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Investigation of fresh concrete behavior under vibration using mass-spring model

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Abstract. This paper deals with the behavior of fresh concrete that is under vibration using mass-spring model (MSM). To this end, behaviors of two different full scale precast concrete molds were investigated experimentally and theoretically. Experiments were performed under vibration with the use of a computer-based data acquisition system. Transducers were used to measure time-dependent lateral displacements at some points on mold while mold is empty and full of fresh concrete. Analytical modeling of molds used in experiments were prepared by three dimensional finite element method (3D FEM) using software. Modeling of full mold, using MSM, was made to solve the problem of dynamic interaction between fresh concrete and mold. Numerical displacement histories obtained from time history analysis were compared with experimental results. The comparisons show that the measured and computed results are compatible.

Keywords: mass spring model (MSM); precast concrete mold; compaction of fresh concrete; vibration; modeling of full scale mold; 3D finite element method; dynamic interaction of fresh concrete-mold

1. Introduction

External vibrators are commonly used in compacting fresh concrete in the production of precast concrete elements. With the effect of vibration that is transmitted to the concrete mold by the operation of vibrators, vibration is also applied to the fresh concrete inside the mold.

The purpose of compaction is to get rid of the air voids that are trapped in loose concrete. Two types of vibrators are common on building sites-poker (immersion) vibrators and surface vibrators. The poker vibrator is the most popular of the appliances used for compacting concrete. This is because it works directly in the concrete. A third type is clamp-on vibrators (external vibrators). External vibrators consist of an electrically or pneumatically operated motor with an eccentric component. They work by vibrating the formwork to which they are fixed. These vibrations are transmitted to the concrete. This vibrator is mainly for precast concrete work, but it is sometimes also used in cast-in-place concrete, especially where there is congested reinforcement.

Wenzel (1986) investigated principles, practices and some specific problems related to compaction of fresh concrete. The study revealed that, vibrations of external vibrators used for concrete compaction in production of precast concrete products cannot penetrate deeper than 200 mm from the mold surface, thus vibrators should be placed on both sides in cross-sections wider

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than this.

In the literature, there exist a limited number of theoretical and/or experimental studies aiming to determine the behavior of fresh concrete under vibration. In most of the studies, fresh concrete has been described to be a non-Newtonian fluid and commit Bingham model without vibration. Tattersall and Baker (1988) defined flow behavior of non-vibrated fresh concrete by the Bingham model given below as

$$\tau = \tau_o + \mu \dot{\gamma} \tag{1}$$

where τ is shear stress, τ_o is yield stress, μ is plastic viscosity and $\dot{\gamma}$ is shear rate. Moreover; they indicated that yield stress lost its value through measurements, therefore; fresh concrete gained Newtonian fluid (zero yield stress) properties, and its plastic viscosity decreased. Larrard *et al.* (1997), using a device called 'BTRHEOM', showed that yield stress of fresh concrete was halved under vibration, in some cases even was close to zero. It also pointed out that plastic viscosity was not affected by vibration. Alexandridis and Gardner (1981) investigated shear strength characteristics of fresh concrete by using a three-axis compression device. Experimental results were analyzed by Mohr-Coulomb and Rowe shear strength theories. Analysis of "angle of internal friction" of fresh concrete by Mohr-Coulomb theory gave the result as a constant for concrete mix between 37^o-41^o. Analysis by Rowe theory showed that this parameter is between 18^o-21^o.

United States Department of Transportation (2003) described the "Poisson's ratio" of fresh concrete by an equation using a software called HIPERPAV. Poisson's ratio was found between 0.40-0.42 in the plastic state. As a function of time, Poisson's ratio was described by using following equation

$$\upsilon(t) = -0.05 \ln(t + 1.11) + 0.425 \leq 0.42 \tag{2}$$

where t is time passed after preparation of concrete (hour).

Thomas and Harilal (2014) have determined the properties (slump, water absorption, compressive strength and splitting tensile strength of concrete) of fresh and hardened concrete made using three types of artificial cold bonded aggregates.

Liquid-tank interaction in liquid storage tank were inspired to solve the problem of fresh concrete-mold interaction. Generally, researches on the seismic response of liquid storage tanks have been performed over the past 50 years. Housner (1954, 1957) proposed a simple MSM for computing the seismic response of liquid storage tanks that is still widely used with certain modifications for the analysis of rectangular and cylindrical tanks. His simplified MSM is a two degree-of freedom (DOF) system for a rigid tank; one DOF containing the motion of the tank-liquid system, in which a part of the contained fluid being rigidly attached to the tank wall (impulsive mode) and the other DOF for the motion of the sloshing fluid effect on the tank wall (convective mode).

In subsequent studies, Housner's simplified MSM has been modified to clarify the elasticity of the tank wall. Veletsos and Yang (1976) used one mass for the impulsive portion and two convective mass in their simplified MSM. Haroun and Housner (1981) divided the impulsive mass into two sections; one section rigidly connected to the ground and one section representing the mass participating in the relative movement due to the deformation of the tank shell. Malhotra *et al.* (2000) modified the features of the simplified MSM proposed by Veletsos and Yang (1976) using one convective mode.

There are some studies performed in recent years regarding liquid storage tank (Goudarzi and Sabbagh-Yazdi 2009, Goudarzi and Alimohammadi 2010, Seleemah and El-Sharkawy 2011).

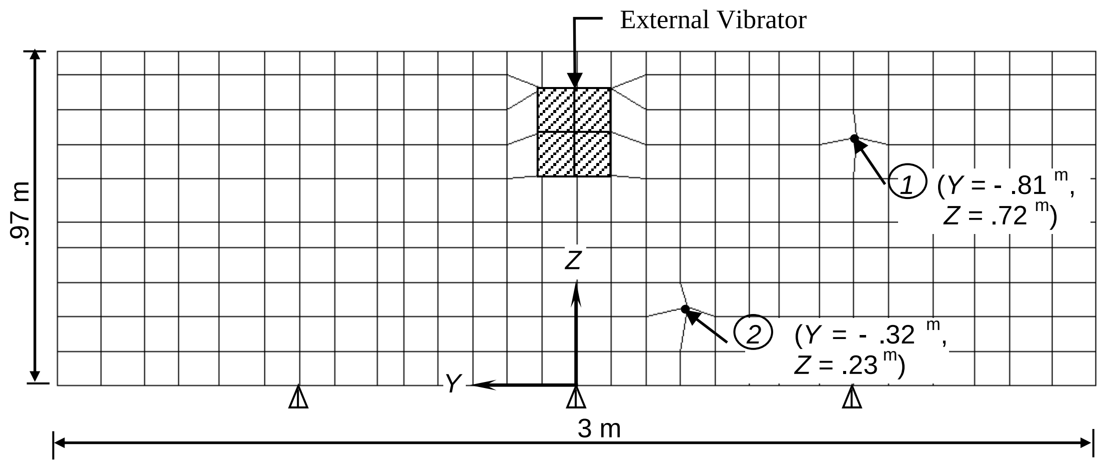

Fig. 1 Measurement surface of finite element mesh of Box culvert mold (Y-Z Plane, *X*=-1.45 m)

