Structural Engineering and Mechanics, *Vol. 56, No. 5 (2015) 871-897*
DOI: http://dx.doi.org/10.12989/sem.2015.56.5.871

Simultaneous identification of moving loads and structural damage by adjoint variable

Reza Abbasnia*, Akbar Mirzaee[a] and Mohsenali Shayanfar[b]

Department of Civil Engineering, Iran University of Science and Technology, Narmak, Tehran, Islamic Republic of Iran

(Received April 15, 2014, Revised October 31, 2015, Accepted November 17, 2015)

Abstract. This paper presents a novel method based on sensitivity of structural response for identifying both the system parameters and input excitation force of a bridge. This method, referred to as “Adjoint Variable Method”, is a sensitivity-based finite element model updating method. The computational cost of sensitivity analyses is the main concern associated with damage detection by these methods. The main advantage of proposed method is inclusion of an analytical method to augment the accuracy and speed of the solution. The reliable performance of the method to precisely indentify the location and intensity of all types of predetermined single, multiple and random damages over the whole domain of moving vehicle speed is shown. A comparison study is also carried out to demonstrate the relative effectiveness and upgraded performance of the proposed method in comparison to the similar ordinary sensitivity analysis methods. Moreover, various sources of error including the effects of noise and primary errors on the numerical stability of the proposed method are discussed.

Keywords: simultaneous damage detection; sensitivity; model updating; Ill posed problem; Inverse problem; regularization; noise

1. Introduction

Bridges have finite life spans and begin to degrade as soon as they are put into service. Processes such as corrosion, fatigue, erosion, wear and overloads degrade them until they are no longer fit for their intended use. Currently, bridges are inspected visually every two years. There is a strong interest to aid these inspection efforts with a more continuous, reliable, physics-based and less subjective procedure. This has led to a great deal of activity in structural health monitoring.

Structural Health Monitoring (SHM) defines the process of assessing the state of health of a structure and of predicting its remaining life. SHM potentially offers increased safety, since faults cannot grow to a dangerous level and reduces ownership costs by replacing pre-planned precautionary servicing with targeted, responsive maintenance.

An SHM system comprises of both hardware and software elements. The hardware elements

*Corresponding author, Associate Professor, E-mail: abbasnia@iust.ac.ir
[a]Ph.D., E-mail: mirzaee_akbar@yahoo.com
[b]E-mail: shayanfar@iust.ac.ir

Copyright © 2015 Techno-Press, Ltd.
http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

are essentially the sensors and the associated instrumentation, while the software components consist of damage modeling and damage detection algorithms (Srinivasan *et al*. 2011). Successful development and implementation of the SHM process involves the understanding of modeling aspects and computing technology.

Damage detection is the primary task of most of SHM systems. In general, all existing methods can be divided into two groups: local and global approaches. Local monitoring methods locate and identify small defects in narrow inspection zones via ultrasonic testing (Staszewski 2003, Ostachowicz *et al*. 2009) or statistical classification techniques (Silva *et al*. 2008, Nair *et al*. 2006). These methods do not require structural modeling and are outside the scope of this paper.

Detecting damages in large structures like bridges usually requires a combination of local and global structural responses recorded on the structure. Also, it is very important that the selected structural responses are sensitive to the possible damages in the structure.

In this regard, vibration measurements on bridges are among the global measurements that can be directly related to the damages occurring in the bridge. The premise of this analogy lies in the fact that changes in the stiffness and mass properties of the bridges can result in changes of dynamic characteristics of bridges.

The developments in the field of System Identification (SI) using vibration data of civil engineering structures have been recently reviewed by several authors.

Doebling *et al*. (1998) provided a comprehensive review on the damage detection methods by examining changes in the dynamic properties of a structure. Zou *et al*. (2000) summarized the methods on vibration-based damage detection and health monitoring for composite structures, especially in delamination modeling techniques and delamination detection.

Damage detection usually requires a mathematical model on the structure in conjunction with experimental modal parameters of the structure. The identification approaches are mainly based on the change in the natural frequencies (Cawley and Adams 1979, Friswell *et al*. 1994, Narkis 1994), mode shapes (Pandey *et al*. 1991, Ratcliffe 1997, Rizos *et al*. 1990) or measured modal flexibility (Pandey and Biswas 1994, Doebling *et al*. 1996, Lim 1991, Wu and Law 2004).

The natural frequency is easy to measure with a high level of accuracy, and is the most common dynamic parameter for damage detection. However, problems may arise in some structures if only natural frequency is used, since the symmetry of the structure would lead to non-uniqueness in the solution in the inverse analysis of damage detection.

For non-stationary and moving loads, the analysis is most often performed in the time domain via a direct comparison of the simulated and measured responses. (Choi and Kim 2005)

Majumder and Manohar (2003, 2004) propose a method for damage identification of linear and non-linear beams excited by a moving oscillator; the beam and the oscillator are treated together as a single coupled and time-varying system. Sieniawska *et al*. (2009) use a static substitute of the equation of motion for identification of parameters of a linear structure from its responses to a moving load of a known constant magnitude.

Identification of moving loads is important not only for assessment of pavements or bridges but also in traffic studies, in design code calibration, for traffic control, etc. Several techniques have been developed, which address both these identification problems separately: either they identify the damage while assuming load characteristics to be known or they identify the moving load, but the structure is assumed to be undamaged.

Identification of moving loads has been studied extensively in the past two decades (Yu and Chan 2007). A direct measurement of the dynamic axle loads of vehicles is expensive, difficult and subject to bias. Therefore, techniques of indirect identification from measured responses have been studied, as they can be performed easier and at lower costs. Chan, Law *et al*. have proposed four general methods for indirect identification, which are the Time-Domain Method (TDM) (Law *et al*. 1997), the Frequency-Time Domain Method (FTDM) (Law *et al*. 1999), Interpretive Method I (IMI) (Chan and O'Connor 1990) and Interpretive Method II (IMII) (Chan *et al*. 1999). All of them require the parameters of the model of the bridge to be known in advance. Each method has its merits and limitations, which are compared in Chan *et al*. (2001). The numerical ill-conditioning of the problem seems to be the main factor that decreases the accuracy of the identification results. To improve the accuracy, techniques based on the Singular Value Decomposition (SVD) have been investigated and adopted for the inverse computation (Yu and Chan 2003). Other regularization methods have been also used, e.g., Law *et al*. (2001) and Zhu and Law (2006) use the Tikhonov regularization, while Law and Fang (2001) and González *et al*. (2008) couple it with the dynamic programming approach. In general, all these methods require a known and well-defined model of the structure in order to build the load–response relation, even if some of them allow for the identification of chosen additional parameters besides the moving load, such as the prestressing force (Law *et al*. 2008) or parameters of the vehicle model (Jiang *et al*. 2004).

In real applications unknown damages and unknown moving loads can coexist and together influence the response of the system; it seems that their simultaneous identification is a relatively unexplored area.

In the case of unknown excitations and unknown structural damages, the related identification problems are inherently coupled: both factors together influence the structural response and cannot be identified independently from each other. Hoshiya and Maruyama (1987) apply a weighted global iteration procedure and the extended Kalman filter for simultaneous identification of a moving force and modal parameters of a simply supported beam. Lu and Law (2007) identify damage and parameters of non-moving impulsive or sinusoidal force excitations in a two-step identification process using a limited number of measurements. Zhang *et al*. (2009) present a method for simultaneous identification of structural physical parameters and an unknown support excitation. Zhu and Law (2007) propose a method for simultaneous identification of moving loads and damages using a two-step approach that separately adjusts the loads and the damage factors in each iteration of the optimization process; the number of sensors is one less than the number of the beam elements.

Zhang *et al*. (2010) addressed simultaneous identification of damages and nonmoving excitation forces in truss structures; a moving force could be identified only by a simultaneous identification of all load-time histories in all involved Degrees Of Freedom (DOF's).

In this paper, a novel sensitivity-based damage detection method referred to as Adjoint Variable Method (AVM), is developed. The sensitivities of dynamic response with respect to the structural physical parameters and the input excitation force are calculated simultaneously and perturbations in the structural parameters are identified together with the input excitation forces using an iterative algorithm.

The outline of the work is as follows: Inverse problems along with model updating are briefly introduced in section2. The basic theory of sensitivity analysis is addressed in section3 and the proposed algorithm will be presented in section4. Numerical simulations are presented in section 5 with studies on the effect of different factors which may affect the accuracy of the proposed analysis in practice. Conclusion will be drawn in the last section.

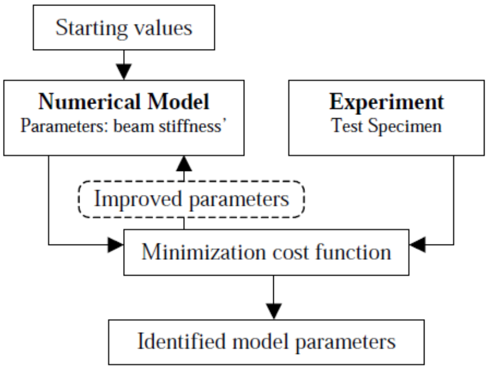

Fig. 1 General flowchart of a FEM-updating

2. Finite element model updating and inverse problem

Since many algorithms of damage detection are based on the difference between modified model before occurrence of damage and after that, problems such as parameter identification and damage detection are closely related to model updating. Discrepancy between two models is used for detection and quantification of damage.

A key step in model-based damage identification is the updating of the finite element model of the structure in such a way that the measured responses can be reproduced by the FE-model. A general flowchart of this operation is given in Fig. 1 the identification procedure presented in this paper is a sensitivity based model updating routine. Sensitivity coefficients are the derivatives of the system responses with respect to the physical parameters or input excitation force, and are needed in the cost function of the flowchart of Fig. 1.

2.1 Finite element modeling of bridge vibration under moving loads

For a general finite element model of a linear elastic time-invariant structure, the equation of motion is given by

$$[\mathrm{M}]\{z_{,tt}\} + [\mathrm{C}]\{z_{,t}\} + [\mathrm{K}]\{z\} = [\mathrm{B}]\{\mathrm{F}\} \tag{1}$$

Where [M] and [K] are mass and stiffness matrices and [C] is damping matrix. $Z_{,tt}$ and $Z_{,t}$ and Z are the respective acceleration, velocity and displacement vectors for the whole structure and {F} is a vector of applied forces with matrix [B] mapping these forces to the associated DOF's of the structure. A proportional damping is assumed to show the effect of damping ratio on the dynamic magnification factor. Rayleigh damping, in which the damping matrix is proportional to the combination of the mass and stiffness matrices, is used.

$$[\mathrm{C}] = a_0[\mathrm{M}] + a_1[\mathrm{K}] \tag{2}$$

Where a_0 and a_1 are constants to be determined from two modal damping ratios. If a more accurate estimation of the actual damping is required, a more general form of Rayleigh damping, the Caughey damping model can be adopted. The dynamic responses of the structures can be obtained by direct numerical integration using Newmark method.

