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Static and dynamic behavior of FGM plate using a new first shear deformation plate theory

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Abstract. In this paper, a new first shear deformation plate theory based on neutral surface position is developed for the static and the free vibration analysis of functionally graded plates (FGPs). Moreover, the number of unknowns of this theory is the least one comparing with the traditional first-order and the other higher order shear deformation theories. The neutral surface position for a functionally graded plate which its material properties vary in the thickness direction is determined. The mechanical properties of the plate are assumed to vary continuously in the thickness direction by a simple power-law distribution in terms of the volume fractions of the constituents. Based on the present shear deformation plate theory and the neutral surface concept, the governing equations are derived from the principle of Hamilton. There is no stretching-bending coupling effect in the neutral surface based formulation. Numerical illustrations concern flexural and dynamic behavior of FG plates with Metal-Ceramic composition. Parametric studies are performed for varying ceramic volume fraction, length to thickness ratios. The accuracy of the present solutions is verified by comparing the obtained results with the existing solutions.

Keywords: functionally graded material; first shear deformation theory; neutral surface position; volume fraction

1. Introduction

Nowadays functionally graded materials (FGMs) are an alternative materials widely used in aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical and shipbuilding industries. In fact, FGMs have been proposed, developed and successfully used in industrial applications since 1980’s (Koizumi 1993). Classical composites structures suffer from discontinuity of material properties at the interface of the layers and constituents of the composite. Therefore the stress fields in these regions create interface problems and thermal stress concentrations under high temperature environments. Furthermore, large plastic deformation of the interface may trigger the initiation and propagation of cracks in the material (Vel 2004). These

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problems can be decreased by gradually changing the volume fraction of constituent materials and tailoring the material for the desired application. In fact, FGMs are materials with spatial variation of the material properties. However, in most of the applications available in the literature, as in the present work, the variation is through the thickness only. Therefore, the early state development of improved production techniques, new applications, introduction to effective micromechanical models and the development of theoretical methodologies for accurate structural predictions, encourage researchers in this field. Many papers, dealing with static and dynamic behavior of FGMs, have been published recently. An interesting literature review of above mentioned work may be found in the paper of Birman and Byrd (2007). Taj *et al*. (2013) conducted static analysis of FG plates using higher order shear deformation theory. Recently, Tounsi and his co-workers (Hadji *et al.* 2011, Houari *et al.* 2011, El Meiche *et al.* 2011, Bourada *et al.* 2012, Bachir Bouiadjra *et al.* 2012, Fekrar *et al*. 2012, Klouche Djedid *et al*. 2014, Nedri *et al*. 2014, Ait Amar Meziane *et al*. 2014, Draiche *et al*. 2014, Sadoune *et al*. 2014, Ait Yahia *et al*. 2014, Belkorissat *et al*. 2015) developed new shear deformation plates theories involving only four unknown functions.

Belabed *et al.* (2014) presented an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Hamidi *et al.* (2015) investigated a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Hebali *et al.* (2014) studied the static and free vibration analysis of functionally graded plates using a new quasi-3D hyperbolic shear deformation theory. Mahi *et al.* (2015) studied the bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates using a new hyperbolic shear deformation theory. Tounsi *et al*. (2013) use a refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates. Bouderba *et al.* (2013) studied the thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations. Zidi *et al.* (2014) study hygro-thermo-mechanical loading for the Bending of FGM plates using a four variable refined plate theory. Bennoun *et al.* (2016) analyzed the vibration of functionally graded sandwich plates using a novel five variable refined plate theory. Ait Atmane *et al.* (2016), studied the effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations. Belifa *et al.* (2016) studied the bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position.

In the present article, a new first shear deformation plate theory based on neutral surface position is developed for the static and dynamic analysis of functionally graded plates. This theory has number of advantages over the CLPT and FSDPT. In the present theory the governing differential equation is of fourth order and plate physical properties and lateral loading are being used. The governing equations are obtained from the Hamilton principle and Navier solutions of FG simply supported plates are presented. The accuracy and effectiveness of the present theory are established through numerical examples. Numerical results are presented for Ceramic-Metal functionally graded plates.