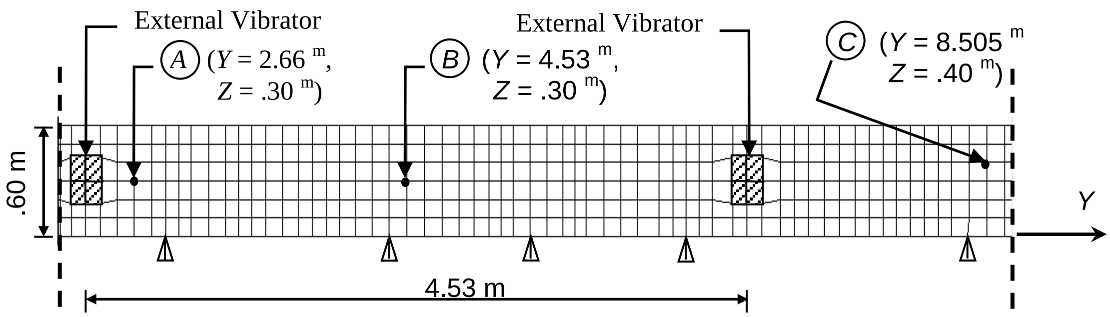

Fig. 2 Measurement surface of finite element mesh of Column mold (Y-Z Plane, *X*=.30 m)

The main aim of this study is to provide a new approach using MSM for behavior of fresh concrete under vibration in precast concrete structures. This paper contains experimental and theoretical studies. The experimental studies are performed in the production workshop of Kambeton Company (Adana/Turkey). Numerical FEM analysis using SAP2000® software, version 16, was used to simulate the behavior of the full scale test specimens. Linear Link/Support object in SAP2000 software was used to model behavior of fresh concrete. In this study, the numerical results for mold deflection histories under vibration are compared with those of test.

2. Experimental investigation

In the content of current study, two full scale molds of precast concrete units having geometry of real sizes were utilized. The views of the two different molds (Box culvert and Column elements) used in the experiments are shown in Figs. 1 and 2. 1 and 2 points are measurement points in Fig. 1, and A, B and C points are measurement points in Fig. 2. The length and height of Column mold is 9.0 m and 600 mm. The size of Box culvert mold with 200 mm thickness in x direction and 250 mm in y direction is 3.0×2.9 m. And, its height is 970 mm. Using these molds, manufacturing were made for real engineering applications in the precast concrete production

Table 1 Specifications of external vibrator

Mechanical Features					Electrical Features	
Vibrat. range	Centrifugal force		Weight	Max. input power	Max. current A	
Vibr./min	kg	kN	kg	*W*	42V	250V
6000 (200 Hz)	1157	11.34	25	1200	23	-

Table 2 The boundary conditions of mold systems

The coordinates of joints where **u**=0 (X-Y plane, *Z*=0)			
Box culvert mold		**Column mold**	
X (mm)	*Y* (mm)	*X* (mm)	*Y* (mm)
-750	-1500	600	100
-50	-1500	300	720
750	-1500	300	1260
-750	-1250	300	1820
-50	-1250	300	2880
750	-1250	300	4410
1450	-800	300	5370
1450	0	300	6440
1450	800	300	8370
1250	-800	300	10080
1250	0	-600	100
1250	800	-300	720
750	1500	-300	1260
-50	1500	-300	1820
-750	1500	-300	2880
750	1250	-300	4410
-50	1250	-300	5370
-750	1250	-300	6440
-1450	800	-300	8370
-1450	0	-300	10080
-1450	-800		
-1250	800		
-1250	0		
-1250	-800		

workshop. The molds used for test specimens are made of steel plates of 5 mm thickness. Steel profiles in various size and sections are connected to the molds in horizontal, vertical and diagonal directions in order to strengthen the system.

The external vibrators used in the experiments are connected to a steel plate having dimensions of 200×250 mm. The features of the external vibrator are introduced in Table 1.

The experimental measurements were taken on the surface of the molds for two different precast concrete structural units by using a computer-based data acquisition system. The location of measurement points were selected near and far from vibration points to see their differences and effect. Data acquisition system including instrumentation, hardware and software used in this study is described in detail (Aktas *et al*. 2014, Aktas and Karasin 2014).

3. Three-dimensional modeling of the empty mold

The molds are made of steel plates of 5 mm thickness. Steel profiles in various size and sections are connected to the molds as horizontal, vertical and diagonal in order to strengthen the system. Reinforcing steel profiles located bottom of the mold are assumed to be simply supported in the model. Boundary conditions of molds for Box culvert and Column units are given in Table 2.

Finite element mesh of molds is formed with Frame and Shell (Area) elements. Area finite elements are formed nearly in dimensions of 100×100 mm as quadrilateral, defined by the four joints as square, rectangular or trapezoidal elements; and triangular, defined by three joints, as transition aimed triangular elements depending on the mold geometry. Frame elements are associated with the Area elements by defining them on the same joints.

Dynamic equilibrium equation of a structural system is written in terms of joint displacements as

$$\mathbf{M}\ \mathbf{a}(t)+\mathbf{C}\ \mathbf{v}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{r}(s,t) \tag{3}$$

where **M**, **C** and **K** are given $N\times N$ mass, damping and stiffness matrices respectively, N is the number of degrees of freedom of the system. The time-dependent vectors **a**, **v** and **u** are $N\times 1$ the joint accelerations, velocities and displacements respectively. $\mathbf{r}$ (s, t) is $N\times 1$ applied load vector varying with position (s) and time (t).

Mass participation ratio is a common measure for determining whether or not there are enough modes. The modes used in the analysis can be un-damped free-vibration modes (eigenvectors) or load-dependent Ritz vectors (mode shapes). In this study, load-dependent Ritz vectors are used in dynamic time-history analysis (Wilson *et al.* 1982).

