Structural Engineering and Mechanics, *Vol. 56, No. 4 (2015) 649-665*
DOI: http://dx.doi.org/10.12989/sem.2015.56.4.649

The effect of rotation on piezo-thermoelastic medium using different theories

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(Received May 11, 2015, Revised October 5, 2015, Accepted November 6, 2015)

Abstract. The present paper attempts to investigate the propagation of plane waves in generalized piezo-thermoelastic medium under the effect of rotation. The normal mode analysis is used to obtain the expressions for the displacement components, the temperature, the stress and the strain components. Comparisons are made with the results predicted by different theories (Coupled theory, Lord-Schulman, Green-Lindsay) in the absence and presence of rotation.

Keywords: rotation; piezo-thermoelasticity; relaxation time; normal mode analysis; generalized thermoelasticity

1. Introduction

Piezoelectric materials are used for transducer electrical and mechanical energy. Piezoelectric material technology has enabled a wide variety of commercially successful sensors and actuators. These devices range in complexity and sophistication from the ultrasound arrays used in biomedical imaging to the tonal buzzers used in automobile horns, from high intensity focused ultrasound (HIFU) arrays which can thermally ablate tumors to the speaker elements in talking greeting cards.

The theory of thermo-piezoelectricity was first proposed by Mindlin (1961). He also derived governing equations of a thermo-piezoelectric plate (1974). Nowacki (1978, 1979) has studied the physical laws for the thermo-piezoelectric materials. Chandrasekharaiah (1988) has generalized Mindlin's theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances. The propagation of Rayleigh waves in generalized piezo-thermoelastic half space is investigated by Sharma and Walia (2008). Abd-alla and Alsheikh (2009) studied the reflection and refraction of plane quasi-longitudinal waves at an interface of two piezoelectric media under initial stresses. Abd-alla *et al*. (2012, 2011) investigated the reflection phenomena of quasi-vertical transverse and longitudinal waves in the piezoelectric medium under initial stresses. Abd-alla *et al*. (2014) studied the phenomena of reflection and transmission waves in smart nano-materials. Hou *et al*. (2008) studied three-dimensional fundamental solutions for transversely isotropic piezo-thermoelastic material. The effect of rotation on wave characteristics in

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

piezoelectric crystals has been discussed by various authors such as Gates (1968) and Soderkvist (1994). Sharma and Walia (2007) studied the effect of rotation on Rayleigh waves in piezo-thermoelastic half space. Othman (2004) studied the effect of rotation on plane waves in generalized thermoelasticity with two relaxation times. Othman *et al*. (2008) studied the generalized magneto-thermo-viscoelastic plane waves under the effect of rotation without energy dissipation. Othman and Sarhan (2014) studied the effect of rotation on a fibre-reinforced thermo-elastic under Green-Naghdi theory and the influence of gravity. Othman *et al*. (2014) studied the effect of rotation on micropolar generalized thermoelasticity with two-temperature using a dual-phase-lag model. The development of the effect of rotation is available in many studies, such as Ellahi and Ashgar (2007), Hayat *et al*. (2004a, b, 2007, 2003).

The generalized theories of thermoelasticity have been developed to overcome the infinite propagation speed of thermal signals predicted by the classical coupled dynamical theory of thermoelasticity Biot (1956). The subject of generalized thermoelasticity covers a wide range of extensions of the classical theory of thermoelasticity. We recall the first two earliest and well-known generalized theories proposed by Lord and Shulman (1967) and Green and Lindsay (1972). In the model (1967) the Fourier law of heat conduction is replaced by Maxwell-Cattaneo law that introduces one thermal relaxation time parameter in the Fourier law, whereas in the model of Green and Lindsay (1972) of two relaxation parameters are introduced in the constitutive relations for the stress tensor and the entropy. Othman *et al*. (2002) studied the generalized thermo-viscoelastic plane waves with two relaxation times. Othman (2002) studied Lord-Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two-dimensional generalized thermoelasticity. Othman and Sarhan (2014) studied propagation of plane waves of a mode-I crack for a generalized thermoelasticity under influence of gravity for the different theories.

In this paper, we have investigated the effect of rotation of piezo-thermoelastic medium based on three theories (CT, L-S, G-L) by applying the normal mode analysis. Also, the effect of rotation on the physical quantities is discussed numerically and illustrated graphically.

2. Basic equations

The basic governing field equations of generalized hexagonal piezo-thermoelastic in homogeneous anisotropic solid for two dimensional motions in *x-z* plane are

2.1 Strain-displacement-relation

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{1}$$

2.2 Stress-strain-temperature

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{kij}E_k - \beta_{ij}(1 + t_1\frac{\partial}{\partial t})T\,\delta_{ij}\,. \tag{2}$$

2.3 Equation of motion

$$\sigma_{ij,j} = \rho\,[\ddot{u}_i + \{\Omega\times(\Omega\times u)\}_i + (2\Omega\times\dot{u})_i\,], \tag{3}$$

2.4 Gauss equation and electric field relation

$$D_{i,i} = 0 \tag{4}$$

$$D_i = e_{ijk}\,\varepsilon_{jk} + \in_{ij}\,E_j + p_i\,(1+t_1\frac{\partial}{\partial t})T\,, \tag{5}$$

Where $E_i = -\varphi_{,i}$

2.5 Heat conduction equation

$$K_{ij}T_{,ij} = \rho C_e\,(1+t_0\frac{\partial}{\partial t})\dot{T} + T_0[\beta_{ij}\,(1+n_1 t_0\frac{\partial}{\partial t})\dot{u}_{i,j} - p_i\,(1+n_1 t_0\frac{\partial}{\partial t})\dot{\varphi}_{,i}\,]. \tag{6}$$

Where i, j, k, l=1,2,3

Eqs. (2)-(6) are the field equations of the generalized thermoelastic solid, can be defined in terms of the three theories (CT), (L-S) and (G-L) as follows:

1. The coupled (CT) theory, when $t_1=t_0=0$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - e_{kij}E_k - \beta_{ij}T\,\delta_{ij}\,, \tag{7}$$

$$D_i = e_{ijk}\,\varepsilon_{jk} + \in_{ij}\,E_j + p_iT\,, \tag{8}$$

$$K_{ij}T_{,ij} = \rho C_e\dot{T} + T_0[\beta_{ij}\dot{u}_{i,j} - p_i\,\dot{\varphi}_{,i}\,]. \tag{9}$$

2. Lord-Shulman (L-S) theory, when $n_1=1$, $t_1=0$, $t_0>0$

Eqs. (7) and (8) remain unchanged and (6) has the form

$$K_{ij}T_{,ij} = (1+t_0\frac{\partial}{\partial t})[\,\rho C_e\dot{T} + T_0(\beta_{ij}\dot{u}_{i,j} - p_i\,\dot{\varphi}_{,i}\,)]. \tag{10}$$