2. Theoretical formulations

2.1 Physical neutral surface

Consider a rectangular plate made of FGMs of thickness *h*, length *a*, and width *b*, referred to

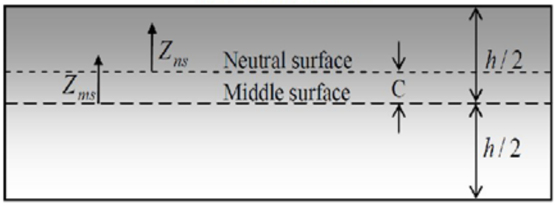

Fig. 1 The position of middle surface and neutral surface for a functionally graded plate.

the rectangular cartesian coordinates (x, y, z). The x-y plane is taken to be the undeformed mid-plane of the plate, and the z axis is perpendicular to the x-y plane. Due to asymmetry of material properties of FG plates with respect to middle plane, the stretching and bending equations are coupled. But, if the origin of the coordinate system is suitably selected in the thickness direction of the FG plate so as to be the neutral surface, the properties of the FG plate being symmetric with respect to it. To specify the position of neutral surface of FG plates, two different planes are considered for the measurement of z namely, z_{ms} and z_{ns} measured from the middle surface and the neutral surface of the plate, respectively, as depicted in Fig. 1.

The volume-fraction of ceramic V_C is expressed based on z_{ms} and z_{ns} coordinates as

$$V_C = \left(\frac{z_{ms}}{h} + \frac{1}{2}\right)^n = \left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^n \tag{1}$$

where n is the power law index which takes the value greater or equal to zero and C is the distance of neutral surface from the mid-surface. Material non-homogeneous properties of a functionally graded material plate may be obtained by means of the Voigt rule of mixture (Suresh and Mortensen 1998). Thus, using Eq. (1), the material non-homogeneous properties of FG plate P, as a function of thickness coordinate, become

$$P(z) = P_M + P_{CM}\left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^n, \quad P_{CM} = P_C - P_M \tag{2}$$

where P_M and P_C are the corresponding properties of the metal and ceramic, respectively. In the present work, we assume that the elasticity modules E and the mass density ρ are described by Eq. (2), while Poisson's ratio ν, is considered to be constant across the thickness (Benachour *et al*. 2011, Larbi Chaht *et al*. 2014).

The position of the neutral surface of the FG plate is determined to satisfy the first moment with respect to Young's modulus being zero as follows (Bourada *et al*. 2015, Bousahla *et al*. 2014, Al-Basyouni *et al*. 2015, Fekrar *et al*. 2014, Tounsi *et al*. 2015).

$$C = \frac{\int_{-h/2}^{h/2} E(z_{ms}) z_{ms} dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms}) dz_{ms}} \tag{3}$$

2.2 Basic assumptions

The assumptions of the present theory are as follows:

1. The origin of the Cartesian coordinate system is taken at the neutral surface of the FG plate.
2. The displacements are small in comparison with the height of the plate and, therefore, strains involved are infinitesimal.
3. The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.
4. This theory assumes constant transverse shear stress and it needs a shear correction factor to satisfy the plate boundary conditions on the lower and upper surface.

2.3 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained as follows

$$\begin{aligned} u(x,y,z_{ns},t) &= u_0(x,y,t) - z_{ns}\frac{\partial \phi}{\partial x} \\ v(x,y,z_{ns},t) &= v_0(x,y,t) - z_{ns}\frac{\partial \phi}{\partial y} \\ w(x,y,z_{ns},t) &= w(x,y,t) \end{aligned} \tag{4}$$

where u, v, w are displacements in the x, y, z directions, u_0 and v_0 are the neutral surface displacements. ϕ is function of coordinates x, y and time t.

The strains associated with the displacements in Eq. (4) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z_{ns}\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \gamma_{yz}^S \\ \gamma_{xz}^S \end{Bmatrix} \tag{5}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 \phi}{\partial x^2} \\ -\dfrac{\partial^2 \phi}{\partial y^2} \\ -2\dfrac{\partial^2 \phi}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^S \\ \gamma_{xz}^S \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w}{\partial y} - \dfrac{\partial \phi}{\partial y} \\ \dfrac{\partial w}{\partial x} - \dfrac{\partial \phi}{\partial x} \end{Bmatrix} \tag{6}$$

The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^S \\ \gamma_{xz}^S \end{Bmatrix} \tag{7}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. k_s is a shear correction factor which is analogous to shear correction factor proposed by Mindlin (1951).