3.1 The load applied by external vibrator on the mold

For most loadings in Eq. (3), the arbitrary time-dependent load can be further factored into a sum of position vector **f** (s) multiplied by time function g (t)

$$\mathbf{r}(s,t) = \sum_j \mathbf{f}_j(s)\ g_j(t)=\mathbf{f}(s)\ g(t) \tag{4}$$

The load in Eq. (4) will be harmonic if the structure is loaded by vibrators and can be written as

$$\mathbf{r}(s,t) = \mathbf{f}_o\,.Sin(\omega.t) \tag{5a}$$

$$\omega=2\ \pi\ f \tag{5b}$$

$$T=\frac{2\pi}{\omega}=\frac{1}{f} \tag{5c}$$

where $\mathbf{f}_o$ is constant centrifugal force (the amplitude of the load, see Table 1,), ω is angular frequency of the vibrator load, f is cyclic frequency of the vibrator load, t is time (sec), T is period (sec) and Sin(ωt) is the time-dependent sinusoidal function.

It should be noted that, the load applied by the vibrator is normal to the surface of the mold.

In analyses on empty mold, vibrator load is accepted to act as pressure load uniformly distributed on Area (shell) element surfaces in contact with plates to which vibrators are attached to.

It is assumed that the motion of the mold starts from the rest, and then the initial conditions for the mold are

$$\mathbf{u}(t=0)=0,\ \mathbf{v}(t=0)=0 \tag{6}$$

Eq. (3) can be solved for **u** (nodal displacement), subject to boundary conditions (Table 2) and initial conditions Eq. (6).

Three-dimensional view of molds obtained from FEM analysis are illustrated in Figs. 3 and 4.

First, the dynamic analysis was performed only for the empty mold (in the absence of fresh concrete) to verify the reliability of the finite element mesh used in the model and the methods used in the analyses. Modeling of empty mold is given in detail (Aktas and Karasin 2014). Modeling of full mold is described below.

4. Three-dimensional modeling of the full mold

Behavior of fresh concrete was considered as the superposition of static and dynamic (MSM) analysis. Static analysis is defined below.

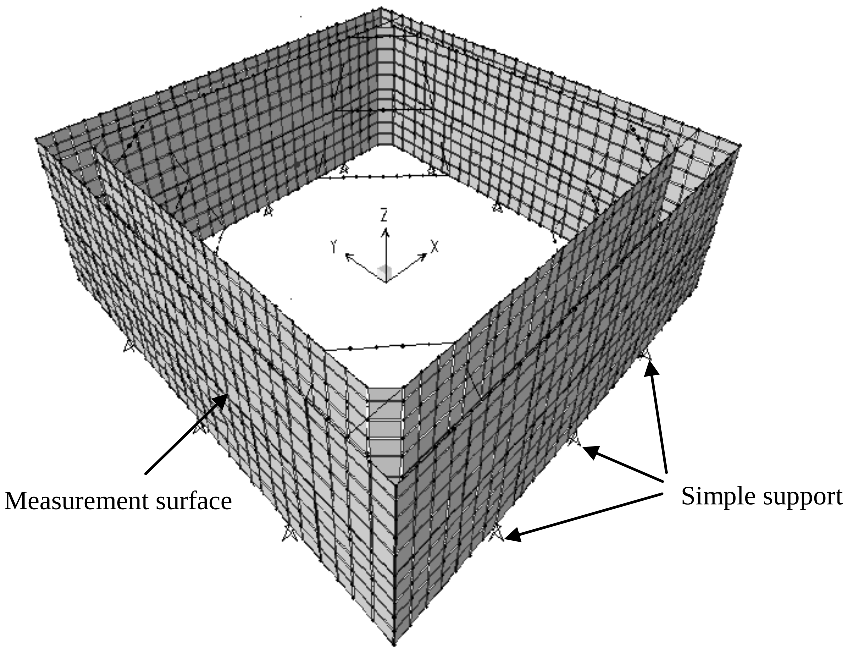

Fig. 3 Measurement surface of finite element mesh of Box culvert mold (Y-Z Plane, X=.30 m)

