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Hygrothermal effects on the vibration and stability of an initially stressed laminated plate

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Abstract. The influence of hygrothermal effects on the vibration frequency and buckling load of a shear deformable composite plate with arbitrary initial stresses was investigated. The governing equations of the effects of humid, thermal and initial stresses are established using the variational method. The material properties of the composite plate are affected by both temperature and moisture. The initial stress is taken to be a combination of uniaxial load and pure bending in a hygrothermal environment. The influence of various parameters, such as the fiber volume fraction, temperature, moisture concentration, length/thickness ratios, initial stresses and bending stress ratio on the vibration and stability of the response of a laminated plate are studied in detail. The behavior of vibration and stability are sensitive to temperature, moisture concentration, fiber volume fraction and initial stresses.

Keywords: hygrothermal effect; laminated plates; initial stress

1. Introduction

Lightweight high-strength composite plates have been successfully used in many engineering fields. In most actual situations, an increase in temperature or moisture will induce hygrothermal-elastic stresses and consequently change the properties of the composite plates, affecting their stability and vibration behavior. The environment of the hygrothermal application significantly effects on the performance of composites. The vibration and buckling behaviors of composite laminated plates in hygrothermal environments must be accurately determined for industrial applications. The efficient use of laminated composite plates depends on a deep understanding of stability and vibration behavior in various hygrothermal environments.

A number of studies of the hygrothermal effects of composite plates have been published. Shen studied the influence of hygrothermal effects on the postbuckling and nonlinear bending of laminated plates (2001, 2002). They found that temperature and moisture affect the material

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properties of the composites. They considered the initial geometric imperfection of the plate and used perturbation technique to determine the postbuckling equilibrium paths and load-bending moment curves. The influences of temperature, moisture concentration, fiber volume fraction and initial geometric imperfections were studied. Shen *et al*. (2004) subsequently studied the hygrothermal effect on the dynamic response of laminated plates that were resting on a Pasternak elastic foundation. The material properties of the composites were affected by the variation of temperature and moisture based on a micromechanical model. The governing equations included the plate-foundation interaction and hygrothermal effects. The numerical study concerned the free vibration and dynamic response of laminated plates that were resting on elastic foundations. The influences of temperature, moisture concentration, fiber volume fraction and foundation stiffness were studied. Shen and Wang (2012) extended that study to large-amplitude vibrations of hybrid laminated plates that contained piezoelectric layers and rested on an elastic foundation in a thermal environment. Their numerical illustrations concerned the nonlinear vibration characteristics of unsymmetric cross-ply and antisymmetric angle-ply laminated plates with fully covered or embedded piezoelectric actuators under different sets of thermal and electrical loading conditions. The results showed that the stiffness of the foundation and the stacking sequence significantly affected the nonlinear vibration characteristics of the hybrid laminated plate.

Rao and Sinha (2004) studied the effects of moisture and temperature on the bending characteristics of composite plates. They performed finite element analysis to elucidate the hygrothermal strains and reduced elastic properties of multidirectional composites at an elevated moisture concentration and temperature. They evaluated the deflections and stresses of composite plates with uniform and linearly varying through-the-thickness temperature and moisture content. Panda and Singh (2010, 2011) employed the finite element method to address the nonlinear free vibration behavior of a thermally post-buckled laminated composite spherical shallow shell panel and a single/doubly curved shell panel. They derived a system of governing differential equations from Hamilton's principle and converted these differential equations to algebraic equations using the nonlinear finite elements, which they solved numerically using a direct iterative method. The effects of amplitude ratios, aspect ratios, support conditions, thickness ratios, curvature ratios and the material properties on the nonlinear free vibration frequencies were studied. Bahrami and Nosier (2007) presented a general displacement field for hygrothermal problems of long laminated composite plates. They used Reddy's layerwise theory to determine the local deformation parameters to predict the interlaminar normal and shear stress distributions.

Singh and Verma (2009) studied the effects of temperature and moisture on the buckling of laminated composite plates with random geometric and material properties. They used a Taylor series, based on the mean-centered first order perturbation technique, to find mean and standard derivation of the hygrothermal buckling loads of laminated composite plates subjected to a uniform rise in temperature and moisture. They found that small amount of material and geometric variations of the composite plate significantly affected the hygrothermal buckling load. Lo *et al*. (2010) proposed a global-local higher order theory to study the response of laminated composite plates to variations in temperature and moisture concentration. They studied various materials that are sensitive to the hygrothermal environment. Their numerical results suggested that temperature-dependent material properties ought to be used in the analysis of laminated plates that are subjected to hygrothermal loads. Kumar and Singh (2010) investigated the effect of the randomness of the material properties on the buckling of laminated composite plate in hygrothermal environments, using the micromechanical model in their analysis. Their results revealed that the plate is significantly affected by the hygrothermal buckling load. Zenkour (2012) proposed the sinusoidal shear deformation plate theory to study the response of multilayered angle-ply composite plates to variations in temperature and moisture concentration. They considered classical, uniform and parabolic shear deformation plate theories for various materials that are sensitive to hygrothermal environment. Rajasekaran and Wilson (2013) studied the buckling and vibration of isotropic plates with variable thickness. They considered plates that were subjected to uniaxial and biaxial compressions and to shear loadings. They found that buckling load as the in-plane load that makes the determinant of the stiffness matrix equal to zero and the natural frequencies by carrying out an eigenvalue analysis of the stiffness and mass matrices.

Panda and Singh (2009, 2013a, b) investigated the thermal post-buckling of a laminated composite cylindrical/hyperboloid shell panel, a laminated composite shell panel and a doubly curved laminated composite panel with shape memory alloy fibers, using the nonlinear finite element method. They derived governing equation by minimizing the total potential energy of the system. They introduced geometric nonlinearity in the Green-Lagrange sense, and studied the influence of the layup sequences, thickness ratios, amplitude ratios and aspect ratios. Panda and Mahapatra (2014, 2015) investigated the large amplitude vibration of laminated composite shells and laminated composite curved panels with various geometries at various temperature environments. Recently, Mahapatra *et al*. (2016) further analyzed the large amplitude vibration of a laminated composite spherical shell panel at various temperatures and with various moisture contents. They derived the governing equations using Hamilton's principle and solved them using a direct iterative method. The mid-plane kinematics of the laminated shell were evaluated using a higher order plate theory to count accurately the out-of-plane shear stresses and strains. The authors discussed effect of different parameters on the nonlinear vibration. The most recent studies of the effect of hygrothermal environments on composite plates have tended to focus on the plate materials. Wosu *et al*. (2012) considered the effects of temperature and moisture on the response of graphite/epoxy laminated composites to high strain rate penetration loading. Their results revealed that loading in the thickness direction under extreme temperature, moisture and combined moisture and temperature conditions exponentially reduced the compressive strength, elastic modulus, and energy absorbed. The combined effect of temperature and moisture on the damage process was more apparent than the effect of temperature or moisture alone. Lee and Kim (2013) studied the postbuckling behavior of functionally graded material plate in hygrothermal environments. They used first-order shear deformation theory, and von Karman strain–displacement relations. Their results revealed that significant effects of moisture on the model owing to an increase in the volume fraction index of the materials. As far as the present authors know, the literature includes very little discussion of initially stressed laminated composite plates with temperature and moisture dependent properties.

Many methods for solving partial differential equations, which arise in many practical engineering problems, have been developed; they include finite elements, finite difference, differential quadrature, boundary elements and the meshless methods. Recently, Civalek (2008) used the discrete singular convolution method to discretize the spatial derivatives and reduce partial differential equations into an eigenvalue problem. Civalek and Emsen (2009) subsequently investigated the bending of Reissner-Mindlin plates using four-node quadrilateral elements with discrete singular convolution. The results in both studies agree closely with those obtained by other researchers. However, the numerical results seem to be significantly affected by the number of grid cells used in the discrete singular convolution method and the number of nodes used in finite element methods. This work utilizes another method for solving the partial differential equations: the Galerkin method is applied to the governing partial differential equations to yield ordinary differential equations, providing a simple, accurate and efficient method of numerical analysis to evaluate natural frequencies and buckling loads. In addition, residual stresses, which are encountered under many practice conditions, are modeled using an arbitrary initial stress in this work.