3. Green-Lindsay (G-L) theory, when $n_1=0$, $t_1\geq t_0>0$

Eqs. (2) and (5) remain unchanged and (6) has the form

$$K_{ij}T_{,ij} = \rho C_e\,(1+t_0\frac{\partial}{\partial t})\dot{T} + T_0[\beta_{ij}\,\dot{u}_{i,j} - p_i\,\dot{\varphi}_{,i}\,]. \tag{11}$$

The constitutive relation and electric displacement of the hexagonal (6mm) crystal symmetry given by

$$\sigma_{xx} = C_{11}\varepsilon_{xx} + C_{13}\varepsilon_{zz} - e_{31}E_z - \beta_1(1+t_1\frac{\partial}{\partial t})T\,, \tag{12}$$

$$\sigma_{zz} = C_{13}\varepsilon_{xx} + C_{33}\varepsilon_{zz} - e_{33}E_z - \beta_3(1+t_1\frac{\partial}{\partial t})T\,, \tag{13}$$

$$\sigma_{zx} = 2C_{44}\varepsilon_{zx} - e_{15}E_x\,, \tag{14}$$

$$D_{\mathrm{x}} = e_{15}(u_{,z} + w_{,x}) + \in_{11} E_x\,, \tag{15}$$

$$D_{\mathrm{z}} = e_{31}u_{,x} + e_{33}w_{,z} + \in_{33} E_z + p_3(1 + t_1\frac{\partial}{\partial t})T\,. \tag{16}$$

3. Formulation of the problem

We consider a homogeneous, anisotropic, piezo-thermoelastic half space of hexagonal type. We chose x-axis in the direction of wave propagation so that all particles on a line parallel to y-axis are equally displaced. Therefore, all the field quantities will be independent of y coordinate. The medium is assumed to be rotating with angular frequency Ω=(0, Ω, 0) that rotate at a constant rate about the y-axis.

The basic governing field Eqs. (3), (4), and (6) for temperature change $T(x,z,t)$, displacement vector $\boldsymbol{u}(x,z,t) = (u,0,w)$, and electric potential $\varphi(x,z,t)$, are given by

$$C_{11}u_{,xx} + C_{44}u_{,zz} + (C_{13} + C_{44})\,w_{,xz} + (e_{31} + e_{15})\,\varphi_{,xz} - \beta_1(1 + t_1\frac{\partial}{\partial t})T_{,x} = \rho\,(\ddot{u} - \Omega^2 u + 2\Omega\dot{w}), \tag{17}$$

$$(C_{44} + C_{13})\,u_{,xz} + C_{44}w_{,xx} + C_{33}w_{,zz} + e_{15}\,\varphi_{,xx} + e_{33}\,\varphi_{,zz} - \beta_3(1 + t_1\frac{\partial}{\partial t})T_{,z} = \rho\,(\ddot{w} - \Omega^2 w - 2\Omega\dot{u}), \tag{18}$$

$$K_1T_{,xx} + K_3T_{,zz} - \rho C_e(1 + t_0\frac{\partial}{\partial t})\dot{T} = T_0[\beta_1(1 + n_1t_0\frac{\partial}{\partial t})\dot{u}_{,x} + \beta_3(1 + n_1t_0\frac{\partial}{\partial t})\dot{w}_{,z} - p_3(1 + n_1t_0\frac{\partial}{\partial t})\dot{\varphi}_{,z}\,], \tag{19}$$

$$(e_{15} + e_{31})u_{,xz} + e_{15}w_{,xx} + e_{33}w_{,zz} - \in_{11}\varphi_{,xx} - \in_{33}\varphi_{,zz} + p_3(1 + t_1\frac{\partial}{\partial t})T_{,z} = 0 \tag{20}$$

For simplification we shall use the following non-dimensional variables

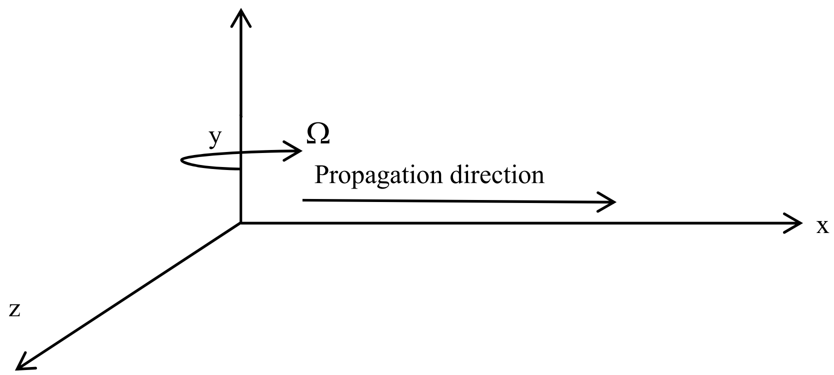

Fig. 1 Geometry of the problem

$$x' = \frac{\omega^*}{v_p}x, \quad z' = \frac{\omega^*}{v_p}z, \quad u' = \frac{\rho\omega^* v_p}{\beta_1 T_0}u, \quad w' = \frac{\rho\omega^* v_p}{\beta_1 T_0}w, \quad T' = \frac{T}{T_0}, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\beta_1 T_0}, \quad \varphi' = \varepsilon_p \varphi, \quad \Omega' = \frac{\Omega}{\omega^*},$$

$$\{t', t_1', t_0'\} = \omega^*\{t, t_1, t_0\}, \; D_i' = \frac{D_i}{e} \; \omega^* = \frac{C_e c_{11}}{K_1}, \; \varepsilon_p = \frac{\omega^* e_{33}}{v_p \beta_1 T_0}, \; \beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_3,$$

$$\beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3. \tag{21}$$

The dimensionless of Eqs. (17)-(20)

$$\delta_1 u_{,xx} + \delta_2 u_{,zz} + \delta_3 w_{,xz} + \delta_4 \varphi_{,xz} - (1 + t_1 \frac{\partial}{\partial t})T_{,x} = (\ddot{u} - \Omega^2 u + 2\Omega\dot{w}), \tag{22}$$

$$\delta_3 u_{,xz} + \delta_2 w_{,xx} + \delta_5 w_{,zz} + \delta_6 \varphi_{,xx} + \varphi_{,zz} + \delta_7 (1 + t_1 \frac{\partial}{\partial t})T_{,z} = (\ddot{w} - \Omega^2 w - 2\Omega\dot{u}), \tag{23}$$

$$\delta_8 u_{,xz} + \delta_9 w_{,xx} + \delta_{10} w_{,zz} + \delta_{11}\varphi_{,xx} + \delta_{12}\varphi_{,zz} + \delta_{13}(1 + t_1 \frac{\partial}{\partial t})T_{,z} = 0, \tag{24}$$

$$\delta_{14} T_{,xx} + \delta_{15} T_{,zz} - (1 + t_0 \frac{\partial}{\partial t})\dot{T} = (1 + n_1 t_0 \frac{\partial}{\partial t})[\delta_{16}\dot{u}_{,x} + \delta_{17}\dot{w}_{,z} + \delta_{18}\dot{\varphi}_{,z}]. \tag{25}$$

Where δ_j, j=1−18 are given in Appendix A.

