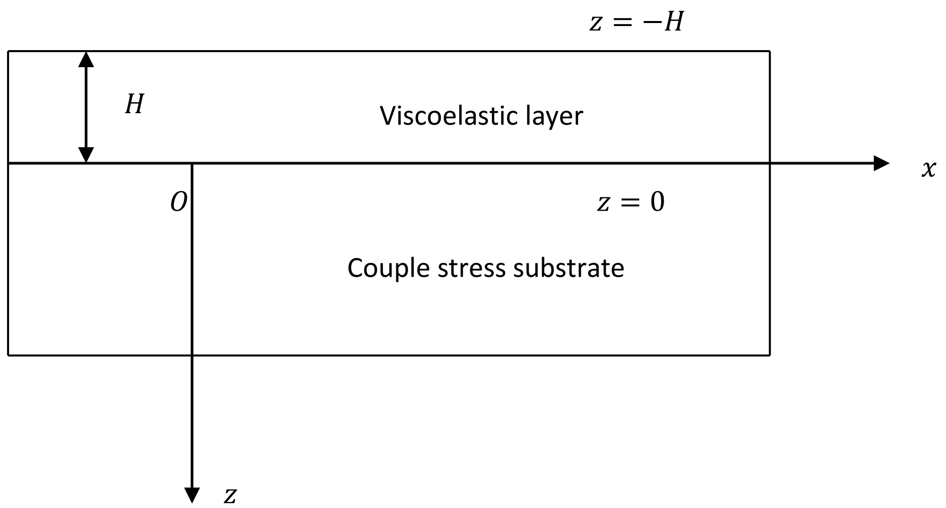

Fig. 1 Geometry of the problem

isotropic material in the absence of body forces (Hadjesfandiari and Dargush 2011) are given by

$$(\lambda + \mu + \eta\nabla^2)\nabla(\nabla.\vec{u}) + (\mu - \eta\nabla^2)\nabla^2\vec{u} = \rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{1}$$

where λ and μ are Lamé constants, $\eta = \mu l^2$ is couple-stress coefficient, l is characteristic length, ρ is the density of the material of elastic half space and $\vec{u}$ is the displacement vector. Let us assume that $\vec{u} = (0, v, 0)$ and $\frac{\partial}{\partial y} \equiv 0$

Imposing above said conditions, equation of motion becomes

$$\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) - l^2\left(\frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial z^4} + 2\frac{\partial^4 v}{\partial x^2 \partial z^2}\right) = \frac{1}{C_2^2}\frac{\partial^2 v}{\partial t^2}, \text{ where } C_2^2 = \frac{\mu}{\rho} \tag{2}$$

Let $v = f(z)e^{-i(\omega t - kx)}$ be the solution, where k is the wave number, $\omega = kc$ is the angular frequency and c is the phase velocity. Eq. (2) reduces to

$$\frac{d^4 f}{dz^4} - S\frac{d^2 f}{dz^2} + Pf = 0, \text{ where } S = 2k^2 + \frac{1}{l^2} \text{ and } P = k^4 + \frac{k^2}{l^2} - \frac{\omega^2}{l^2 C_2^2} \tag{3}$$

Since in the couple stress elastic half space amplitude of waves decreases with increase in depth, so solution of above differential equation becomes

$$f(z) = A_1 e^{-a_1 z} + B_1 e^{-b_1 z}, \text{ where } a_1 = \sqrt{\frac{S + \sqrt{S^2 - 4P}}{2}} \text{ and } b_1 = \sqrt{\frac{S - \sqrt{S^2 - 4P}}{2}} \tag{4}$$

Hence

$$v = (A_1 e^{-a_1 z} + B_1 e^{-b_1 z})e^{-i(\omega t - kx)} \tag{5}$$

The constitutive relations in elastic half space are given by (Hadjesfandiari and Dargush 2011)

$$\sigma_{ji} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) - \eta\nabla^2\left(u_{i\,,j} - u_{j,i}\right) \tag{6}$$

$$\mu_{ji} = 4\eta\left(\omega_{i,j} - \omega_{j,i}\right), \text{ where } \omega_i = \frac{1}{2}\epsilon_{ijk}u_{k\,,j} \tag{7}$$

Here, u_i are the displacement components, σ_{ji} is the non-symmetric force-stress tensor, μ_{ji} is skew symmetric couple-stress tensor, δ_{ij} is Kronecker's delta and ϵ_{ijk} is permutation tensor and $i, j, k = 1, 2, 3$.

$$\sigma_{yz} = \mu\frac{\partial v}{\partial z} + \mu l^2\left(\frac{\partial^3 v}{\partial x^2\partial z} + \frac{\partial^3 v}{\partial z^3}\right) \text{ and } \mu_{xz} = 2\mu l^2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{8}$$