Laminated composite plates are used in various engineering structures; they are unavoidably exposed to humid, thermal and initial stresses. The authors' previous studies (Chen *et al*. 2007, 2009) proposed a higher order theory to examine the buckling and vibration behaviors of initially stressed laminated plates. Nayak *et al*. (2005), Nayak and Shenoi (2005) also analyzed the buckling and vibration behaviors of initially stressed composite sandwich plates using a higher order finite element theory. The results of the two theories both revealed that the initial stress significantly affects the behaviors of laminated plates. Therefore, to study the stability of laminated plates under hygrothermal conditions, the initial stress must be considered. In the present work, the material properties are assumed to be functions of temperature and moisture concentration. As the ambient temperature and moisture concentration are assumed to be uniformly distributed within the plates, variations of temperature and moisture concentration in the laminated plates are independent of time and position. Based on these assumptions, hygrothermal effects on the buckling load and vibration frequency of a shear deformable composite plate with arbitrary initial stresses are investigated. The influence of various parameters, such as fiber volume fraction, temperature, moisture concentration, length/thickness ratios, initial stresses and bending stress ratio on the vibration and stability of a laminated plate with arbitrary initial stresses is studied in detail and discussed.

2. Constitutive equations

This study establishes the governing equations of a composite laminated plate with uniform thickness h under general initial stresses and thermal conditions. Hamilton's principle, as described by Brunelle and Robertson (1974), Chen *et al*. (2006) is applied to derive the equations of motion of the plate.

$$\delta\int_{t_0}^{t_1}(U_s - K_t - W_i - W_e)dt = 0$$

$$U_s = \frac{1}{2}\int_{V_0}(\sigma_{ij}^o + \sigma_{ij}^T + \sigma_{ij}^H)\varepsilon_{ij}dV$$

$$K_t = \frac{1}{2}\int_{V_0}\rho\dot{v}_i\,\dot{v}_i\,dV$$

$$W_e = \int_{S_0} p_i v_i dS$$

$$W_i = \int_{v_o} X_i v_i dV \tag{1}$$

where U_s, K_t, W_e and W_i are the strain energy, kinetic energy and work due to the surface and volume forces, respectively. δ is the variation of the function; σ_{ij}^o, σ_{ij}^T, σ_{ij}^H and ε_{ij} are the mechanical stresses, thermal stresses and humid stresses and strain in the material coordinates, respectively; ρ is the mass density; v_i is the displacement in the spatial frame; p_i is the vector of surface force per unit area; X_i is the vector of body force per unit initial volume, and V_0 and S_0 are the volume and boundary surface, respectively. The application of the minimum total energy principle leads to the general equations and boundary conditions. Assume that the stresses and applied forces are constant, and substitute the integral forms of U_s, K_t, W_e and W_i into Eq. (1). Performing the variation, integrating the kinetic energy term by parts with respect to time, and applying the assumption that δv_i vanishes at time t_0 and t_1, yield

$$\int_{t_0}^{t_1}[\int_{V_0}[(\sigma_{ij}^o+\sigma_{ij}^T+\sigma_{ij}^H)\delta\varepsilon_{ij}-X_i\delta v_i-\rho\ddot{v}_i\delta v_i]dV-\int_{S_0}(p_i\delta v_i)dS]dt=0 \tag{2}$$

Based on the Reissner-Mindlin plate theory, the incremental displacements are assumed to have the following forms.

$$\begin{aligned} v_x(x,y,z,t)&=u_x(x,y,t)+z\varphi_x(x,y,t)\\ v_y(x,y,z,t)&=u_y(x,y,t)+z\varphi_y(x,y,t)\\ v_z(x,y,z,t)&=w(x,y,t) \end{aligned} \tag{3}$$

The temperature and moisture are assumed to be uniform and at equilibrium. Incorporating the effects of temperature and moisture cause, the constitutive relations for the k[th] lamina with respect to the laminate coordinate axes to become

$$\begin{aligned} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\\ \sigma_{zy}\\ \sigma_{zx}\end{bmatrix}^{(k)} &= \begin{bmatrix}\sigma_{xx}^o\\ \sigma_{yy}^o\\ \sigma_{xy}^o\\ \sigma_{zy}^o\\ \sigma_{zx}^o\end{bmatrix}^{(k)} + \begin{bmatrix}\sigma_{xx}^T\\ \sigma_{yy}^T\\ \sigma_{xy}^T\\ \sigma_{zy}^T\\ \sigma_{zx}^T\end{bmatrix}^{(k)} + \begin{bmatrix}\sigma_{xx}^H\\ \sigma_{yy}^H\\ \sigma_{xy}^H\\ \sigma_{zy}^H\\ \sigma_{zx}^H\end{bmatrix}^{(k)} \\ &= \begin{bmatrix}C_{11}&C_{12}&C_{16}&0&0\\ C_{12}&C_{22}&C_{26}&0&0\\ C_{16}&C_{26}&C_{66}&0&0\\ 0&0&0&C_{44}&C_{45}\\ 0&0&0&C_{45}&C_{55}\end{bmatrix}^{(k)} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\\ \varepsilon_{zy}\\ \varepsilon_{zx}\end{bmatrix}^{(k)} + \begin{bmatrix}C_{11}&C_{12}&C_{16}&0&0\\ C_{12}&C_{22}&C_{26}&0&0\\ C_{16}&C_{26}&C_{66}&0&0\\ 0&0&0&C_{44}&C_{45}\\ 0&0&0&C_{45}&C_{55}\end{bmatrix}^{(k)} \begin{bmatrix}-\alpha_{xx}\Delta T\\ -\alpha_{yy}\Delta T\\ -\alpha_{xy}\Delta T\\ 0\\ 0\end{bmatrix}^{(k)} \\ &\quad + \begin{bmatrix}C_{11}&C_{12}&C_{16}&0&0\\ C_{12}&C_{22}&C_{26}&0&0\\ C_{16}&C_{26}&C_{66}&0&0\\ 0&0&0&C_{44}&C_{45}\\ 0&0&0&C_{45}&C_{55}\end{bmatrix}^{(k)} \begin{bmatrix}-\beta_{xx}\Delta C\\ -\beta_{yy}\Delta C\\ -\beta_{xy}\Delta C\\ 0\\ 0\end{bmatrix}^{(k)} \end{aligned} \tag{4}$$

where C_{ij} is the material stiffness of the lamina; ΔT and ΔC are the rise of temperature and moisture concentration, respectively; α_{xx}, α_{yy} and α_{xy} are the transformed thermal expansion coefficients in the principal material directions, and the coefficients β_{xx}, β_{yy} and β_{xy} are the moisture expansion coefficients. The constitutive equation for a lamina using a common structural axis system is then obtained by coordinate transformation.

The material properties are functions of both temperature and moisture content. Based on a micro-mechanical model of laminates (Tsai and Hahn 1980), the material properties can be expressed as

$$E_{xx} = V_f E_F + V_m E_m$$

$$\frac{1}{E_{yy}} = \frac{V_f}{E_f} + \frac{V_m}{E_m} - V_f V_m \frac{\upsilon_f^2 E_m / E_f + \upsilon_m^2 E_f / E_m - 2\upsilon_f \upsilon_m}{V_f E_f + V_m E_m}$$

$$\frac{1}{G_{xy}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \tag{5}$$

In the above equations, E_f, α_f and υ_f are the Young's modulus, thermal expansion coefficient and Poisson ratio of the fiber, respectively. E_m, α_m and υ_m are the corresponding properties in the matrix. V_f and V_m are the fiber and matrix volume fractions of the plate, and are related by

$$V_f + V_m = 1 \tag{6}$$

ρ_f and ρ_m are the fiber and matrix mass densities, and are related to the mass density of a laminate by

$$\rho = V_f \rho_f + V_m \rho_m \tag{7}$$

The thermal expansion coefficients in the longitudinal and transverse directions are written as

$$\alpha_{xx} = \frac{V_f E_f \alpha_f + V_m E_m \alpha_m}{V_f E_f + V_m E_m}$$

$$\alpha_{yy} = (1+\upsilon_f) V_f \alpha_f + (1+\upsilon_m) V_m \alpha_m - \upsilon_{xy} \alpha_{xx} \tag{8}$$

The Poisson ratio υ_{xy} is

$$\upsilon_{xy} = V_f \upsilon_f + V_m \upsilon_m \tag{9}$$

The longitudinal and transverse coefficients of humid expansion of a lamina are written as

$$\beta_{xx} = \frac{V_f E_f c_{fm} \beta_f + V_m E_m \beta_m}{E_{xx}(V_f \rho_f c_{fm} + V_m \rho_m)} \rho$$

$$\beta_{yy} = \frac{V_f (1+\upsilon_f) c_{fm} \beta_f + V_m (1+\upsilon_m) \beta_m}{V_f \rho_f c_{fm} + V_m \rho_m} \rho - \upsilon_{xy} \beta_{xx} \tag{10}$$

where c_{fm}, β_f and β_m are the moisture concentration ratio, and the fiber and matrix humid expansion coefficients, respectively.