2.2 Objective functions

The approach minimizes the difference between response quantities (acceleration response) of the measured data and model predictions. This problem may be expressed as the minimization of *J*, where

$$J(\theta) = \|z_m - z(\alpha)\|^2 = \epsilon^T \epsilon$$

$$\epsilon = z_m - z(\alpha) \tag{3}$$

Here z_m and $z(\alpha)$ are the measured and computed response vectors, α is a vector of all unknown parameters, and ϵ is the response residual vector.

2.3 Nonlinear model updating for damage detection

When the parameters of a model are unknown, they must be estimated using measured data. Since the relationship between the acceleration responses $\ddot{z}_1$ and the fractional stiffness parameter α is nonlinear, a nonlinear model updating technique, like the Gauss-Newton method, is required. This kind of method has the advantage that the second derivatives, which can be challenging to compute, are not required. The Gauss-Newton method in the damage detection procedure can be described in terms of the acceleration response at the i^{th} DOF of the structure as

$$\ddot{z}_{dl}(\alpha_d) = \ddot{z}_{ul}(\alpha^0) + S(\alpha^0) \times \Delta\alpha^1 + S(\alpha^0 + \Delta\alpha^1) \times \Delta\alpha^2 + \cdots \tag{4}$$

The superscript 0, 1, 2 denote the iteration numbers. Index u denotes the initial state or state 0 while index d denotes the final damage state. $\ddot{z}_{dl}$ and $\ddot{z}_{ul}$ are vectors of the acceleration response at the i^{th} DOF of the damaged and intact states, respectively. The damage identification equation for $(k+1)$th iteration is

$$\Delta\ddot{z}^k = S^k \times \Delta\alpha^{k+1} \tag{5}$$

Where S^k and $\delta\ddot{z}^k$ are obtained from the k^{th} iteration. The iteration in Eq. (5) starts with an initial value α^0 leading to $\Delta\ddot{z}^0 = \ddot{z}_{dl} - \ddot{z}_{ul}(\alpha^0)$ and $S^0 = S(\alpha^0)$. The parameter vector $\alpha^k = \alpha^0 + \sum_{i=1}^{k} \Delta\alpha^i$. Sensitivity matrix $S^k = S(\alpha^k)$, And the residual vector $\Delta\ddot{z}^k = \ddot{z}_{dl} - \ddot{z}_{ul}(\alpha^0) - \sum_{i=0}^{k-1} S^i \Delta\alpha^{i+1}$, $(k = 1,2,\dots)$ of the next iteration are then computed from results in the previous iterations.

The acceleration response vector $\ddot{z}_{ul}$ from the physical intact structure is computed, in general, from the associated analytical model via dynamic analysis. $\ddot{z}_{dl}$ is the acceleration response of the model of the damaged structure. In general, the measured acceleration responses (including measurement errors) from the damaged structure are obtained for $\ddot{z}_{dl}$.

The iteration is terminated when a pre-selected criterion is met. The final identified damaged vector becomes (Ratcliffe 1997).

$$\Delta\alpha = \Delta\alpha^1 + \Delta\alpha^2 + \cdots + \Delta\alpha^n \tag{6}$$

2.4 Regularization

Like many other inverse problems, the solution of Eq. (5) is often ill-conditioned and regularization techniques are needed to provide bounds to the solution. The aim of regularization in the inverse analysis is to promote certain regions of parameter space where the model realization should exist. The two most widely used regularization methods are Tikhonov regularization (Friswell and Penny 1994) and truncated singular value decomposition (Friswell and Mottershead 1995, Ricles and Kosmatka 1992). In the Tikhonov regularization, the new cost function is defined as

$$J(\Delta\alpha^{k+1},\lambda) = \left\|S^k.\Delta\alpha^{k+1} - \Delta\ddot{z}^k\right\|_2^2 + \lambda^2\left\|\Delta\alpha^{k+1}\right\|_2^2 \tag{7}$$

The regularization parameter $\lambda \geq 0$ controls the extent of contribution of the two errors to the cost function in Eq. (7) and the fractional stiffness change increment $\Delta\alpha^{k+1}$ is obtained by minimizing the cost function in Eq. (7)

The regularized solution from minimizing the function in Eq. (7) can be written in the following form as

$$\Delta\alpha^{k+1} = ((S^k)^T S^k + \lambda^2 I)^{-1}(S^k)^T\Delta\ddot{z}^k \tag{8}$$

To express the contribution of the singular values and the corresponding vectors in the solution clearly and to show how the regularization parameter plays the role as the filter factor, the sensitivity matrix is singular value decomposed and Singular Value Decomposition (SVD) applies to the sensitivity matrix S^k to obtain

$$S^k = U\Sigma V^T \tag{9}$$

Where $U \in R^{nt\times nt}$ and $V \in R^{m\times m}$ are orthogonal matrices satisfying $U^TU = I_{nt}$ and$V^TV = I_m$, and matrix $\sum$ has the size of $nt \times m$ with the singular values $\sigma_i(i = 1,2,\ldots,m)$ on the diagonal arranged in a decreasing order such that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m \geq 0$ and zeros elsewhere.

The regularized solution in Eq. (8) can be written as

$$\Delta\alpha^{k+1} = \sum_{i=1}^{m} f_i \frac{U_i^T\Delta\ddot{z}^k}{\sigma_i} V_i \tag{10}$$

Where $f_i = \sigma_i^2/(\sigma_i^2 + \lambda^2)$ $(i = 1,2,\ldots,m)$ are referred as filter factors. The solution norm $\left\|\Delta\alpha^{k+1}\right\|_2^2$ and the residual norm $\left\|S^k.\Delta\alpha^{k+1} - \Delta\ddot{z}^k\right\|_2^2$ can be expressed as

$$\eta^2 = \left\|\Delta\alpha^{k+1}\right\|_2^2 = \sum_{i=1}^{m}\left(\frac{\sigma_i^2}{\sigma_i^2+\lambda^2}\frac{U_i^T\Delta\ddot{z}^k}{\sigma_i}\right)^2 \tag{11}$$

$$\rho^2 = \left\|S^k.\Delta\alpha^{k+1} - \Delta\ddot{z}^k\right\|_2^2 = \sum_{i=1}^{m}\left(\frac{\lambda^2}{\sigma_i^2+\lambda^2}U_i^T\Delta\ddot{z}^k\right)^2 \tag{12}$$

These two quantities represent the smoothness and goodness-of-fit of the solution and they should be balanced by choosing an appropriate regularization parameter.

2.5 Element dmage idex

In the inverse problem of damage identification, it is assumed that the stiffness matrix of the whole element decreases uniformly with damage, and the flexural rigidity, EI_i of the i^{th} finite element of the beam becomes $\beta_i EI_i$,when there is damage. The fractional change in stiffness of an element can be expressed as (Zhu and Hao 2007).

$$\Delta K_{bi} = \left(K_{bi} - \widetilde{K}_{bi}\right) = (1 - \beta_i)K_{bi} \quad (13)$$

Where K_{bi} and $\widetilde{K}_{bi}$ are the i^{th} element stiffness matrices of the undamaged and damaged beam, respectively. ΔK_{bi} is the stiffness reduction of the element. A positive value of $\beta_i \in [0,1]$ will indicate a loss in the element stiffness. The i^{th} element is undamaged when $\beta_i = 1$ and the stiffness of the i^{th} element is completely lost when $\beta_i = 0$

The stiffness matrix of the damaged structure is the assemblage of the entire element stiffness matrix $\widetilde{K}_{bi}$

$$K_b = \sum_{i=1}^{N} A_i^T \widetilde{K}_{bi} A_i = \sum_{i=1}^{N} \beta_i A_i^T K_{bi} A_i \quad (14)$$

Where A_i is the extended matrix of element nodal displacement that facilitates assembling of global stiffness matrix from the constituent element stiffness matrix.

2.6 Input force identification

The sensitivity-based analysis method without considering the second and higher order effects is adopted in this study. In the Gauss-Newton method, we have

$$\{\delta \ddot{z}\} = [S_F]\{\delta P\} \quad (15)$$

The physical parameters of the intact structure are used, as an approximation, to calculate the matrix $[S_F]$ as we are not certain about the true state of the damage structure. $[S_F]$ is the sensitivity matrix, which is the change of acceleration response with respect to the force parameters in time domain and $\{\delta P\}$ is the vector of perturbation in the force parameters. Eq. (15) can be solved by Tikhonov method as

$$\delta P = ((S_F)^T S_F + \lambda^2 I)^{-1} (S_F)^T \delta \ddot{z} \quad (16)$$

2.7 Damage identification

Once the forces have been obtained from above, we can move on to the local damage identification. Again by using Gauss-Newton method, we have

$$\{\delta \ddot{z}\} = [S_S]\{\delta \alpha\} \quad (17)$$

$[S_S]$ is the sensitivity matrix, which is the change of acceleration response with respect to the physical parameter in time domain. $\{\delta\alpha\}$ is the vector of perturbation of the parameter. The physical parameter can also be obtained using Tikhonov method as

$$\delta \alpha = ((S_S)^T S_S + \lambda^2 I)^{-1} (S_S)^T \delta \ddot{z} \quad (18)$$

3. Sensitivity analysis of transient dynamic response

The objective of sensitivity analysis is to quantify the effects of parameter variations on calculated results. Terms such as influence, importance, ranking by importance and dominance are

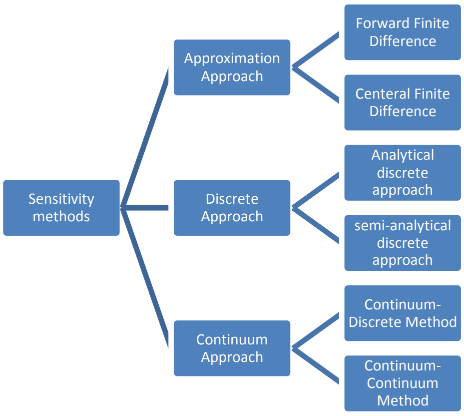

Fig. 2 Different approaches to sensitivity analysis

all related to sensitivity analysis.

The most important difficulty in sensitivity base SI methods is calculation of sensitivity matrix. Calculation of this massive matrix is repeated in each iteration and according to its dimensions, is so time-consuming and has a significant effect on the efficiency of method.

3.1 Methods of structural sensitivity analysis

When the parameter variations are small, the traditional way to assess their effects on calculated responses is by using perturbation theory, either directly or indirectly, via variational principles. The basic aim of perturbation theory is to predict the effects of small parameter variations without actually calculating the perturbed configuration but rather by using solely unperturbed quantities.

Various methods employed in sensitivity analysis are listed in Fig. 2 three approaches are used to obtain the sensitivity matrix: the approximation, discrete, and continuum approaches.

3.2 Approximation approach

In the approximation approach, sensitivity matrix is obtained by either the forward finite difference or the central finite difference method.