Using the material properties defined in Eq. (2), stiffness coefficients, Q_{ij} can be expressed as

$$Q_{11} = Q_{22} = \frac{E(z_{ns})}{\left(1-\nu^2\right)}, \tag{8a}$$

$$Q_{12}(z_{ns}) = \frac{\nu E\left(z_{ns}\right)}{1-\nu^2}, \tag{8b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z_{ns})}{2\left(1+\nu\right)} \tag{8c}$$

2.4 Governing equations

The equations of motion are obtained using the principle of variational energy and virtual work.

$$\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{\phi}}{\partial x} \\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{\phi}}{\partial y} \\
&\delta \phi: \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x^2 \partial y^2} + \frac{\partial^2 M_y}{dy^2} - \frac{\partial Q_{xz}}{\partial x} - \frac{\partial Q_{yz}}{\partial y} = I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_2 \nabla^2 \ddot{\phi} \\
&\delta w: -\frac{\partial \mathrm{Q}_{\mathrm{xz}}}{\partial \mathrm{x}} - \frac{\partial \mathrm{Q}_{\mathrm{yz}}}{\partial \mathrm{y}} + q = I_0 \ddot{w}
\end{aligned} \tag{9}$$

where

$$\left(N_i, M_i\right) = \int_{-h/2-C}^{h/2-C} \left(1, z_{ns}\right)\sigma_i dz_{ns}, \quad \left(i = x, y, xy\right)$$

$$\text{and} \quad \left(Q_{xz}, Q_{yz}\right) = \int_{-h/2-C}^{h/2-C} \left(\tau_{xz}, \tau_{yz}\right) dz_{ns}, \tag{10}$$

By substituting Eq. (5) into Eq. (7) and the subsequent results into Eq. (10), the stress resultants are obtained as

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k \end{Bmatrix}, \quad Q = A^S \gamma \tag{11}$$

where

$$N = \{N_x, N_y, N_{xy}\}^t, \quad M = \{M_x, M_y, M_{xy}\}^t \tag{12a}$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}, k^s = \{k_x^s, k_y^s, k_{xy}^s\} \tag{12b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{12c}$$

$$Q = \{Q_{xz}, Q_{yz}\}^t, \quad \gamma = \{\gamma_{xz}, \gamma_{yz}\}^t, \quad A = \begin{bmatrix} A_{44}^S & 0 \\ 0 & A_{55}^S \end{bmatrix}, \tag{12d}$$

where A_{ij}, D_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & D_{11} \\ A_{12} & D_{12} \\ A_{66} & D_{66} \end{Bmatrix} = \int_{-h/2-C}^{h/2-C} Q_{11}\left(1, z_{ns}^2\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz_{ns}, \tag{13a}$$

and

$$\left(A_{22}, D_{22}\right) = \left(A_{11}, D_{11}\right) \tag{13b}$$

$$A_{44}^S = A_{55}^S = k_S \int_{-h/2-C}^{h/2-C} \frac{E(z_{ns})}{2(1+\nu)} dz_{ns} \tag{13c}$$

By substituting Eq. (11) into Eq. (9), the equations of motion can be expressed in terms of displacements (u_0, v_0, ϕ, w) as

$$A_{11} d_{11} u_0 + A_{66} d_{22} u_0 + \left(A_{12} + A_{66}\right) d_{12} v_0 = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{\phi}}{\partial x} \tag{14a}$$

$$A_{22} d_{22} v_0 + A_{66} d_{11} v_0 + \left(A_{12} + A_{66}\right) d_{12} u_0 = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{\phi}}{\partial y} \tag{14b}$$

$$\begin{aligned} & - D_{11} d_{1111} \phi - 2\left(D_{12} + 2 D_{66}\right) d_{1122} \phi \\ & - D_{22} d_{2222} \phi - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} - A_{55}^S d_{11} \left(w - \phi\right) \\ & - A_{44}^S d_{22} \left(w - \phi\right) = I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{\phi} \end{aligned} \tag{14c}$$

$$A_{55}^S d_{11} \left(w - \phi\right) + A_{44}^S d_{22} \left(w - \phi\right) + q = I_0 \ddot{w} \tag{14d}$$

where d_{ij} and d_{ijl} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial y_j}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1,2). \tag{15}$$