Using Eq. (5) in Eq. (8), we get

$$\sigma_{yz} = [\,\mu(-a_1A_1e^{-a_1z} - b_1B_1e^{-b_1z}) + \mu l^2(a_1A_1k^2e^{-a_1z} + b_1B_1k^2e^{-b_1z} - a_1^3A_1e^{-a_1z} - b_1^3B_1e^{-b_1z})]e^{-i(\omega t - kx)} \tag{9}$$

$$\mu_{xz} = 2\mu l^2[(a_1^2A_1e^{-a_1z} + b_1^2B_1e^{-b_1z}) - (A_1e^{-a_1z} + B_1e^{-b_1z})k^2]e^{-i(\omega t - kx)} \tag{10}$$

2.2 Heterogeneous viscoelastic layer

For the heterogeneity of the layer, we have assumed that properties of the medium change only in *z*-direction. For SH waves propagation in the *x*-direction and causing displacement in *y*-direction only, we shall assume that $\vec{u}_1 = (0, v_1, 0)$ and $\frac{\partial}{\partial y} \equiv 0$

Equation of motion in the absence of body forces and under the above mentioned assumptions (Ravindra 1968) is given by

$$\frac{\partial P_{xy}}{\partial x} + \frac{\partial P_{yz}}{\partial z} = \rho_1\frac{\partial^2 v_1}{\partial t^2} \tag{11}$$

where $P_{xy} = \left(\mu_1 + \eta_1\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial x}$ and $P_{yz} = \left(\mu_1 + \eta_1\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial z}$

In the upper viscoelastic layer μ_1, η_1 and ρ_1 are assumed to be function of depth only and are given by

$$\left.\begin{aligned}\mu_1 &= \mu_0(1 - sin\alpha z)\\ \eta_1 &= \eta_0(1 - sin\alpha z)\\ \rho_1 &= \rho_0(1 - sin\alpha z)\end{aligned}\right\} \tag{12}$$

where μ_0, η_0, ρ_0 are the constant values of μ_1, η_1 and ρ_1 at the interface of layer and half space and α is an arbitrary constant having dimensions of inverse of length.

For heterogeneous viscoelastic layer, Eq. (11) becomes

$$\left(\mu_1 + \eta_1\frac{\partial}{\partial t}\right)\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial}{\partial z}\left[\left(\mu_1 + \eta_1\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial z}\right] = \rho_1\frac{\partial^2 v_1}{\partial t^2} \tag{13}$$

Now, assuming the solution $v_1 = v_L(z)e^{-i(\omega t - kx)}$, equation of motion becomes

$$\frac{d^2v_L}{dz^2}+\frac{1}{\overline{\mu_1}}(\overline{\mu_1})'\frac{dv_L}{dz}+\left(\frac{\rho_1\omega^2}{\overline{\mu_1}}-k^2\right)v_L=0 \tag{14}$$

where $\overline{\mu_1}=\mu_1-i\omega\eta_1$ and $(\overline{\mu_1})'=\frac{d\overline{\mu_1}}{dz}$

Taking $v_L(z)=\frac{Y_1(z)}{\sqrt{\overline{\mu_1}}}$, Eq. (14) reduces to

$$\frac{d^2Y_1}{dz^2}+\left[\frac{1}{4(\overline{\mu_1})^2}\left(\frac{d\overline{\mu_1}}{dz}\right)^2-\frac{1}{2\overline{\mu_1}}\frac{d^2\overline{\mu_1}}{dz^2}+\frac{\rho_1\omega^2}{\overline{\mu_1}}-k^2\right]Y_1=0 \tag{15}$$

Solving this equation further, gives us

$$\frac{d^2Y_1}{dz^2}+\left[\frac{\alpha^2}{4}+\frac{\rho_0\omega^2}{\overline{\mu_0}}-k^2\right]Y_1=0,\ \text{where } \overline{\mu_0}=\mu_0-i\omega\eta_0 \tag{16}$$

Solution to above differential equation is
$Y_1=A\cos(mz)+B\sin(mz)$, where A and B are the arbitrary constants and

$$m^2=\left(\frac{\alpha^2}{4}+\frac{\rho_0\omega^2}{\overline{\mu_0}}-k^2\right)$$

Hence $v_1=v_L(z)e^{-i(\omega t-kx)}=\frac{Y_1(z)}{\sqrt{\overline{\mu_1}}}e^{-i(\omega t-kx)}$

$$v_1=\frac{1}{\sqrt{\overline{\mu_0}}}(1-\sin\alpha z)^{-1/2}(A\cos(mz)+B\sin(mz))e^{-i(\omega t-kx)} \tag{17}$$

2.3 Boundary conditions

Boundary conditions to be satisfied at the free surface of the viscoelastic layer and at the interfacial surface between viscoelastic layer and couple stress half space are

$P_{yz}=\left(\mu_1+\eta_1\frac{\partial}{\partial t}\right)\frac{\partial v_1}{\partial z}=0$ at $z=-H$, $v_1=v$, $P_{yz}=\sigma_{yz}$ and $\mu_{xz}=0$ at $z=0$