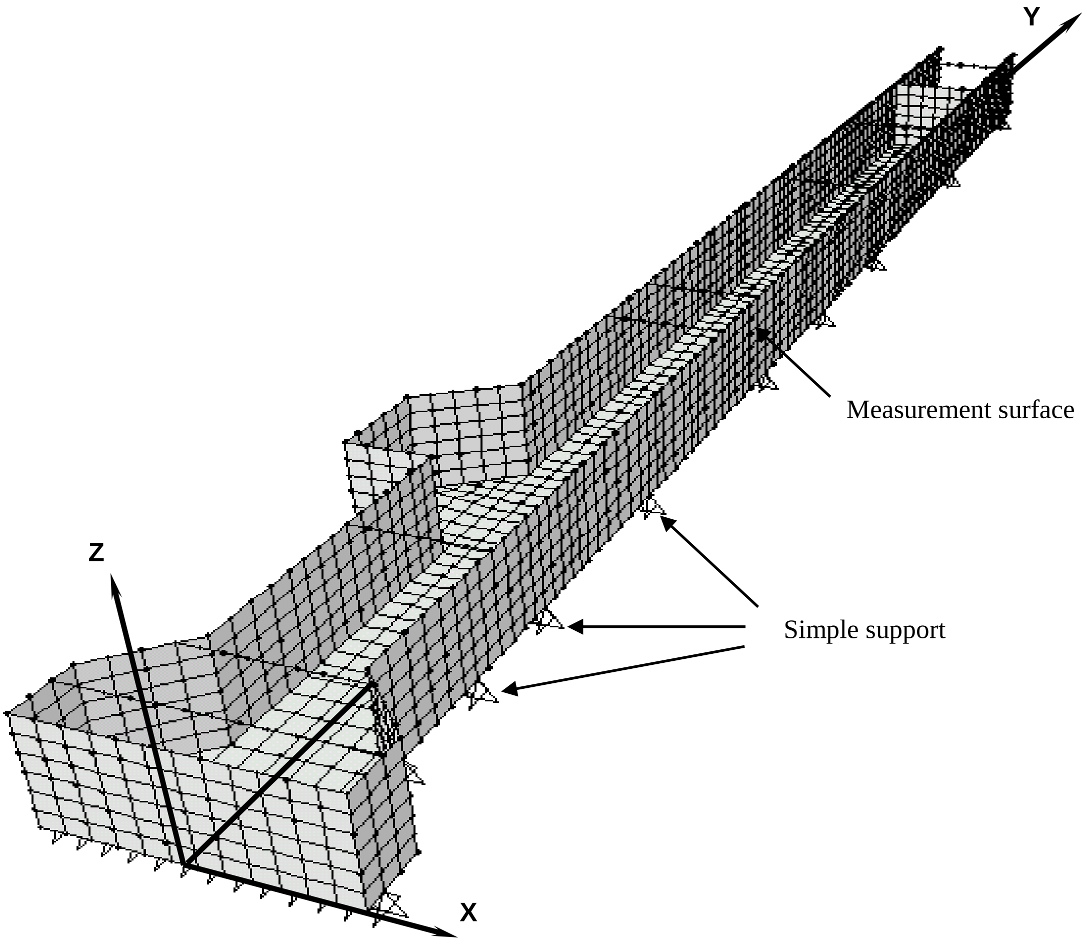

Fig. 4 Three-dimensional view of finite element mesh of Column mold

4.1 Static analysis: pressure load applied by fresh concrete on the mold

Fresh concrete exerts a pressure load, magnitude of which changes with position, on the mold surface. Pressure load applied by fresh concrete on the mold can be stated as follows.

$$b(s) = K_o \, \gamma \, h \tag{7a}$$

where $b(s)$ is a function of position. $b(s)$ function shows lateral static pressure exerted by non-solid fresh concrete on the mold. K_o is known in soil mechanics in two different ways as follows.

$$K_o = 1 - Sin\,\phi \tag{7b}$$

$$K_o = \frac{\nu}{1-\nu} \tag{7c}$$

in which K_o is lateral pressure coefficient, γ is specific weight of material, h is height, ϕ is angle of internal friction of material and ν is Poisson's ratio of material.

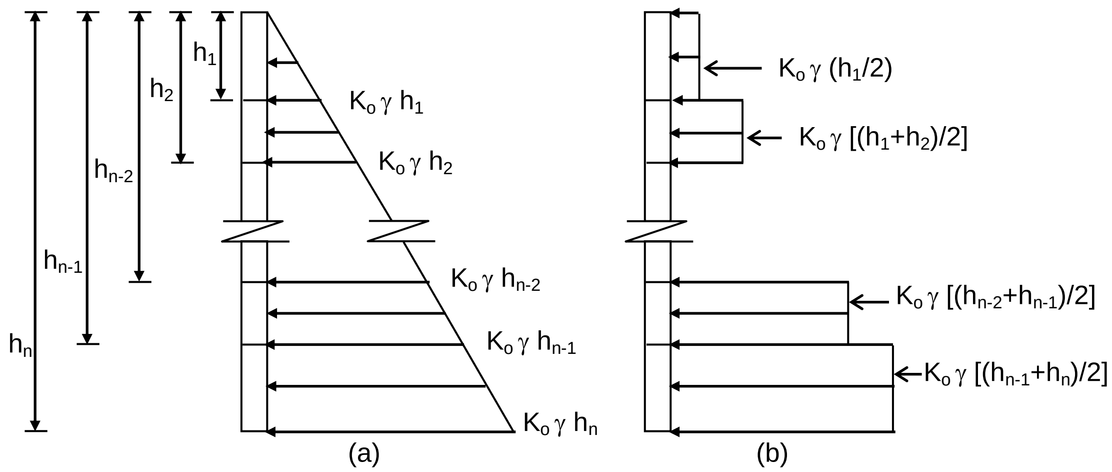

Fig. 5 (a) Real distribution of lateral pressure, (b) Simulation of lateral pressure

K_o is a coefficient changing between 0-1, and this coefficient in Eq. (7a) can be stated in two different forms, depending either on angle of internal friction of material as in Eq. (7b), or on Poisson's ratio of material as in Eq. (7c). Eq. (7b) yields K_o as 0.69 for internal friction of 18^o, and Eq. (7c) yields K_o as 0.72 for Poisson's ratio of 0.42. Due to being more liquid the fresh concrete during vibration, K_o was assumed as a value of 0.75 greater than the value calculated by Eq. (7b) and (7c) (Aktas *et al.* 2014).

Fig. 5(a) shows real distribution of static lateral pressure along mold height. Assuming mold is divided into *n* area elements along the mold height, uniform pressure load values to be applied on each element is computed as described in Fig. 5(b).

4.2 Dynamic analysis: Mass Spring Model (MSM) - Parametric study

In the context of this study, Haroun and Housner (1981) is based. Housner's two mass approximations consist of two parts. The first component of liquid mass is impulse mass which moves with the tank. And second part of liquid mass is convective mass that moves separate from the tank. In the accordance of Housner's formulas, the height of impulse mass is three times the height of liquid divided by eight, that is 37.5%.

In this study, impulsive mass was applied at the level of 400 mm for box culvert element and 200 mm for column. Convective mass is exerted at the level of 700 mm for box culvert element and 400 mm for column. In determination of the height of impulsive and convective mass, 37.5% and 70% of the height of structure was assumed approximately, respectively. 60% of the mass quantity of concrete was appointed to the impulse mass, and 40% to the convective mass.

Link elements in SAP2000 software is used in a wide range especially at the interaction of composite structural elements, the structure - soil load transfer model. Linear Link/Support object in SAP2000 was used to model fresh concrete in MSM. All directional properties -DOF (three translational, three rotational)- are fixed in impulsive part. In convective (sloshing) component, the direction of Link element is active while the others are fixed. MSM is shown for box culvert and column mold in Figs. 6 and 7. As seen in Figures, in this study, a single mass has been used for the part of convective (oscillating).