If the design is perturbed to u+Δu, where Δu represents a small change in the design, then the sensitivity of $\psi(u)$ can be approximated as

$$\frac{d\psi}{du} \approx \frac{\psi(u+\Delta u)-\psi(u)}{\Delta u} \tag{19}$$

Eq. (19) is called the forward difference method since the design is perturbed in the direction of +Δu. If $-\Delta u$ is substituted in Eq. (19) for Δu, and then the equation is defined as the backward difference method. Additionally, if the design is perturbed in both directions, such that the design sensitivity is approximated by Eq. (20), the equation is defined as the central difference method.

$$\frac{d\psi}{du} \approx \frac{\psi(u+\Delta u)-\psi(u-\Delta u)}{2\Delta u} \tag{20}$$

3.3 Discrete approach

In the discrete method, sensitivity matrix is obtained by design derivatives of the discrete governing equation. For this process, it is necessary to take the derivative of the stiffness matrix. If this derivative is obtained analytically using the explicit expression of the stiffness matrix with respect to the variable, it is an analytical method, since the analytical expressions of stiffness matrix are used. However, if the derivative is obtained using a finite difference method, the method is called a semi analytical method. The design represents a structural parameter that can affect the results of the analysis.

The design sensitivity information of a general performance measure can be computed either with the direct differentiation method or with the adjoint variable method.

3.3.1 Direct differentiation method

The Direct Differentiation Method (DDM) is a general, accurate and efficient method to compute finite element response sensitivities to the model parameters. This method directly solves for the design dependency of a state variable, and then computes performance sensitivity using the chain rule of differentiation. This method clearly shows the implicit dependence on the design, and a very simple sensitivity expression can be obtained.

Consider a structure in which the generalized stiffness and mass matrices have been reduced by accounting for boundary conditions. Performing differentiation to both sides of Eq. (1) with respect to the i^{th} excitation force, P_i , we have

$$[\mathrm{M}]\left\{\frac{\partial \mathrm{z}_{,\mathrm{tt}}}{\partial \mathrm{P_i}}\right\}+[\mathrm{C}]\left\{\frac{\partial \mathrm{z}_{,\mathrm{t}}}{\partial \mathrm{P_i}}\right\}+[\mathrm{K}]\left\{\frac{\partial \mathrm{z}}{\partial \mathrm{P_i}}\right\}=[\mathrm{B}] \tag{21}$$

Performing differentiation to both sides of Eq. (1) again with respect to the j^{th} physical parameter, α_j, of the j^{th} element, and assuming Rayleigh damping in the system, we have

$$[\mathrm{M}]\left\{\frac{\partial \mathrm{z}_{,\mathrm{tt}}}{\partial \alpha_\mathrm{j}}\right\}+[\mathrm{C}]\left\{\frac{\partial \mathrm{z}_{,\mathrm{t}}}{\partial \alpha_\mathrm{j}}\right\}+[\mathrm{K}]\left\{\frac{\partial \mathrm{z}}{\partial \alpha_\mathrm{j}}\right\}=-\frac{\partial[\mathrm{K}]}{\partial \alpha_\mathrm{j}}\{\mathrm{z}\}-\mathrm{a}_1\frac{\partial[\mathrm{K}]}{\partial \alpha_\mathrm{j}}\{\mathrm{z}_{,\mathrm{t}}\} \tag{22}$$

Where a_1 is the coefficient for Rayleigh damping. Note that Eqs. (20) and (21) have the same form as Eq. (1). The response sensitivities can also be obtained by Newmark method. The initial values of the dynamic responses and the sensitivities can be taken equal to zero.

3.4 Continuum approach

In the continuum approach, the design derivative of the variational equation is taken before it is discretized. If the structural problem and sensitivity equations are solved as a continuum problem, then it is called the continuum-continuum method. The continuum sensitivity equation is solved by discretization in the same way that structural problems are solved. Since differentiation is taken at the continuum domain and is then followed by discretization, this method is called the continuum-discrete method.

3.5 Adjoint variable method

Sensitivity analysis can be performed efficiently by using deterministic methods based on adjoint functions. The use of adjoint functions for analyzing the effects of small perturbations in a linear system was introduced by Wigner (1945).

This method, constructs an adjoint problem that solves for the adjoint variable, which contains all implicitly dependent terms.

For the dynamic response of structure, the following form of a general performance measure will be considered

$$\psi = g(z(T), b) + \int_0^T G(z, b)dt \tag{23}$$

Where the final time *T* is determined by a condition in the form

$$\Omega\big(z(T), z_{,t}(T), b\big) = 0 \tag{24}$$

It is presumed that Eq. (24), uniquely determines *T*, at least locally. This requires that the time derivative of Ω is nonzero at *T*, as

$$\Omega_{,t} = \frac{\partial\Omega}{\partial z} z_{,t}(T) + \frac{\partial\Omega}{\partial z} z_{,tt}(T) \neq 0 \tag{25}$$

To obtain the design sensitivity of *Ψ*, define a design variation in the form

$$b_\tau = b + \tau\delta b \tag{26}$$

Design *b* is perturbed in the direction of *δb* with the parameter *τ*. Substituting b_τ into Eq. (23), the derivative of Eq. (23), can be evaluated with respect to *τ* at *τ*=0. Leibnitz's rule of differentiation of an integral may be used to obtain the following expression

$$\psi' = \frac{\partial g}{\partial b}\delta b + \frac{\partial g}{\partial z}\left[z'(T) + z_{,t}(T)T'\right] + G(z(T), b)T' + \int_0^T \left[\frac{\partial G}{\partial z} z' + \frac{\partial G}{\partial b}\delta b\right] dt$$

$$z' = z'(b, \delta b) \equiv \frac{d}{d\tau} z(t, b + \tau\delta b)|_{\tau=0} = \frac{d}{db}[z(t, b)]\delta b$$

$$T' = T'(b, \delta b) \equiv \frac{d}{d\tau} T(b + \tau\delta b)\Big|_{\tau=0} = \frac{dT}{db}\delta b \tag{27}$$

Note that since the expression in Eq. (23), that determines *T* depends on the design, *T* will also depend on the design. Thus, terms arise in Eq. (27), which involves the derivative of *T* with respect to the design. In order to eliminate these terms, differentiate Eq. (24), with respect to *τ* and evaluate it at *τ*=0 in order to obtain

$$\frac{\partial\Omega}{\partial z}\left[z'(T) + z_{,t}(T)T'\right] + \frac{\partial\Omega}{\partial z_{,t}}\left[z'_{,t}(T) + z_{,tt}(T)T'\right] + \frac{\partial\Omega}{\partial b}\delta b = 0 \tag{28}$$

This equation may also be written as

$$\Omega_{,t}T' = \left[\frac{\partial\Omega}{\partial z} z_{,t}(T) + \frac{\partial\Omega}{\partial z_{,t}} z_{,tt}(T)\right] T' = -\left(\frac{\partial\Omega}{\partial z} z'(T) + \frac{\partial\Omega}{\partial z_{,t}} z'_{,t}(T) + \frac{\partial\Omega}{\partial b}\delta b\right) \tag{29}$$

Since it is presumed by Eq. (25), that $\Omega_{,t} \neq 0$, then

$$T' = -\frac{1}{\Omega_{,t}}\left(\frac{\partial\Omega}{\partial z} z'(T) + \frac{\partial\Omega}{\partial z_{,t}} z'_{,t}(T) + \frac{\partial\Omega}{\partial b}\delta b\right) \tag{30}$$

Substituting the result of Eq. (30), into Eq. (27), the following is obtained:

$$\psi' = [\frac{\partial g}{\partial z} - \left(\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T), b) \frac{1}{\Omega_{,t}} \frac{\partial \Omega}{\partial z}\right] z'(T) - \left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T), b)\right] \frac{1}{\Omega_{,t}} \frac{\partial \Omega}{\partial z_{,t}} z'_{,t}(T) +$$

$$\int_0^T \left[\frac{\partial G}{\partial z} z' + \frac{\partial G}{\partial b} \delta b\right] dt + \frac{\partial g}{\partial b} \delta b - \left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T), b)\right] \frac{1}{\Omega_{,t}} \frac{\partial \Omega}{\partial b} \delta b \qquad (31)$$

Note that *Ψ'* depends on *z'* and $z'_{,t}$ at *T*, as well as on *z'* within the integration.

In order to write *Ψ'* in Eq. (30), explicitly in terms of a design variation, the adjoint variable technique can be used. In the case of a dynamic system, all terms in Eq. (1), can be multiplied by $\lambda^T(t)$ and integrated over the interval [0, *T*], to obtain the following identity in *λ*

$$\int_0^T \lambda^T \left[M(b) z_{,tt} + C(b) z_{,t} + K(b) z - F(t, b)\right] dt = 0 \qquad (32)$$

Since this equation must hold for arbitrary *λ*, which is now taken to be independent of the design, substitute b_τ into Eq. (32), and differentiate it with respect to *τ* in order to obtain the following relationship

$$\int_0^T \left[\lambda^T M(b) z'_{,tt} + \lambda^T C(b) z'_{,t} + \lambda^T K(b) z' - \frac{\partial R}{\partial b} \delta b\right] dt = 0 \qquad (33)$$

Where

$$R = \tilde{\lambda}^T F(t, b) - \tilde{\lambda}^T M(b) \tilde{z}_{,tt} - \tilde{\lambda}^T C(b) \tilde{z}_{,t} - \tilde{\lambda}^T K(b) \tilde{z} \qquad (34)$$

With the superposed tilde (~) denoting variables that are held constant during the differentiation with respect to the design in Eq. (33).

Since Eq. (33), contains the time derivatives of *z'*, integrate the first two integrands by parts in order to move the time derivatives to *λ*, as

$$\lambda^T M(b) z'_{,t}(T) - \lambda_{,t}^T(T) M(b) z'(T) + \lambda^T C(b) z'(T) +$$

$$\int_0^T \left\{\left[\lambda_{,tt}^T M(b) - \lambda_{,t}^T C(b) + \lambda^T K(b)\right] z' - \frac{\partial R}{\partial b} \delta b\right\} dt = 0 \qquad (35)$$

The adjoint variable method expresses the unknown terms in Eq. (31), in terms of the adjoint variable (*λ*). Since Eq. (35), must hold for arbitrary functions *λ*(*t*), *λ* may be chosen so that the coefficients of terms involving *z'*(*T*),$z'_{,t}(T)$ and *z'* in Eq. (31), and Eq. (35), are equal. If such a function *λ*(*t*) can be found, then the unwanted terms in Eq. (31), involving *z'*(*T*), $z'_{,t}(T)$ and *z'* can be replaced by terms that explicitly depend on *δb* in Eq. (35), To be more specific, choose a *λ*(*t*) that satisfies the following

$$M(b)\lambda(T) = -\left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T), b)\right] \frac{1}{\Omega_{,t}} \frac{\partial \Omega^T}{\partial z_{,t}} \qquad (36)$$

$$M(b)\lambda_{,t}(T) = C^T(b)\lambda(T) - \frac{\partial g^T}{\partial z} + \left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T), b)\right] \frac{1}{\Omega_{,t}} \frac{\partial \Omega^T}{\partial z} \qquad (37)$$

$$M(b)\lambda_{,tt} - C^T(b)\lambda_{,t} + K(b)\lambda = \frac{\partial G^T}{\partial z}, 0 \le t \le T \qquad (38)$$

Note that once the dynamic equation of Eq. (1), is solved and Eq. (24), is used to determine *T*, then *z*(*T*), $z_{,t}(T)$, $\frac{\partial \Omega}{\partial z}$, $\frac{\partial \Omega}{\partial z_{,t}}$ and $\Omega_{,t}$ may be evaluated. Eq. (25), can then be solved for *λ*(*T*) since the mass matrix *M*(*b*) is nonsingular. Having determined *λ*(*T*), all terms on the right of Eq. (37), can be evaluated, and the equation can be solved for $\lambda_{,t}(T)$. Thus, a set of terminal conditions on λ has been determined. Since $M(b)$ is nonsingular, Eq. (38), may then be integrated from T to 0, yielding the unique solution $\lambda(t)$. Taken as a whole, Eq. (36), through Eq. (38), may be thought of as a terminal value problem.