Using above mentioned boundary conditions we get following four equations

$$\begin{aligned}&\sqrt{\overline{\mu_0}}[2m(1+\sin(\alpha H))\sin(mH)+\alpha\cos(\alpha H)\cos(mH)]A\\&\quad+\sqrt{\overline{\mu_0}}[2m(1+\sin(\alpha H))\cos(mH)-\alpha\cos(\alpha H)\sin(mH)]B=0\end{aligned} \tag{18}$$

$$-\frac{A}{\sqrt{\overline{\mu_0}}}+A_1+B_1=0 \tag{19}$$

$$\frac{\alpha\sqrt{\overline{\mu_0}}A}{2}+\sqrt{\overline{\mu_0}}mB+\mu a_1[1+(a_1^2-k^2)l^2]A_1+\mu b_1[1+(b_1^2-k^2)l^2]B_1=0 \tag{20}$$

$$(a_1^2-k^2)A_1+(b_1^2-k^2)B_1=0 \tag{21}$$

Eqs. (18)-(21) will have a non-trivial solution, if determinant of coefficients of unknowns A, B, A_1, B_1 vanishes. After applying this condition to the above system of equations, we obtain the following secular equations for the SH waves in an heterogeneous viscoelastic layer over a couple stress half space with microstructures

$$2T_1\overline{\mu_0}m(a_1^2-b_1^2)+2T_2[\mu(k^2-a_1^2)(k^2-b_1^2)(a_1-b_1)l^2+\mu b_1(k^2-a_1^2)-\mu a_1(k^2-b_1^2)]+T_2\overline{\mu_0}\alpha(b_1^2-a_1^2)]=0 \tag{22}$$

where $T_1=[2m\{1+sin(\alpha H)\}sin(mH)+\alpha\,cos(\alpha H)\,cos(mH)]$
and $T_2=[2m\{1+sin(\alpha H)\}cos(mH)-\alpha cos(\alpha H)sin(mH)]$
Separating the real and imaginary parts of dispersion equation,

$$2R_1(\mu_0m_1+\omega\eta_0m_2)(a_1^2-b_1^2)+2I_1(\omega\eta_0m_1-\mu_0m_2)(a_1^2-b_1^2)+2R_2Q+\alpha(R_2\mu_0+\omega\eta_0I_2)(b_1^2-a_1^2)=0 \tag{23}$$

from the real part of dispersion equation of SH waves and

$$2R_1(\mu_0m_2-\omega\eta_0m_1)(a_1^2-b_1^2)+2I_1(\mu_0m_1+\omega\eta_0m_2)(a_1^2-b_1^2)+2I_2Q+\alpha(I_2\mu_0-R_2\omega\eta_0)(b_1^2-a_1^2)=0 \tag{24}$$

from the imaginary part of dispersion equation of SH waves.
where $T_1=R_1+iI_1, T_2=R_2+iI_2$, $m=m_1+im_2$ and

$$Q=\mu(k^2-a_1^2)(k^2-b_1^2)(a_1-b_1)l^2+\mu b_1(k^2-a_1^2)-\mu a_1(k^2-b_1^2)$$
$$R_1=2(1+sin(\alpha H))(m_1S_1-m_2S_2)+\alpha\,cos(\alpha H)\,E_1$$
$$I_1=2(1+sin(\alpha H))(m_1S_2+m_2S_1)-\alpha\,cos(\alpha H)\,E_2$$
$$R_2=2(1+sin(\alpha H))(m_1E_1+m_2E_2)-\alpha\,cos(\alpha H)\,S_1$$
$$I_2=2(1+sin(\alpha H))(m_2E_1-m_1E_2)-\alpha\,cos(\alpha H)\,S_2$$
$$S_1=sin(m_1H)cosh(m_2H),\ S_2=cos(m_1H)sinh(m_2H)$$
$$E_1=cos(m_1H)cosh(m_2H),\ E_2=sin(m_1H)sinh(m_2H)$$
$$m_1=r^{\frac{1}{2}}cos\left(\frac{\theta}{2}\right),\ m_2=r^{\frac{1}{2}}sin\left(\frac{\theta}{2}\right)$$
$$F_1=\frac{\rho_0\omega^3\eta_0}{\mu_0^2+\omega^2\eta_0^2},\ F_2=\frac{\alpha^2}{4}-k^2+\frac{\rho_0\omega^2\mu_0}{\mu_0^2+\omega^2\eta_0^2},\ r=(F_1^2+F_2^2)^{\frac{1}{2}},\ tan(\theta)=\left(\frac{F_1}{F_2}\right)$$

3. Numerical results and discussion

For viscoelastic layer the various material parameters (Gubbins 1990), are $\rho_0=4705\ kg/m^3$, $\mu_0=1.987\times10^{10}\ N/m^2$, $\frac{\mu_0}{\eta_0}=10^6\ s^{-1}$, $\beta_1^2=\frac{\mu_0}{\rho_0}$. The material properties for couple stress half space made of dionysos Marble (Vardoulakis and Georgiadis 1997), are $\mu=30.5\times10^9\,N/m^2$, Density = ρ = 2717 kg/m^3, the value of shear velocity comes out to be, C_2 = 3350 m/s. To find the impact of characteristic length, different cases of characteristic length (l), comparable with the internal cell size of granular macromorphic rock ($O(10^{-4})$) such as l = 0.0001 m, l = 0.0004 m, l = 0.0008 m are considered.