4. Normal mode analysis.

The solution of the considered physical variables can be decomposed in terms of normal modes in the following form

$$[u, w, \varphi, T](x, z, t) = [u^*, w^*, \varphi^*, T^*](z)\, e^{ia(x - ct)}. \tag{26}$$

Where $D = \frac{d}{dz}$, $c = \frac{\omega}{a}$, ω is the complex time constant (frequency), i is the imaginary unit, a is the wave number in the x-direction, and u^*, w^*, φ^* and T^* are the amplitudes of the functions, then

$$(D^2 + A_1)u^* + (A_2 D + A_3)w^* + A_4 D\varphi^* + A_5 T^* = 0, \tag{27}$$

$$(A_6 D + A_7)u^* + (D^2 + A_8)w^* + (A_9 D^2 + A_{10})\varphi^* + A_{11} D T^* = 0, \tag{28}$$

$$A_{12} D u^* + (D^2 + A_{13})w^* + (A_{14} D^2 + A_{15})\varphi^* + A_{16} D T^* = 0, \tag{29}$$

$$A_{17} u^* + A_{18} D w^* + A_{19} D\varphi^* + (D^2 + A_{20})T^* = 0. \tag{30}$$

Where A_j, j=1−20 are given in Appendix A.

$$(D^8 - A\, D^6 + B\, D^4 - C\, D^2 + E)\{u^*(z), w^*(z), \varphi^*(z), T^*(z)\} = 0, \tag{31}$$

Where A, B, C, E are given in Appendix A.

Eq. (31) can be factored as

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)(\mathrm{D}^2 - k_4^2)\,\{u^*(z), w^*(z), \varphi^*(z), T^*(z)\} = 0, \tag{32}$$

The solution of Eq. (31), which is bounded as $z\to\infty$, is given by

$$u^* = \sum_{n=1}^{4} M_n e^{-k_n z}, \tag{33}$$

In a similar manner, we get

$$w^* = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z}, \tag{34}$$

$$\varphi^* = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z}, \tag{35}$$

$$T^* = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z}. \tag{36}$$

Where $k_n^2 (n = 1,2,3,4)$ are the roots of the characteristic equation of Eq. (32).

By taking dimensionless and normal mode to Eqs. (12)-(16) then substituting from Eqs. (33)-(36), we obtain

$$\sigma_{xx}^* = \sum_{n=1}^{4} H_{4n} M_n e^{-k_n z}, \tag{37}$$

$$\sigma_{zz}^* = \sum_{n=1}^{4} H_{5n} M_n e^{-k_n z}, \tag{38}$$

$$\sigma_{xz}^* = \sum_{n=1}^{4} H_{6n} M_n e^{-k_n z}, \tag{39}$$

$$D_x^* = \sum_{n=1}^{4} H_{7n} M_n e^{-k_n z}, \tag{40}$$

$$D_z^* = \sum_{n=1}^{4} H_{8n} M_n e^{-k_n z}, \tag{41}$$

Where H_{jn}, j=1−8, i=1,2,3,4 are given in Appendix B.

5. Boundary conditions

The parameters M_n (n=1,2,3,4) have to be chosen such that the boundary conditions on the surface z=0 take the form

$$\sigma_{zz}(x,0,t) = -f_1^* \mathrm{e}^{ia(x-ct)}, \quad \sigma_{xz}(x,0,t) = 0, \quad T = f_2^* \mathrm{e}^{ia(x-ct)}, \quad \frac{\partial \varphi}{\partial z} = 0 \tag{42}$$

Where f_1^*, f_2^* are constant.

Using the expressions of the variables considered into the above boundary conditions (42), we can obtain the following equations satisfied by the parameters

$$\sum_{n=1}^{4} H_{5n} M_n = -f_1^*, \tag{43}$$

$$\sum_{n=1}^{4} H_{6n} M_n = 0, \tag{44}$$

$$\sum_{n=1}^{4} H_{3n} M_n = f_2^*, \tag{45}$$

$$\sum_{n=1}^{4} k_n H_{2n} M_n = 0. \tag{46}$$

Solving Eqs. (43) - (46) for M_n (n=1,2,3,4) by using the inverse of matrix method as follows

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{51} & H_{52} & H_{53} & H_{54} \\ H_{61} & H_{62} & H_{63} & H_{64} \\ H_{31} & H_{32} & H_{33} & H_{34} \\ k_1 H_{21} & k_2 H_{22} & k_3 H_{23} & k_4 H_{24} \end{pmatrix}^{-1} \begin{pmatrix} -f_1^* \\ 0 \\ f_2^* \\ 0 \end{pmatrix}. \tag{47}$$

6. Numerical results and discussions

The material chosen for the purpose of numerical calculations is taken as Cadmium Selenide (CdSe) having hexagonal symmetry (6 mm class)

$$C_{11} = 7.41\times10^{10}\,Nm^{-2},\quad C_{12} = 4.52\times10^{10}\,Nm^{-2},\quad C_{13} = 3.93\times10^{10}\,Nm^{-2},\quad C_{33} = 8.36\times10^{10}\,Nm^{-2},$$

$$C_{44} = 1.32\times10^{10}\,Nm^{-2},\quad T_0 = 298K,\quad \rho = 5504\,Kgm^{-3},\quad e_{13} = -0.160\,Cm^{-2},\quad e_{33} = 0.347\,Cm^{-2},$$

$$e_{15} = -0.138\,Cm^{-2},\quad \beta_1 = 0.621\times10^{6}\,Nk^{-1}.m^{-2},\ \beta_3 = 0.551\times10^{6}\,NK^{-1}.m^{-2},\quad p_3 = -2.94\times10^{-6}\,CK^{-1}.m^{-2},$$

$$K_1 = K_3 = 9\,Wm^{-1}.K^{-1},\quad \in_{11} = 8.26\times10^{-11}\,C^2N^{-1}.m^{-2},\quad \in_{33} = 9.03\times10^{-11}\,C^2N^{-1}.m^{-2},\quad C_e = 260\,J.Kg^{-1}K^{-1}$$

Figs. 2-10 depict the variety of the displacement components u, w the stress components σ_{xx}, σ_{xz}, σ_{zz} the temperature T, the electric potential φ and the electric displacements D_x and D_z in the absence and presence of rotation (i.e., Ω=0, 0.5). The computations are carried out for the non-dimensional time t=0.3 on the surface plane x=1.5. Fig. 2 depicts that the distribution of the horizontal displacement u always begins from positive values. In the context of the (CT), (L-S) and (G-L) theory, the values of u decreases in the range $0\leq z\leq 1.3$, for Ω=0, 0.5. It is also clear that the rotation acts to increase the values of u. Fig. 3 is plotted for the variation in vertical displacement w with distance z. Here, we can observe that w increases and finally goes to zero in the context of (CT), (L-S) and (G-L) theories for Ω=0 while the value of w decrease in the range