Substituting Eqs. (3)-(10) into Eq. (2); performing all necessary partial integrations, and grouping terms according to displacement variation, δu_x, δu_y, δw, $\delta \varphi_x$ or $\delta \varphi_y$, yields the following five governing equations.

$$(L_1 + P_1 + T_1 + H_1)_{,x} + (L_2 + P_2 + T_2 + H_2)_{,y} + f_x = I_1 \ddot{u}_x \tag{11}$$

$$(L_2 + P_3 + T_3 + H_3)_{,x} + (L_3 + P_4 + T_4 + H_4)_{,y} + f_y = I_1 \ddot{u}_y \tag{12}$$

$$(L_4+P_5+T_5+H_5)_{,x}+(L_5+P_6+T_6+H_6)_{,y}+f_z=I_1\ddot{w} \tag{13}$$

$$(L_6+P_7+T_7+H_7)_{,x}+(L_7+P_8+T_8+H_8)_{,y}-L_8-P_9+m_x=I_3\ddot{\varphi}_x \tag{14}$$

$$(L_7+P_{10}+T_9+H_9)_{,x}+(L_9+P_{11}+T_{10}+H_{10})_{,y}-L_{10}-P_{12}+m_y=I_3\ddot{\varphi}_y \tag{15}$$

where f_x, f_y and f_z are the lateral loadings, and m_x and m_y are the body forces. The Appendix provides the other coefficients in the above equations.

3. Example problem

Many parameters affect the stability and vibration behaviors of an arbitrarily initially stressed laminated plate in a hygrothermal environment, and not all possible cases can be reasonably considered. The buckling and vibration behaviors of a simply supported eight-layered cross-ply laminate with a $(0°/90°)_{4s}$ stacking sequence that is subjected to longitudinal normal and pure bending stresses are investigated herein.

The lateral loads and body forces are taken to be zero (f_x, f_y, f_z, m_x, m_y=0). The initial stress is assumed to be

$$\sigma_{xx}=\sigma_n+2z\sigma_m/h \tag{16}$$

The bending stress σ_m and the uniaxial stress σ_n are taken to be constants and all the other initial stresses are set to zero. Therefore, the only nonzero initial stress resultants are $N_{xx}=h\sigma_n$, $M_{xx}=Sh^2\sigma_n/6$, and $M^*_{xx}=h^3\sigma_n/12$. The factor $S=\sigma_m/\sigma_n$ is the ratio of a bending stress to a normal stress. When S=0, the initial bending stress is zero. For a cross-ply plate, the stiffness values C_{16}, C_{26} and C_{45} in Eq. (3) equal zero. The governing Eqs. (11)-(15) ignore hygrothermal coupling. For a simply supported laminated plate of length a and width b, the mode shapes that satisfy the geometric boundary conditions have the following forms.

$$u_x=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\cos(m\pi x/a)\sin(n\pi y/b)e^{i\omega_{mn}t}$$

$$u_y=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\sin(m\pi x/a)\cos(n\pi y/b)e^{i\omega_{mn}t}$$

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(m\pi x/a)\sin(n\pi y/b)e^{i\omega_{mn}t}$$

$$\varphi_x=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Phi_{xmn}\cos(m\pi x/a)\sin(n\pi y/b)e^{i\omega_{mn}t}$$

$$\varphi_y=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Phi_{ymn}\sin(m\pi x/a)\cos(n\pi y/b)e^{i\omega_{mn}t} \tag{17}$$

in which all summations are from m, n=1 to ∞. Substituting the assumed displacement field Eq. (17) into the governing Eqs. (11)-(15) and grouping the coefficients, yielding the governing matrix equation of motion

$$\{[C]-\lambda[G]\}\{\Delta\}=\{0\} \tag{18}$$

where $\{\Delta\}=[U_{mn}, V_{mn}, W_{mn}, \Phi_{xmn}, \Phi_{ymn}]^T$ and λ refers to the corresponding vibration frequency or buckling load coefficient.

For a vibration problem, the governing vibration equations are further obtained by setting $\lambda=\omega^2$ in Eq. (18). The natural frequencies can then be evaluated by solving the eigenvalue equation

$$\{[C]-\omega^2[G]\}\{\Delta\}=\{0\} \tag{19}$$

The smallest root of Eq. (18) is the vibration frequency of the laminated plate.

For a buckling problem, $e^{i\omega_{mn}t}$ is neglected in Eq. (17). The static buckling behavior is obtained by setting $\lambda=N_{ij}$ in Eq. (18). The smallest applied load N_{ij} that causes buckling is determined by solving the eigenvalue problem given as

$$\{[C]-N_{ij}[G]\}\{\Delta\}=\{0\} \tag{20}$$

For the vibration problem of a laminated plate under hygrothermal environmental conditions, the matrix $[G]$ is the mass matrix and the matrix $[C]$ includes the effects of initial stresses and hygrothermal stresses.

The associated coefficients of the symmetric matrices $[C]$ and $[G]$ are expressed as,

$$C_{1,1}=(-A_{11}\alpha^2-N_{xx}\alpha^2-A_{66}\beta^2+\alpha^2N_{xx}^T+\beta^2N_{yy}^T+\alpha^2N_{xx}^H+\beta^2N_{yy}^H)/h$$

$$C_{1,2}=-(A_{12}+A_{66})\alpha\beta/h,\quad C_{1,5}=-(B_{11}+B_{66})\alpha\beta/h$$

$$C_{1,4}=(-M_{xx}\alpha^2-B_{66}\beta^2-B_{11}\alpha^2+\alpha^2M_{xx}^T+\beta^2M_{yy}^T+\alpha^2M_{xx}^H+\beta^2M_{yy}^H)/h$$

$$C_{2,2}=(-A_{66}\alpha^2-N_{xx}\alpha^2-A_{22}\beta^2+\alpha^2N_{xx}^T+\beta^2N_{yy}^T+\alpha^2N_{xx}^H+\beta^2N_{yy}^H)/h,\quad C_{2,4}=C_{1,5}$$

$$C_{2,5}=(-M_{xx}\alpha^2+B_{66}\alpha^2+B_{22}\beta^2+\alpha^2M_{xx}^T+\beta^2M_{yy}^T+\alpha^2M_{xx}^H+\beta^2M_{yy}^H)/h$$

$$C_{3,4}=-A_{55}\alpha/h,\quad C_{3,5}=-A_{44}\beta/h$$

$$C_{3,3}=(-A_{55}\alpha^2-N_{xx}\alpha^2-A_{44}\beta^2+\alpha^2N_{xx}^T+\beta^2N_{yy}^T+\alpha^2N_{xx}^H+\beta^2N_{yy}^H)/h$$

$$C_{4,4}=(-D_{11}\alpha^2-M_{xx}^*\alpha^2-D_{66}\beta^2-A_{55}+\alpha^2M_{xx}^{T*}+\beta^2M_{yy}^{T*}+\alpha^2M_{xx}^{H*}+\beta^2M_{yy}^{H*})/h^2$$

$$C_{4,5}=-(D_{12}+D_{66})\alpha\beta/h^2,\quad \alpha=m\pi/a,\quad \beta=n\pi/b$$

$$C_{5,5}=(-D_{66}\alpha^2-M_{xx}^*\alpha^2+D_{22}\beta^2+A_{44}+\alpha^2M_{xx}^{T*}+\beta^2M_{yy}^{T*}+\alpha^2M_{xx}^{H*}+\beta^2M_{yy}^{H*})/h^2$$

$$G_{1,1}=G_{2,2}=G_{3,3}=-I_1,\quad G_{4,4}=G_{5,5}=-I_3/h^2$$

In the buckling load problems, the initial stresses N_{ij}, M_{ij} and M^*_{ij} are deleted from the matrix $[C]$, and the coefficients of matrix $[G]$ are,

$$G_{1,1}=G_{2,2}=G_{3,3}=\alpha^2,\quad G_{1,4}=G_{2,5}=S\alpha^2/6h,\quad G_{4,4}=G_{5,5}=\alpha^2/12h^2\,.$$

