Structural Engineering and Mechanics, *Vol. 56, No. 1 (2015) 85-106*
DOI: http://dx.doi.org/10.12989/sem.2015.56.1.085

Thermo-mechanical post-buckling behavior of thick functionally graded plates resting on elastic foundations

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(Received February 2, 2015, Revised July 30, 2015, Accepted September 15, 2015)

Abstract. Postbuckling of thick plates made of functionally graded material (FGM) subjected to in-plane compressive, thermal and thermomechanical loads is investigated in this work. It is assumed that the plate is in contact with a Pasternak-type elastic foundation during deformation. Thermomechanical non-homogeneous properties are considered to be temperature independent, and graded smoothly by the distribution of power law across the thickness in the thickness in terms of the volume fractions of constituents. By employing the higher order shear deformation plate theory together the non-linear von-Karman strain-displacement relations, the equilibrium and compatibility equations of imperfect FGM plates are derived. The Galerkin technique is used to determine the buckling loads and postbuckling equilibrium paths for simply supported plates. Numerical examples are presented to show the influences of power law index, foundation stiffness and imperfection on the buckling and postbuckling loading capacity of the plates.

Keywords: functionally graded materials; postbuckling; higher order shear deformation theory; elastic foundation; imperfection

1. Introduction

Buckling and post-buckling behaviors of functionally graded (FG) plates as a major branch of solid structures is of interest in design and has attracted the attention of the researchers in recent years (Talha and Singh 2010, Akavci 2013, Chakraverty and Pradhan 2014, Pradhan and Chakraverty 2015, Mantari and Granados 2015). Eslami and his co-workers employed analytical formulation, classical and higher order plate theories in conjunction with adjacent equilibrium criterion to study the buckling of FG plates with and without imperfection under thermal and mechanical loads (Javaheri and Eslami 2002a, b, Samsam Shariat and Eslami 2006, 2007). Lanhe (2004) used the first order shear deformation theory to determine the critical buckling temperatures for simply supported FG plates. Zhao *et al*. (2009) investigated the mechanical and thermal buckling of FG plates using element-free Ritz method. Tung and Duc (2010) developed a simple

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

analytical solution to predict the buckling and post-buckling behavior of thin FG plates. Duc and Tung (2011) presented an analytical investigation on the buckling and post-buckling responses of thick FG plates resting on elastic foundations and subjected to in-plane compressive, thermal and thermo-mechanical loads. El Meiche *et al*. (2011) proposed a new hyperbolic shear deformation theory for buckling and vibration of FG sandwich plate. Bourada *et al*. (2012) presented a novel four-variable refined plate theory for thermal buckling analysis of FG sandwich plates. Bachir Bouiadjra *et al*. (2012) presented a four-variable refined plate theory for buckling analysis of FG plates. Bachir Bouiadjra *et al*. (2013) investigated the nonlinear thermal buckling behavior of FG plates using an efficient sinusoidal shear deformation theory. Sobhy (2013) examined the vibration and the buckling responses of exponentially graded sandwich plates resting on Pasternak elastic foundation. Ait Amar Meziane *et al*. (2014) developed an efficient and simple refined theory to study the buckling and free vibration responses of exponentially graded sandwich plates under various boundary conditions. Khalfi *et al*. (2014) presented a refined and simple shear deformation theory for thermal buckling of solar FG plates resting on elastic foundation. Swaminathan and Naveenkumar (2014) presented a higher order refined computational models for the buckling analysis of FG sandwich plates. Akil (2014) presented a higher order theory for the buckling and post buckling response of sandwich beams having functionally graded faces. Bennai *et al.* (2015) developed a new refined hyperbolic shear and normal deformation beam theory for the free vibration and buckling of FG sandwich beams under various boundary conditions.

These structural components like plates supported on an elastic foundation often find applications in the construction of nuclear, mechanical, aerospace, and civil engineering structures (Houari *et al*. 2013, Bessaim *et al*. 2013, Hebali *et al*. 2014, Zidi *et al*. 2014, Fekrar *et al*. 2014, Bousahla *et al*. 2014, Ramu and Mohanty 2015, Hamidi *et al*. 2015, Ait Yahia *et al*. 2015, Mahi *et al*. 2015). To describe the interaction between the plate and foundation, several foundation models have been developed. The simplest one is the Winkler or one-parameter model (Winkler, 1867) which models the foundation as a series of separated springs without coupling effects between each other. This model was improved by Pasternak (1954) by considering a shear spring to simulate the interactions between the separated springs in the Winkler model. The Pasternak or two-parameter model is widely used to describe the mechanical behavior of structure–foundation interactions and will be used here to simulate the interactions between the plate and foundation. Consequently, it is necessary to account for effects of elastic foundation for a better understanding of the postbuckling response of structures such as plates and shells. Librescu and Lin have extended previous works (Librescu and Lin 1997, Lin and Librescu 1998) to consider the postbuckling behavior of flat and curved laminated composite panels resting on Winkler elastic foundations (Librescu and Lin 1997, Lin and Librescu 1998). However, investigation on FG plates and shells supported by elastic foundation are limited in number. Bouderba *et al.* (2013) studied the thermo-mechanical bending response of thick FG plate resting on two-parameter elastic foundations. Duc and Tung (2011) presented an analytical formulation for the buckling and post-buckling responses of thick FG plates resting on elastic foundations and subjected to in-plane compressive, thermal and thermomechanical loads. Yaghoobi and Torabi (2013a) studied the buckling behavior of FG plates supported by two-parameter Pasternak's foundations under thermal loads. An analytical approach was applied for solving the problem. Moreover, Yaghoobi and Yaghoobi (2013) investigated the buckling response of symmetric sandwich plates with FGM face sheets resting on an elastic foundation based on the first-order shear deformation plate theory and subjected to mechanical, thermal and thermo-mechanical loads. Yaghoobi and Torabi (2013b) analyzed post-buckling and nonlinear vibration behavior of geometrically imperfect FG beams resting on nonlinear elastic foundation subjected to axial force.

This work presents an analytical formulation to study the buckling and post-buckling response of simply supported FG thick plates resting on two-parameter elastic foundations and subjected to in-plane compressive, thermal, and thermo-mechanical loads. Shi's higher order shear deformation theory (Shi 2007) is employed to establish governing equations considering into account geometrical nonlinearity and initial geometrical imperfection. Analytical expressions of buckling loads and post-buckling load–deflection curves for simply supported FG plates are determined by Galerkin technique. Numerical examples are presented to show the influences of geometrical and material properties, in-plane restraint, foundation stiffness and imperfection on the response of the FG plates.