3.1 Effects of heterogeneity parameter

To study the role of heterogeneity parameter on the characteristics of SH waves in the viscoelastic layer, dispersion curves are provided for three different values of heterogeneity

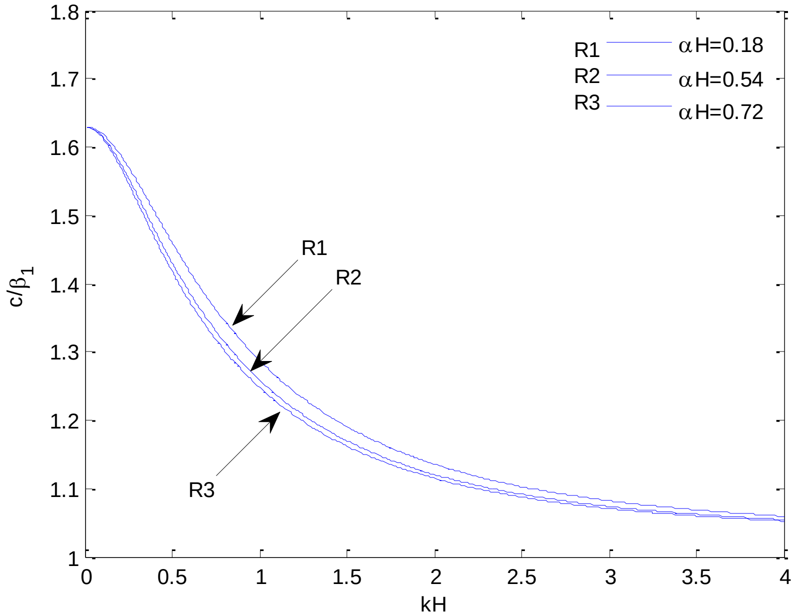

Fig. 2 Variation of normalized real phase velocity (c/β_1) of SH waves against normalized wave number (kH) for different values of heterogeneity parameter (αH)

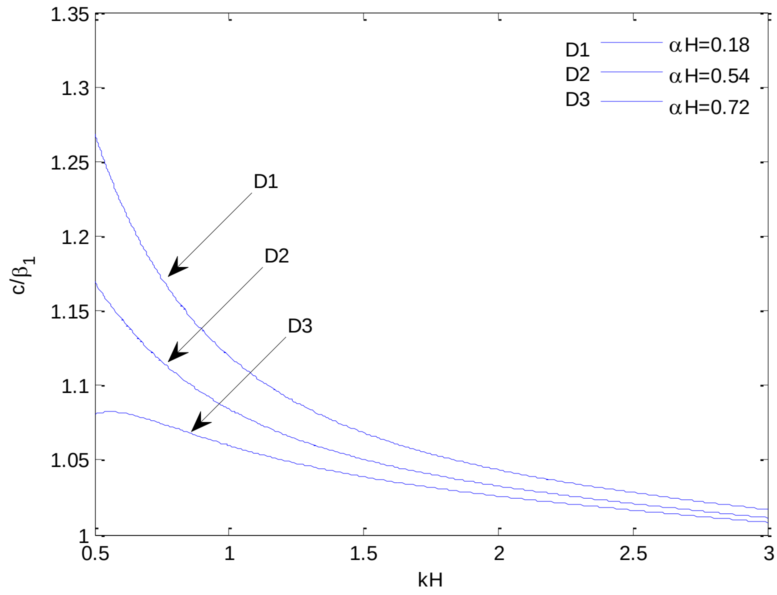

Fig. 3 Variation of normalized damping phase velocity (c/β_1) of SH waves against normalized wave number (kH) for different values of heterogeneity parameter (αH)

parameter (αH=0.18, 0.54 and 0.72 for the curves R1, R2 and R3 respectively), by keeping fixed values of $H = 0.09m$, characteristic length parameter $l = 0.0004\,m$ and friction parameter $\mu_1/\eta_1 = 10^6$.

From Fig. 2, it can be observed that normalized real phase velocity decreases with the increase in normalized wave number before becoming asymptotically closer to 1, for all the considered values of heterogeneity parameter. It can be further seen that with the increase in heterogeneity parameter, real phase velocity decreases. Similar kind of trends, as seen for real phase velocity are also observed for normalized damping phase velocity of SH waves (Fig. 3). A closed survey of the demonstrated results shows that damping phase velocity is more sensitive to heterogeneity parameter (αH).