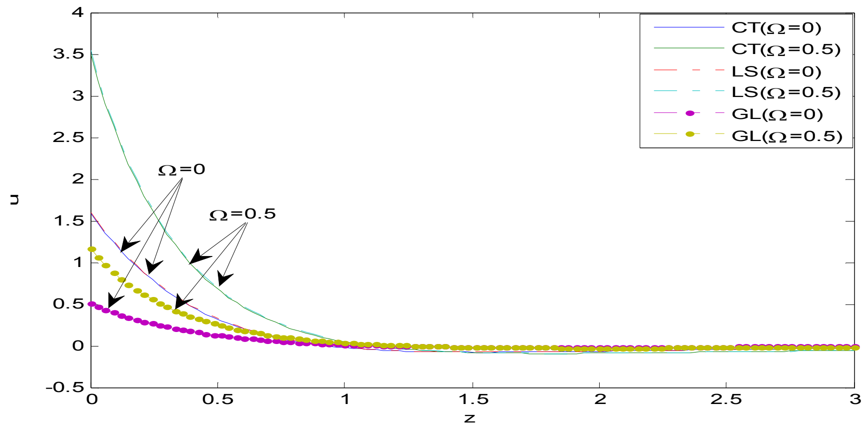

Fig. 2 Horizontal displacement distribution *u* in the absence and presence of rotation

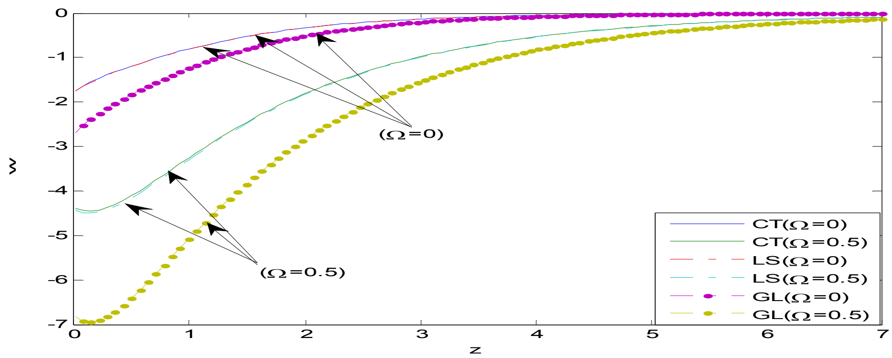

Fig. 3 Vertical displacement distribution *w* in the absence and presence of rotation

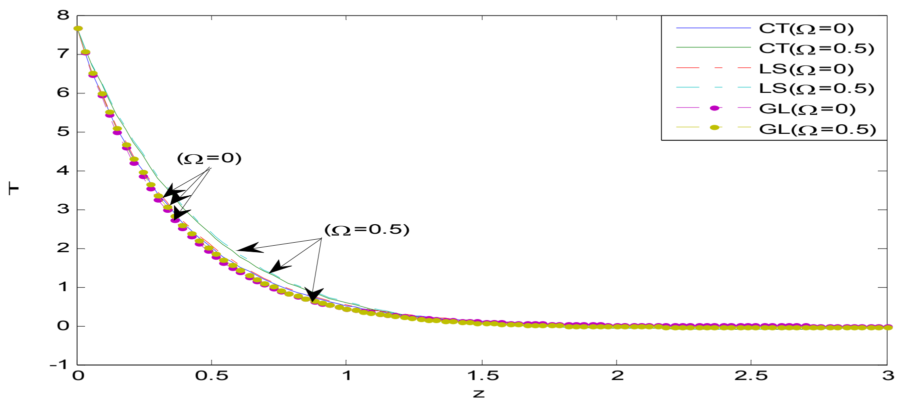

Fig. 4 Temperature distribution *T* in the absence and presence of rotation

$0 \le z \le 0.2$ then increase in the range $0.2 \le z \le 7$, then w converges to zero with increasing of the distance $z \ge 7$ for $\Omega = 0.5$. It is observed that the two curves of the (CT) and (L-S) theories are coinciding for $\Omega = 0$. Fig. 4 demonstrates the behavior of the temperature T based on three theories.

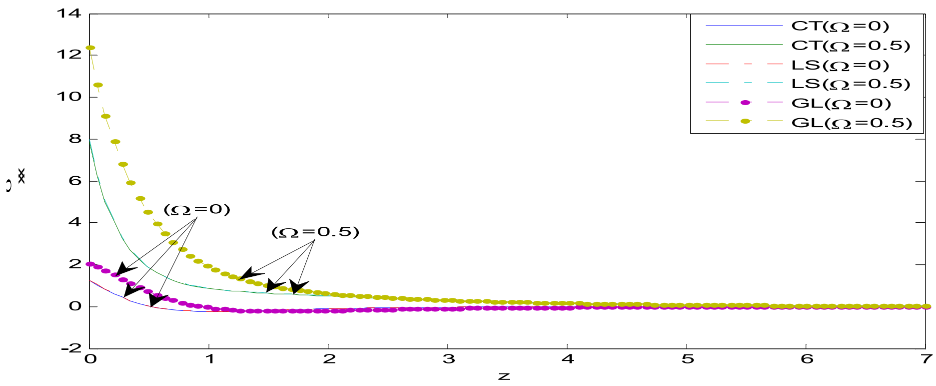

Fig. 5 Distribution of stress component σ_{xx} in the absence and presence of rotation

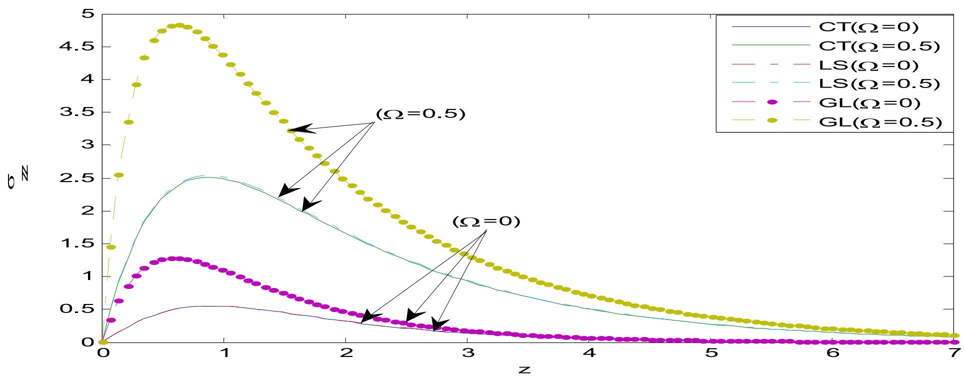

Fig. 6 Distribution of stress component σ_{zz} in the absence and presence of rotation