Since the terms involving a variation in the state variable in Eq. (31), and Eq. (35), are identical, substitute Eq. (35), into Eq. (31), to obtain

$$\psi' = \frac{\partial g}{\partial b}\delta b + \int_0^T \left[\frac{\partial G}{\partial b} + \frac{\partial R}{\partial b}\right] dt\delta b - [\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T), b)] \frac{1}{\Omega_{,t}} \frac{\partial \Omega}{\partial b}\delta b \equiv \frac{\partial \psi}{\partial b}\delta b \tag{39}$$

Every term in this equation can now be calculated. The terms $\frac{\partial g}{\partial b}$, $\frac{\partial G}{\partial b}$ and $\frac{\partial \Omega}{\partial b}$ represent explicit partial derivatives with respect to the design. The term $\frac{\partial R}{\partial b}$, however, must be evaluated from Eq. (34), thus requiring $\lambda(t)$. Note also that since design variation δb does not depend on time, it is taken outside the integral in Eq. (39).

Since Eq. (39), must hold for all δb, the design derivative vector of Ψ is

$$\frac{d\psi}{db} = \frac{\partial g}{\partial b}(z(T), b) + \int_0^T [\frac{\partial G}{\partial b}(z, b) + \frac{\partial R}{\partial b}(\lambda(t), z(t), z_{,t}(t), z_{,tt}(t), b)]dt$$
$$- \frac{1}{\Omega_{,t}}\left[\frac{\partial g}{\partial z} z_{,t}(T) + G(z(T), b)\right]\frac{\partial \Omega}{\partial b} \tag{40}$$

3.6 Sensitivity method selection

The advantage of the finite difference method is obvious. If structural analysis can be performed and the performance measure can be obtained as a result of structural analysis, then the expressions in Eq. (15) and Eq. (16) are virtually independent of the problem types considered.

Major disadvantage of the finite difference method is the accuracy of its sensitivity results. Depending on perturbation size, sensitivity results are quite different. For a mildly nonlinear performance measure, relatively large perturbation provides a reasonable estimation of sensitivity results. However, for highly nonlinear performances, a large perturbation yields completely inaccurate results. Thus, the determination of perturbation size greatly affects the sensitivity result. Although it may be necessary to choose a very small perturbation, numerical noise becomes dominant for a too-small perturbation size. That is, with a too-small perturbation, no reliable difference can be found in the analysis results.

The continuum-continuum approach is so limited and is not applicable in complex engineering structures because very simple, classical problems can be solved analytically.

The discrete and continuum-discrete methods are equivalent under the conditions given below. (Choi and Kim 2005).

First, the same discretization (shape function) used in the FE method must be used for continuum design sensitivity analysis. Second, an exact integration (instead of a numerical integration) must be used in the generation of the stiffness matrix and in the evaluation of continuum-based design sensitivity expressions. Third, the exact solution (and not a numerical solution) of the FE matrix equation and the adjoint equation should be used to compare these two methods. Fourth, the movement of discrete grid points must be consistent with the design parameterization method used in the continuum method.

In this paper two different analytical discrete methods, including DDM and AVM are presented and efficiency of the proposed method is investigated with compared to DDM.

4. Proposed method

While structural vibration responses are used for damage detection, assuming *G*=0, Eq. (38), is a free vibration of beam with terminal conditions. Solving Eq. (38), for a single degree of freedom system is as follow

$$m\lambda_{,tt} - c\lambda_{,t} + k\lambda = 0 \text{ with terminal conditions: } \lambda(T), \dot{\lambda}(T)$$

$$\lambda_T(t) = e^{\xi\omega(t-T)}\left(A_1 \sin(\omega_D t) + B_1 \cos(\omega_D t)\right) \tag{41}$$

$$\begin{cases} A_1 = \left(\dfrac{\lambda_{,t}(T)}{\omega_D} - \dfrac{\xi}{\sqrt{1-\xi^2}}\lambda(T)\right)\cos(\omega_D T) + \lambda(T)\sin(\omega_D T) \\ B_1 = \dfrac{\lambda(T)}{\cos(\omega_D t)} - A_1 \tan(\omega_D T) \end{cases} \tag{42}$$

In which

$$\xi = {}^{c}\!/_{2m\omega} = {}^{c}\!/_{c_{cr}} < 1 \text{ and } \omega_D = \omega\sqrt{1-\xi^2}$$

When time *T* is known, the coefficients of the characteristic equation of *T'* and thereupon Ω will be zero, so the terminal conditions are as follow

$$\lambda(T) = 0 \tag{43}$$

$$\lambda_{,t}(T) = M^{-1}(b) \times \left(-\frac{\partial g^T}{\partial z}\right) \tag{44}$$

Substitute Eq. (43), into Eq. (44), to obtain

$$A_1 = \frac{\lambda_{,t}(T)}{\omega_D}\cos(\omega_D T) \;\; \& \; B_1 = -\frac{\lambda_{,t}(T)}{\omega_D}\sin(\omega_D T) \tag{45}$$

Note that $\frac{\partial g}{\partial z}$ like A_1 and B_1 is dependent to time *T*, so terminal values for different amounts of *T* are not similar and adjoint equation should be calculated for all amounts of *T* separately. So

$$\lambda_T(t) = e^{\xi\omega(t-T)}\left(\frac{\lambda_{,t}(T)}{\omega_D}\cos(\omega_D T)\sin(\omega_D t) - \frac{\lambda_{,t}(T)}{\omega_D}\sin(\omega_D T)\cos(\omega_D t)\right) = P_T f(t) + Q_T g(t)$$

$$P_T = e^{-\xi\omega T}\frac{\lambda_{,t}(T)}{\omega_D}\cos(\omega_D T) \;\; f(t) = e^{\xi\omega t}\sin(\omega_D t) \;\; Q_T = -e^{-\xi\omega T}\frac{\lambda_{,t}(T)}{\omega_D}\sin(\omega_D T) \;\; g(t) = e^{\xi\omega t}\cos(\omega_D t) \tag{46}$$

4.1 Sensitivity matrix for physical parameter

Using Eq. (40), assuming *T* is known and *G*=0 because of using structural vibration data, Eq. (47), can be obtained

$$\frac{d\psi}{db} = \int_0^T \frac{\partial R}{\partial b} dt \tag{47}$$

In this equation:

$R = \tilde{\lambda}^T F(t) - \tilde{\lambda}^T M \tilde{z}_{,tt} - \tilde{\lambda}^T C(b) \tilde{z}_{,t} - \tilde{\lambda}^T K(b) \tilde{z}$ and $C = a_0 K(b) + a_1 M$ is Rayleigh damping matrix.

Performing differentiation to Eq. (34) with respect to the j^{th} physical parameter, α_j, of the j^{th} element, we have

$$\frac{\partial R}{\partial b} = \frac{\partial R}{\partial \alpha_j} = -\widetilde{\lambda^T} a_0 \frac{\partial K}{\partial \alpha_j} \widetilde{z_{,t}} - \widetilde{\lambda^T} \frac{\partial K}{\partial \alpha_j} \tilde{z} \tag{48}$$

And component of sensitivity matrix in time T is

$$\frac{d\psi}{d\alpha_j}(T) = \int_0^T (-\widetilde{\lambda^T} a_0 \frac{\partial K}{\partial \alpha_j} \widetilde{z_{,t}} - \widetilde{\lambda^T} \frac{\partial K}{\partial \alpha_j} \tilde{z}) dt \tag{49}$$

In a multi degree of freedom problem, solving the above equations directly is not possible. For this purpose, change the variables as follow

$$\{\lambda\} = [\phi]\{Y\} \tag{50}$$

In this equation matrix $[\varphi]$ forms vibration modes (modal matrix) and terminal conditions of above equations are

$$\{Y(T)\} = M^{-1}[\phi]^T[m]\{\lambda(T)\} \tag{51}$$

$$\{Y_{,t}(T)\} = M^{-1}[\phi]^T[m]\{\lambda_{,t}(T)\} \tag{52}$$

By inserting Eq. (50) in Eq. (38) and multiplying $[\phi]^T$ in both sides, the new equation in modal space is

$$[M]\{Y_{,tt}\} - [C]\{Y_{,t}\} + [K]\{Y\} = \{0\} \tag{53}$$

Each of $[M]$, $[C]$ and $[K]$ matrices is diagonal, so

$$M_i\{Y_{,tt_i}\} - C_i\{Y_{,t_i}\} + K_i\{Y_i\} = \{0\} \tag{54}$$

$$\frac{d\psi}{d\alpha_j}(T) = -\int_0^T \langle Y\rangle \times [\phi]^T \times a_0 \left[\frac{\partial k}{\partial \alpha_j}\right] \times \{z_{,t}\} + \langle Y\rangle \times [\phi]^T \times \left[\frac{\partial k}{\partial \alpha_j}\right] \times \{z\} dt \tag{55}$$

Consider: $[\phi]^T \times a_0 \left[\frac{\partial k}{\partial \alpha_j}\right] \times \{z_{,t}\} = \{zz_{,t}\}$ and $[\phi]^T \times \left[\frac{\partial k}{\partial \alpha_j}\right] \times \{z\} = \{zz\}$

Eq. (55) can be reduced to Eq. (56)

$$\frac{d\psi}{d\alpha_j}(T) = -\int_0^T \langle Y\rangle \times \{zz_{,t}\} + \langle Y\rangle \times \{zz\} dt \tag{56}$$

From Eq. (46) variable Y in modal space can be written as

$$\{Y\} = \{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\} \tag{57}$$

Replacing Eq. (57) in Eq. (56) a new expression is derived to calculate the sensitivity.

$$\frac{\mathrm{d}\psi}{\mathrm{d}\alpha_j}(\mathrm{T}) = -\int_0^{\mathrm{T}} (\{\mathrm{P(T)}\}.\{\mathrm{f(t)}\} + \{\mathrm{Q(T)}\}.\{\mathrm{g(t)}\})^{\mathrm{T}} \times \{\mathrm{zz}_{,t}\} + (\{\mathrm{P(T)}\}.\{\mathrm{f(t)}\} + \{\mathrm{Q(T)}\}.\{\mathrm{g(t)}\})^{\mathrm{T}} \times \{\mathrm{zz}\}\mathrm{dt} \quad (58)$$