2. Theoretical formulations

2.1 Kinematics

In this work, the Shi's higher order shear deformation plate theory (Shi 2007) is used. Shi (2007) employed the improved third order shear deformation theory based on a more rigorous kinematics of displacements defined as TSDT to investigate static analysis of isotropic and orthotropic beams and plates. The author concluded that the proposed new higher order shear deformation theory provided more accuracy than other higher order shear deformable theory, especially when the transverse shear plays a very important role. Because of the kinematics of displacement in TSDT derived from an elasticity formulation rather than the hypothesis of displacements, it is interesting to use this theory for the buckling and post-buckling of the FG plates. The displacement field can then be expressed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w(x,y)}{\partial x} + \Psi(z)\ \phi_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w(x,y)}{\partial y} + \Psi(z)\ \phi_y(x,y) \\ w(x,y,z) &= w_0(x,z) \end{aligned} \tag{1a}$$

with

$$\Psi(z) = \frac{5}{4}\left(z - \frac{4}{3h^2}z^3\right) \tag{1b}$$

where u_0, v_0 and w_0 are generalized displacement at the mid-plane of the plate in the x, y, and z directions, respectively; ϕ_x, ϕ_y are the slope rotations in the (x, z) and (y, z) planes, respectively; and h is the plate thickness.

The non-linear von Karman strain–displacement equations are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + \Psi(z)\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \Psi'(z)\begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix}, \tag{2}$$

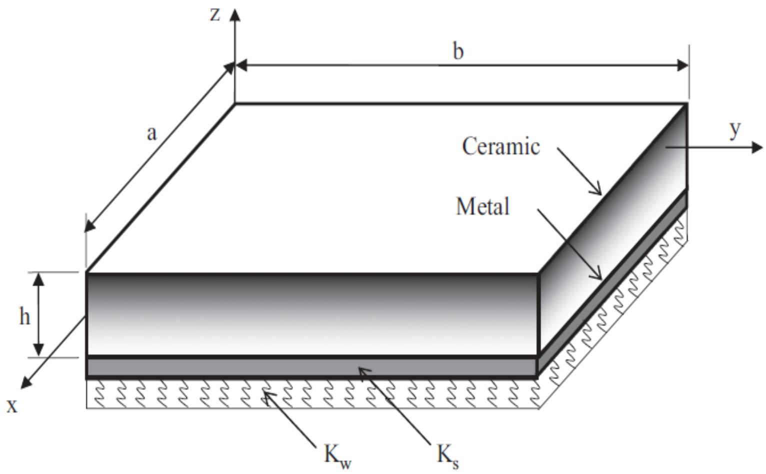

Fig. 1 Coordinate system and geometry for FG plates on Pasternak elastic foundation

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} u_{0,x} + (w_{0,x})^2/2 \\ v_{0,x} + (w_{0,y})^2/2 \\ u_{0,y} + v_{0,x} + w_{0,x} w_{0,y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -w_{0,xx} \\ -w_{0,yy} \\ -2w_{0,xy} \end{Bmatrix}, \quad \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix}, \tag{3}$$

2.2 Constitutive equations

Consider a ceramic-metal FG plate of length *a*, width *b* and thickness *h* resting on an elastic foundation as shown in Fig. 1. The properties of FG plate (*P*) are assumed to vary through the plate thickness with a power law distribution of the volume fraction of the two materials as (Benachour *et al*. 2011, Ould Larbi *et al*. 2013, Tounsi *et al*. 2013, Belabed *et al*. 2014, Bourada *et al*. 2015, Al-Basyouni *et al*. 2015, Zemri *et al*. 2015, Bouchafa *et al*. 2015)

$$P(z) = P_M + P_{CM}\left(\frac{z}{h} + \frac{1}{2}\right)^N, \quad P_{CM} = P_C - P_M \tag{4}$$

where P_M and P_C are the corresponding properties of the metal and ceramic, respectively, and *N* is the power law index which takes the value greater or equal to zero. In the present work, we assume that the elasticity modules *E*, thermal conductivity *K*, and the thermal expansion coefficient α, are described by Eq. (4).

The linear constitutive relations of a FG plate can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha\,\Delta T \\ \varepsilon_y - \alpha\,\Delta T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{5}$$

where ΔT is temperature rise from stress free initial state or temperature difference between two surfaces of the FG plate.

By employing the virtual work principle to minimize the functional of total potential energy function result in the expressions for the nonlinear equilibrium equations of a perfect plate resting on two parameters elastic foundation as

$$N_{x,x} + N_{xy,y} = 0 \tag{6a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{6b}$$

$$(M_{x,xx} + 2M_{xy,xy} + M_{y,yy}) + N_x w_{,xx} + 2\ N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w = 0 \tag{6c}$$

$$S_{x,x} + S_{xy,y} - Q_x = 0 \tag{6d}$$

$$S_{xy,x} + S_{y,y} - Q_y = 0 \tag{6e}$$

where k_w is the Winkler foundation stiffness and k_g is a constant showing the effect of the shear interactions of the vertical elements. The force and moment resultants (N, Q, S and M) of the FG plate are determined by

$$(N_i,\ M_i,\ S_i) = \int_{-h/2}^{h/2} \sigma_i\,(1, z, \Psi(z))\,dz, \quad (i = x, y, xy) \tag{7a}$$

$$Q_i = \int_{-h/2}^{h/2} \sigma_j\ \Psi'(z)\,dz, \qquad (\,i = x, y\,);\ \ (\,j = xz, yz) \tag{7b}$$

Substitution of Eqs. (2) and (5) into Eqs. (7) yields the constitutive relations as

$$\begin{aligned}(N_x,\ M_x, S_x) = \frac{1}{1-\nu^2}[&(E_1, E_2, E_3)\,(\varepsilon_x^0 + \nu\,\varepsilon_y^0) + (E_2, E_4, E_5)\,(k_x + \nu\,k_y) \\ &+ (E_3, E_5, E_7)\,(\eta_x + \nu\,\eta_y) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)]\end{aligned} \tag{8a}$$

$$\begin{aligned}(N_y,\ M_y,\ S_y) = \frac{1}{1-\nu^2}[&(E_1, E_2, E_3)\,(\varepsilon_y^o + \nu\,\varepsilon_x^0) + (E_2, E_4, E_5)\,(k_y + \nu\,k_x) \\ &+ (E_3, E_5, E_7)\,(\eta_y + \nu\,\eta_x) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)]\end{aligned} \tag{8b}$$

$$(N_{xy},\, M_{xy},\, S_{xy}) = \frac{1}{2(1+\nu)}[(E_1, E_2, E_3)\,\gamma^0_{xy} + (E_2, E_4, E_5)\,k_{xy} + (E_3, E_5, E_7)\,\eta_{xy}] \tag{8c}$$

$$(Q_x, Q_y) = \frac{1}{2(1+\nu)}\,E_8\left(\gamma^0_{xz}, \gamma^0_{yz}\right) \tag{8d}$$

where

$$(E_1, E_2, E_3, E_4, E_5, E_7) = \int_{-h/2}^{h/2} (1, z, \Psi,\ z^2, z\Psi, \Psi^2) E(z) dz\,,\quad E_8 = \int_{-h/2}^{h/2} (\Psi'(z))^2\, E(z)\, dz \tag{9a}$$

$$(\Phi_1, \Phi_2, \Phi_3) = \int_{-h/2}^{h/2} (1, z, \Psi) E(z)\, \alpha(z)\, \Delta T(z)\, dz \tag{9b}$$