3.2 Effects of friction parameter

Dispersion curves, to demonstrate the role of friction parameter on SH waves in the viscoelastic layer are provided for three different values of friction parameter $\mu_1/\eta_1 = 7 \times 10^5\, s^{-1}, 10 \times 10^5\, s^{-1}, 80 \times 10^5\, s^{-1}$, by keeping fixed value of heterogeneity parameter $\alpha H = 0.54$ and characteristic length parameter $l = 0.0004\, m$.

It is observed from Figs. 4-5 that normalized real phase velocity tends to decrease with the increase in friction parameter, but has an inverse impact on the normalized damping phase velocity and in both the cases effects are seen for the normalized wave number greater than 1.5.

3.3 Effects of thickness of viscoelastic layer

The influence of thickness(H), of viscoelastic layer on normalized real and damping phase velocities of propagation of SH waves are shown in Figs. 6-7. Fig. 6, shows the variation in normalized real phase velocity (c/β_1), against the normalized wave number (kH), for three

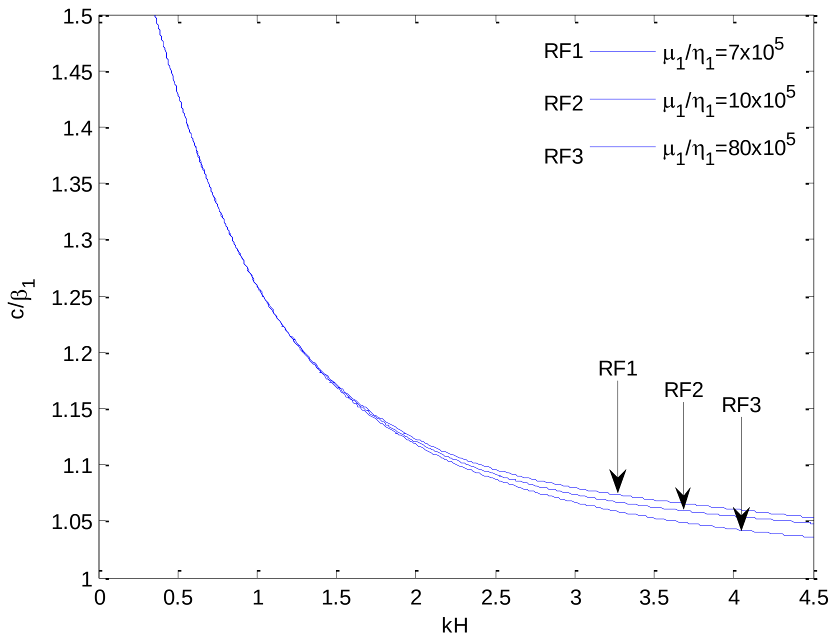

Fig. 4 Variation of normalized real phase velocity (c/β_1) of SH waves against normalized wave number (kH) for different values of internal friction parameter (μ_1/η_1)

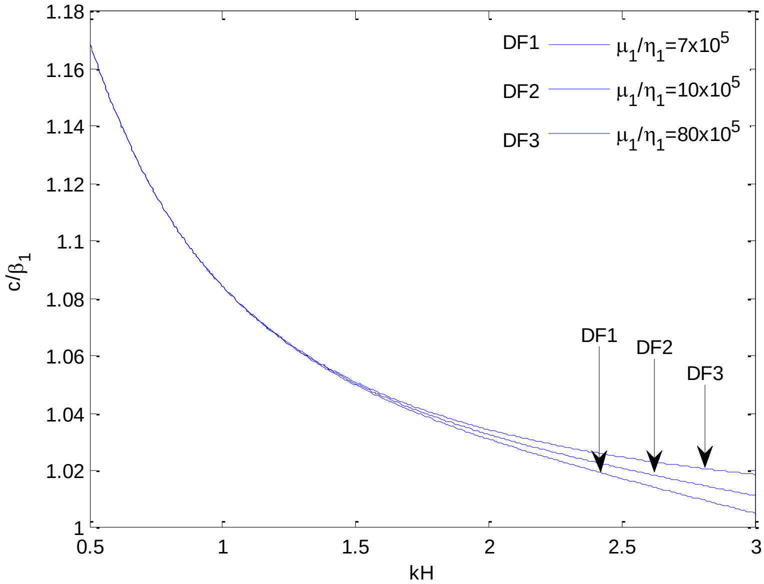

Fig. 5 Variation of normalized damping phase velocity (c/β_1) of SH waves against normalized wave number (kH) for different values of internal friction parameter (μ_1/η_1)