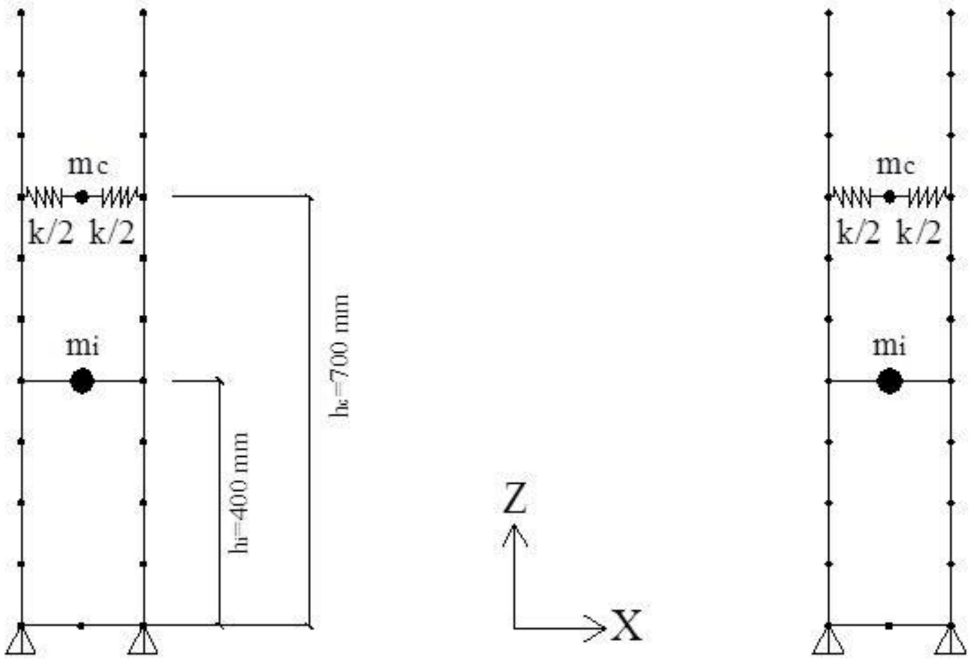

Fig. 6 Defining of concrete behavior with Mass Spring Model (MSM) for box culvert mold

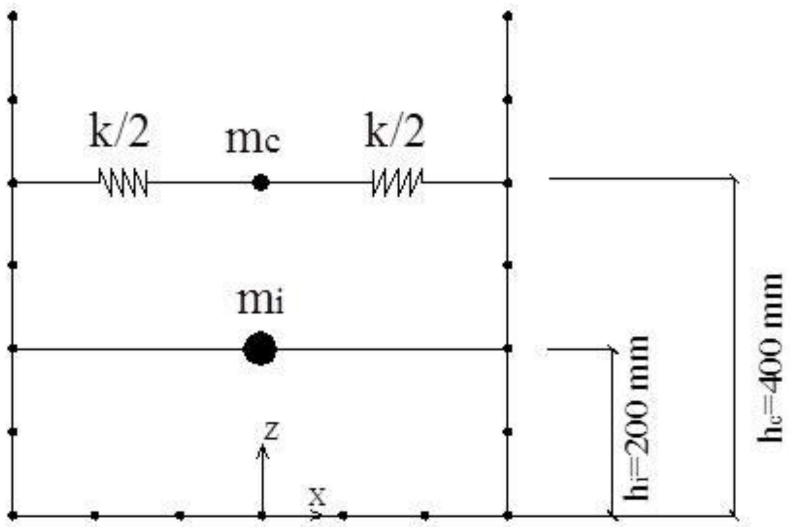

Fig. 7 Defining of concrete behavior with Mass Spring Model (MSM) for column mold

5. Case studies

In this section, findings of performed experimental and analytical studies on two different steel mold are presented and discussed. In all figures of examples, positive displacement histories are in the direction from the mold surface towards concrete.

Displacement histories are in the normal direction to the surface of the mold. In the figures of exercise, the comparisons are illustrated in the steady state part of the behavior. Although some irregularities in the behavior are indicated in the earlier time interval, it disappears shortly and behavior becomes uniform. Thus, graphics are represented for a typical time period where behavior is uniform. In the theoretical analyses, FEM results were compared with the test results. Damping ratio (0.05 in all modes) and mass participation ratio (at least 90% of the total mass of the system) are assumed in accordance with Turkish Earthquake Code (TEC). In determination of number of vibration modes in the analyses, the basis was to pass 90% of the modal mass participation ratio of the structure in the global axes. The time step, number of time steps, and time span in the analyses were selected respectively as 0.5 ms, 8192, and 4.096 sec, which were the same as in the experimental records.

In case studies, the mold systems for Box culvert and Column elements were taken into consideration. Cyclic frequency of the vibrator load was selected as 100 Hz, which was the same as in the test records. An external vibrator was employed in the Box culvert element. Two external vibrators were used for the Column element.

The parameters (specific weight, Young's modulus and Poisson's ratio) of steel was assumed the same as in software. Specific weight of fresh concrete was measured in workshop. Lateral pressure coefficient of fresh concrete was assumed taking into account the Eqs. 7(b) and 7(c) specified in section of 4.1. Impulsive mass, convective mass ratios and spring constant for *Link object* were taken as assumptions in this study.

The value of spring constant (k) is calculated as 0.97 N/mm for water in a master thesis performed in Cukurova University (Hakkak, 2012). Value of spring constant was assumed for fresh concrete taking into consideration the above mentioned study. In mold systems, some parameters of mass, spring, steel and concrete are as follows:

Impulsive mass ratio (m_i)	: 0.6
Convective mass ratio (m_c)	: 0.4
Spring constant (k, stiffness)	: 2.4 N/mm
Specific weight of steel (γ_s)	: 7.682×10^{-5} N/mm^3
Young's modulus of steel (E_s)	: 199948 N/mm^2
Poisson's ratio of steel (ν_s)	: 0.3
Specific weight of fresh concrete (γ_c)	: 24×10^{-6} N/mm^3
Lateral pressure coefficient of fresh concrete (K_o)	: 0.75

5.1 Application 1

In this application, computations of joint displacement histories of mold for the Box culvert were carried out and compared with experimental results. Comparison of time histories was made at two measurement points named 1 and 2 (Fig. 1). 90 modes were used in time-history analysis. The computed numerical results were compared with measured results in Figs. 8 and 9.

5.2 Application 2

In this exercise, numerical analysis is performed for Column mold, and results are compared with experimental data. Comparison of time histories was made at three measurement points called A, B and C (Fig. 2). 60 modes were used in time-history analysis. The computational results were compared with experimental ones in Figs. 10 to 12.