The last three equations of Eqs. (6) may be rewritten into two equations in terms of variables w_0 and $\phi_{x,x}+\phi_{y,y}$ by substituting Eqs. (3) and (8) into Eqs. (6c)-(6e). Subsequently, elimination of the variable $\phi_{x,x}+\phi_{y,y}$ from two the resulting equations leads to the following system of equilibrium equations

$$N_{x,x} + N_{xy,y} = 0 \tag{10a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{10b}$$

$$\begin{aligned}&(D_1 D_3 - D_2^2)\nabla^6 w - D_1 D_4\ \nabla^4 w - D_3 \nabla^2 (N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w)\\ &+ D_4\,(N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w) = 0\end{aligned} \tag{10c}$$

where

$$D_1 = \frac{E_1\, E_4 - E_2^2}{E_1(1-\nu^2)}\,,\quad D_2 = \frac{E_1\, E_5 - E_2\, E_3}{E_1(1-\nu^2)}\,,\quad D_3 = \frac{E_1\, E_7 - E_3^2}{E_1(1-\nu^2)}\,,\quad D_4 = \frac{E_8}{2(1+\nu)} \tag{11}$$

For an imperfect FG plate, Eq. (10) are modified into form as

$$\begin{aligned}&(D_1 D_3 - D_2^2)\nabla^6 w - D_1 D_4 \nabla^4 w\ - D_3 \nabla^2 [\,f_{,yy}(w_{,xx} + w^*_{,xx}) - 2\,f_{,xy}(w_{,xy} + w^*_{xy})\\ &\qquad + f_{,xx}(w_{,yy} + w^*_{,yy}) - k_w w + k_g \nabla^2 w]\\ &\qquad + D_4\,[\,f_{,yy}(w_{,xx} + w^*_{,xx}) - 2\,f_{,xy}(w_{,xy} + w^*_{xy}) + f_{,xx}(w_{,yy} + w^*_{,yy}) - k_w w + k_g \nabla^2 w] = 0\end{aligned} \tag{12}$$

in which $w^*_0(x, y)$ is a known function representing initial small imperfection of the plate. Note that the terms $\nabla^6 w_0$ and $\nabla^4 w_0$ are unchanged because these terms are obtained from the expressions for bending moments M_i and higher order moments S_i and these moments depend not on the total curvature but only on the change in curvature of the plate (Samsam Shariat and Eslami 2006). Also, $f(x, y)$ is stress function defined by

$$N_x = f_{,yy}\ ,\ N_y = f_{,xx}\ ,\ N_{xy} = -f_{,xy} \tag{13}$$

The geometrical compatibility equation for an imperfect plate is expressed as

$$\varepsilon^0_{x,yy} + \varepsilon^0_{y,xx} + \gamma^0_{xy,xy} = w^2_{0,xy} - w_{0,xx}w_{0,yy} + 2w_{0,xy}w^*_{0,xy} - w_{0,xx}w^*_{0,yy} - w_{0,yy}w^*_{0,xx} \tag{14}$$

From the constitutive relations (8) and Eq. (13) one can write

$$\begin{aligned}(\varepsilon^0_x, \varepsilon^0_y) &= \frac{1}{E_1}[(f_{,yy}, f_{,xx}) - \nu\,(f_{,xx}, f_{,yy}) - E_2\,(k_x, k_y) - E_3\,(\eta_x, \eta_y)\\ &\qquad + \Phi_1\,(1,1)]\\ \gamma^0_{xy} &= -\frac{1}{E_1}[2\,(1+\nu)\,f_{,xy} + E_2\,k_{xy} + E_3\,\eta_{xy}]\end{aligned} \tag{15}$$

Substituting Eq. (15) into Eq. (14), the compatibility equation of an imperfect FG plate becomes

$$\nabla^4 f - E_1\,(w^2_{0,xy} - w_{0,xx}\,w_{0,yy} + 2\,w_{0,xy}\,w^*_{0,xy} - w_{0,xx}\,w^*_{0,yy} - w_{0,yy}\,w^*_{0,xx}) = 0 \tag{16}$$

It is noted that Eqs. (12) and (16) are nonlinear equations employed to study the stability of thick FG plates resting on elastic foundations subjected to mechanical, thermal and thermo-mechanical loads.

Three cases of boundary conditions are considered in this work, referred to as Cases 1, 2 and 3 (Librescu and Lin 1997, Lin and Librescu 1998).

• *Case 1:* Four edges of the plate are simply supported and freely movable (FM). The associated boundary conditions are

$$w = N_{xy} = \varphi_y = M_x = S_x = 0, \qquad N_x = N_{x0} \ \text{ at } \ x = 0, a \tag{17a}$$

$$w = N_{xy} = \varphi_x = M_y = S_y = 0, \qquad N_y = N_{y0} \ \text{ at } \ y = 0, b \tag{17b}$$

• *Case 2:* Four edges of the plate are simply supported and immovable (IM). In this case, boundary conditions are

$$w = u_0 = \varphi_y = M_x = S_x = 0, \qquad N_x = N_{x0} \ \text{ at } \ x = 0, a \tag{18a}$$

$$w = v_0 = \varphi_x = M_y = S_y = 0, \qquad N_y = N_{y0} \ \text{ at } \ y = 0, b \tag{18b}$$

• *Case 3:* All edges are simply supported. Two edges x=0, a are freely movable and subjected to compressive load in the x direction, whereas the remaining two edges y=0, b are unloaded and immovable. For this case, the boundary conditions are defined as

$$w = N_{xy} = \varphi_y = M_x = S_x = 0, \qquad N_x = N_{x0} \ \text{ at } \ x = 0, a \tag{19a}$$

$$w = v_0 = \varphi_x = M_y = S_y = 0, \qquad N_y = N_{y0} \ \text{ at } \ y = 0, b \tag{19b}$$

where N_{x0}, N_{y0} are axial compressive loads at movable edges (i.e., Case 1 and the first of Case 3) or are fictitious compressive edge loads at immovable edges (i.e., Case 2 and the second of Case 3).