Eq. (58) can be rewritten as follow

$$\frac{\mathrm{d}\psi}{\mathrm{d}\alpha_j}(\mathrm{T}) = -\int_0^{\mathrm{T}} \langle \mathrm{P(T)}\rangle \times (\{\mathrm{f(t)}\}.\{\mathrm{zz}_{,t}\} + \{\mathrm{f(t)}\}.\{\mathrm{zz}_{,t}\}) + \langle \mathrm{Q(T)}\rangle \times \left(\{\mathrm{g(t)}\}.\{\mathrm{zz}_{,t}\} + \{\mathrm{g(t)}\}.\{\mathrm{zz}_{,t}\}\right)\mathrm{dt} \quad (59)$$

Consider following parameters

$$\mathrm{A} = \int_0^{\mathrm{T}} \{\mathrm{f(t)}\}.\{\mathrm{zz}_{,t}\}\,\mathrm{dt} \quad \mathrm{B} = \int_0^{\mathrm{T}} \{\mathrm{g(t)}\}.\{\mathrm{zz}_{,t}\}\,\mathrm{dt} \quad \mathrm{C} = \int_0^{\mathrm{T}} \{\mathrm{f(t)}\}.\{\mathrm{zz}\}\,\mathrm{dt} \quad \mathrm{D} = \int_0^{\mathrm{T}} \{\mathrm{g(t)}\}.\{\mathrm{zz}\}\,\mathrm{dt}$$

So, Eq. (59) is presented as

$$\frac{\mathrm{d}\psi}{\mathrm{d}\alpha_j}(\mathrm{T}) = -\langle \mathrm{P(T)}\rangle \times (\{\mathrm{A}\} + \{\mathrm{C}\}) - \langle \mathrm{Q(T)}\rangle \times (\{\mathrm{B}\} + \{\mathrm{C}\}) \quad (60)$$

Solution of Eq. (60) directly is too time consuming, because in each time step all terms in Eq. (60) should be recalculted. Therefore, an incremental solution is developed as follow

$$\{\mathrm{A}_{\mathrm{T}+\Delta\mathrm{T}}\} = \int_0^{\mathrm{T}+\Delta\mathrm{T}} \{\mathrm{f(t)}\}.\{\mathrm{zz}_{,t}\}\,dt = \int_0^{\mathrm{T}} \{\mathrm{f(t)}\}.\{\mathrm{zz}_{,t}\}\,\mathrm{dt} + \int_{\mathrm{T}}^{\mathrm{T}+\Delta\mathrm{T}} \{\mathrm{f(t)}\}.\{\mathrm{zz}_{,t}\}\,\mathrm{dt} \quad (61)$$

$$\{\mathrm{A}_{\mathrm{T}+\Delta\mathrm{T}}\} = \{\mathrm{A}_{\mathrm{T}}\} + \{\delta\mathrm{A}\}, \{\delta\mathrm{A}\} = \int_{\mathrm{T}}^{\mathrm{T}+\Delta\mathrm{T}} \{\mathrm{f(t)}\}.\{\mathrm{zz}_{,t}\}\,\mathrm{dt} \cong \left\{\mathrm{f}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\}.\left\{\mathrm{zz}_{,t}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\} \quad (62)$$

Similar to Eq. (60) for other parameters we have

$$\{\delta\mathrm{B}\} = \int_{\mathrm{T}}^{\mathrm{T}+\Delta\mathrm{T}} \{\mathrm{g(t)}\}.\{\mathrm{zz}_{,t}\}\,\mathrm{dt} \cong \left\{\mathrm{g}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\}.\left\{\mathrm{zz}_{,t}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\} \quad (63)$$

$$\{\delta\mathrm{C}\} = \int_{\mathrm{T}}^{\mathrm{T}+\Delta\mathrm{T}} \{\mathrm{f(t)}\}.\{\mathrm{zz}\}\,\mathrm{dt} \cong \left\{\mathrm{f}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\}.\left\{\mathrm{zz}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\} \quad (64)$$

$$\{\delta\mathrm{D}\} = \int_{\mathrm{T}}^{\mathrm{T}+\Delta\mathrm{T}} \{\mathrm{g(t)}\}.\{\mathrm{zz}\}\,\mathrm{dt} \cong \left\{\mathrm{g}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\}.\left\{\mathrm{zz}\left(\mathrm{T} + \frac{\Delta\mathrm{T}}{2}\right)\right\} \quad (65)$$

And finally the sensitivity expression in time $T + \Delta T$ is

$$\frac{\mathrm{d}\psi}{\mathrm{d}\alpha_j}(\mathrm{T} + \Delta\mathrm{T}) = -\langle \mathrm{P}(\mathrm{T} + \Delta\mathrm{T})\rangle \times (\{\mathrm{A}_{\mathrm{T}+\Delta\mathrm{T}}\} + \{\mathrm{C}_{\mathrm{T}+\Delta\mathrm{T}}\}) - \langle \mathrm{Q}(\mathrm{T} + \Delta\mathrm{T})\rangle \times (\{\mathrm{B}_{\mathrm{T}+\Delta\mathrm{T}}\} + \{\mathrm{D}_{\mathrm{T}+\Delta\mathrm{T}}\}) \quad (66)$$

4.2 Sensitivity matrix for excitation force

Performing differentiation to Eq. (34) with respect to the parameters of the i^{th} excitation force, we have

$$\frac{\partial R}{\partial b} = \frac{\partial R}{\partial P_i} = \widetilde{\lambda^T} B \tag{67}$$

And component of sensitivity matrix in time T is

$$\frac{d\psi}{dP_i}(T) = \int_0^T \widetilde{\lambda^T} B\, dt \tag{68}$$

Using modal space and Eq. (57) we have

$$\frac{d\psi}{dP_i}(T) = \int_0^T \langle Y \rangle \times [\phi]^T \times [B] dt \tag{69}$$

Consider: $[\phi]^T \times [B] = \{BB\}$ using Eq. (57) in Eq. (69) a new expression is derived to calculate the sensitivity.

$$\frac{d\psi}{dP_i}(T) = \int_0^T (\{P(T)\}.\{f(t)\} + \{Q(T)\}.\{g(t)\})^T \times \{BB\}\, dt \tag{70}$$

Eq. (70) can be rewritten as follow

$$\frac{d\psi}{dP_i}(T) = \int_0^T \langle P(T) \rangle \times (\{f(t)\}.\{BB\}) + \langle Q(T) \rangle \times (\{g(t)\}.\{BB\}) dt \tag{71}$$

Consider following parameters: $E = \int_0^T \{f(t)\}.\{BB\}\, dt$ $F = \int_0^T \{g(t)\}.\{BB\}\, dt$

So, Eq. (71) is presented as

$$\frac{d\psi}{dP_i}(T) = \langle P(T) \rangle \times \{E\} + \langle Q(T) \rangle \times \{F\} \tag{72}$$

Using incremental solution is as follow

$$\{E_{T+\Delta T}\} = \int_0^{T+\Delta T} \{f(t)\}.\{BB\}\, dt = \int_0^T \{f(t)\}.\{BB\}\, dt + \int_T^{T+\Delta T} \{f(t)\}.\{BB\}\, dt \tag{73}$$

$$\{E_{T+\Delta T}\} = \{E_T\} + \{\delta E\}\,, \{\delta E\} = \int_T^{T+\Delta T} \{f(t)\}.\{BB\}\, dt \cong \left\{f\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{BB\left(T + \frac{\Delta T}{2}\right)\right\} \tag{74}$$

Similar to Eq. (74) for other parameter we have

$$\{\delta F\} = \int_T^{T+\Delta T} \{g(t)\}.\{BB\}\, dt \cong \left\{g\left(T + \frac{\Delta T}{2}\right)\right\}.\left\{BB\left(T + \frac{\Delta T}{2}\right)\right\} \tag{75}$$

And finally the sensitivity expression in time $T + \Delta T$ is

$$\frac{d\psi}{dP_i}(T + \Delta T) = \langle P(T + \Delta T) \rangle \times \{E_{T+\Delta T}\} + \langle Q(T + \Delta T) \rangle \times \{F_{T+\Delta T}\} \tag{76}$$

4.3 Computational algorithm

The computational algorithm that leads to the determination of sensitivity matrix is as follow:

- Step1: Calculate $\lambda_{,t}(T)$ from Eq. (44)

- Step2: Calculate ω, ω_D *and* ϕ from and consider *i*=1
- Step3: For the i th element calculate $\frac{\partial K}{\partial b}$ and $zz_{,t}, zz$ and consider *j*=1
- Step4: For the j th sensor and corresponding Dof's calculate $\lambda_{,t}(T)$ from step1 and $Y_{,t}(T)$ from Eq. (52) and T_n=Δt and T_o=0
- Step5: Consider *A*=*B*=*C*=*D*=*E*=*F*=0
- Step6: Calculate $T_m = T_0 + \frac{\Delta t}{2}$ and Calculate $P(T_n) - Q(T_n) - f(T_m) - g(T_m)$ from Eq. (46)
- Step7: Calculate $\{\delta A\}, \{\delta B\}, \{\delta C\}, \{\delta D\}, \{\delta E\}$ and $\{\delta F\}$ from Eq. (61~64 and 74~75)
- Step8: Calculate $\frac{d\psi}{d\alpha_i}(T_n)$ from Eq. (66) and $\frac{d\psi}{dP_i}$ from Eq. (76)
- Step9: If $T_n < T_{final}$ Consider $T_0 = T_n$ and $T_n = T_n + \Delta t$ and go to step5 otherwise go to next step
- Step10: If $j < number\ of\ sensors$ Consider j=j+1 and go to step 4 otherwise go to next step
- Step11: If $i < number\ of\ elements$ Consider i=i+1 and go to step 3 otherwise finish.

4.4 Procedure of iteration for force identification and damage detection

The proposed method requires measurement from two states of the structure. The first set of measurement from the undamaged structure serves to update the system parameters with a known set of force input. While in the measurement on the second state of damage, both the excitation force and the damaged structure are unknown, and the following iterative algorithm is used in the identification (Lu and Law 2007). The updated finite element model and excitation force in ith iteration step serves as the reference model in the subsequent comparison.

(A) Iteration for excitation force parameters

Starting with an initial guess on the unknown force parameter vector P_0 and the set of physical parameter α_0 from the updated FE model of the structure, the procedure of iteration is given as:

- Step 1: With the initial force vector and vector of the undamaged system, Eq. (1) is solved at $j = k + 1$ iteration step for the displacement vector $\{z\}$ using Newmark method and subsequently for the acceleration vector $\{\ddot{z}\}$ and the error vector $\{\delta\ddot{z}\}$.
- Step 2: The sensitivity matrix $[S_F]$ of the response with respect to the force is obtained from Eq. (76) and proposed algorithm for $j = k + 1$ iteration step with the force vector $\{P_k\}$ obtained from a previous step.
- Step 3: Find $\{P_{k+1}\}$ from Eq. (16).
- Step 4: Repeat steps 1~3 until the following convergence criteria are satisfied.

$$\frac{\|P_{k+1} - P_k\|}{\|P_{k+1}\|} \times 100\% \ \leq \mathrm{Tol1} \tag{77}$$

- Step 5: The final vector $\{P_{k+1}\}$ obtained is taken as the modified set of force $\{P\}$ for the second stage of iteration.