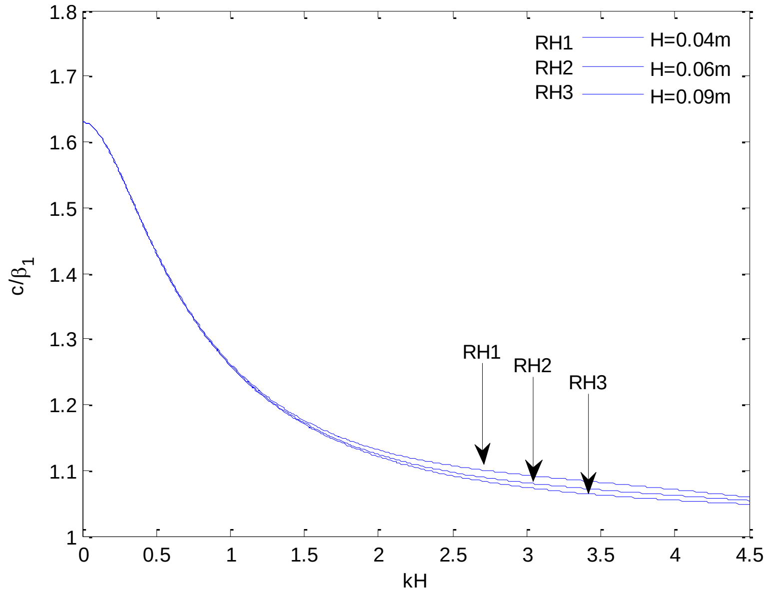

Fig. 6 Variation of normalized real phase velocity (c/β_1) of SH waves against normalized wave number (kH) for varying thickness of viscoelastic layer (H)

different values of thickness, H=0.04m, 0.06m and 0.09m keeping fixed value of heterogeneity parameter $\alpha H = 0.54$, friction parameter $\mu_1/\eta_1 = 10^6$ and characteristic length parameter $l = 0.0004$. It is observed that with the increase in thickness parameter H, normalized real phase velocity decreases. Normalized damping velocity of SH waves increases with increase in this

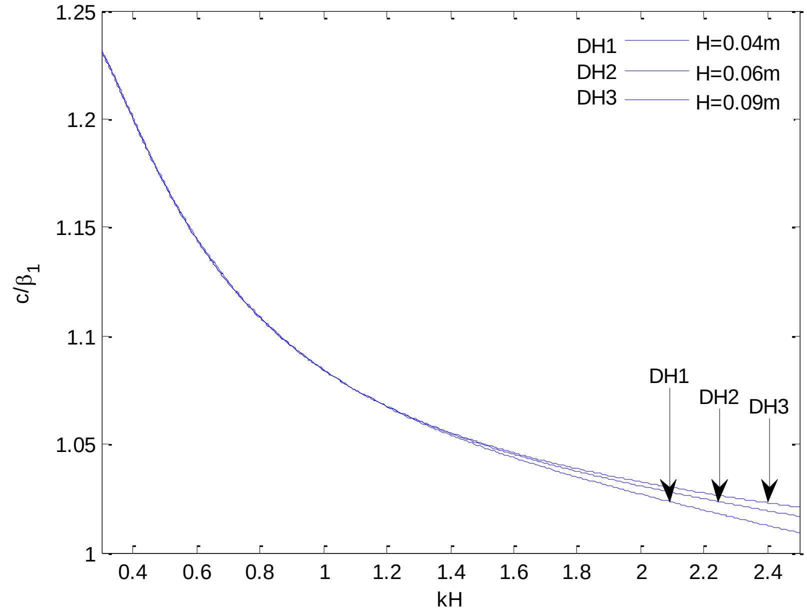

Fig. 7 Variation of normalized damping phase velocity (c/β_1) of SH waves against normalized wave number (*kH*) for varying thickness of viscoelastic layer (*H*)

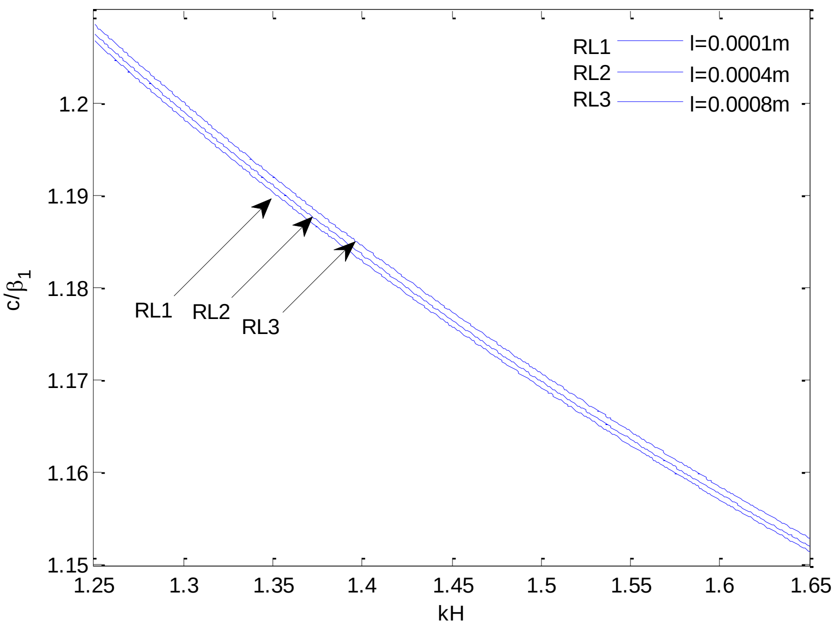

Fig. 8 Variation of normalized real phase velocity (c/β_1) of SH waves against normalized wave number (*kH*) for different values of characteristic length parameter (*l*)