The proposed solutions of w and f respecting boundary conditions (17)-(19) are considered to be (Librescu and Lin 1997, Lin and Librescu 1998)

$$(w, w^*) = (W, \mu h)\sin(\lambda_m x)\sin(\delta_n y) \tag{20a}$$

$$\begin{aligned} f &= A_1\cos(2\lambda_m x) + A_2\cos(2\delta_n y) + A_3\sin(\lambda_m x)\sin(\delta_n y) \\ &\quad + \frac{1}{2}N_{x0}y^2 + \frac{1}{2}N_{y0}x^2 \end{aligned} \tag{20b}$$

$$\varphi_x = B_1\cos(\lambda_m x)\sin(\delta_n y), \quad \varphi_y = B_2\sin(\lambda_m x)\cos(\delta_n y) \tag{20c}$$

where $\lambda_m = m\pi/a$, $\delta_n = n\pi/b$, m, n are odd numbers, W is amplitude of the deflection and μ is imperfection parameter. The coefficients $\boldsymbol{A_i}$ (i=1, 2, 3) are determined by substitution of Eqs. (20a), (20b) into Eq. (16) as

$$A_1 = \frac{E_1\delta_n^2}{32\lambda_m^2}W(W+2\mu h), \quad A_2 = \frac{E_1\lambda_m^2}{32\delta_n^2}W(W+2\mu h), \quad A_3 = 0 \tag{21}$$

Using Eqs. (3) and (8) in Eqs. (6d), (6e) and substituting Eqs. (20a), (20c) into the resulting equations, the coefficients B_1 and B_2 are determined as

$$B_1 = \frac{a_{12}a_{23} - a_{22}a_{13}}{a_{12}^2 - a_{11}a_{22}}W, \quad B_2 = \frac{a_{12}a_{13} - a_{11}a_{23}}{a_{12}^2 - a_{11}a_{22}}W \tag{22}$$

in which

$$(a_{11}, a_{22}, a_{12}) = D_3(\lambda_m^2, \delta_m^2, \nu\lambda_m\delta_n) + \frac{1-\nu}{2}D_3(\lambda_m^2, \delta_m^2, \lambda_m\delta_n) + D_4(1,1,0) \tag{23a}$$

$$(a_{13}, a_{23}) = -D_2(\lambda_m^3 + \lambda_m\delta_n^2, \delta_n^3 + \delta_n\lambda_m^2) \tag{23b}$$

Then, setting Eqs. (20a), (20b) into Eq. (12) and employing the Galerkin method for the resulting equation yield

$$\begin{aligned} &(-(D_1D_3 - D_2^2)(\lambda_m^2+\delta_n^2)^3 - D_1D_4(\lambda_m^2+\delta_n^2)^2 - [k_w + k_g(\lambda_m^2+\delta_n^2)]\,[(D_3(\lambda_m^2+\delta_n^2) + D_4])\,W \\ &-\frac{E_1}{16}(D_3(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4 + \lambda_m^6 + \delta_n^6) + D_4(\lambda_m^4+\delta_n^4)) \times W(W+\mu h)\,(W+2\mu h) \\ &-(D_3(\lambda_m^2+\delta_n^2) + D_4) \times (N_{x0}\lambda_m^2 + N_{y0}\delta_n^2)\,(W+\mu h) = 0 \end{aligned} \tag{24}$$

This equation will be employed to investigate the buckling and post-buckling responses of thick FG plates under mechanical, thermal and thermomechanical loads.

2.3 Mechanical post-buckling analysis

A simply supported FG plate with all movable edges is considered and this plate is supported by elastic foundations and subjected to axial edge compressive loads ($\boldsymbol{F_x}$, $\boldsymbol{F_y}$) uniformly distributed on edges $\boldsymbol{x}$=0, $\boldsymbol{a}$ and $\boldsymbol{y}$=0, $\boldsymbol{b}$, respectively. In this case, prebuckling force resultants are (Samsam Shariat and Eslami 2007)

$$N_{x0} = -F_x h\,,\ N_{y0} = -F_y h \tag{25}$$

and Eq. (24) leads to

$$F_x = e_1^1 \frac{W}{h(W+\mu h)} + e_2^1 \frac{W}{h}(W + 2\mu h) \tag{26}$$

where

$$e_1^1 = \frac{(D_1 D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^3 + D_1 D_4 (\lambda_m^2 + \delta_n^2)^2}{(\lambda_m^2 + \beta\,\delta_n^2)[D_3(\lambda_m^2 + \delta_n^2) + D_4]} + \frac{[K_w + K_g a^2 (\lambda_m^2 + \delta_n^2)]\,D_1}{a^4(\lambda_m^2 + \beta\,\delta_n^2)} \tag{27a}$$

$$e_2^1 = \frac{E_1}{16(\lambda_m^2 + \beta\,\delta_n^2)[D_3(\lambda_m^2 + \delta_n^2) + D_4]} \times [D_3(\lambda_m^4 \delta_n^2 + \lambda_m^2 \delta_n^4 + \lambda_m^6 + \delta_n^6) + D_4(\lambda_m^4 + \delta_n^4)] \tag{27b}$$

in which

$$\beta = F_y / F_x\ ,\ K_w = \frac{k_w a^4}{D_1}\ ,\ K_g = \frac{k_g a^2}{D_1} \tag{28}$$

For a perfect FG plate, Eq. (26) reduces to an equation from which buckling compressive load may be determined as $F_{xb} = e_1^1$.