It shows that the values of T decrease in the range $0 \leq z \leq 2$ for $\Omega=0$, 0.5. We notice that the rotation has no great effect on the distribution of the temperature T. Fig. 5 exhibits that the distribution of the stress component σ_{xx} always begin from a positive value in the context of three theories for $\Omega=0$, 0.5. It decreases to the range $0 \leq z \leq 1$, for $\Omega=0$ and in the range $0 \leq z \leq 2$ for $\Omega=0.5$ then σ_{xx} converge to zero with increasing of the distance z at $z \geq 4$ for $\Omega=0$, 0.5. Fig. 6 shows the variation of the stress component σ_{zz} with distance z. The behavior of σ_{zz} for the three theories is almost same for $\Omega=0$, 0.5 and satisfy the boundary conditions. It is an increasing function in the domain $0 \leq z \leq 0.8$ and a decreasing function in the domain $0.8 \leq z \leq 7$ for $\Omega=0.5$ while σ_{zz} start with increasing to a maximum value in the range $0 \leq z \leq 0.5$ then decreasing in the range $0.5 \leq z \leq 7$ for $\Omega=0$ Fig. 7 exhibits the stress component σ_{xz} and demonstrates that it reaches a zero value and satisfies the boundary conditions at $z=0$. In the context of the three theories, the values of σ_{xz} increase in the beginning to a maximum value in the range $0 \leq z \leq 0.8$, then decrease and converge to zero in the range $0.8 \leq z \leq 7$ for $\Omega=0$, 0.5. We can also observed from Figs. 5-7 that the rotation acts to increase the magnitude of the real part of the stress components σ_{xx}, σ_{zz} and σ_{xz}.

Fig. 8 depicts that the distribution of the electric potential φ, in the context of three theories, increases in the range $0 \leq z \leq 1.8$ then decreases and converges to zero for $\Omega=0$ while increases in the range $0 \leq z \leq 1.6$ then decreases for $\Omega=0.5$. Fig. 9 is plotted to show the variation of the electric displacement component D_x in the context of the three theories for $\Omega=0$, 0.5. Here, we can observe

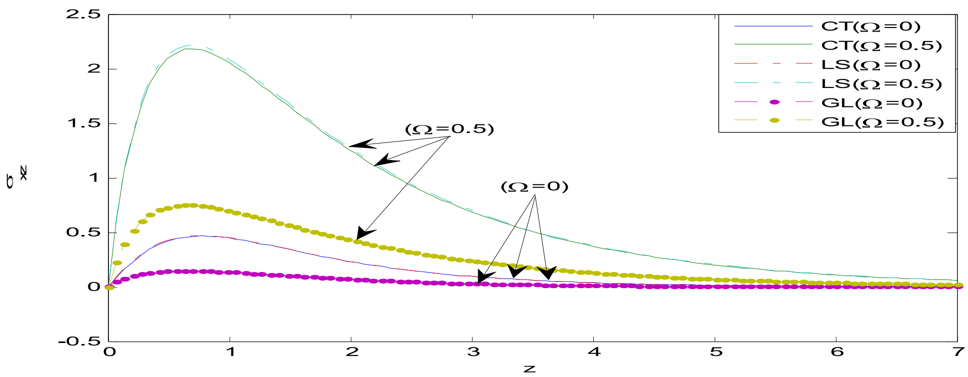

Fig. 7 Distribution of stress component σ_{xz} in the absence and presence of rotation

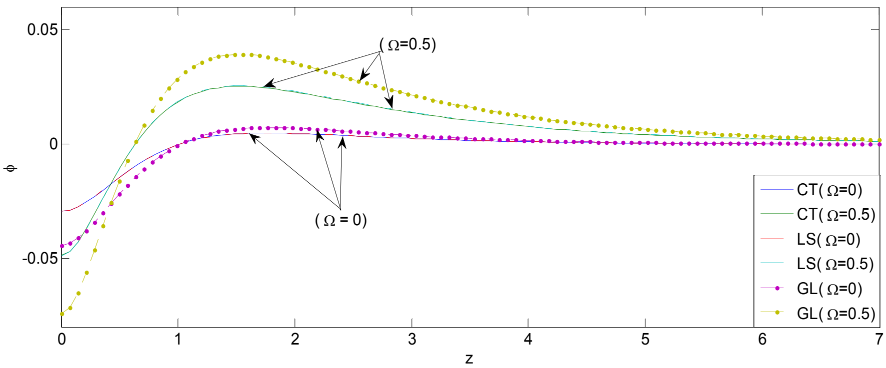

Fig. 8 Distribution of electric potential ϕ in the absence and presence of rotation

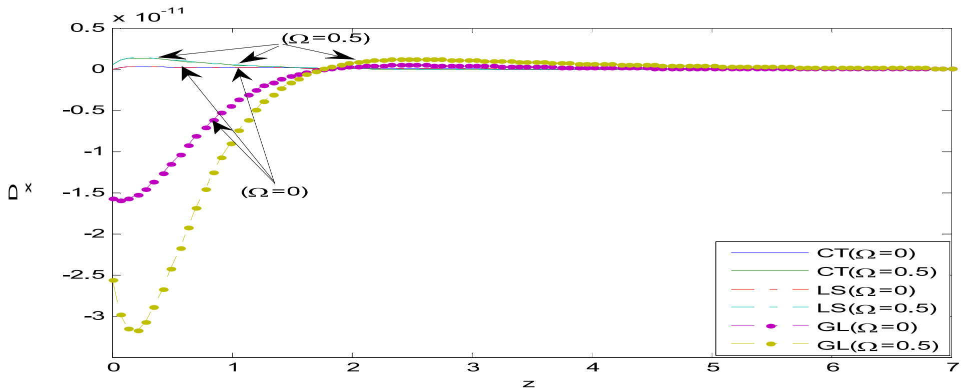

Fig. 9 Distribution of electric displacement component D_x in the absence and presence of rotation

that D_x increases and finally goes to zero in the context of (CT) and (L-S) theories for Ω=0, 0.5. The values of D_x based on G-L theory, decrease then increase for Ω=0 while D_x decreases to a minimum value in the range $0 \le z \le 0.3$ increasing in the range $0.3 \le z \le 2.2$ then decreasing and converges to zero with increasing the distance $z \ge 4.3$ for Ω=0.5. Fig. 10 shows that the electric

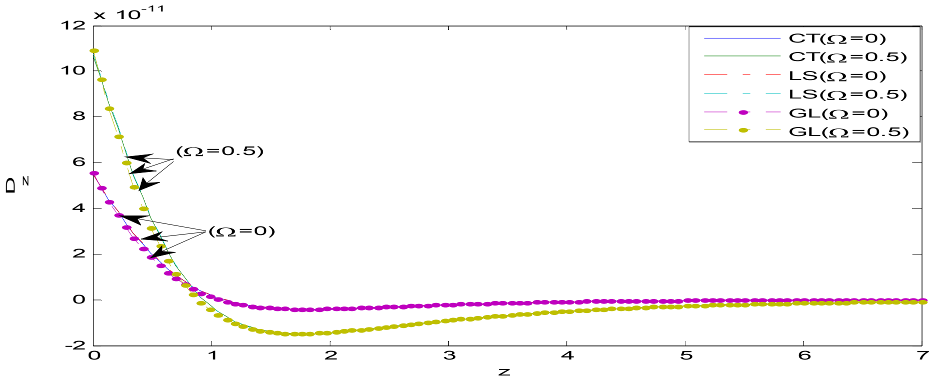

Fig. 10 Distribution of electric displacement component D_z in the absence and presence of rotation