4. Numerical results and discussion

To evaluate the numerical accuracy of the present method, a validation study is firstly performed. Tables 1 and 2 compare the buckling load ratios P_{cr}^*/P_{cr} of an eight-layered cross-ply laminated plate at various temperatures and moisture concentrations that are obtained with those obtained by Patel *et al*. (2002). The buckling load ratio P_{cr}^*/P_{cr} represents the influence of moisture or temperature on the critical buckling load as the buckling load P_{cr} is due to an applied un-axial load without moisture/temperature, and P_{cr}^* is the buckling load in an environment of uniform moisture/temperature. The results reveal that the present solutions agree closely with those of Patel *et al*. (2002). The vibration frequencies of a four-layered cross-ply laminated plate in a thermal environment are also considered to determine the accuracy of the proposed methodology. Table 3 compares the results obtained herein at various temperatures and ratios of longitudinal to transverse thermal expansion coefficients are compared with those of Liu and Huang (1996). Again, the presented results concerning the vibration of a cross-ply laminated plate agree closely with those of Liu and Huang (1996). Table 4 compares the linear vibration frequencies at α_{xx}/α_{yy}=0.1 and T_0=0 and 15 with of Nanda and Pradyumna (2011), Naidu and Sinha (2007) and close agreement is observed.

As the numerous combinations of parameters that affect the vibration and stability of an initially stressed laminated plate in hygrothermal environment are complex, not all such factors can be discussed. In the following, the influence of various parameters, such as the fiber volume fraction, temperature, moisture concentration, length/thickness ratios, initial stresses and bending stress ratio on the vibration and stability response of a symmetric eight-layers cross-ply laminated plate with a (0°/90°)4s stacking sequence in an hygrothermal environment under arbitrary initial

Table 1 Comparison of critical load ratio of an eight-layered cross-ply laminated plate for various temperatures

Source	Temperature (℃)				
	300	325	350	375	400
Patel *et al*. (2002) results	1	0.9954	0.9313	0.8650	0.8300
Present results	1	0.9933	0.9422	0.8885	0.8567

Table 2 Comparison of critical load ratio of an eight-layered cross-ply laminated plate for various moisture concentrations

Source	Moisture concentration (%)						
	0	0.25	0.5	0.75	1	1.25	1.5
Patel *et al*. (2002) results	1	0.9863	0.9731	0.9606	0.9487	0.9380	0.9273
Present results	1	0.9863	0.9732	0.9607	0.9489	0.9383	0.9277

Table 3 Comparison of vibration frequencies of a four-layered cross-ply laminated plate in thermal environment

T_0	Source	α_{xx}/α_{yy}			
		-0.05	0.1	0.2	0.3
-50	Liu and Huang (1996) results	15.149	15.247	15.320	15.394
	Present results	15.165	15.277	15.351	15.425
0	Liu and Huang (1996) results	15.150	15.150	15.150	15.150
	Present results	15.179	15.179	15.179	15.179
50	Liu and Huang (1996) results	15.164	15.052	14.978	14.902
	Present results	15.193	15.081	15.006	14.930

Table 4 Comparison of linear vibration frequencies of a simple supported four-layered cross-ply laminated plate in thermal environment

T_0	Source	α_{xx}/α_{yy}
		0.1
0	Nanda and Pradyumna (2011) results	15.1556
	Naidu and Sinha (2007) results	15.1480
	Liu and Huang (1996) results	15.1500
	Present results	15.1790
50	Nanda and Pradyumna (2011) results	15.0326
	Naidu and Sinha (2007) results	15.0985
	Liu and Huang (1996) results	15.0520
	Present results	15.0810

stress conditions is studied in detail and discussed. The temperature/moisture dependent material properties of the graphite fiber reinforced polymer that is adopted in this study are (Adams and Miller 1977, Bowles and Tompkins 1989, Shen 2001) as follows.

$$E_f=230\text{ GPa},\ G_f=9\text{ GPa},\ \alpha_f=-0.54\times10^{-6}/{}^{o}C,\ \upsilon_f=0.203,\ \rho_f=1750\,kg/m^3,\ \beta_f=0,\ c_{fm}=0,$$
$$\alpha_m=45\times10^{-6}/{}^{o}C,\ \upsilon_m=0.34,\ \rho_m=1200\,kg/m^3,\ \beta_m=2.68\times10^{-3}/wt\%H_2O,\ E_m=$$
$$(3.51-0.003\times(25+\varDelta T)-0.142\times\varDelta C)$$

The nondimensional vibration frequency ($\Omega=10\omega b^2\sqrt{\rho_f/h^2E_f}$) and buckling coefficient (K=100b^2N_{xx}/E_f) are defined and used throughout the study. The terms ω_{nf} and K_{cr} are the dimensionless fundamental natural frequency and critical buckling load. A positive buckling coefficient K denotes a tensile stress. When S=0 and K=0, no initial stress exists. The following tables and figures provide the effects of moisture, temperature, fiber volume fraction and initial stress on vibration frequency and buckling load.

Tables 5 and 6 present the buckling load and vibration frequency of laminated plates that are subjected to a uniform temperature and moisture concentration. To understand better the effects of fiber volume fraction, temperature and moisture concentration on the buckling load and vibration frequency of laminated plates, the buckling load or vibration frequency of laminated plates at 0℃ with 0% moisture and no fiber reinforcement are established as a base. These values of buckling load and vibration frequency of the laminated plates are 5.0687 and 8.5414, respectively, as indicated in Tables 5 and 6. The buckling and frequency of the laminated plate with various fiber volume fractions, temperatures and moisture concentrations are divided by the base values to obtain the data in parentheses. The buckling load and vibration frequency and the data in parentheses increase with the fiber volume fraction, but decrease as the moisture or temperature increases. Additionally, the plate without graphite fiber reinforcement (V_f=0) is more sensitive to variations in temperature and moisture ratio than are the graphite fiber reinforced polymer plates. Indeed, at high temperature and high moisture concentration, the matrix is greatly degraded, worsening the mechanical characteristics of the laminated plate. This behavior is a manifestation of the excellent resistance of the graphite fiber to hygrothermal stress. The buckling or/and frequency of laminated plates that are unaffected by moisture or/and temperature is/are always higher than that/those of laminated plates in a hygrothermal environment. Thus, the laminated plate has a lower buckling/frequency if it has a lower fiber volume fraction and a higher moisture content or/and temperature. Tables 7 and 8 show the effects of temperature, moisture and fiber volume fraction V_f (=0, 0.2 and 0.4) on the buckling load and vibration frequency of laminated plates. The buckling load and vibration frequency decrease as the uniform temperature or/and moisture content increases. Since a plate with a higher fiber volume fraction has a higher stiffness, a plate with a lower fiber volume fraction and a higher temperature/moisture content has a lower critical buckling load and a lower natural frequency.