2.4 Thermal post-buckling analysis

A simply supported FG plate with all immovable edges is assumed here. The plate is also rested by an elastic foundation and exposed to temperature environments or subjected to through the thickness temperature gradient. The in-plane condition on immovability at all edges, i.e., $\boldsymbol{u}_0$=0 at $\boldsymbol{x}$=0, $\boldsymbol{a}$ and $\boldsymbol{v}_0$=0 at $\boldsymbol{y}$=0, $\boldsymbol{b}$, is given in an average sense as (Tung and Duc 2010, Shen 2007)

$$\int_0^b\int_0^a \frac{\partial u_0}{\partial x}\,dxdy = 0\,,\ \int_0^a\int_0^b \frac{\partial v_0}{\partial x}\,dydx = 0 \tag{29}$$

From Eqs. (3) and (8) one can determine the following expressions in which Eq. (13) and imperfection have been included

$$\frac{\partial u_0}{\partial x} = \frac{1}{E_1}(f_{,yy} - \nu\, f_{,xx}) + \frac{E_2}{E_1} w_{,xx} - \frac{E_3}{E_1}\varphi_{x,x} - \frac{1}{2} w_{,x}^2 - w_{,x} w_{,x}^* + \frac{\Phi_1}{E_1} \tag{30a}$$

$$\frac{\partial v_0}{\partial y} = \frac{1}{E_1}(f_{,xx} - \nu\, f_{,yy}) + \frac{E_2}{E_1} w_{,yy} - \frac{E_3}{E_1}\varphi_{y,y} - \frac{1}{2} w_{,y}^2 - w_{,y} w_{,y}^* + \frac{\Phi_1}{E_1} \tag{30b}$$

Introduction of Eqs. (20) into Eqs. (30) and then the result into Eq. (29) give

$$N_{x0} = -\frac{\Phi_1}{1-\nu} + \frac{4}{mn\pi^2(1-\nu^2)}\left[E_3(\lambda_m B_1 + \nu\,\delta_n B_2) + E_2(\lambda_m^2 + \nu\,\delta_n^2)\right] W + \frac{E_1}{8(1-\nu^2)}(\lambda_m^2 + \nu\,\delta_n^2)\,W(W+2\mu h) \tag{31a}$$

$$N_{y0} = -\frac{\Phi_1}{1-\nu} + \frac{4}{mn\pi^2(1-\nu^2)}\left[E_3(\nu\,\lambda_m B_1 + \delta_n B_2) + E_2(\nu\,\lambda_m^2 + \delta_n^2)\right] W + \frac{E_1}{8(1-\nu^2)}(\nu\,\lambda_m^2 + \delta_n^2)\,W(W+2\mu h) \tag{31b}$$

When the deflection dependence of fictitious edge loads is ignored, i.e., $W = 0$, Eqs. (31) becomes

$$N_{x0} = N_{y0} = -\frac{\Phi_1}{1-\nu} \tag{32}$$

Substituting Eqs. (31) into Eq. (24) yields the expression of thermal parameter as

$$\frac{\Phi_1}{1-\nu} = \left[\frac{(D_1D_3 - D_2^2)(\lambda_m^2+\delta_n^2)^2 + D_1D_4(\lambda_m^2+\delta_n^2)}{D_3(\lambda_m^2+\delta_n^2)+D_4} + \frac{k_w + k_g(\lambda_m^2+\delta_n^2)}{(\lambda_m^2+\delta_n^2)}\right]\frac{W}{W+\mu h}$$
$$+\frac{4}{mn\pi^2(1-\nu^2)(\lambda_m^2+\delta_n^2)}\times\left[E_3(\lambda_m^3B_1 + \nu\,\lambda_m^2\delta_n B_2 + \nu\,\lambda_m\delta_n^2 B_1 + \delta_n^3 B_2) + E_2\,(\lambda_m^4 + 2\nu\,\lambda_m^2\delta_n^2 + \delta_n^4)\right]W$$
$$+\left[\frac{E_1\left[D_3(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4 + \lambda_m^6 + \delta_n^6) + D_4(\lambda_m^4 + \delta_n^4)\right]}{16\left[D_3(\lambda_m^2+\delta_n^2)+D_4\right](\lambda_m^2+\delta_n^2)} + \frac{E_1\left[(\lambda_m^4 + 2\,\nu\,\lambda_m^2\delta_n^2 + \delta_n^4)\right]}{8(1-\nu^2)(\lambda_m^2+\delta_n^2)}\right]W(W+2\mu h) \tag{33}$$

2.4.1 Uniform temperature rise

The FG plate is subjected to temperature environments uniformly raised from stress free initial state T_i to final value T_f, and temperature change $\Delta T = T_f - T_i$ is assumed to be independent from thickness variable. The thermal parameter Φ_1 is obtained from Eq. (9b), and substitution of the result into Eq. (33) yields

$$\Delta T = e_1^2\,\frac{W}{W+\mu h} + e_2^2\,W + e_3^2\,W(W+2\mu h) \tag{34}$$

where

$$e_1^2=\frac{(1-\nu)}{L\,[D_3(\lambda_m^2+\delta_n^2)+D_4]}\times\left[(D_1D_3-D_2^2)(\lambda_m^2+\delta_n^2)^2+D_1D_4(\lambda_m^2+\delta_n^2)\right]$$
$$+\frac{\left[K_w+K_g\,a^2(\lambda_m^2+\delta_n^2)\right](1-\nu)\,D_1}{a^4\,L\,(\lambda_m^2+\delta_n^2)}\,,$$
$$e_2^2=\frac{4}{m\,n\,L\,\pi^2(1+\nu)(\lambda_m^2+\delta_n^2)}\times$$
$$\left[E_3(\lambda_m^3\,B_1+\nu\,\lambda_m^2\,\delta_n\,B_2+\nu\,\lambda_m\,\delta_n^2\,B_1+\delta_n^3\,B_2)+E_2(\lambda_m^4+2\nu\,\lambda_m^2\delta_n^2+\delta_n^4)\right], \tag{35}$$
$$e_3^2=\frac{E_1(1-\nu)}{16L\,(\lambda_m^2+\delta_n^2)[D_3(\lambda_m^2+\delta_n^2)+D_4]}\times[D_3(\lambda_m^4\delta_n^2+\lambda_m^2\delta_n^4+\lambda_m^6+\delta_n^6)+D_4(\lambda_m^4+\delta_n^4)]$$
$$+\frac{E_1(\lambda_m^4+2\nu\,\lambda_m^2\,\delta_n^2+\delta_n^4)}{8\,L\,(1+\nu)(\lambda_m^2+\delta_n^2)}$$

in which

$$L=E_m\alpha_m+\frac{E_m\alpha_{cm}+E_{cm}\alpha_m}{N+1}+\frac{E_{cm}\alpha_{cm}}{2N+1} \tag{36}$$

By Setting μ=0, Eq. (34) leads to an equation from which buckling temperature change of the perfect FG plates may be obtained as $\Delta T_b=e_1^2$.