3. Analytical solution

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eqs. (14a)-(14d) for a simply supported FG plate. The following boundary conditions are imposed at the side edges

$$v_0 = w = \phi = N_x = M_x = 0 \quad \text{at } x=0,\ a \tag{16a}$$

$$u_0 = w = \phi = N_y = M_y = 0 \quad \text{at } y=0,\ b \tag{16b}$$

The equations of motion admit the Navier solutions for simply supported plates. The variables u_0, v_0, ϕ, w can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ v_0 \\ \phi \\ w \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\lambda x)\sin(\mu y) \\ V_{mn} \sin(\lambda x)\cos(\mu y) \\ \psi_{mn} \sin(\lambda x)\sin(\mu y) \\ W_{mn} \sin(\lambda x)\sin(\mu y) \end{Bmatrix} \tag{17}$$

U_{mn}, V_{mn}, ψ_{mn}, and W_{mn} are arbitrary parameters to be determined, and $\lambda=m\pi/a$ and $\mu=n\pi/b$.

For the case of a sinusoidally distributed load, we have

$$q = q_0 \sin\left(\frac{\pi}{a}x\right)\left(\frac{\pi}{b}y\right) \tag{18}$$

where q_0 represents the intensity of the load at the plate centre.

Substituting Eq. (17) into Eq. (14a)-(14d), the closed form solutions can be obtained from

$$\left([C] - \omega^2 [M]\right)\{\Delta\} = \{F\}, \tag{19}$$

where $\{\Delta\}=\{U_{mn}, V_{mn}, \psi_{mn}, W_{mn}\}^t$, and $[C]$ and $[M]$ are the symmetric matrixes given by

$$[C] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix}, \quad [M] = \begin{bmatrix} m_{11} & 0 & m_{13} & 0 \\ 0 & m_{22} & m_{23} & 0 \\ m_{13} & m_{23} & m_{33} & 0 \\ 0 & 0 & 0 & m_{44} \end{bmatrix} \tag{20}$$

4. Results and discussion

The study has been focused on the static behavior of functionally graded plate based on the present new first shear deformation plate theory and based on neutral surface position. For verification purpose, the obtained results are compared with those reported in the literature. In all

Table 1 Effects of volume fraction exponent on the dimensionless displacements of a FGM square plate subjected to sinusoidal loading (*a/h*=10)

n	Model	$\bar{u}$	$\bar{v}$	$\bar{w}$
Ceramic	**Present**	**0.22012**	**0.14674**	**0.29607**
	HSDT#	0.21805	0.14493	0.29423
	HSDPT*	0.21815	0.144885	0.29604
0.2	**Present**	**0.30688**	**0.21739**	**0.36013**
	HSDT#	0.28172	0.19820	0.33767
	HSDPT*	0.30479	0.21538	0.35988
0.5	**Present**	**0.44102**	**0.32783**	**0.45405**
	HSDT#	0.42131	0.31034	0.44407
	HSDPT*	0.43859	0.32549	0.45369
1	**Present**	**0.64442**	**0.49722**	**0.58897**
	HSDT#	0.64137	0.49438	0.58895
	HSDPT*	0.64112	0.49408	0.58893
2	**Present**	**0.90332**	**0.71468**	**0.75522**
	HSDT#	0.89858	0.71035	0.75747
	HSDPT*	0.89793	0.70968	0.75733
5	**Present**	**1.07519**	**0.85212**	**0.90144**
	HSDT#	1.06297	0.84129	0.90951
	HSDPT*	1.06620	0.84399	0.91171
Metallic	**Present**	**1.19492**	**0.79661**	**1.60722**
	HSDT#	1.18373	0.78677	1.59724
	HSDPT*	1.18428	0.78652	1.60709

#Results form Ref (Reddy 2000)
*Results from Ref (Merazi *et al*. 2015)

examples of the present work, a shear correction factor of 5/6 (Benguediab 2014) is used for the present theory. The Poisson's ratio of the plate is assumed to be constant through the thickness and equal to 0.3.