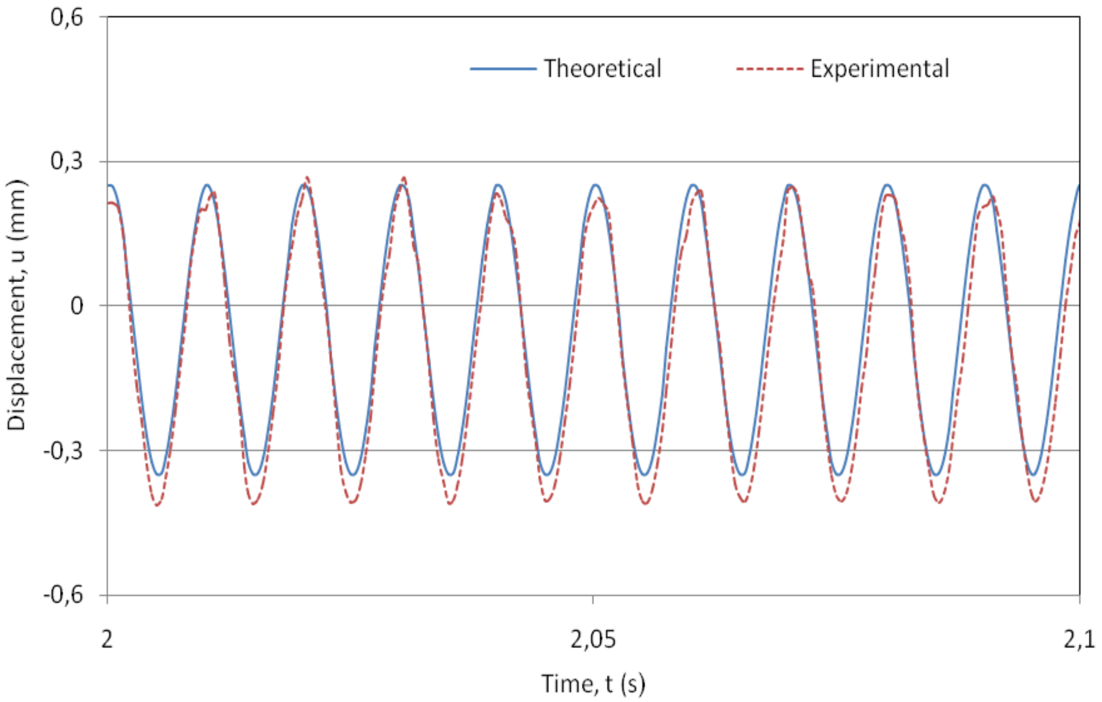

Fig. 8 Theoretical and experimental displacement histories of the Box culvert mold (point 1)

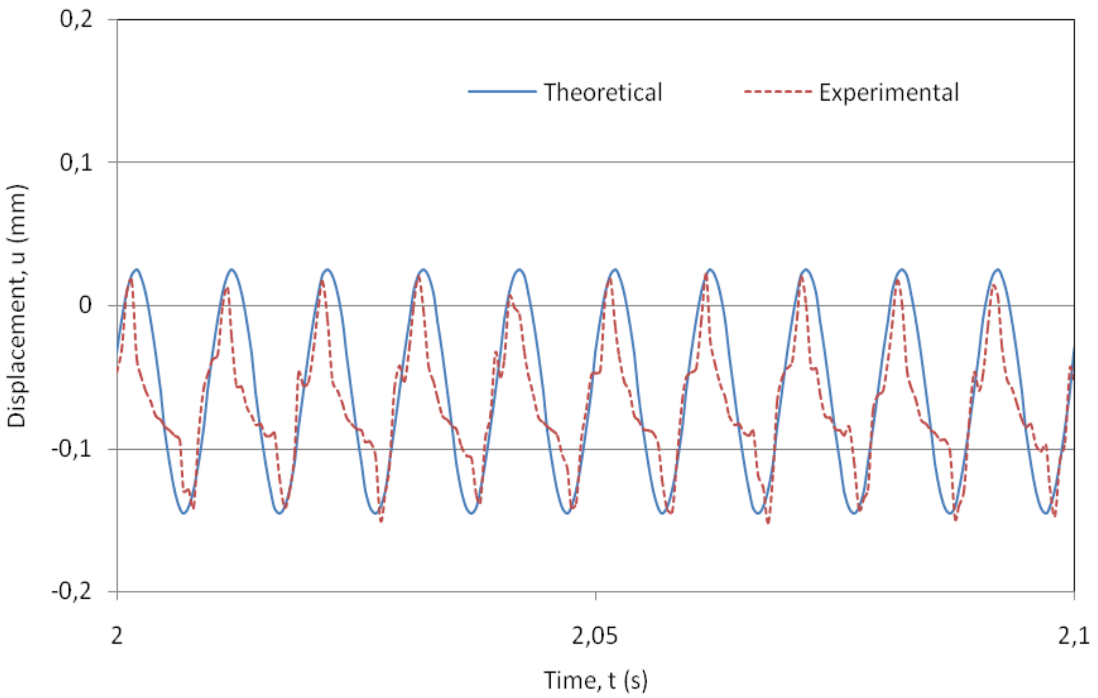

Fig. 9 Theoretical and experimental displacement histories of the Box culvert mold (point 2)

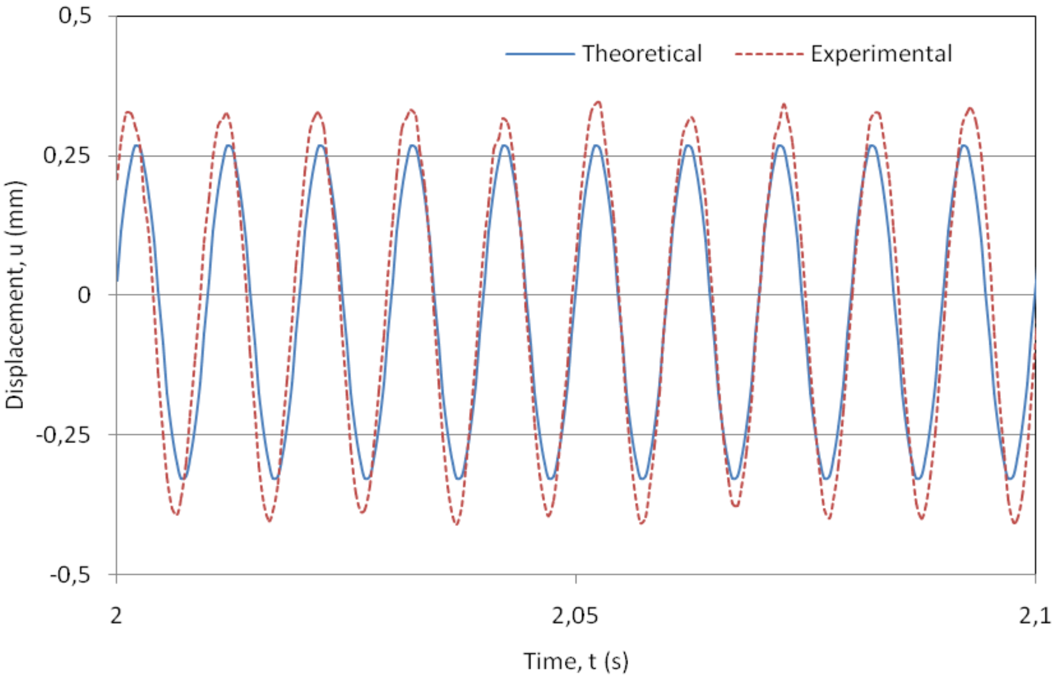

Fig. 10 Theoretical and experimental displacement histories of the Column mold (point A)