parameter (Fig. 7) and again in both the cases, results are more prominent for wave number (kH) greater than 1.5.

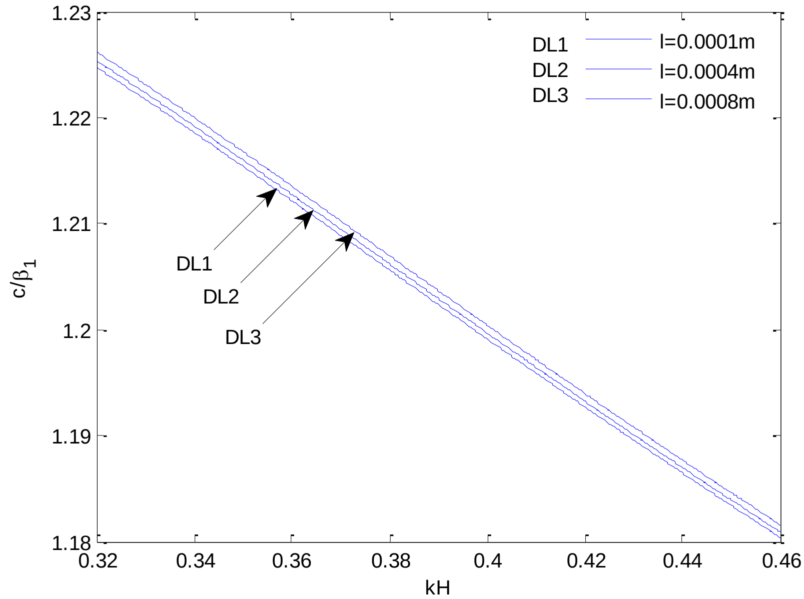

Fig. 9 Variation of normalized damping phase velocity (c/β_1) of SH waves against normalized wave number (kH) for different values of characteristic length parameter (l)

3.4 Effects of internal microstructure of the substrate

For observing the effects of internal microstructure of the underlying substrate, the variation in normalized real and damping phase velocities (c/β_1), against the normalized wave number (kH), are provided in Figs. 8-9, for three different values of characteristic length parameter l = 0.0001m, 0.0004m 0.0008m, keeping fixed values of heterogeneity parameter $\alpha H = 0.54$, thickness of viscoelastic layer $H = 0.09m$ and friction parameter $\mu_1/\eta_1 = 10^6$. It can be observed that both real (Fig. 8) and damping (Fig. 9) phase velocity of SH waves, increases with the increase in characteristic length (l) of the material.

4. Conclusions

The propagation of SH waves is studied in viscoelastic layer overlying a couple stress elastic half space. It is observed that the microstructural parameter characteristic length, heterogeneity, internal friction and thickness of layer have significant effects on the propagation of SH waves. The numerical results are represented graphically for various combinations of parameters involved in the problem. Following conclusions can be drawn from the present analysis

- Increase in characteristic length parameter involved in couple stress theory results in increase of phase velocity of the SH waves. The study has been made for both real and damping phase velocity.
- Heterogeneity parameter involved in viscoelastic layer has prominent effects on the phase velocity profiles of SH waves. Both real and damped phase velocities decrease with the increase in this parameter.

• Real phase velocity is observed to be decreasing with the increase in both internal friction parameter and thickness of the viscoelastic layer for the wave number greater than 1.5. On the other side, these two parameters have an inverse effect on damping phase velocity for the same range.

The theoretical consideration of study concerning microstructural effects of the substrate and effects of other parameters of viscoelastic layer on propagation of SH waves, may find possible applications in seismology, exploration geophysics, non destructive testing and in designing chemical and biochemical sensors coated with surface bound receptive layers, possessing viscoelastic properties which are used to detect compounds in the liquid or gases.

References

Akgoz, B. and Civalek, O. (2013), "Modeling and analysis of micro-sized plates resting on elastic medium using the modified couple stress theory", *Meccanica*, **48**, 863-873.

Bhattacharya, S.N. (1970), "Exact solution of SH-wave equation for inhomogeneous media", *Bull. Seismol. Soc. A.*, **60**(6), 1847-1859.

Borcherdt, R.D. (2009), *Viscoelastic Waves in Layered Media*, Cambridge University Press, New York.