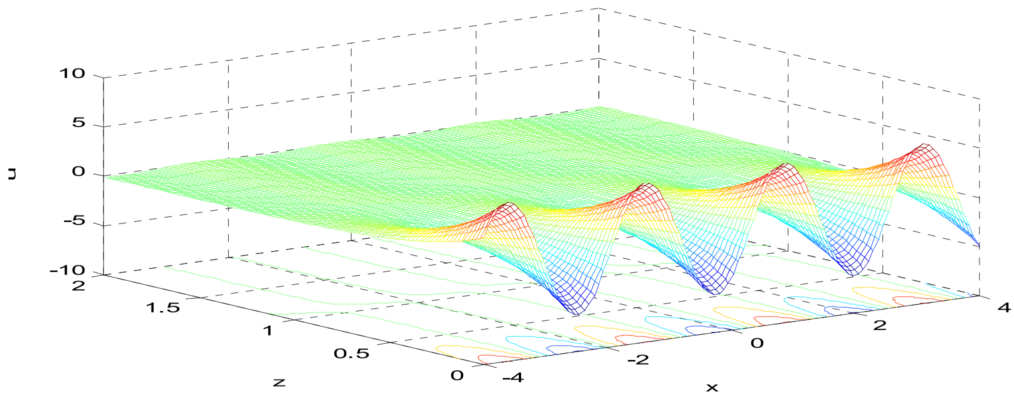

Fig. 11 (3D) Horizontal component of displacement *u* against both components of distance based on G-L model at Ω=0.5

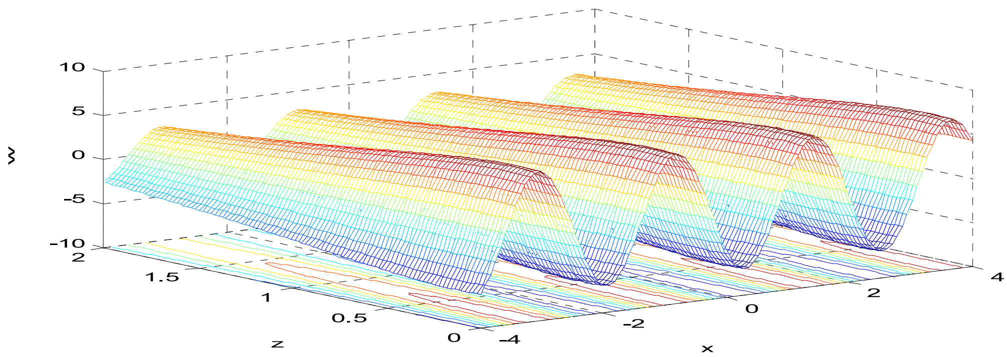

Fig. 12 (3D) Vertical component of displacement *w* against both components of distance based on G-L model at Ω=0.5

displacement component D_z in the context of the three theories for Ω=0, 0.5, always begin from positive value. It is shown that the value of D_z based in the three theories decreases in the range

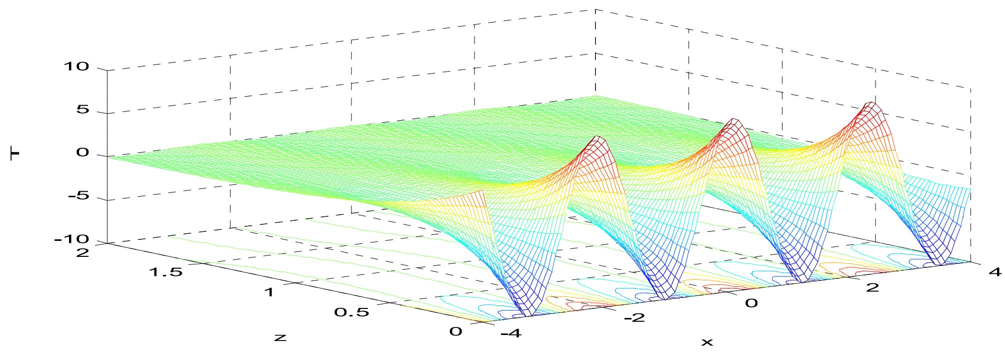

Fig. 13 (3D) Temperature against both components of distance based on G-L model at Ω=0.5

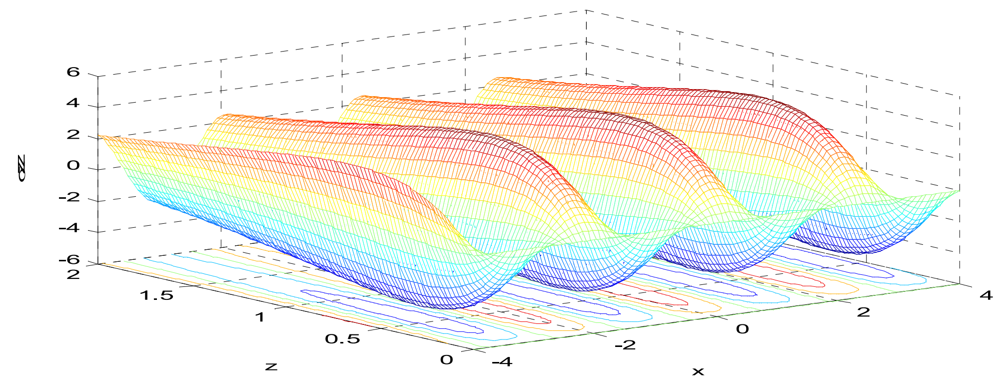

Fig. 14 (3D) Distribution of stress component σ_{zz} ainst both components of distance based on G-L model at Ω=0.5

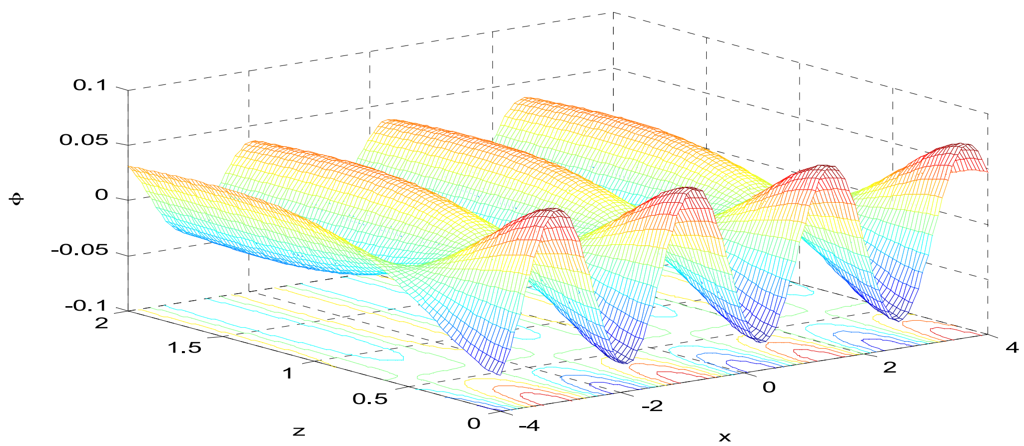

Fig. 15 (3D) Electric potential against both components of distance based on G-L model at Ω=0.5

$0 \leq z \leq 1.5$ then converge to zero for $\Omega=0$, while D_z decreases to the minimum value in the range $0 \leq z \leq 1.7$ then increase in the range $1.7 \leq z \leq 5$ and converges to zero with increasing the distance $z \geq 5$.