A functionally graded material consisting of Aluminum-Alumina is considered. The following material properties are used in computing the numerical values.

And their properties change through the thickness of the plate according to power-law. The bottom surfaces of the FG plate are aluminium rich, whereas the top surfaces of the FG plate are alumina rich.

4.1 Bending analysis

The first example is performed for isotropic Al/Al2O3 square plates under sinusoidal loads. The Young's modulus and Poisson's ratio are, for aluminium (Al): 70 GPa, 0,3 and for alumina (Al_2O_3) : 380 GPa, 0,3, respectively. The various non-dimensional parameters used are

$$\bar{w} = 10\frac{E_C h^3}{q_0 a^4} w\left(\frac{a}{2},\frac{b}{2}\right),\quad \bar{u} = 100\frac{E_C h^3}{q_0 a^4} u\left(0,\frac{b}{2},\frac{-h}{4}-c\right),\quad \bar{v} = 100\frac{E_C h^3}{q_0 a^4} v\left(\frac{a}{2},0,\frac{-h}{6}-c\right)$$

$$\overline{\sigma_x} = \frac{h}{hq_0}\sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}-c\right),\quad \overline{\sigma_y} = \frac{h}{hq_0}\sigma_y\left(\frac{a}{2},\frac{b}{2},\frac{h}{3}-c\right),\quad \overline{\tau_{xy}} = \frac{h}{hq_0}\tau_{xy}\left(0,0,-\frac{h}{3}-c\right)$$

$$\overline{\tau_{xz}} = \frac{h}{hq_0}\tau_{xz}\left(0,\frac{b}{2},-c\right),\quad \overline{\tau_{yz}} = \frac{h}{hq_0}\tau_{yz}\left(\frac{a}{2},0,\frac{h}{6}-c\right).$$

Results are tabulated in Table 1 and Table 2. The tables contain the non dimensionalised deflections and stresses respectively.

The results obtained are compared with the Shear Deformation Theory of Reddy (2000) and a new hyperbolic shear deformation plate theory used by Merazi *et al* (2015). It can be observed that the values obtained using the present theory (NFSDPT) are in good agreement with the those given by the theory of Reddy (2000) and the model used by Merazi *et al.* (2015).

The table shows the effect of volume fraction exponent (V_f) on the stresses and displacements of a functionally graded square plate with *a/h*=10. It can be observed that as the plate becomes

Table 2 Effects of Volume fraction exponent on the dimensionless stresses of a FGM square plate subjected to sinusoidal loading (*a/h*=10)

n	Model	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
	Present	**1.97576**	**1.31718**	**0.70925**	**0.19099**	**0.19099**
Ceramic	HSDT#	1.98915	1.31035	0.70557	0.23778	0.19051
	HSDPT*	1.99515	1.31219	0.70656	0.24406	0.21289
	Present	**2.23747**	**1.39229**	**0.72254**	**0.19769**	**0.20699**
0.2	HSDT#	2.12671	1.30958	0.66757	0.22532	0.18045
	HSDPT*	2.26002	1.38706	0.72053	0.24805	0.22655
	Present	**2.59253**	**1.46451**	**0.69276**	**0.19964**	**0.22305**
0.5	HSDT#	2.61051	1.47147	0.66668	0.23817	0.19071
	HSDPT*	2.61929	1.45863	0.69119	0.24945	0.24311
	Present	**3.05366**	**1.49683**	**0.61251**	**0.19099**	**0.23484**
1	HSDT#	3.08501	1.4898	0.61111	0.23817	0.19071
	HSDPT*	3.08640	1.48954	0.611061	0.24406	0.26178
	Present	**3.56565**	**1.40565**	**0.54590**	**0.16252**	**0.22894**
2	HSDT#	3.60664	1.39575	0.54434	0.22568	0.1807
	HSDPT*	3.60856	1.39561	0.54413	0.22427	0.27558
	Present	**4.18482**	**1.11758**	**0.57827**	**0.12509**	**0.17396**
5	HSDT#	4.24293	1.10539	0.57368	0.21609	0.17307
	HSDPT*	4.24758	1.10329	0.57553	0.19919	0.24164
	Present	**1.97576**	**1.31718**	**0.70925**	**0.19099**	**0.19099**
Metallic	HSDT#	1.98915	1.31035	0.70557	0.23778	0.19051
	HSDPT*	1.99515	1.31219	0.70656	0.24406	0.21289