Table 5 Effect of temperature rise on critical buckling load and vibration frequency of laminated plates (a/b=1, a/h=10, K=0, S=0, ΔC=0)

	V_f	Temperature rise (℃)			
		0	50	100	150
K_{cr}	0	5.0687 (1)	4.8377 (0.9544)	4.6075 (0.9090)	4.3782 (0.8638)
	0.1	11.6571 (2.2998)	11.3883 (2.2468)	11.1178 (2.1934)	10.8453 (2.1397)
	0.2	17.4845 (3.4495)	17.1472 (3.3830)	16.8038 (3.3152)	16.4534 (3.2461)
	0.3	23.0311 (4.5438)	22.6150 (4.4617)	22.1880 (4.3775)	21.7487 (4.2908)
	0.4	28.6316 (5.6487)	28.1347 (5.5507)	27.6222 (5.4496)	27.0925 (5.3451)
	0.5	34.5830 (6.8229)	34.0079 (6.7094)	33.4128 (6.5920)	32.7955 (6.4702)
ω_{nf}	0	8.5414 (1)	8.3444 (0.9769)	8.1435 (0.9534)	7.9382 (0.9294)
	0.1	12.6661 (1.4829)	12.5192 (1.4657)	12.3696 (1.4482)	12.2171 (1.4303)
	0.2	15.1831 (1.7776)	15.0359 (1.7604)	14.8846 (1.7426)	14.7286 (1.7244)
	0.3	17.0710 (1.9986)	16.9161 (1.9805)	16.7556 (1.9617)	16.5890 (1.9422)
	0.4	18.6615 (2.1848)	18.4989 (2.1658)	18.3297 (2.1460)	18.1530 (2.1253)
	0.5	20.1235 (2.3560)	19.9555 (2.3363)	19.7801 (2.3158)	19.5966 (2.2943)

Table 6 Effect of moisture concentration on critical buckling load and vibration frequency of laminated plates (a/b=1, a/h=10, K=0, S=0, ΔT=0)

	V_f	Moisture concentration (%)			
		0	1	2	3
K_{cr}	0	5.0687 (1)	3.6978 (0.7295)	2.4270 (0.4788)	1.2564 (0.2479)
	0.1	11.6571 (2.2998)	10.1178 (1.9961)	8.6877 (1.7140)	7.3665 (1.4533)
	0.2	17.4845 (3.4495)	15.7196 (3.1013)	14.0734 (2.7765)	12.5453 (2.4751)
	0.3	23.0311 (4.5438)	20.9919 (4.1415)	19.0840 (3.7651)	17.3065 (3.4144)
	0.4	28.6316 (5.6487)	26.2585 (5.1805)	24.0343 (4.7417)	21.9577 (4.3320)
	0.5	34.5830 (6.8229)	31.7873 (6.2713)	29.1655 (5.7540)	26.7160 (5.2708)
ω_{nf}	0	8.5414 (1)	7.2954 (0.8541)	5.9103 (0.6920)	4.2524 (0.4979)
	0.1	12.6661 (1.4829)	11.8002 (1.3815)	10.9345 (1.2802)	10.0688 (1.1788)
	0.2	15.1831 (1.7776)	14.3964 (1.6855)	13.6217 (1.5948)	12.8610 (1.5057)
	0.3	17.0710 (1.9986)	16.2978 (1.9081)	15.5395 (1.8193)	14.7982 (1.7325)
	0.4	18.6615 (2.1848)	17.8714 (2.0923)	17.0978 (2.0018)	16.3425 (1.9133)
	0.5	20.1235 (2.3560)	19.2930 (2.2588)	18.4802 (2.1636)	17.6872 (2.0708)

Table 7 Hygrothermal effect on the critical buckling load of a laminated plate with various fiber volume fractions (a/b=1, a/h=10, K=0, S=0)

ΔT	V_f	Moisture concentration (%)			
		0	1	2	3
0	0	5.0687 (1)	3.6978 (0.7295)	2.4270 (0.4788)	1.2564 (0.2479)
	0.2	17.4845 (3.4495)	15.7196 (3.1013)	14.0734 (2.7765)	12.5453 (2.4751)
	0.4	28.6316 (5.6487)	26.2585 (5.1805)	24.0343 (4.7417)	21.9577 (4.3320)
75	0	4.7225 (0.9317)	3.4315 (0.6770)	2.2407 (0.4421)	1.1500 (0.2269)
	0.2	16.9763 (3.3492)	15.3008 (3.0187)	13.7430 (2.7113)	12.3021 (2.4271)
	0.4	27.8805 (5.5005)	25.6138 (5.0533)	23.4936 (4.6350)	21.5184 (4.2453)
150	0	4.3782 (0.8638)	3.1672 (0.6249)	2.0564 (0.4057)	1.0457 (0.2063)
	0.2	16.4534 (3.2461)	14.8655 (2.9328)	13.3940 (2.6425)	12.0377 (2.3749)
	0.4	27.0925 (5.3451)	24.9282 (4.9181)	22.9077 (4.5194)	21.0288 (4.1488)

Table 8 Hygrothermal effect on the natural vibration frequency of a laminated plate with various fiber volume fractions (a/b=1, a/h=10, K=0, S=0)

ΔT	V_f	Moisture concentration (%)			
		0	1	2	3
0	0	8.5414 (1)	7.2954 (0.8541)	5.9103 (0.6920)	4.2524 (0.4979)
	0.2	15.1831 (1.7776)	14.3964 (1.6855)	13.6217 (1.5948)	12.8610 (1.5057)
	0.4	18.6615 (2.1848)	17.8714 (2.0923)	17.0978 (2.0018)	16.3425 (1.9133)
75	0	8.2445 (0.9652)	7.0278 (0.8228)	5.6789 (0.6649)	4.0685 (0.4763)
	0.2	14.9608 (1.7516)	14.2033 (1.6629)	13.4609 (1.5760)	12.7357 (1.4911)
	0.4	18.4151 (2.1560)	17.6507 (2.0665)	16.9044 (1.9791)	16.1782 (1.8941)
150	0	7.9382 (0.9294)	6.7517 (0.7905)	5.4404 (0.6369)	3.8796 (0.4542)
	0.2	14.7286 (1.7244)	13.9998 (1.6391)	13.2889 (1.5558)	12.5981 (1.4749)
	0.4	18.1530 (2.1253)	17.4129 (2.0386)	16.6923 (1.9543)	15.9931 (1.8724)

Fig. 1 presents the effect of the buckling coefficient on the vibration frequency of laminated plates (V_f=0.5) at various uniform temperatures and moisture contents. The vibration frequency decreases as the initial compressive stress, temperature and moisture content increase. The critical buckling load is obtained when the vibration frequency equals zero. A laminated plate at a higher temperature or/and moisture content has a smaller buckling coefficient. Fig. 2 plots the effect of fiber volume fraction on the vibration frequency of an initially-stressed laminated plate under hygrothermal conditions. The laminate plate with a higher fiber volume fraction has a larger vibration frequency and higher buckling load. Tables 6 and 7 similarly show that a laminated plate with a lower fiber volume fraction has a lower buckling load.

Tables 9 and 10 present the effect of the length-to-thickness ratio (a/h) on the critical buckling coefficient and natural vibrations frequency vibration frequency of a laminated plate. Both the buckling load and vibration frequency increase with the length-to-thickness ratio. Since a plate with a higher fiber volume fraction has a higher stiffness, one with a smaller length-to-thickness ratio and a higher temperature/moisture content has a lower critical buckling load and a lower natural frequency.