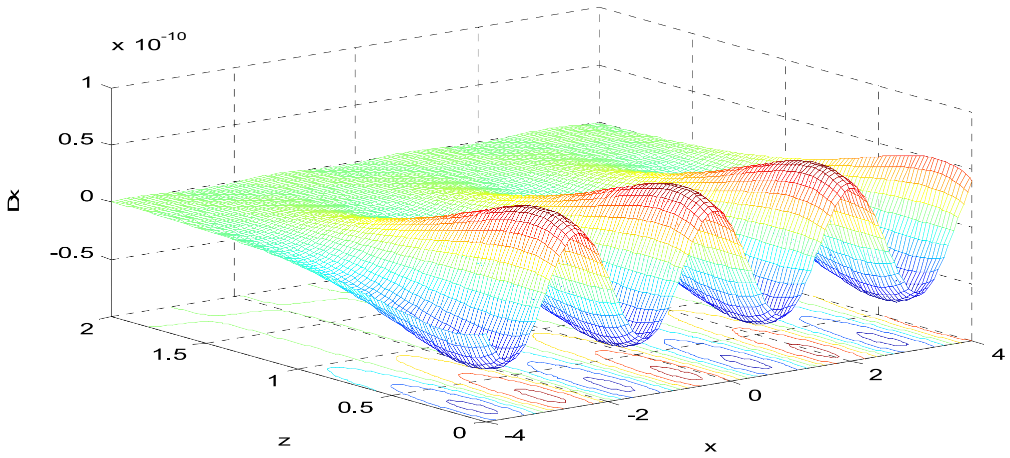

Fig. 16 (3D) Electric displacement against both components of distance based on G-L model at Ω=0.5

3D Figs. 11-16 are representing the relation between the physical variables and both components of distance, in the presence of rotation (Ω=0.5), in the context of the Green-Lindsay theory (G-L). The curves obtained are highly depending on the vertical, distance and all the physical quantities are moving in wave propagation.

7. Conclusions

By comparing the figures which obtained under the three theories, important phenomena are observed that:

• Analytical solutions based upon normal mode analysis for thermoelastic problem in solids have been developed and utilized.

• The method which used in the present article is applicable to a wide range of problems in hydro-dynamics and thermoelasticity.

• The value of all physical quantities converges to zero with an increase in distance *z* and all functions are continuous.

• All the physical quantities satisfy the boundary conditions. From all most figures it is clear that the rotation acts to increase the magnitude of the real part of the physical quantities.

• The comparisons of different theories of thermoelasticity, (CT), (L-S), (G-L) theories are carried out.

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CC

Nomenclature

u_i	is the mechanical displacement
φ	is the electric potential
T	is an absolute temperature
ε_{ij}	is the strain tensor
σ_{ij}	is the stress tensor
β_{ij}	is the thermal elastic coupling tensor
E_i	is the electric field
D_i	is the electric displacement
C_{ijkl}	is the elastic parameters tensor
e_{jik}	is the piezoelectric moduli
$\in_{ij}$	is the dielectric moduli
p_i	is the pyroelectric moduli
ρ	is the mass density
t_0, t_1	is the thermal relaxation time parameters
K_{ij}	is the heat conduction tensor
T_0	is the reference temperature
C_e	is the specific heat at constant strain
n_1	is non-dimensional parameter

Appendix A

$$\delta_1=\frac{C_{11}}{\rho v_p^2},\ \delta_2=\frac{C_{44}}{\rho v_p^2},\ \ \delta_3=\frac{C_{13}+C_{44}}{\rho v_p^2},\ \ \delta_4=\frac{(e_{31}+e_{15})}{e_{33}},\ \ \delta_5=\frac{C_{33}}{\rho v_p^2},\ \ \delta_6=\frac{e_{15}}{e_{33}},\ \ \delta_7=-\frac{\beta_3}{\beta_1},$$

$$\delta_8=\frac{(e_{15}+e_{31})}{\rho v_p^2},\ \ \delta_9=\frac{e_{15}}{\rho v_p^2},\ \ \delta_{10}=\frac{e_{33}}{\rho v_p^2},\ \ \delta_{11}=-\frac{\in_{11}}{e_{33}},\ \ \delta_{12}=-\frac{\in_{33}}{e_{33}},\ \ \delta_{13}=\frac{p_3}{\beta_1},\ \ \delta_{14}=\frac{k_1\omega^*}{\rho C_e v_p^2},$$

$$\delta_{15}=\frac{k_3\omega^*}{\rho C_e v_p^2},\ \delta_{16}=\frac{\beta_1^2 T_0}{\rho^2 C_e v_p^2},\ \ \delta_{17}=\frac{\beta_1\beta_3 T_0}{\rho^2 C_e v_p^2},\ \delta_{18}=-\frac{p_3\beta_1 T_0}{\rho C_e e_{33}},\ A_1=-\frac{(a^2c^2-a^2\delta_1+\Omega^2)}{\delta_2},\ A_2=\frac{ia\delta_3}{\delta_2},$$

$$A_3=\frac{2iac\Omega}{\delta_2},\ A_4=\frac{ia\delta_4}{\delta_2},\ A_5=-\frac{ia(1-iact_1)}{\delta_2},\ \ A_6=\frac{ia\delta_3}{\delta_5},\ \ A_7=-\frac{2iac\Omega}{\delta_5},\ \ A_8=-\frac{(a^2\delta_2-a^2c^2-\Omega^2)}{\delta_5},$$

$$A_9=\frac{1}{\delta_5},\ \ A_{10}=-\frac{a^2\delta_6}{\delta_5},\ A_{11}=\frac{\delta_7(1-iact_1)}{\delta_5},\ \ A_{12}=\frac{ia\delta_8}{\delta_{10}},\ \ A_{13}=-\frac{a^2\delta_9}{\delta_{10}},\ A_{14}=\frac{\delta_{12}}{\delta_{10}},\ A_{15}=-\frac{a^2\delta_{11}}{\delta_{10}},$$

$$A_{16}=\frac{\delta_{13}(1-iact_1)}{\delta_{10}},\ A_{17}=\frac{ia\delta_{16}}{\delta_{15}}(iac+a^2c^2n_1t_0),\ A_{18}=\frac{\delta_{17}}{\delta_{15}}(iac+a^2c^2n_1t_0),\ \ A_{19}=\frac{\delta_{18}}{\delta_{15}}(iac+a^2c^2n_1t_0),$$