2.4.2 Nonlinear temperature

The metal-rich surface temperature $\boldsymbol{T_M}$ is maintained at reference value while ceramic-rich surface temperature $\boldsymbol{T_C}$ is enhanced and steadily conducted across the thickness direction according to one-dimensional Fourier equation

$$\frac{d}{dz}\left[K(z)\frac{dT}{dz}\right]=0\,,\quad T(z=-h/2)=T_M\,,\quad T(z=h/2)=T_C\,. \tag{37}$$

Using $K(z)$ according to Eq. (4), the solution of Eq. (37) may be found in terms of polynomial series, and the first seven terms of this series gives the following approximation (Duc and Tung 2011, Shahrjerdi *et al*. 2011)

$$T(z)=T_M+\Delta T\;\frac{r\displaystyle\sum_{j=0}^{5}\frac{(-r^N K_{CM}/K_M)^j}{jN+1}}{\displaystyle\sum_{j=0}^{5}\frac{(-K_{CM}/K_M)^j}{jN+1}} \tag{38}$$

where r=(2z+h)/2h and, in this case of thermal loading, $\Delta T=T_C-T_M$ is defined as the temperature difference between two surfaces of the FG plate.

Substitution of Eq. (38) into Eq. (9b) and setting the result Φ_1 into Eq. (33) yield a closed-form expression of temperature–deflection curves which is similar to Eq. (34), providing L is replaced by H defined as

$$H=\frac{\sum_{j=0}^{5}\frac{(-K_{CM}/K_M)^j}{jN+1}\left[\frac{E_M\alpha_M}{jN+2}+\frac{E_M\alpha_{CM}+E_{CM}\alpha_M}{(j+1)N+2}+\frac{E_{CM}\alpha_{CM}}{(j+2)N+2}\right]}{\sum_{j=0}^{5}\frac{(-K_{CM}/K_M)^j}{jN+1}} \tag{39}$$

2.5 Thermo-mechanical post-buckling analysis

The FG plate supported by the elastic foundation is uniformly compressed by F_x on two movable edges x=0, a and simultaneously exposed to elevated temperature environments or subjected to nonlinear temperature distribution. The two edges y=0, b are considered to be immovable. In this case, $N_{x0}=-F_x h$ and fictitious compressive load on immovable edges is obtained by setting the second of Eq. (30) in the second of Eq. (29) as

$$N_{y0}=\nu N_{x0}-\Phi_1+\frac{4\delta_n}{mn\pi^2}\left[E_3B_2+E_2\delta_n\right]W+\frac{E_1\delta_n^2}{8}W(W+2\mu h) \tag{40}$$

Then, $\boldsymbol{N}_{x0}$ and $\boldsymbol{N}_{y0}$ are placed in Eq. (24) to give

$$F_x=e_1^3\frac{W}{h\,(W+\mu h)}+e_2^3\frac{W}{h}+e_3^3W\frac{(W+2\mu h)}{h}+\frac{L\,\delta_n^2\,\Delta T}{h(\lambda_m^2+\nu\delta_n^2)} \tag{41}$$

where the coefficients $\boldsymbol{e}_1^3;\ \boldsymbol{e}_2^3;\ \boldsymbol{e}_3^3$ are defined as follows

$$e_1^3=\frac{(D_1D_3-D_2^2)(\lambda_m^2+\delta_n^2)^3+D_1D_4(\lambda_m^2+\delta_n^2)^2}{(\lambda_m^2+\nu\delta_n^2)[D_3(\lambda_m^2+\delta_n^2)+D_4]}+\frac{[K_w+K_g\,a^2(\lambda_m^2+\delta_n^2)]D_1}{a^4(\lambda_m^2+\nu\delta_n^2)}, \tag{42a}$$

$$e_2^3=\frac{4\delta_n^3}{mn\pi^2(\lambda_m^2+\nu\delta_n^2)}\times\left[E_3B_2+E_2\,\delta_n\right] \tag{42b}$$

$$e_3^3=\frac{E_1}{16(\lambda_m^2+\nu\delta_n^2)[D_3(\lambda_m^2+\delta_n^2)+D_4]}\times[D_3(\lambda_m^4\delta_n^2+\lambda_m^2\delta_n^4+\lambda_m^6+\delta_n^6)+D_4(\lambda_m^4+\delta_n^4)]+\frac{E_1\delta_n^4}{8(\lambda_m^2+\nu\delta_n^2)} \tag{42c}$$

and L is replaced by H in the case of the FG plates subjected to combined action of uniaxial compressive load and temperature gradient.

Eqs. (26), (34) and (41) are explicit expressions of load–deflection curves for thick FG plates supported by Pasternak elastic foundations and subjected to axial compressive, thermal and thermo-mechanical loads, respectively.

3. Results and discussion

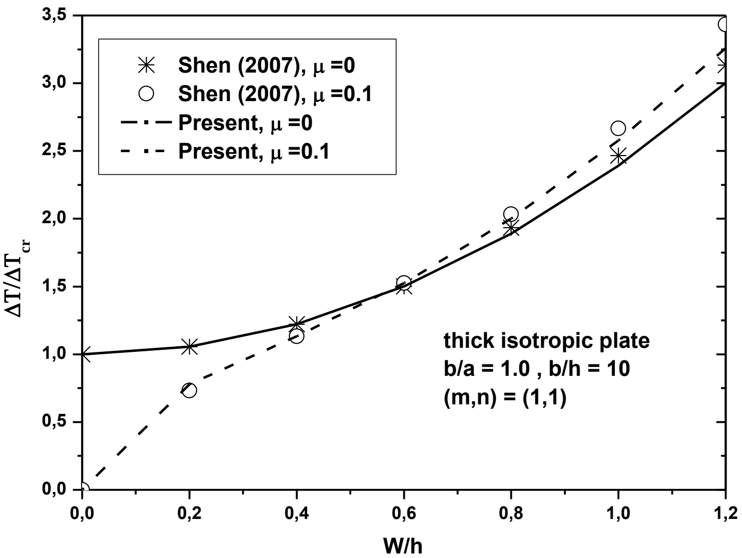

Fig. 2 Comparisons of thermal post-buckling load-deflection curves for isotropic plates