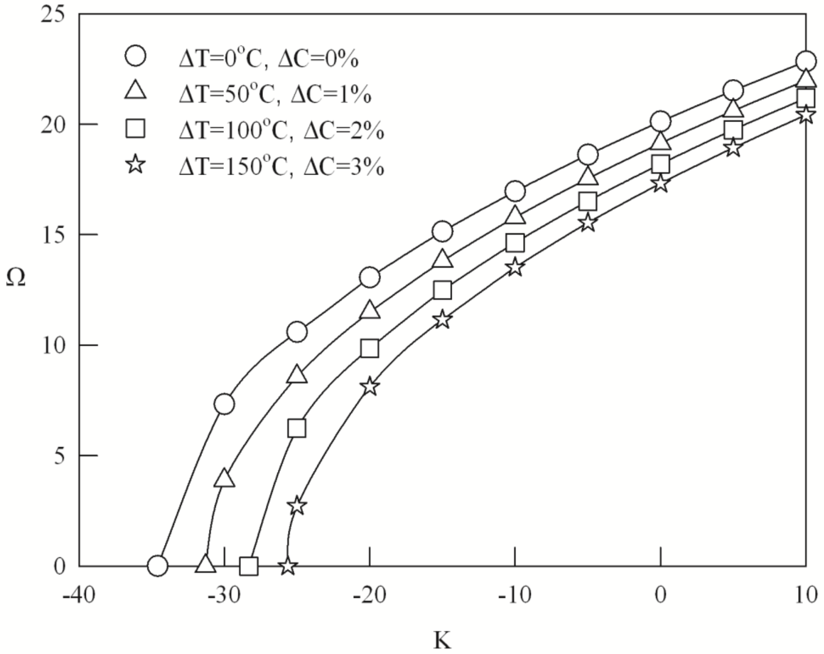

Fig. 1 The vibration frequency versus the buckling coefficient under various hygrothermal conditions (a/b=1, a/h=10, S=0, V_f=0.5)

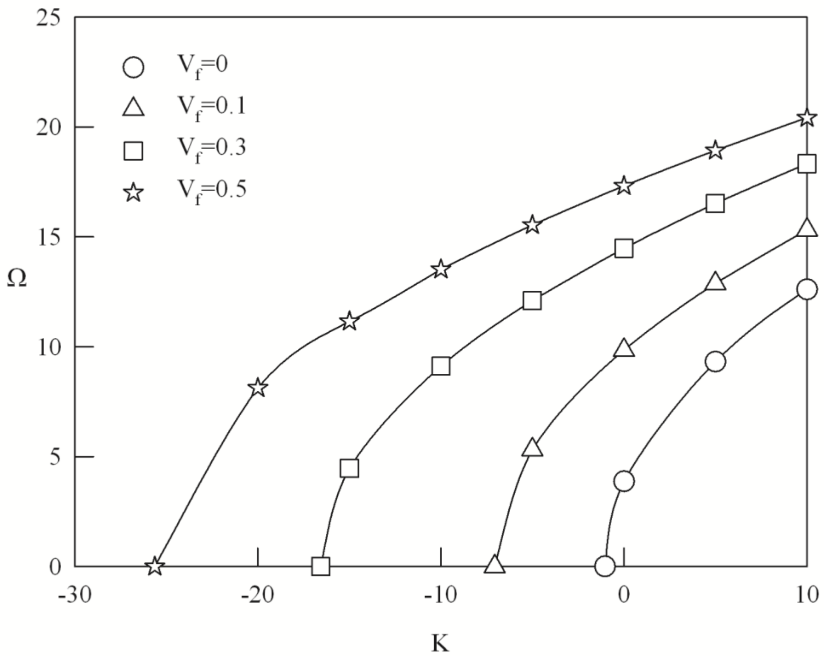

Fig. 2 Effect of initial stress on the natural frequency of a laminated plate with various fiber volume fractions (a/b=1, a/h=10, S=0, ΔT=150℃, ΔC=3%)

Table 9 Hygrothermal effect on the critical buckling coefficient of a laminated plate with various length to thickness ratios (a/b=1, K=0, S=0, V_f=0.4)

ΔT	a/h	Moisture concentration (%)			
		0	1	2	3
0	10	28.6316 (1)	26.2585 (0.9171)	24.0343 (0.8394)	21.9577 (0.7669)
	20	36.5207 (1.2755)	34.2348 (1.1957)	32.1043 (1.1213)	30.1283 (1.0523)
	40	39.2900 (1.3723)	37.0590 (1.2943)	34.9885 (1.2220)	33.0782 (1.1553)
	80	40.0550 (1.3990)	37.8415 (1.3217)	35.7902 (1.2500)	33.9010 (1.1840)
75	10	27.8805 (0.9738)	25.6138 (0.8946)	23.4936 (0.8205)	21.5184 (0.7516)
	20	35.9134 (1.2543)	33.7442 (1.1786)	31.7289 (1.1082)	29.8665 (1.0431)
	40	38.7732 (1.3542)	36.6672 (1.2807)	34.7212 (1.2127)	32.9349 (1.1503)
	80	39.5671 (1.3819)	37.4813 (1.3091)	35.5577 (1.2419)	33.7963 (1.1804)
150	10	27.0925 (0.9462)	24.9282 (0.8707)	22.9077 (0.8001)	21.0288 (0.7345)
	20	35.2857 (1.2324)	33.2306 (1.1606)	31.3276 (1.0942)	29.5752 (1.0330)
	40	38.2495 (1.3359)	36.2675 (1.2667)	34.4451 (1.2030)	32.7817 (1.1449)
	80	39.0768 (1.3648)	37.1186 (1.2964)	35.3225 (1.2337)	33.6886 (1.1766)

Table 10 Hygrothermal effect on the natural vibration frequency of a laminated plate with various length to thickness ratios (a/b=1, K=0, S=0, V_f =0.4)

ΔT	a/h	Moisture concentration (%)			
		0	1	2	3
0	10	18.6615 (1)	17.8714 (0.9577)	17.0978 (0.9162)	16.3425 (0.8757)
	20	21.0763 (1.1294)	20.4060(1.1294)	19.7609 (1.0589)	19.1431 (1.0258)
	40	21.8608 (1.1714)	21.2311 (1.1377)	20.6294 (1.1055)	20.0584 (1.0749)
	80	22.0726 (1.1828)	21.4540 (1.1496)	20.8644 (1.1180)	20.3063 (1.0881)
75	10	18.4151 (0.9868)	17.6507 (0.9458)	16.9044 (0.9058)	16.1782 (0.8669)
	20	20.9003 (1.1200)	20.2593 (1.0856)	19.6450 (1.0527)	19.0597 (1.0213)
	40	21.7165 (1.1637)	21.1185 (1.1317)	20.5505 (1.1012)	20.0149 (1.0725)
	80	21.9377 (1.1756)	21.3517 (1.1442)	20.7966 (1.1144)	20.2749 (1.0865)
150	10	18.1530 (0.9728)	17.4129 (0.9331)	16.6923 (0.8945)	15.9931 (0.8570)
	20	20.7169 (1.1101)	20.1045 (1.0773)	19.5204 (1.0460)	18.9665 (1.0163)
	40	21.5694 (1.1558)	21.0031 (1.1255)	20.4686 (1.0968)	19.9683 (1.0700)
	80	21.8014 (1.1683)	21.2481 (1.1386)	20.7277 (1.1107)	20.2426 (1.0847)

Table 11 presents the effects of initial stresses on the vibration frequencies of a laminated plate in various hygrothermal environments. Evidently, the compressive load (K<0) softens the vibration frequency and a tensile load has the reverse effect. At a high moisture concentration, the drop in the vibration frequency of the laminated plates under compressive stress is greater than that under tensile stress. Table 12 presents the effects of initial stresses on the higher vibration modes of a laminated plate at various temperatures. A compressive load (K<0) reduces the vibration frequency and a tensile load has the opposite effect at both the fundamental and higher vibration modes.