(B) Iteration for the physical parameters of the structure

With the modified excitation force parameter vector $\{P\}$ obtained from (A) above, the set of physical parameters is then obtained as below:

- Step 6: The vector of physical parameter $\{\alpha_s\}$ from the updated finite element model of the structure is taken as the set of initial values. Eq. (1) is solved at $j = k + 1$iteration step for the displacement vector {z} by Newmark method and subsequently for the acceleration vector and the error vector $\{\ddot{z}\}$.
- Step 7: The sensitivity matrix $[S_s]$ of the response with respect to the different physical parameters of the structure is obtained from Eq. (66) and proposed method at $j = k + 1$ iteration step with the initial

physical parameter vector $\{\alpha_k\}$ obtained from a previous step.
- Step 8: Find $\{\alpha_{k+1}\}$ from Eq. (18).
- Step 9: Repeat steps 6~8 until the following convergence criteria are reached.

$$\frac{||\alpha_{k+1} - \alpha_k||}{||\alpha_k||} \times 100\% \leq \text{Tol2} \tag{78}$$

$$\frac{||\text{Response}_{k+1} - \text{Response}_k||}{||\text{Response}_k||} \times 100\% \leq \text{Tol3} \tag{79}$$

- Step 10: The final vector $\{\alpha_{k+1}\}$ obtained is taken as the modified set of physical parameters $\{\alpha_k\}$ for the next cycle of iteration on the force parameters.

The identified excitation force obtained in (*A*) should be further improved using the updated physical parameters obtained in (*B*) and repeating steps 1~5 and the vector of physical parameters should also be further improved using the modified excitation force and repeating steps 6~10.

This iteration procedure continuous until the following convergence criteria is reached.

$$\frac{||P_{i+1} - P_i||}{||P_{i+1}||} \times 100\% \leq \text{Tol4} \tag{80}$$

$$\frac{||\alpha_{i+1} - \alpha_i||}{||\alpha_i||} \times 100\% \leq \text{Tol5} \tag{81}$$

The convergence of this computation strategy has been proved by Li and Chen (1999). All tolerances is set equal to 1×10^{-6} in this study except otherwise specified.

5. Numerical results

To illustrate the formulations presented in the previous sections, we consider the system shown in Figs. 3 and 7, and capabilities of proposed method are investigated.

The Relative Percentage Error for Physical parameter (RPEP) and Excitation Force (RPEF) in the identified results is calculated from Eq. (82), where $||.||$ is the norm of matrix, $E_{Identified}$ and E_{True} are the identified and the true elastic modulus respectively and $P_{Identified}$ and P_{True} are the identified and the true excitation force respectively.

$$\text{RPEP} = \frac{||E_{\text{Identified}} - E_{\text{True}}||}{||E_{\text{True}}||} \times 100\% \;\&\; RPEF = \frac{||P_{\text{Identified}} - P_{\text{True}}||}{||P_{\text{True}}||} \times 100\% \tag{82}$$

Since the true value of elastic modulus is unknown, RPEP and RPEF can just be used for investigating the efficiency of method.

5.1 Multi span model

A two-span bridge as shown in Fig. 3 is studied to illustrate the proposed method. It consists of 20 Euler-Bernoulli beam elements with 21 nodes each with two DOF's. The mass density of material is 7.8×10^3 kg/m^3 and the elastic modulus of material is 2.1×10^7 N/ cm^2. The total length of bridge is 20 m and height and width of the frame section are respectively 200 and 200 mm. The first five un-damped natural frequencies of the intact bridge are 29.3829, 45.8299, 117.3834, 148.1623 and 265.0938 Hz. Rayleigh damping model is adopted with the damping ratios of the first two modes taken equal to 0.05. The equivalent Rayleigh coefficients a_0 and a_1 are respectively 0.1 and 4.6413×10^{-6}.

The transverse point load P has a constant velocity, $= L/T$, Where T is the traveling time across the bridge and L is the total length of the bridge.

The integration parameters $\beta = 1/4$ and $\gamma = 1/2$ is used for Newmark method, which lead to constant-average acceleration approximation. Speed ratio is defined as

$$\alpha = \frac{V}{V_{cr}} \tag{83}$$

In which V_{cr} is critical speed ($V_{cr} = \frac{\pi}{l}\sqrt{\frac{EI}{\rho}}$), V is moving load speed and ρ is mass per unit length of beam.

5.1.1 Damage scenarios

Five damage scenarios of single, multiple and random damages in the bridge without measurement noise are studied and they are shown in Table 1.

Local damage is simulated with a reduction in the elastic modulus of material of an element. The sampling rate is 10000 Hz and 600 data of the acceleration response (degree of indeterminacy is 20) collected along the z-direction at nodes 2, 8, 12 and 18 are used in the identification.

Scenario 1 studies the single damage scenario. The proposed method converges in all speed ranges. Maximum relative percentage of error for physical parameter (RPEP) is 0.5972 and

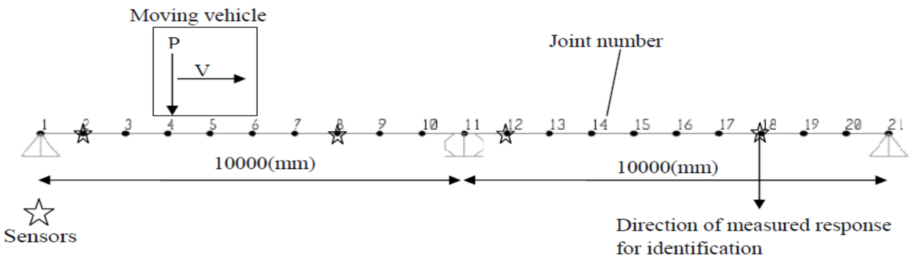

Fig. 3 Multi span bridge model used in detection procedure

Table1 Damage scenarios for multi-span bridge

Damage scenario	Damage type	Damage location	Reduction in elastic modulus	Noise
M1-1	Single	14	20%	Nil
M1-2	Multi	8,13,17	11%,4%,7%	Nil
M1-3	Multi	3,7,11,15,18	2%,6%,5%,2%,8%	Nil
M1-4	Random	All elements	Random damage in all elements with an average of 5%	Nil
M1-5	Random	All elements	Random damage in all elements with an average of 10%	Nil
M1-6	Estimation of undamaged state	All elements	5% reduction in all elements	Nil

Table 2 RPEP of AVM method for model1

Damage scenario	Speed ratio								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M1-1	0.5031	0.4890	0.4168	0.4931	0.4737	0.4226	0.5031	0.4890	0.4168
M1-2	0.5465	0.4159	0.4145	0.4492	0.4833	0.4180	0.5465	0.4159	0.4145
M1-3	0.5972	0.4129	0.4185	0.4153	0.4471	0.3441	0.5972	0.4129	0.4185
M1-4	0.4445	0.4156	0.4186	0.3852	0.3951	0.3026	0.4445	0.4156	0.4186
M1-5	0.2876	0.3624	0.2888	0.3736	0.2124	0.3048	0.2876	0.3624	0.2888
M1-6	0.2812	0.2729	0.3078	0.2870	0.2774	0.3044	0.2812	0.2729	0.3078

Table 3 RPEF of AVM method for model1

Damage scenario	Speed ratio								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M1-1	0.0039	0.0110	0.0105	0.0115	0.0112	0.0133	0.0039	0.0110	0.0105
M1-2	0.0044	0.0095	0.0084	0.0097	0.0131	0.0114	0.0044	0.0095	0.0084
M1-3	0.0049	0.0091	0.0112	0.0095	0.0085	0.0098	0.0049	0.0091	0.0112
M1-4	0.0030	0.0093	0.0105	0.0025	0.0107	0.0061	0.0030	0.0093	0.0105
M1-5	0.0026	0.0027	0.0025	0.0025	0.0021	0.0027	0.0026	0.0027	0.0025
M1-6	0.0025	0.0024	0.0025	0.0024	0.0025	0.0026	0.0025	0.0024	0.0025

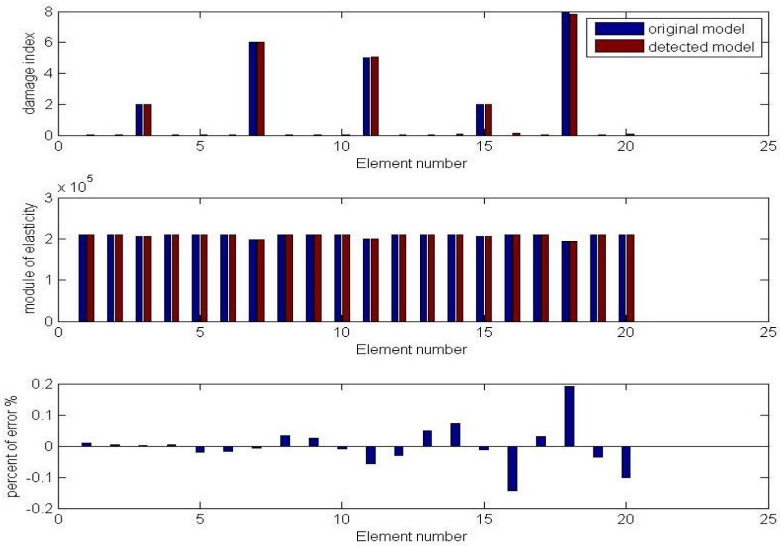

Fig. 4 Detection of damage location and amount in elements 3, 7, 11, 15 and 18 and distribution of error in different elements with AVM scheme

maximum relative percentage of error for input excitation force (RPEF) is 0.0049.

Scenarios 2 and 3 are on multiple damages with different amount of measured responses for the identification and scenarios 4-5 are on random damages with different average for the identification. These scenarios also converge in all speed ranges with maximum RPEP of 0.4931 and maximum RPEF of 0.0131. One more scenario with model error is also included as scenario 6. This scenario consists of no simulated damage in the structure, but with the initial elastic modulus of material of all the elements under-estimated by 5% in the inverse identification.

Using proposed method, the damage locations and amount are identified correctly in all the scenarios (Fig. 4) and the RPEP and RPEF parameters are shown in Tables 2 and 3.