Chakraborty, M. (1985), "Reflection and transmission of SH waves from an inhomogeneous half space", *Proc. Indian Natn. Sci. Acad.*, **51**(4), 716-723.

Chattopadhyay, A., Gupta, S., Singh, A.K. and Sahu, S.A. (2010), "Propagation of SH waves in an irregular non homogeneous monoclinic crustal layer over a semi-infinite monoclinic medium", *Appl. Math. Sci.*, **4**(44), 2157-2170.

Chaudhary, S., Kaushik, V.P. and Tomar, S.K. (2010), "Transmission of plane SH-waves through a monoclinic layer embedded between two different self-reinforced elastic solid half spaces", *Int. J. Appl. Math. Mech.*, **6**(19), 22-43.

Chen, W. and Li, X. (2014), "A new modified couple stress theory for anisotropic elasticity and microscale laminated Kirchhoff plate model", *Arch. Appl. Mech.*, **84**, 323-341.

Cosserat, E. and Cosserat, F. (1909), "Théorie des corps déformables (Theory of Deformable Bodies)", A. Hermann et Fils, Paris.

Das, T.K., Sengupta, P.R. and Debnath, L. (1991), "Thermo-visco-elastic Rayleigh waves under the influence of couple stress and gravity", *Int. J. Math. Math. Sci*., **14**(3), 553-560.

Eringen, A.C. (1968), *Theory of Micropolar Elasticity*, Ed. Liebowitz, H., Fracture, Vol. 2, Academic Press, New York.

Georgiadis, H.G. and Velgaki, E.G. (2003), "High-frequency Rayleigh waves in materials with micro-structure and couple-stress effects", *Int. J. Solid. Struct.*, **40**, 2501-2520.

Gubbins, D. (1990), *Seismology and Plate Tectonics*, Cambridge University Press, Cambridge.

Hadjesfandiari, A.R. and Dargush, G.F. (2011), "Couple stress theory for solids", *Int. J. Solid. Struct.*, **48**, 2496-2510.

Kakar, R. (2015), "SH-wave propagation in a heterogeneous layer over an inhomogeneous isotropic elastic half-space", *Earthq. Struct.*, **9**(2), 305-320.

Kaushik, V.P. and Chopra, S.D. (1984), "Transmission and reflection of inhomogeneous plane SH- waves at an interface between two horizontally and vertically heterogeneous viscoelastic solids", *Proc. Indian Natn. Sci. Acad*,, **50**(4), 291-311.

Koiter, W.T. (1964), "Couple stresses in the theory of elasticity, I and II", *Proc. Ned. Akad. Wet B*, **67**, 17-44.

Lakes, R.S. (1991), "Experimental micro mechanics methods for conventional and negative poisson's ratio cellular solids as cosserat continua", *J. Eng. Mater. Tech.*, **113**, 148-155.

Mindlin, R.D. and Tiersten, H.F. (1962), "Effects of couple-stresses in linear elasticity", *Arch. Ration.*

Mech. Anal., **11**, 415-488.
Nowacki, W. (1974), *Micropolar Elasticity*, International Center for Mechanical Sciences, Courses and Lectures No. 151, Udine, Springer-Verlag, Wien-New York.
Ottosen, N.S., Ristinmaa, M. and Ljung, C. (2000), "Rayleigh waves by the indeterminate couple-stress theory", *Eur. J. Mech. A/Solid.*, **19**, 929-947.
Ravindra, R. (1968), "Usual assumptions in the treatment of wave propagation in heterogeneous elastic media", *Pure Appl. Geophys*., **70**(1), 12-17.
Sahu, S.A., Saroj, P.K. and Dewangan, N. (2014), "SH-waves in viscoelastic heterogeneous layer over half-space with self-weight", *Arch. Appl. Mech*., **84**, 235-245.
Schoenberg, M. (1971), "Transmission and reflection of plane waves at an elastic-viscoelastic interface", *Geophys. J. Int.*, **25**, 35-47.
Sengupta, P.R. and Ghosh, B. (1974), "Effects of couple stresses on the propagation of waves in an elastic layer", *Pure Appl. Geophy*., **112**, 331-338.
Toupin , R.A. (1962), "Elastic materials with couple-stresses", *Arch. Ration. Mech. Anal.*, **11**, 385-414.
Vardoulakis, I. and Georgiadis, H.G. (1997), "SH surface waves in a homogeneous gradient-elastic half-space with surface energy", *J. Elast*., **47**, 147-165.
Voigt, W. (1887), "Theoretische Studien fiber die Elastizitatsverhiltnisse der Kristalle (Theoretical Studies on the Elasticity Relationships of Crystals)", Abh Gesch Wissenschaften 34.
Yang, P.S., Liu, S.W. and Sung, J.C. (2008), "Transient response of SH waves in a layered half-space with sub-surface and interface cracks", *Appl. Math Model.*, **32**, 595-609.

CC