Table 11 Hygrothermal effect on the vibration frequency of a laminated plate under various initial stresses (a/b=1, a/h=10, S=0, V_f=0.5%)

ΔT	K	Moisture concentration (%)						
		0	0.5	1	1.5	2	2.5	3
0	20	25.2814	24.9505	24.6255	24.3066	23.9940	23.6880	23.3887
	10	22.8485	22.4818	22.1205	21.7649	21.4153	21.0719	20.7348
	0	20.1235	19.7061	19.2930	18.8843	18.4802	18.0811	17.6872
	-10	16.9664	16.4692	15.9726	15.4764	14.9807	14.4855	13.9907
	-20	13.0676	12.4152	11.7484	11.0644	10.3598	9.6298	8.8681
75	20	25.0791	24.7601	24.4472	24.1406	23.8405	23.5472	23.2607
	10	22.6244	22.2703	21.9219	21.5794	21.2432	20.9134	20.5904
	0	19.8688	19.4645	19.0649	18.6702	18.2805	17.8962	17.5176
	-10	16.6635	16.1793	15.6963	15.2144	14.7336	14.2541	13.7757
	-20	12.6718	12.0280	11.3700	10.6949	9.9992	9.2780	8.5248
150	20	24.8640	24.5564	24.2550	23.9601	23.6718	23.3903	23.1158
	10	22.3858	22.0435	21.7073	21.3773	21.0537	20.7367	20.4265
	0	19.5966	19.2047	18.8178	18.4362	18.0599	17.6894	17.3247
	-10	16.3379	15.8658	15.3952	14.9263	14.4590	13.9934	13.5295
	-20	12.2406	11.6029	10.9506	10.2809	9.5899	8.8724	8.1211

Table 12 Effect of initial stresses on the higher vibration modes of a laminated plate at different temperatures. (a/b=1, a/h=10, S=0, ΔC=3%, V_f=0.3%)

ΔT	K	(m,n)						
		(1, 1)	(1, 2)	(2, 1)	(2, 2)	(1, 3)	(3, 1)	(3, 3)
0	10	18.5881	38.2762	39.4883	52.0666	61.5812	65.0739	88.0620
	5	16.8004	37.4406	36.1423	49.5769	61.0654	60.5409	84.7674
	0	14.7982	36.5860	32.4530	46.9553	60.5451	55.6400	81.3395
	-5	12.4787	35.7109	28.2866	44.1784	60.0204	50.2634	77.7606
	-10	9.6152	34.8138	23.3894	41.2149	59.4910	44.2381	74.0088
75	10	18.4701	37.4712	38.9866	51.0825	59.8599	63.8756	85.9130
	5	16.6696	36.6173	35.5933	48.5423	59.3291	59.2511	82.5327
	0	14.6496	35.7430	31.8405	45.8616	58.7935	54.2336	79.0079
	-5	12.3022	34.8467	27.5817	43.0142	58.2529	48.7020	75.3183
	-10	9.3849	33.9268	22.5319	39.9644	57.7074	42.4557	71.4384
150	10	18.3388	36.6015	38.4469	50.0294	58.0238	62.6003	83.6364
	5	16.5240	35.0013	35.7267	47.4329	57.4760	57.8739	80.1601
	0	14.4837	34.8301	31.1774	44.6857	52.7256	56.9230	76.5261
	-5	12.1042	33.9097	26.8134	41.7582	47.0169	56.3645	72.7107
	-10	9.1238	32.9636	21.5846	38.6093	40.5116	55.8005	68.6837

However, at higher vibration modes, the reduction in the vibration frequency is not significant. High temperature raise also reduces the vibration frequency at the fundamental and higher

Table 13 Hygrothermal effect on the vibration frequency of a laminated plate under various bending stress ratios (a/b=1, a/h=10, K=10, V_f=0.3%)

ΔT	ΔC	*S*						
		0	5	10	15	20	25	30
0	0	23.0311	23.0090	22.9433	22.8350	22.6864	22.4999	22.2790
	1	20.9919	20.9736	20.9192	20.8297	20.7065	20.5516	20.3676
	2	19.0840	19.0690	19.0244	18.9507	18.8492	18.7215	18.5692
	3	17.3065	17.2943	17.2579	17.1977	17.1146	17.0099	16.8848
75	0	22.4029	22.3822	22.3206	22.2190	22.0794	21.9042	21.6962
	1	20.4596	20.4424	20.3914	20.3071	20.1912	20.0454	19.8719
	2	18.6459	18.6318	18.5897	18.5203	18.4245	18.3039	18.1600
	3	16.9607	16.9492	16.9147	16.8577	16.7791	16.6798	16.5611
150	0	21.7487	21.7294	21.6720	21.5774	21.4472	21.2836	21.0891
	1	19.8984	19.8824	19.8348	19.7562	19.6479	19.5116	19.3491
	2	18.1756	18.1625	18.1231	18.0582	17.9686	17.8556	17.7206
	3	16.5788	16.5680	16.5357	16.4823	16.4085	16.3152	16.2036

Table 14 Effect of bending stress ratio on the vibration frequency of a laminated plate with different initial stresses. (a/b=1, a/h=10, ΔT=150℃, ΔC=3%, V_f=0.3%)

K	*S*						
	0	5	10	15	20	25	30
25	24.3493	24.3459	24.3358	24.3189	24.2952	24.2645	24.2269
15	21.8126	21.8113	21.8072	21.8004	21.7910	21.7787	21.7637
5	18.9392	18.9391	18.9385	18.9377	18.9365	18.9349	18.9330
-5	15.5434	15.5432	15.5426	15.5415	15.5400	15.5381	15.5358
-15	11.1580	11.1553	11.1474	11.1341	11.1155	11.0915	11.0621
-25	2.7209	2.6906	2.5975	2.4341	2.1843	1.8118	1.2066

vibration modes.

Table 13 presents the effects of bending stress ratio on the vibration frequency of initially stressed laminated plates in different hygrothermal environments. The bending stress has a weak effect on the natural frequency of a plate under a tensile stress, but a significant effect on the natural frequency of a plate under a higher compressive stress (that approaches the buckling load). The effect becomes considerable when the compressive initial stress is high. Table 14 presents the effect of the bending stress ratio on the vibration frequency of the laminated plate. The vibration frequency declines as the bending stress ratio increases.

5. Conclusions

The vibration and stability of initially stressed laminated plates in hygrothermal environments are investigated. The effects of moisture, temperature, fiber volume fraction and initial stress on the vibration and buckling behaviors of laminated plates are elucidated. The following conclusions are drawn.

• Moisture, temperature, fiber volume fraction and initial stress significantly affect the vibration frequency and buckling load, which are only weakly affected by bending under tensile stress.

• The buckling load and vibration frequency decrease as the moisture content and temperature increase and the fiber volume fraction decreases. The buckling load and vibration frequency of a laminated plate is lower if the fiber volume fraction is lower or moisture content or temperature is higher.

• Compressive stress significantly reduces the vibration frequency of laminated plates; tensile stress has the opposite effect.

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CC

Appendix

$$L_1 = A_{11}\, u_{x,x} + A_{16}(u_{x,y} + u_{y,x}) + A_{12}\, u_{y,y} + B_{11}\, \varphi_{x,x} + B_{16}(\varphi_{x,y} + \varphi_{y,x}) + B_{12}\, \varphi_{y,y}$$

$$L_2 = A_{16}\, u_{x,x} + A_{26}\, u_{y,y} + A_{66}(u_{x,y} + u_{y,x}) + B_{16}\, \varphi_{x,x} + B_{66}(\varphi_{x,y} + \varphi_{y,x}) + B_{26}\, \varphi_{y,y}$$

$$L_3 = A_{12}\, u_{x,x} + A_{26}(u_{x,y} + u_{y,x}) + A_{22}\, u_{y,y} + B_{12}\, \varphi_{x,x} + B_{26}(\varphi_{x,y} + \varphi_{y,x}) + B_{22}\, \varphi_{y,y}$$

$$L_4 = A_{55}(w_{,x} + \varphi_x) + A_{45}(w_{,y} + \varphi_y)$$

$$L_5 = A_{45}(w_{,x} + \varphi_x) + A_{44}(w_{,y} + \varphi_y)$$

$$L_6 = B_{11} u_{x,x} + B_{16}(u_{x,y} + u_{y,x}) + B_{12}\, \varphi_{y,y} + D_{11}\varphi_{x,x} + D_{16}(\varphi_{x,y} + \varphi_{y,x}) + D_{12}\, \varphi_{y,y}$$

$$L_7 = B_{16}\, u_{x,x} + B_{66}(u_{x,y} + u_{y,x}) + B_{26}\, u_{y,y} + D_{16}\, \varphi_{x,x} + D_{66}(\varphi_{x,y} + \varphi_{y,x}) + D_{22}\, \varphi_{y,y}$$