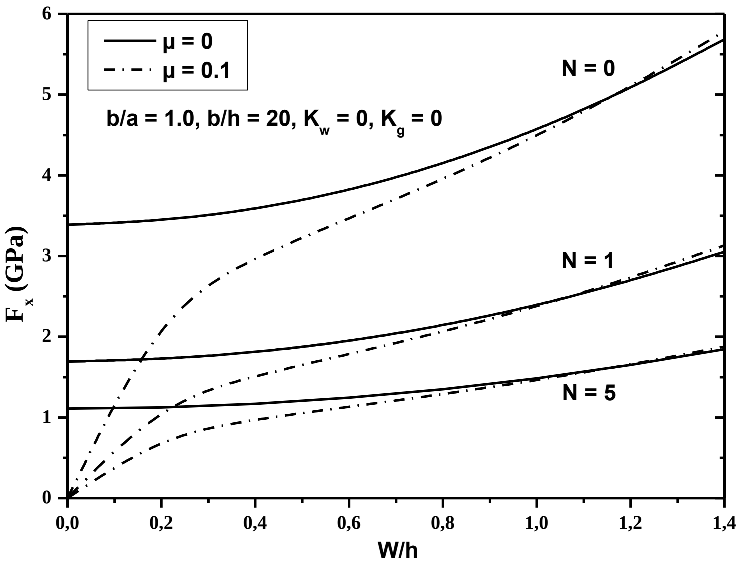

Fig. 3 Effects of the power law index on the post-buckling of FG plates under uniaxial compressive load (all movable edges)

For the validation purpose, the thermal post-buckling of a simply supported square thick isotropic plate is examined. The plate is subjected to uniform temperature field with all immovable edges and without elastic foundation. The thermal post-buckling load-deflection curves for perfect and imperfect isotropic plates (v=0.3) obtained using the present formulation are compared to those of Shen (2007) in Fig. 2. In general a good agreement is observed in this comparison. However, for high values of the thermal load, significant differences between the results from the proposed method and Shen's method are appeared. These differences are due to the used methods.

In the following, we consider a square ceramic-metal plate with the following properties (Javaheri and Eslami 2002b, Samsam Shariat and Eslami 2006, 2007, Lanhe 2004, Duc and Tung 2011):

- $E_M = 70\,\text{GPa}$, $\nu_M = 0.3$, $\alpha_M = 23\times10^{-6}\,°\text{C}^{-1}$, $K_M = 204\,\text{W/mK}$
- $E_C = 380\,\text{GPa}$, $\nu_C = 0.3$, $\alpha_C = 7.4\times10^{-6}\,°\text{C}^{-1}$, $K_C = 10.4\,\text{W/mK}$

It is noted that the buckling of perfect plates occurs for m=n=1, and these values of half waves are also employed to plot load-deflection equilibrium paths for both perfect and imperfect plates. In graphs, *W/h* presents the dimensionless maximum deflection and the FG plate-foundation interaction is ignored, unless otherwise stated.

Fig. 3 proves that the increase of the power law index (N) causes the decreasing trend of postbuckling curves of the FG plates with movable edges under uniaxial compressive load. It can be also seen that when N is varied from 0 to 1, both critical buckling loads and post-buckling carrying capacity are strongly dropped. However, a slower variation is remarked when N is greater than 1.

Fig. 4 compares the postbuckling response of compressed FG plates under two types of in-plane boundary restraint. The plate is considered to be freely movable (FM) on all edges (Case 1) and immovable (IM) on two unloaded edges y=0, b (Case 2). As can be observed, in spite of lower critical buckling loads, the postbuckling equilibrium paths for Case 2 become higher than those for Case 1 in deep region of postbuckling response.

Figs. 5 and 6 show the variation of thermal postbuckling load-deflection curves for FG plates with all immovable edges subjected to uniform temperature rise and nonlinear temperature distribution, respectively, with various values of N. It can be seen that, the reduction of volume fraction percentage of ceramic constituent leads to a decrease in the capability of temperature

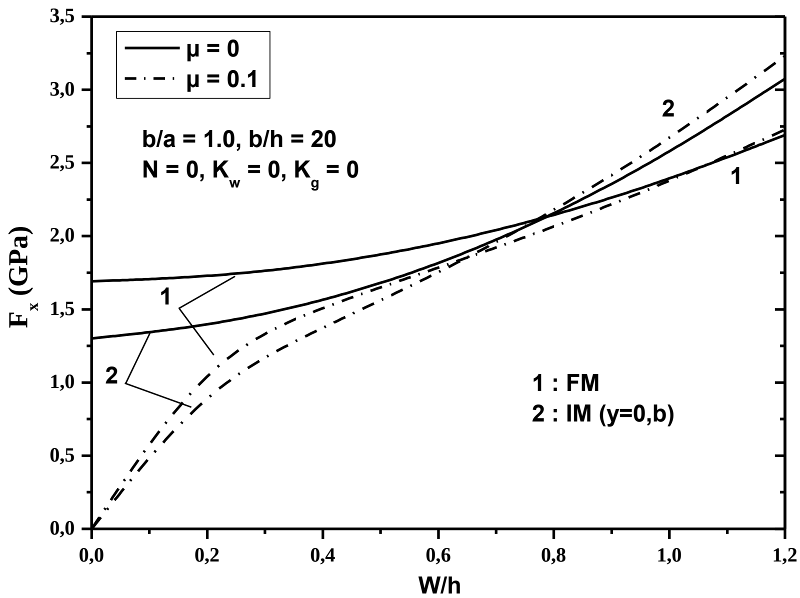

Fig. 4 Effects of in-plane restraint on the post-buckling of FG plate under uniaxial compression

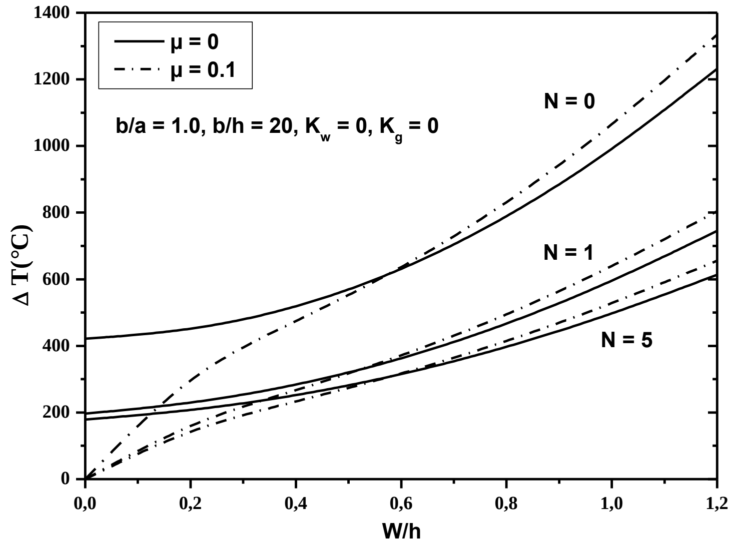

Fig. 5 Effects of the power law index on the post-buckling of FG plates under uniform temperature rise (all IM edges)