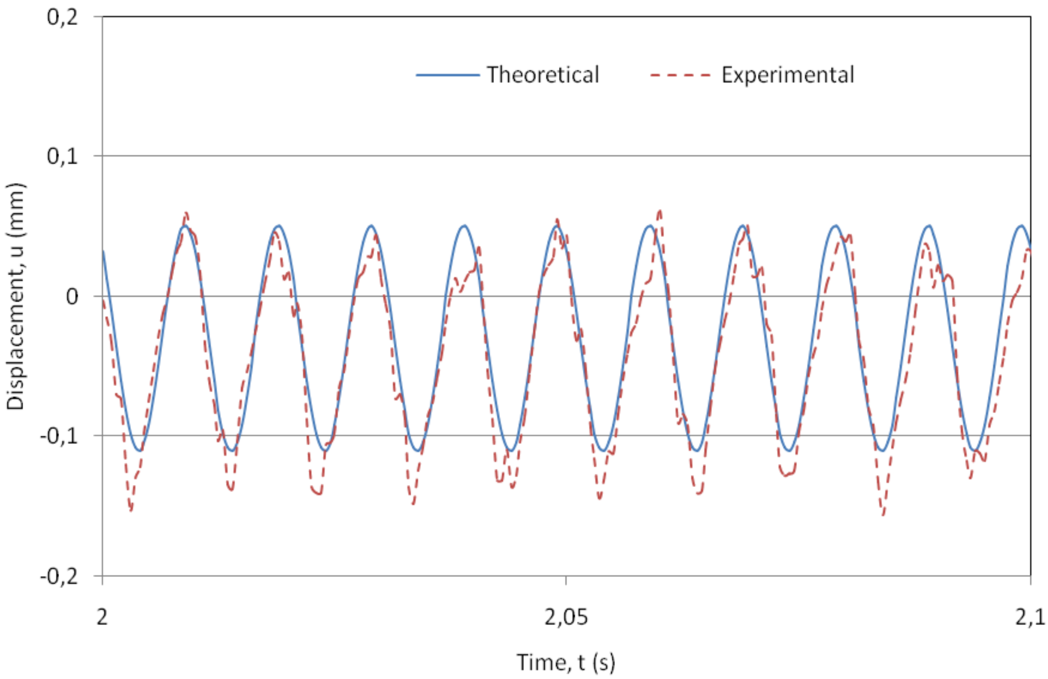

Fig. 11 Theoretical and experimental displacement histories of the Column mold (point B)

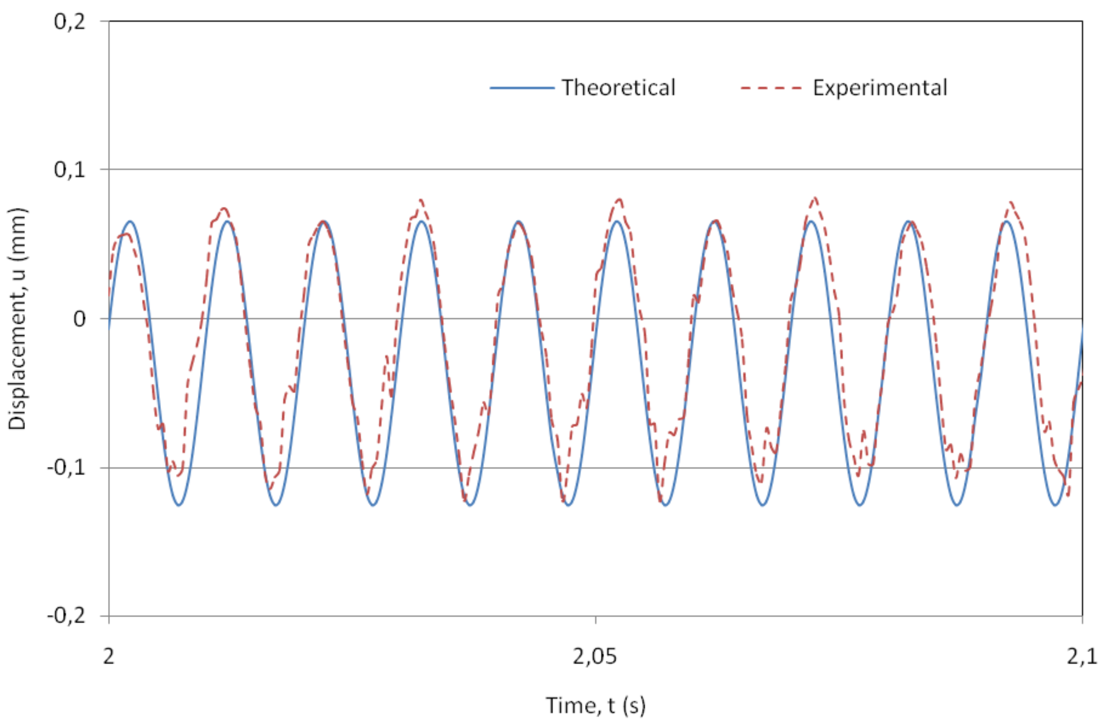

Fig. 12 Theoretical and experimental displacement histories of the Column mold (point C)

6. Results and discussion

The aim of this section of the paper is to examine, discuss and compare the results obtained from experiments and FEM model. As previously mentioned, developed FEM model were tested with experimental data. In the content of theoretical study, three-dimensional model of the mold was developed using FEM. First, a theoretical three-dimensional simulation model of the empty mold (without fresh concrete in it) was performed with the aim of verification of the adequacy of the FEM model (Aktas and Karasin 2014). Then, a theoretical three-dimensional model of the full mold (with fresh concrete in it) was carried out using FEM (Aktas *et al.* 2014). In the mentioned article, the fresh concrete is modeled using mass and time-dependent function for computer-aided mold design (CAMD).