5.1.2 Effect of noise

To evaluate the sensitivity of results to measurement noise, noise-polluted measurements are simulated by adding to the noise-free acceleration vector a corresponding noise vector whose Root-Mean-Square (RMS) value is equal to a certain percentage of the RMS value of the noise-free data vector. The components of all the noise vectors are of Gaussian distribution, uncorrelated and with a zero mean and unit standard deviation. Then on the basis of the noise-free acceleration $\ddot{z}_{,tt_{nf}}$; the noise-polluted acceleration $\ddot{z}_{,tt_{np}}$ of the bridge at location *x* can be simulated by

$$\ddot{z}_{,tt_{np}} = \ddot{z}_{,tt_{nf}} + \mathrm{RMS}\left(\ddot{z}_{,tt_{nf}}\right) \times N_{level} \times N_{unit} \tag{84}$$

Where RMS ($\ddot{z}_{,tt_{nf}}$) is the RMS value of the noise-free acceleration vector $\ddot{z}_{,tt_{nf}}$ $\times N_{level}$ is the noise level, and N_{unit} is a randomly generated noise vector with zero mean and unit standard deviation. (Jiang *et al*. 2004)

In order to study effect of noise in stability of sensitivity methods, scenario2 (speed ratio of moving load is considered to be fix and equal with 0.3) is considered and different levels of noise pollution are investigated, and RPEP changes with increasing number of loops for iterative procedure has been studied.

Results are illustrated in Fig. 5 for DDM and AVM methods respectively. These contours show that both AVM and DDM methods are sensitive to noise and if noise level becomes greater than 1.4% these methods lose their effectiveness and are not able to detect damage. So, in cases with noise level greater than 1.4%, a denoising tool alongside sensitivity methods should be used.

5.1.3 Efficiency of proposed method

In order to compare and quantification the performance of different methods and evaluate the proposed method, Relative Efficiency Parameter (REP) is defined as

$$\mathrm{REP} = \mathrm{ST}_{\mathrm{DDM}}/\mathrm{ST}_{\mathrm{AVM}} \tag{85}$$

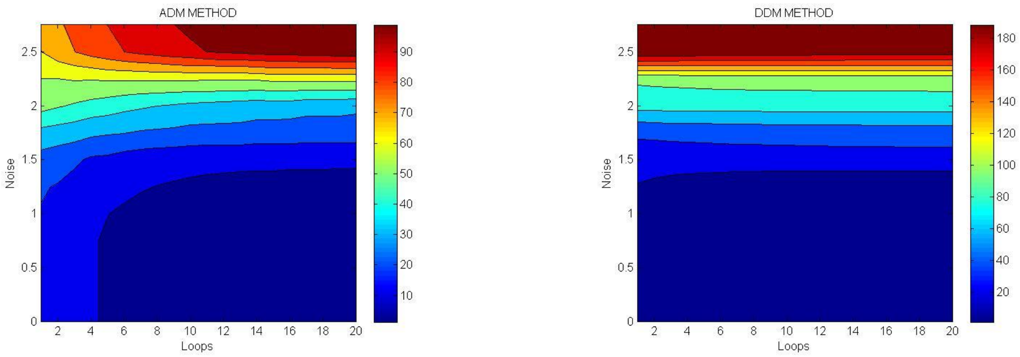

Fig. 5 RPE contours with respect to noise level and loops

Table 4 REP ranges in different scenarios

Damage scenario	Max REP	Min REP	average
M1-1	4.722087	2.367644	3.457256
M1-2	3.884751	2.192863	2.827125
M1-3	3.453202	2.016751	2.615719
M1-4	4.028332	2.211189	2.854083
M1-5	3.179909	1.831374	2.245494
M1-6	2.735529	1.336874	1.915498
Total	4.722087	1.831374	2.652529

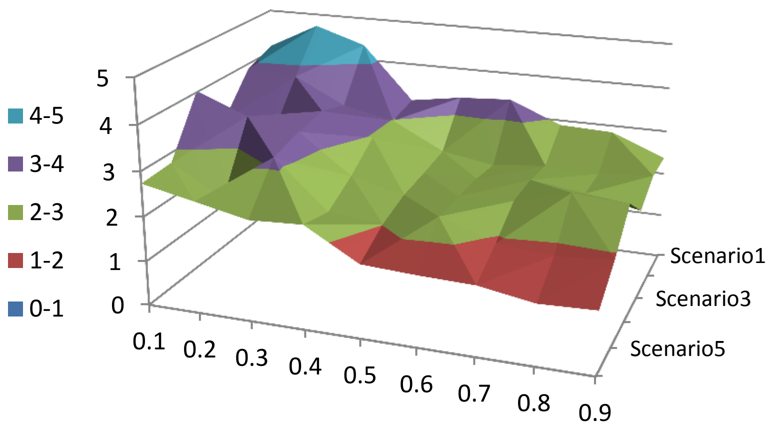

Fig. 6 REP changes in different scenarios with respect to speed ratio

In which, ST is the solution time of SI method. In fact this parameter represents the computation cost of method. Table 4 shows that in different scenarios and for different speed ratio, the efficiency parameter is between 1.831 to 4.722 and its average is 2.653, therefore the AVM method is extremely successful and the computational cost for this method is about 37.7% of other sensitivity base finite element model updating method. Fig. 6 shows the REP changes with respect to speed ratio in different scenarios.

5.2 Plane grid model

A plane grid model of bridge is studied as another numerical example to illustrate the effectiveness of the proposed method. The finite element model of the structure is shown in Fig. 7 the structure is modeled by 46 frame elements and 32 nodes with three DOF's at each node for the translation and rotational deformations. The mass density of material is 7.8×10^3 kg/m^3 and the elastic modulus of material is 2.1×10^7 N/ cm^2. The first five un-damped natural frequencies of the intact bridge are 45.59, 92.77, 181.74, 259.73 and 399.07 Hz. Rayleigh damping model is adopted with the damping ratios of the first two modes taken equal to 0.05. The equivalent Rayleigh coefficients a_0 and a_1 are respectively 0.1 and 2.364×10^{-5}.

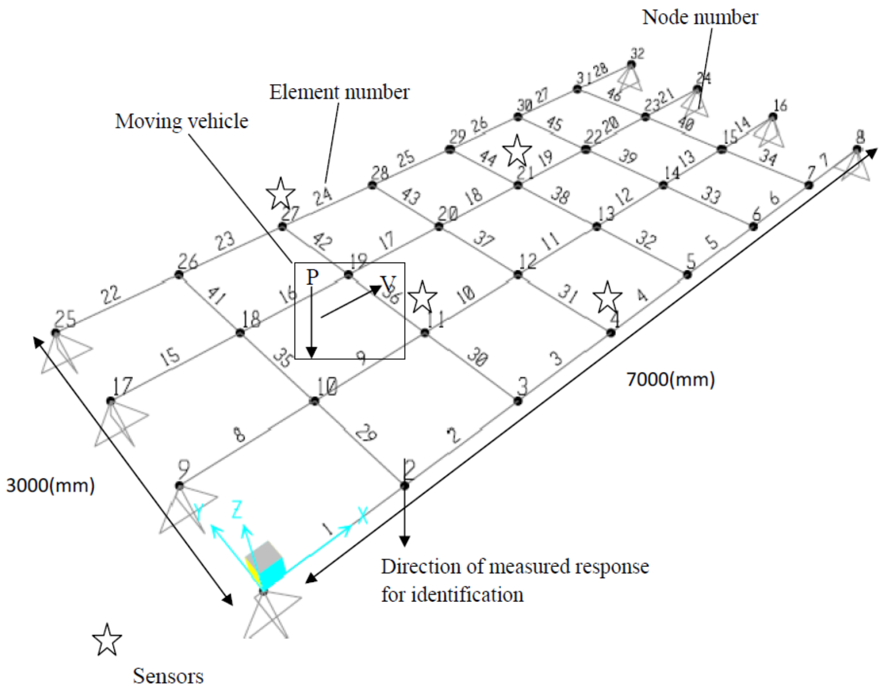

Fig. 7 Plane grid bridge model used in detection procedure

Table 5 Damage scenarios for grid model

Damage scenario	Damage type	Damage location	Reduction in elastic modulus	Noise
M2-1	Single	41	7%	Nil
M2-2	Multi	4,14,20	14%,8%,11%	Nil
M2-3	Multi	5,7,12,15,24,37	4%,11%,6%,2%,10%,16%	Nil
M2-4	Random	All elements	Random damage in all elements with an average of 5%	Nil
M2-5	Random	All elements	Random damage in all elements with an average of 10%	Nil

5.2 Plane grid model

A plane grid model of bridge is studied as another numerical example to illustrate the effectiveness of the proposed method. The finite element model of the structure is shown in Fig. 7 the structure is modeled by 46 frame elements and 32 nodes with three DOF's at each node for the translation and rotational deformations. The mass density of material is 7.8×10^3 kg/m^3 and the elastic modulus of material is 2.1×10^7 N/ cm^2. The first five un-damped natural frequencies of the intact bridge are 45.59, 92.77, 181.74, 259.73 and 399.07 Hz. Rayleigh damping model is adopted with the damping ratios of the first two modes taken equal to 0.05. The equivalent Rayleigh coefficients a_0 and a_1 are respectively 0.1 and 2.364×10^{-5}.

5.2.1 Damage scenarios

Five damage scenarios of single, multiple and random damages in the bridge without measurement noise are studied and they are shown in Table 5.

The sampling rate is 14000 Hz and 460 data of the acceleration response (degree of indeterminacy is 10) collected along the z-direction at nodes 4, 11, 21 and 27 are used.

Similar to the previous model, scenario 1 studies the single damage scenario. The proposed method converges in all speed ranges. The RPEP is 0.1271 and the RPEF is 0.0020.

Scenarios 2 and 3 are on multiple damages with different amount of measured responses for the identification and scenarios 4-5 are on random damages with different average for the identification. These scenarios also converge in all speed ranges with maximum RPEP of 0.1161 and maximum RPEF of 0.0018. One more scenario with model error is also included as scenario 6. Using proposed method, the damage locations and amount are identified correctly in all the scenarios and the RPEP and RPEF parameters are shown in Tables 6 and 7.

5.2.2 Effect of noise

In order to study effect of noise in stability of sensitivity methods, scenario3 (speed ratio of moving load is considered to be fix and equal with 0.3) is considered and different levels of noise pollution are investigated and RPE changes with increasing number of loops for iterative procedure has been studied.