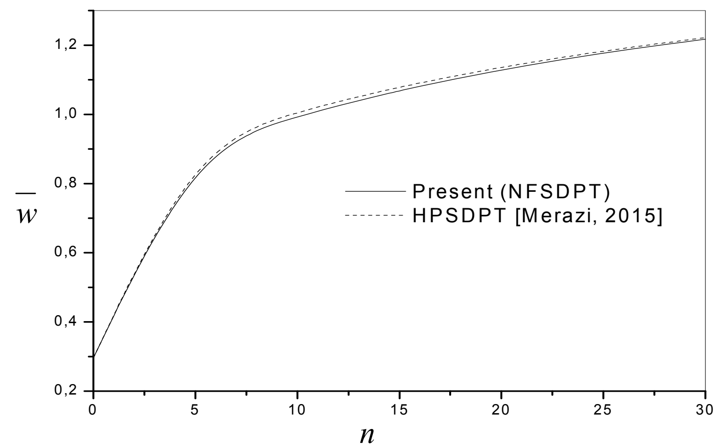

Fig. 2 Comparison of the variation of non-dimensional deflection $\overline{w}$ of Al/Al_2O_3 square plate under sinusoidally distributed load versus power law index n

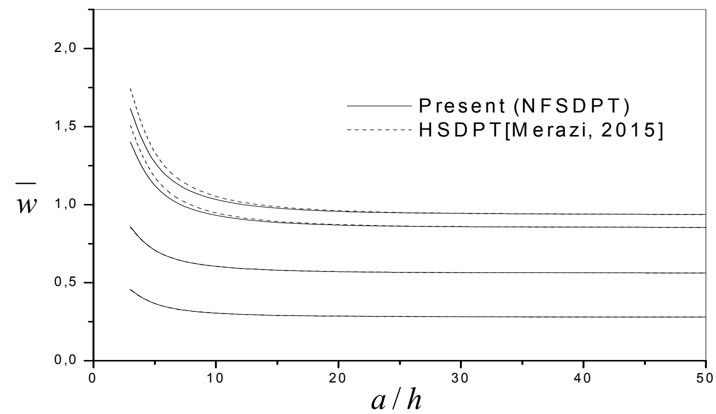

Fig. 3 Comparison of the variation of non-dimensional deflection $\overline{w}$ of square Al/Al2O3 plate under sinusoidally distributed load versus thickness ratio *a/h*

more and more metallic the deflection $\overline{w}$ and normal stress $\bar{\sigma}_x$ increases but normal stress $\bar{\sigma}_y$ decreases. It is very interesting to note that the stresses for a fully ceramic plate are the same as that of a fully metal plate. This is due to the fact that in these two cases the plate is fully homogeneous and stresses do not depend on the Modulus of elasticity.

To further illustrate the accuracy of present theory for wide range of power law index n and tickness ratio *a/h*, the variations of dimensionless deflection $\overline{w}$ with respect to power law index n and thickness ratio *a/h* are illustrated in Figs. 2 and 3, respectively. The obtained results are compared with those computed using the conventional HSDPT (Merazi 2015) with four unknowns. It can be seen that the results of present theory and the conventional HSDPT are almost identical.

To further illustrate the accuracy of present theory for wide range of power law index n and

Table 3 Comparison of First Three natural frequencies of Al/ZrO$_2$ FG square plates for various *a/h* ratio $\bar{w} = \left(w\left(\frac{a^2}{h}\right)\sqrt{\frac{\rho_m}{E_m}} \right)$, $n = 1$.