$$A_{20}=-\frac{(a^2\delta_{14}-iac-a^2c^2t_0)}{\delta_{15}}.$$

$$A=-(\frac{1}{A_{14}-A_9})(A_{14}A_{20}+A_{15}-A_{16}A_{19}+A_8A_{14}-A_9A_{13}-A_9A_{20}+A_9A_{16}A_{18}-A_{10}+A_{11}A_{19}-A_{11}A_{14}A_{18}$$
$$+A_1A_{14}-A_1A_9-A_2A_6A_{14}+A_2A_9A_{12}+A_4A_6-A_4A_{12})$$

$$B=(\frac{1}{A_{14}-A_9})(A_{15}A_{20}+A_8A_{14}A_{20}+A_8A_{15}-A_8A_{16}A_{19}-A_9A_{13}A_{20}-A_{10}A_{13}-A_{10}A_{20}+A_{10}A_{16}A_{18}+A_{11}A_{13}A_{19}-A_{11}A_{15}A_{18}$$
$$+A_1A_{14}A_{20}+A_1A_{15}-A_1A_{16}A_{19}+A_1A_8A_{14}-A_1A_9A_{13}-A_1A_9A_{20}+A_1A_9A_{16}A_{18}-A_1A_{10}+A_1A_{11}A_{19}-A_1A_{11}A_{14}A_{18}$$
$$-A_2A_6A_{14}A_{20}\ \ -A_2A_6A_{15}+A_2A_6A_{16}A_{19}+A_2A_9A_{12}A_{20}-A_2A_9A_{16}A_{17}+A_2A_{10}A_{12}-A_2A_{11}A_{12}A_{19}+A_2A_{11}A_{14}A_{17}$$
$$-A_3A_7A_{14}+A_4A_6A_{13}+A_4A_6A_{20}-A_4A_6A_{16}A_{18}-A_4A_{12}A_{20}+A_4A_{16}A_{17}-A_4A_8A_{12}+A_4A_{11}A_{12}A_{18}-A_4A_{11}A_{17}$$
$$-A_5A_6A_{19}+A_5A_6A_{14}A_{18}+A_5A_{12}A_{19}\ \ -A_5A_{14}A_{17}-A_5A_9A_{12}A_{18}+A_5A_9A_{17})$$

$$C=-(\frac{1}{A_{14}-A_9})(A_8A_{15}A_{20}-A_{10}A_{13}A_{20}+A_1A_{15}A_{20}+A_1A_8A_{14}A_{20}+A_1A_8A_{15}-A_1A_8A_{16}A_{19}-A_1A_9A_{13}A_{20}-A_1A_{10}A_{13}$$
$$-A_1A_{10}A_{20}+A_1A_{10}A_{16}A_{18}+A_1A_{11}A_{13}A_{19}-A_1A_{11}A_{15}A_{18}-A_2A_6A_{15}A_{20}+A_2A_{10}A_{12}A_{20}-A_2A_{10}A_{16}A_{17}+A_2A_{11}A_{15}A_{17}$$
$$-A_3A_7A_{14}A_{20}-A_3A_7A_{15}+A_3A_7A_{16}A_{19}+A_4A_6A_{13}A_{20}-A_4A_8A_{12}A_{20}+A_4A_8A_{16}A_{17}-A_4A_{11}A_{13}A_{17}-A_5A_6A_{13}A_{19}$$
$$+A_5A_6A_{15}A_{18}\ \ -A_5A_{15}A_{17}+A_5A_8A_{12}A_{19}-A_5A_8A_{14}A_{17}+A_5A_9A_{13}A_{17}-A_5A_{10}A_{12}A_{18}+A_5A_{10}A_{17})$$

$$E=(\frac{1}{A_{14}-A_9})(A_1A_8A_{15}A_{20}-A_1A_{10}A_{13}A_{20}-A_3A_7A_{15}A_{20}-A_5A_8A_{15}A_{17}+A_5A_{10}A_{13}A_{17})$$

Appendix B

$$H_{1n} = -\frac{s_{1n}}{s_{2n}}, \quad H_{2n} = -\frac{(q_{4n} + q_{5n} H_{1n})}{q_{6n}}, \quad H_{3n} = -\frac{1}{A_5}[(k_n^2 + A_1) - (A_2 k_n - A_3) H_{1n} - A_4 k_n H_{2n}],$$

$$H_{4n} = r_1 - l_1 k_n H_{1n} - l_2 k_n H_{2n} + r_2 H_{3n}, \quad H_{5n} = r_3 - \delta_5 k_n H_{1n} - k_n H_{2n} + r_4 H_{3n}, \quad H_{6n} = -\delta_2 k_n + r_5 H_{1n} + r_6 H_{2n},$$

$$H_{7n} = -\delta_9 k_n + r_7 H_{1n} + r_8 H_{2n}, \quad H_{8n} = r_9 - \delta_{10} k_n H_{1n} - \delta_{12} k_n H_{2n} + r_{10} H_{3n}, \quad \text{n= 1,2,3,4.}$$

$$q_{1n} = A_{11} k_n^3 + (A_1 A_{11} - A_5 A_6) k_n + A_5 A_7, \; q_{2n} = (-A_2 A_{11} + A_5) k_n^2 + A_3 A_{11} k_n + A_5 A_8, \quad q_{3n} = (-A_4 A_{11} + A_5 A_9) k_n^2 + A_5 A_{10},$$

$$q_{3n} = (-A_4 A_{11} + A_5 A_9) k_n^2 + A_5 A_{10}, \; q_{4n} = A_{16} k_n^3 + (A_1 A_{16} - A_5 A_{12}) k_n, \quad q_{5n} = (A_5 - A_2 A_{16}) k_n^2 + A_3 A_{16} k_n + A_5 A_{13},$$

$$q_{6n} = (A_5 A_{14} - A_4 A_{16}) k_n^2 + A_5 A_{15}, \; \mathrm{s}_{1n} = q_{1n} q_{6n} - q_{3n} q_{4n}, \quad \mathrm{s}_{2n} = q_{2n} q_{6n} - q_{3n} q_{5n}, \quad l_1 = \frac{C_{13}}{\rho v_p^2}, \quad l_2 = \frac{e_{31}}{e_{33}},$$

$$l_3 = \frac{e_{31}}{\rho v_p^2}, \quad r_1 = ia\delta_1, \quad r_2 = -(1 - i\,act_1), \quad r_3 = i\,a l_1, \quad r_4 = \delta_7 (1 - i\,act_1), \quad r_5 = i\,a\delta_2, \quad r_6 = i\,a\delta_6,$$

$$r_7 = ia\delta_9, \quad r_8 = ia\delta_{11}, \quad r_9 = ial_3, \quad r_{10} = \delta_{13}(1 - iact_1)$$