$$L_8 = A_{45}(w_{,y} + \varphi_y) + A_{55}(w_{,x} + \varphi_x)$$

$$L_9 = B_{12} u_{x,x} + B_{22}\, u_{y,y} + B_{26}(u_{x,y} + u_{y,x}) + D_{12}\, \varphi_{x,x} + D_{26}(\varphi_{x,y} + \varphi_{y,x}) + D_{22}\, \varphi_{y,y}$$

$$L_{10} = A_{44}(w_{,y} + \varphi_y) + A_{45}(w_{,x} + \varphi_x)$$

$$P_1 = N_{xx} u_{x,x} + M_{xx}\varphi_{x,x} + N_{xy} u_{x,y} + M_{xy}\varphi_{x,y} + N_{xz} u_{z,x}$$

$$P_2 = N_{yy} u_{x,y} + M_{yy}\varphi_{x,y} + N_{xy} u_{x,x} + M_{xy}\varphi_{x,x} + N_{yz} u_{z,x}$$

$$P_3 = N_{xx} u_{y,x} + M_{xx}\varphi_{y,x} + N_{xy} u_{y,y} + M_{xy}\varphi_{y,y} + N_{xz} u_{z,y}$$

$$P_4 = N_{yy} u_{y,y} + M_{yy}\varphi_{y,y} + N_{xy} u_{y,x} + M_{xy}\varphi_{y,x} + N_{xz} u_{z,y}$$

$$P_5 = N_{xx} w_{,x} + N_{xy} w_{,y}$$

$$P_6 = N_{xy} w_{,x} + N_{yy} w_{,y}$$

$$P_7 = M_{xx} u_{x,x} + M^*_{xx}\varphi_{x,x} + M_{xy} u_{x,y} + M^*_{xy}\varphi_{x,y} + M_{xz} u_{z,x}$$

$$P_8 = M_{yy} u_{x,y} + M^*_{yy}\varphi_{x,y} + M_{xy} u_{x,x} + M^*_{xy}\varphi_{x,x} + M_{yz} u_{z,x}$$

$$P_9 = N_{xz} u_{x,x} + M_{xz}\varphi_{x,x} + N_{zz}\varphi_x + N_{zy} u_{x,y} + M_{zy}\varphi_{x,y}$$

$$P_{10} = M_{xx} u_{y,x} + M^*_{xx}\varphi_{y,x} + M_{xy} u_{y,y} + M^*_{xy}\varphi_{y,y} + M_{xz} u_{z,y}$$

$$P_{11} = M_{yy} u_{y,y} + M^*_{yy}\varphi_{y,y} + M_{xy}\varphi_{y,x} + M^*_{xy} u_{y,x} + M_{xz} u_{z,y}$$

$$P_{12} = N_{xz} u_{y,x} + M_{xz}\varphi_{y,x} + N_{zz}\varphi_y + N_{zy} u_{y,y} + M_{zy}\varphi_{y,y}$$

$$T_1 = N^T_{xx} u_{x,x} + M^T_{xx}\varphi_{x,x} + N^T_{xy} u_{x,y} + M^T_{xy}\varphi_{x,y}$$

$$T_2 = N^T_{yy} u_{x,y} + M^T_{yy}\varphi_{x,y} + N^T_{xy} u_{x,x} + M^T_{xy}\varphi_{x,x}$$

$$T_3 = N^T_{xx} u_{y,x} + M^T_{xx} \varphi_{y,x} + N^T_{xy} u_{y,y} + M^T_{xy} \varphi_{y,y}$$

$$T_4 = N^T_{yy} u_{y,y} + M^T_{yy} \varphi_{y,y} + N^T_{xy} u_{y,x} + M^T_{xy} \varphi_{y,x}$$

$$T_5 = N^T_{xx} w_{,x} + N^T_{xy} w_{,y}$$

$$T_6 = N^T_{xy} w_{,x} + N^T_{yy} w_{,y}$$

$$T_7 = M^T_{xx} u_{x,x} + M^{T*}_{xx} \varphi_{x,x} + M^T_{xy} u_{x,y} + M^{T*}_{xy} \varphi_{x,y}$$

$$T_8 = M^T_{yy} u_{x,y} + M^{T*}_{yy} \varphi_{x,y} + M^T_{xy} u_{x,x} + M^{T*}_{xy} \varphi_{x,x}$$

$$T_9 = M^T_{xx} u_{y,x} + M^{T*}_{xx} \varphi_{y,x} + M^T_{xy} u_{y,y} + M^{T*}_{xy} \varphi_{y,y}$$

$$T_{10} = M^T_{yy} u_{y,y} + M^{T*}_{yy} \varphi_{y,y} + M^T_{xy} \varphi_{y,x} + M^{T*}_{xy} u_{y,x}$$

$$H_1 = N^H_{XX} u_{x,x} + M^H_{xx} \varphi_{x,x} + N^H_{xy} u_{x,y} + M^H_{xy} \varphi_{x,y}$$

$$H_2 = N^H_{yy} u_{x,y} + M^H_{yy} \varphi_{x,y} + N^H_{xy} u_{x,x} + M^H_{xy} \varphi_{x,x}$$

$$H_3 = N^H_{xx} u_{y,x} + M^H_{xx} \varphi_{y,x} + N^H_{xy} u_{y,y} + M^H_{xy} \varphi_{y,y}$$

$$H_4 = N^H_{yy} u_{y,y} + M^H_{yy} \varphi_{y,y} + N^H_{xy} u_{y,x} + M^H_{xy} \varphi_{y,x}$$

$$H_5 = N^H_{xx} w_{,x} + N^H_{xy} w_{,y}$$

$$H_6 = N^H_{xy} w_{,x} + N^H_{yy} w_{,y}$$

$$H_7 = M^H_{xx} u_{x,x} + M^{H*}_{xx} \varphi_{x,x} + M^H_{xy} u_{x,y} + M^{H*}_{xy} \varphi_{x,y}$$

$$H_8 = M^H_{yy} u_{x,y} + M^{H*}_{yy} \varphi_{x,y} + M^H_{xy} u_{x,x} + M^{H*}_{xy} \varphi_{x,x}$$

$$H_9 = M^H_{xx} u_{y,x} + M^{H*}_{xx} \varphi_{y,x} + M^H_{xy} u_{y,y} + M^{H*}_{xy} \varphi_{y,y}$$

$$H_{10} = M^H_{yy} u_{y,y} + M^{H*}_{yy} \varphi_{y,y} + M^H_{xy} \varphi_{y,x} + M^{H*}_{xy} u_{y,x}$$

The arbitrary initially stress resultants are included in N_{ij}, M_{ij} and M^*_{ij}. N^T_{ij} , M^T_{ij} and M^{T*}_{jj} are thermal stresses resultants, N^H_{ij} , M^H_{ij} and M^{H*}_{jj} are humid stresses resultants. The coefficients associated with laminate stiffness, initial stress resultants, hygrothermal stress resultants and rotary inertia in a laminate are obtained by integration of the stiffness and stresses in each lamina through the laminate thickness h (Jones 1975), and are defined as:

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} C_{ij} \, (1, z, z^2) \, dz \quad (i, j = 1, 2, 4, 5, 6)$$

$$(N_{ij}, M_{ij}, M^*_{ij}) = \int_{-h/2}^{h/2} \sigma^o_{ij} \, (1, z, z^2) \, dz \quad (i, j = x, y, z)$$

$$(N_{ij}^{T}, M_{ij}^{T}, M_{ij}^{T*}) = -\int_{-h/2}^{h/2} \sigma_{ij}^{T} \, (1, z, z^2) \, dz \quad (i = x, y)$$

$$(N_{ij}^{H}, M_{ij}^{H}, M_{ij}^{H*}) = -\int_{-h/2}^{h/2} \sigma_{ij}^{H} \, (1, z, z^2) \, dz \quad (i = x, y)$$

$$(I_1, I_3) = \int_{-h/2}^{h/2} \rho(z) \, (1, z^2) \, dz$$

A constant shear correction factor is included in C_{ij} (i, j=4,5).