Fig. 8 shows that both AVM and DDM methods are sensitive to noise and if noise level becomes greater than 1.8% for these methods loses their effectiveness and are not able to detect damage. So, in cases with noise level greater than mentioned value, a de-noising tool such as

Table 6 RPEP of AVM method for model2

Damage scenario	Speed ratio								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M2-1	0.0236	0.0239	0.0264	0.0259	0.0408	0.0266	0.0236	0.0239	0.0264
M2-2	0.0236	0.0239	0.0257	0.0267	0.0336	0.0279	0.0236	0.0239	0.0257
M2-3	0.0224	0.0206	0.0176	0.0194	0.0173	0.0191	0.0224	0.0206	0.0176
M2-4	0.0189	0.0191	0.0134	0.0129	0.0171	0.0162	0.0189	0.0191	0.0134
M2-5	0.0165	0.0221	0.0105	0.0126	0.0156	0.0154	0.0165	0.0221	0.0105
M2-6	0.1271	0.1161	0.0142	0.0123	0.0161	0.0166	0.1271	0.1161	0.0142

Table 7 RPEF of AVM method for model2

Damage scenario	Speed ratio								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M2-1	0.0005	0.0005	0.0007	0.0006	0.0008	0.0006	0.0005	0.0005	0.0007
M2-2	0.0005	0.0005	0.0009	0.0008	0.0008	0.0007	0.0005	0.0005	0.0009
M2-3	0.0006	0.0005	0.0005	0.0007	0.0005	0.0006	0.0006	0.0005	0.0005
M2-4	0.0007	0.0007	0.0004	0.0004	0.0005	0.0005	0.0007	0.0007	0.0004
M2-5	0.0006	0.0005	0.0003	0.0004	0.0004	0.0004	0.0006	0.0005	0.0003
M2-6	0.0020	0.0018	0.0003	0.0004	0.0003	0.0004	0.0020	0.0018	0.0003

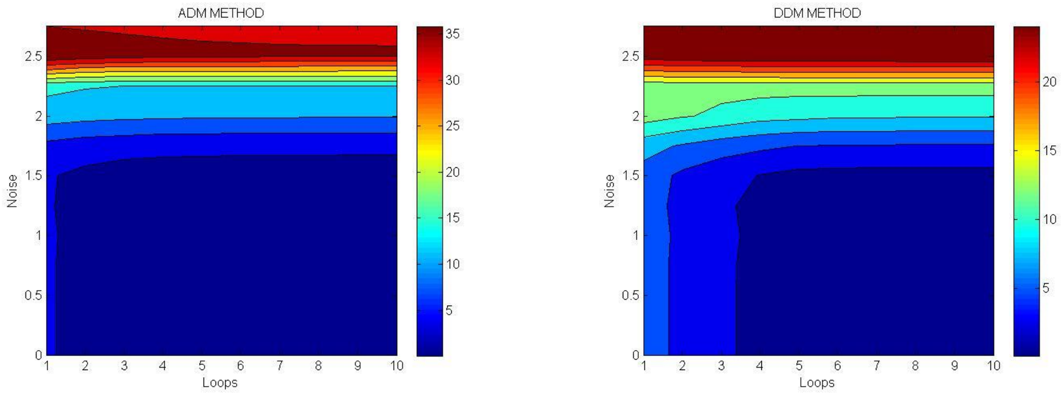

Fig. 8 RPE contours with respect to noise level and loops

Table 8 REP ranges in different scenarios for model2

Damage scenario	Max REP	Min REP	average
M2-1	3.746256	2.180788	2.915806
M2-2	3.492461	2.215382	3.051252
M2-3	3.453202	2.016751	2.615719
M2-4	4.028332	2.211189	2.854083
M2-5	3.179909	1.831374	2.245494
M2-6	2.546435	1.720415	2.067321
Total	4.028332	1.720415	2.624946

wavelet transform alongside sensitivity methods should be used. The wavelet transform is mainly attractive because of its ability to compress and encode information to reduce noise or to detect any local singular behavior of a signal. (Solís *et al*. 2013)

5.2.3 Efficiency of proposed method

Table 8 shows that in different scenarios and for different speeds, the efficiency parameter is between 1.720 to 4.028 and its average is 2.625, therefore the AVM method is extremely successful and computational cost for this method is about 38.1% of other sensitivity base finite element model updating method.

6. Conclusions

In this paper an iterative sensitivity-based method has been developed to identify both the input excitation force and the physical parameters of a bridge from the output of the system only. This method can be a good tool in the case when the structure, for example a highway bridge, should not be switched off from the use for a long time.

In the proposed method an incremental solution for adjoint variable equation developed that calculates each elements of sensitivity matrix separately. The main advantage is inclusion of an analytical method to augment the accuracy and speed of the solution.

Numerical simulations demonstrate the efficiency and accuracy of the method to identify location and intensity of single, multiple and random damages and unknown excitation input force simultaneously in different bridge models. Comparison studies confirmed that computational cost for this method is much lower than other traditional sensitivity methods. For modern, practical engineering applications, the cost of damage detection analysis is expensive. So, this method is feasible for large-scale problems.

References

Alampalli, S. and Fu, G. (1994), "Remote monitoring systems for bridge condition", Transportation Research and Development Bureau, New York State Department of Transportation, Client Report 94.

Cawley, P. and Adams, R.D. (1979), "The location of defects in structures from measurements of natural frequencies", *J. Strain Anal.*, **14**, 49-57.

Chan, T., Law, S., Yung, T. and Yuan, X. (1999), "An interpretive method for moving force identification", *J Sound Vib*., **219**, 503-524.

Chan, T., Yu, L., Law, S. and Yung, T. (2001), "Moving force identification studies. II: comparative studies", *J Sound Vib*., **247**(1), 77-95.

Choi, K.K. and Kim, N.H. (2005), *Structural Sensitivity Analysis and Optimization 1, Linear Systems*, Springer, USA.

Doebling, S.W., Peterson, L.D. and Alvin, K.F. (1996), "Estimation of reciprocal residual flexibility from experimental modal data", *Am. Inst. Aeronaut. Astronaut. J*., **34**, 1678-1685.

Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1998), "A review of damage identification methods that examine changes in dynamic properties", *Shock Vib. Dig*., **30**, 91-105.

Friswell, M.I., Penny, J.E.T. and Wilson, D.A.L. (1994), "Using vibration data and statistical measures to locate damage in structures, modal analysis", *Int. J. Anal. Exper. Modal Anal*., **9**, 239-254.

Friswell, M.I. and Mottershead, J.E. (1995), *Finite Element Model Updating in Structural Dynamics*, Kluwer Academic Publishers, Dordrecht.

González, A., Rowley, C. and Obrien, E. (2008), "A general solution to the identification of moving vehicle forces on a bridge", *Int. J. Numer. Meth. Eng*., **75**(3), 335-354.

Hoshiya, M. and Maruyama, O. (1987), "Identification of running load and beam system", *J. Eng. Mech*., ASCE, **113**(6), 813-824.

Jiang, R.J., Au, F.T. and Cheung, Y.K. (2004), "Identification of vehicles moving on continuous bridges with rough surface", *J. Sound Vib*., **274**, 1045-1063.

Law, S., Chan, T. and Zeng, Q. (1999), "Moving force identification-a frequency and time domain analysis", *J. Dyn. Syst. Meas. Control*., **121**(3), 394-401.

Law, S. and Fang, Y. (2001), "Moving force identification: optimal state estimation approach", *J. Sound Vib*., **239**(2), 233-254.

Law, S., Wu, S. and Shi, Z. (2008), "Moving load and prestress identification using wavelet-based method", *J. Appl. Mech*., **75**(2), 021,014.

Li, J. and Chen, J. (1999), "A statistical average algorithm for the dynamic compound inverse problem", *Comput. Mech*., **30**(2), 88-95.

Lim, T.W. (1991), "Structural damage detection using modal test data", *Am. Inst. Aeronaut. Astronaut. J*., **29**, 2271-2274.

Lu, Z.R. and Law, S.S. (2007), "Identification of system parameters and input force from output only", *Mech. Syst. Signal Pr*., **21**, 2099-2111.

Majumder, L. and Manohar, C. (2003), "A time-domain approach for damage detection in beam structures using vibration data with a moving oscillator as an excitation source", *J. Sound Vib*., **268**(4), 699-716.

Majumder, L. and Manohar, C. (2004), "Nonlinear reduced models for beam damage detection using data on moving oscillatorbeam interactions", *Comput. Struct*., **82**(2-3), 301-314.

Mosavi, A. (2010), "Vibration-based damage detection and health monitoring of bridges", PhD Thesis, Faculty of North Carolina State University, Raleigh, North Carolina, USA.

Narkis, Y. (1994), "Identification of crack location in vibrating simply supported beam", *J. Sound Vib*., **172**, 549-558.

Pandey, A.K., Biswas, M. and Samman, M.M. (1991), "Damage detection from change in curvature mode shapes", *J. Sound Vib*., **145**, 321-332.

Pandey, A.K. and Biswas, M. (1994), "Damage detection in structures using change in flexibility", *J. Sound Vib*., **169**, 3-17.

Ratcliffe, C.P. (1997), "Damage detection using a modified Laplacian operator on mode shape data", *J. Sound Vib*., **204**, 505-517.

Ricles, J.M. and Kosmatka, J.B. (1992), "Damage detection in elastic structures using vibration residual forces and weighted sensitivity", *Am. Inst. Aeronaut. Astronaut. J*., **30**, 2310-2316.

Rizos, P.F., Aspragathos, N. and Dimarogonas, A.D. (1990), "Identification of crack location and magnitude in a cantilever beam", *J. Sound Vib*., **138**, 381-388.

Sieniawska, R., Sniady, P. and Zukowski, S. (2009), "Identification of the structure parameters applying a moving load", *J. Sound Vib*., **319**(1-2), 355-365.

Silva, S., Dias, J.M. and Lopes, J.V. (2008), "Structural health monitoring in smart structures through time series analysis", *Struct. Health Monit*., **7**(3), 231-244.

Solís, M., Algaba, M. and Galvín, P. (2013), "Continuous wavelet analysis of mode shapes differences for damage detection", *J. Mech. Syst. Signal Pr*., **40**, 645-666.

Srinivasan, G., Massimo, R. and Sathyaneryona, H. (2011), *Computational Techniques for Structural Health Monitoring*, Springer, Newyork.

Staszewski, W.J. (2003), "Structural health monitoring using guided ultrasonic waves", Eds. Holnicki-Szulc, J. and Soares, C.A.M., *Advances in smart technologies in structural engineering*, Springer, Berlin.

Wu, D. and Law, S.S. (2004), "Model error correction from truncated modal flexibility sensitivity and generic parameters. I: Simulation", *Mech. Syst. Signal Pr*., **18**(6), 1381-1399.

Yu, L. and Chan, T. (2003), "Moving force identification based on the frequency-time domain method", *J. Sound Vib*., **261**(2), 329-349.

Yu, L. and Chan, T. (2007), "Recent research on identification of moving loads on bridges", *J. Sound Vib*., **305**, 3-21.

Zhang, K., Law, S. and Duan, Z. (2009), "Condition assessment of structures under unknown support excitation", *Earthq. Eng. Eng. Vib*., **8**(1), 103-114.

Zhang, Q., Jankowski, L. and Duan, Z. (2010), "Simultaneous identification of moving masses and structural damage", *Struct. Multidisc. Optim*., **42**, 907-922.

Zhang, Q., Jankowski, L. and Duan, Z. (2010), "Identification of coexistent load and damage", *Struct. Multidisc. Optim*., **41**(2), 243-253.

Zhu, X. and Law, S. (2006), "Moving load identification on multispan continuous bridges with elastic bearings", *Mech. Syst. Signal Pr*., **20**(7), 1759-1782.

Zhu, X. and Law, S. (2007), "Damage detection in simply supported concrete bridge structure under moving vehicular loads", *J. Vib. Acoust*., **129**(1), 58-65.

Zhu, X.Q. and Hao, H. (2007), "Damage detection of bridge beam structures under moving loads", Research Program Report, School of Civil and Resource Engineering, the University of Western Australia.

Zou, T., Tong, L. and Steve, G. (2000), "Vibration based model-dependent damage (delamination) identification and health monitoring for composite structures-a review", *J. Sound Vib*., **230**(2), 357-378.

CC