		Mode N°					
a/h	Source	1	% erreur	2	% erreur	3	% erreur
5	**Present**	**5.6908**	**3.84**	**15.3438**	**5.39**	**25.9249**	**6.33**
	HPSDPT*	5.6777	3.59	15.3438	5.39	25.7764	5.72
	HSDT#	5.6914	3.85	15.3408	5.38	25.9257	6.34
	3D$	5.4806	0.00	14.558	0.00	24.381	0.00
20	**Present**	**6.3372**	**3.76**	**61.3751**	**5.36**	**103.7402**	**5.70**
	HPSDPT*	6.3358	3.74	61.3751	5.36	103.7029	5.66
	HSDT#	6.3371	3.76	61.3744	5.36	103.7404	5.70
	3D$	6.1076	0.00	58.25	0.00	98.145	0.00

#Results form Ref (Reddy 2000)
*Results from Ref (Merazi *et al*. 2015)
$Results from Ref (Chi *et al*. 2006)

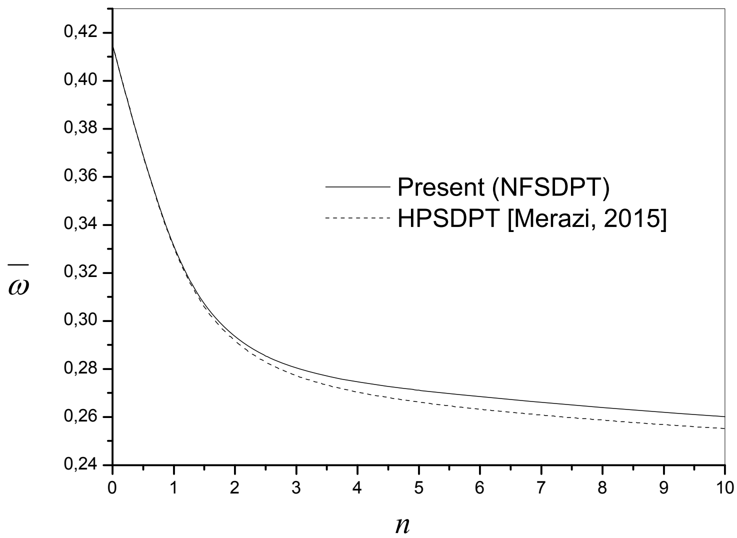

Fig. 4 Comparison of non-dimensional fundamental frequency $\bar{\varpi}$ of Al/Al$_2$O$_3$ square plate versus power law index n (*a/h*=5)

tickness ratio *a/h*, the variations of dimensionless deflection $\bar{w}$ with respect to power law index *n* and thickness ratio *a/h* are illustrated in Figs. 2 and 3, respectively. The obtained results are compared with those computed using the conventional HSDPT (Merazi 2015) with four unknowns. It can be seen that the results of present theory and the conventional HSDPT are almost identical.

4.2 Free vibration analysis

The accuracy of the present theory is also investigated through free vibration analysis of FG plates. The material properties used in the present study are:

Metal (Aluminum Al) : $E_m = 70x10^9 N/m^2$, $\nu = 0.3$, $\rho_m = 2702 Kg/m^3$.

Ceramic (Zerconia ZrO_2) : $E_c = 200x10^9 N/m^2$, $\nu = 0.3$, $\rho_c = 5700 Kg/m^3$.

Several parameters are varied and their dynamic behavior is studied. The first three natural frequencies for the fundamental vibration mode of *m*=*n*=1 of a square Al/Al_2O_3 FG plate are compared with the corresponding results of 3D analysis by Vel *et al* (2004) in Table 3.

The table also presents the results obtained by Merazi theory (Merazi 2015) and the results of HSDT (Reddy 2000). From this Table it is evident that the present theory is in good agreement with the given by the others shear theory.

Fig. 4 shows the variation of nondimensional frequency with respect to power law index *n*. It is observed that the non-dimensional frequencies $\overline{\omega}$ predicted by the NFSDT and the conventional HSDPT are almost identical.

5. Conclusions

An NFSDT is developed for bending and dynamic behavior of FG plates. Based on the present theory and the neutral surface concept, the equations of motion are derived from Hamilton's principle. The accuracy of neutral surface-based model is verified by comparing the obtained results with those reported in the literature. Finally, it can be concluded that the NFSDT is not only accurate but also simple in predicting the bending and dynamic behavior of FG plates.

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