In this study, without making any change in finite element mesh for empty molds, linear Link objects was added throughout surface of molds for MSM. Interaction between fresh concrete and mold, using MSM, was carried out for two different precast concrete mold systems called box culvert and column. The computed numerical displacement histories were compared with experimental test results (Figs. 8 to 12). Although the computed and measured displacement amplitudes are compatible, computed results are smaller than measured data in both mold system (Figs. 8, 10, 11). A reason for this may be that full scale molds were used in theoretical modeling. Another reason may be the applied places of impulsive and convective mass, and their ratios. If more research is done about location and ratios of impulsive and convective mass, better results may be obtained.

7. Conclusions

A theoretical computer modeling for empty mold was developed, and a computed-measured dialogue was carried out (Aktas and Karasin 2014). The accuracy of theoretical model is examined by comparing the numerical results with experimental data. Using this model, it is considered that modeling of fresh concrete can be made more accurately.

MSM was used to model fresh concrete for concrete-mold interaction. The adequacy of the model was checked by comparing numerical results with measured data. There are differences to an extent between computational and measured displacement amplitudes when examined the Figs. 8, 10 and 11.

This study aims to demonstrate that the different materials can be modeled with the use of MSM. Taking advantage of MSM used in this study for modeling fresh concrete, mold design can be performed.

References

Aktas, G., Tanrikulu, A.K. and Baran, T. (2014), "Computer-aided mold design algorithm for precast concrete elements", *ACI Mater. J.*, **111**(1), 77-87.

Aktas, G. and Karasin, A. (2014), "Experimental confirmation for the validity of Ritz method in structural dynamic analysis", *J. Theor. App. Mech.*, **52**(4), 981-993.

Alexsandridis, A. and Gardner, N.J. (1981), "Mechanical behaviour of fresh concrete", *Cement Concete Res.*, **11**(3), 323-339.

Goudarzi, M.A. and Sabbagh-Yazdi, S.R. (2009), "Numerical investigation on accuracy of mass spring models for cylindrical tanks under seismic excitation", *Int. J. Civil Eng.*, **7**(3), 190-202.

Goudarzi, M.A. and Alimohammadi, S. (2010), "Numerical assessment of seismic safety of liquid storage tanks and performance of base isolation system", *Struct. Eng. Mech.*, **35**(6), 759-772.

Hakkak, E.C. (2012), "Dynamic behavior of theoretical and experimental investigation of steel water tank", Master Thesis., Department of Civil Engineering, Cukurova University, Turkey.

Haroun, M.A. and Housner, G.W. (1981), "Seismic design of liquid storage tanks", *J. Tech. Count*., ASCE, **107**, 191-207.

Housner, G.W. (1954), "Earthquake pressures on fluid containers", Eighth Technical Report under Office of Naval Research, Project Designation No. 081-095, California Institute of Technology, Pasadena, California.

Housner, G.W. (1957), "Dynamic pressures on accelerated fluid containers", *Bull. Seis. Soc. Am.*, **47**(1), 15-35.

Larrard, F.D., Hu, C., Sedran. T., Szitkar. J.C., Jolt. M., Claux. F. and Derkx, F. (1997), "New rheometer for soft-to-fluid fresh concrete", *ACI Mater. J.*, **94**(3), 234-243.

Malhotra, P.K., Wenk, T. and Wieland, M. (2000), "Simple Procedures for Seismic Analysis of Liquid Storage Tanks", *Struct. Eng. Int.*, **10**(3), 197-201.

SAP2000® (1999), *Integrated Finite Element Analysis and Design of Structures*, Computers and Structures, Inc., Berkeley, California, USA.

Seleemah, A.A. and El-Sharkawy, M. (2011), "Seismic analysis and modeling of isolated elevated liquid storage tanks", *Earthq. Struct.*, **2**(4), 397-412.

Tattersall, G.H. and Baker, P.H. (1988), "Effect of vibration on the rheological properties of fresh concrete", *Mag. Concete Res.*, **40**(143), 79-89.

TEC Turkish Earthquake Code (2007), *Specification for Structures to be Built in Disaster Areas*, Ministry of Public Works and Settlement, Government of Republic of Turkey, Ankara.

Thomas, J. and Harilal, B. (2014), "Fresh and hardened properties of concrete containing cold bonded aggregates", *Adv. Concete Const.*, **2**(2), 77-89.

U.S. Department of Transportation (2003), "Poission's fatio and temperature gradient adjustments", HIPERPAV Validation Model Summary, Federal Highway Administration Research, Technology, and Development Turner-Fairbank Highway Research Center 6300 Georgetown Pike McLean, Virginia 22101-2296, 1-4.

Veletsos, A.S. and Yang, J.Y. (1976), "Dynamics of Fixed-Base Liquid Storage Tanks", *Proceedings of U.S.-Japan Seminar for Earthquake Engineering Research with Emphasis on Lifeline Systems*, Tokyo, Japan.

Wenzel, D. (1986), "Compaction of concrete - principles, practice, special problems", *Beton. Fert. Tech.*, **52**(3), 153-158.

Wilson, E.L., Yuan, M.W. and Dickens, J.M. (1982), "Dynamic analysis by direct superposition of ritz vectors", *Earthq. Eng. Struct. Dyn.*, **10**, 813-823.

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Notation

b = static lateral pressure
C = damping matrix
E_s = Young's modulus of steel
f = cyclic frequency
f_o = constant centrifugal force (amplitude of the load)
k = spring constant
K = stiffness matrix
K_o = lateral pressure coefficient
m_i = impulsive mass
m_c = convective mass
M = mass matrix
N = number of degrees of freedom
P = pressure load
r = applied load
T = period
u = time-dependent displacement vector
v = time-dependent velocity vector
a = time-dependent acceleration vector
ω = angular frequency
γ_c = specific weight of fresh concrete
γ_s = specific weight of steel
ν_c = Poisson's ratio of fresh concrete
ν_s = Poisson's ratio of steel
ϕ_c = angle of internal friction of fresh concrete
τ = shear stress
τ_o = yield stress
μ = plastic viscosity