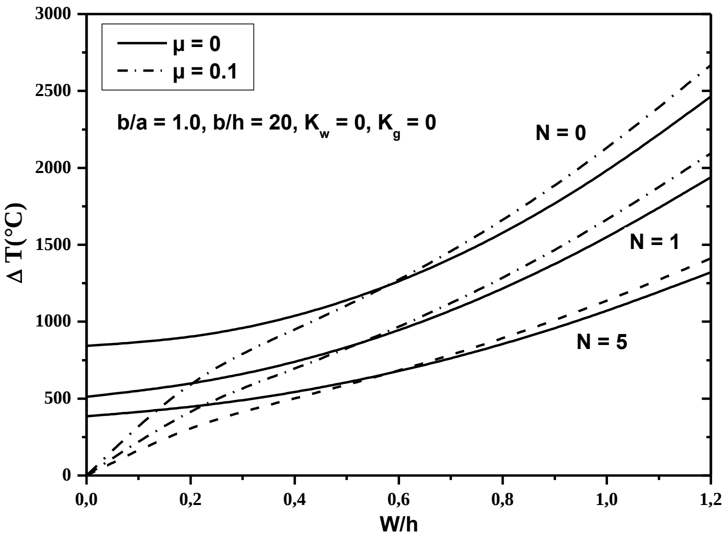

Fig. 6 Effects of the power law index on the post-buckling of FG plates under nonlinear temperature distribution (all IM edges)

resistance of the plates. In addition, it can be deduced that when N varies from 0 to 5, the variation tendency of temperature-deflection curves is not identical for two cases of thermal loading.

The influences of the elastic foundations on the post-buckling response of the FG plates under two types of thermal loads are illustrated in Figs. 7 and 8. As expected, both buckling loads and postbuckling loading bearing capability are amplified because of the presence of elastic foundations. Indeed, for the effect of spring constant factors (K_w, K_g) of the elastic foundation on

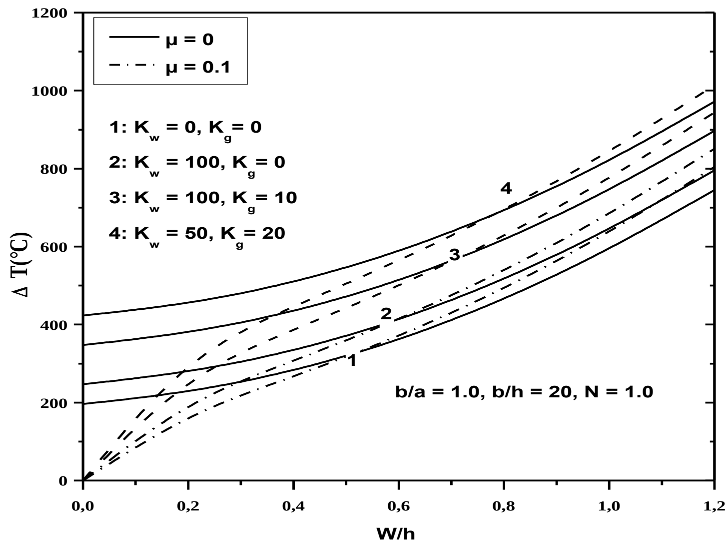

Fig. 7 Effects of the elastic foundations on the post-buckling of FG plates under uniform temperature rise (all IM edges)

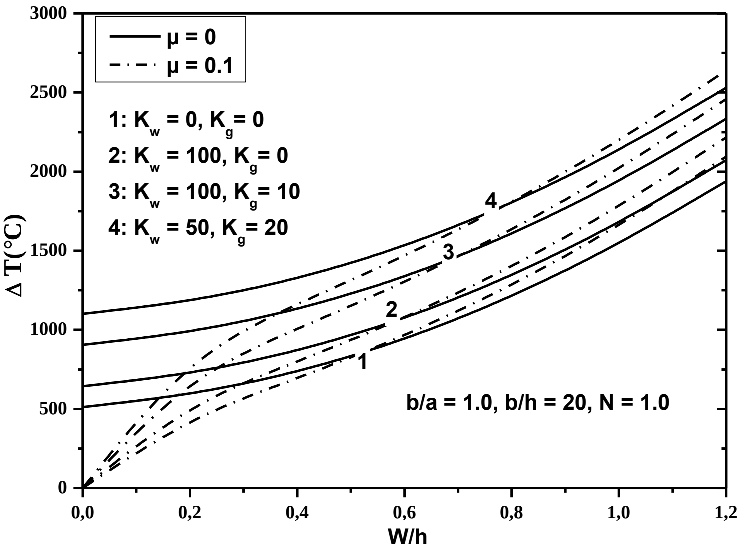

Fig. 8 Effects of the elastic foundations on the post-buckling of FG plates under temperature gradient (all IM edges)

the stability of the plate, it is found that decreasing the spring constant factors leads to the reduction in the stability of the plate for every considered case. This is because the system becomes stiffer when the springs are harder. Furthermore, the shear layer stiffness K_g of Pasternak model has more pronounced effects in comparison with foundation modulus K_w of Winkler model.

The effects of thickness ratios (*a/h*) on the post-buckling behaviour of the FG plates under two types of thermal loads are depicted in Figs. 9 and 10. It can be demonstrated from Figs. 9 and 10

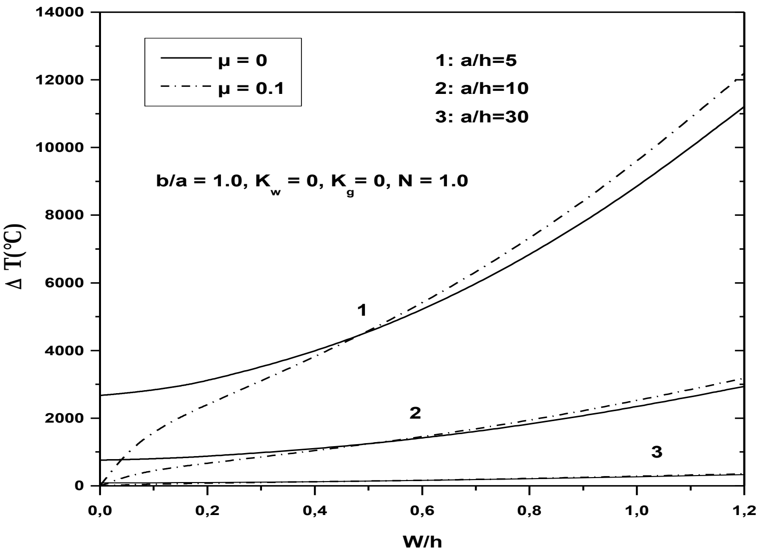

Fig. 9 Effects of the thickness ratios on the post-buckling of FG plates under uniform temperature rise (all IM edges)

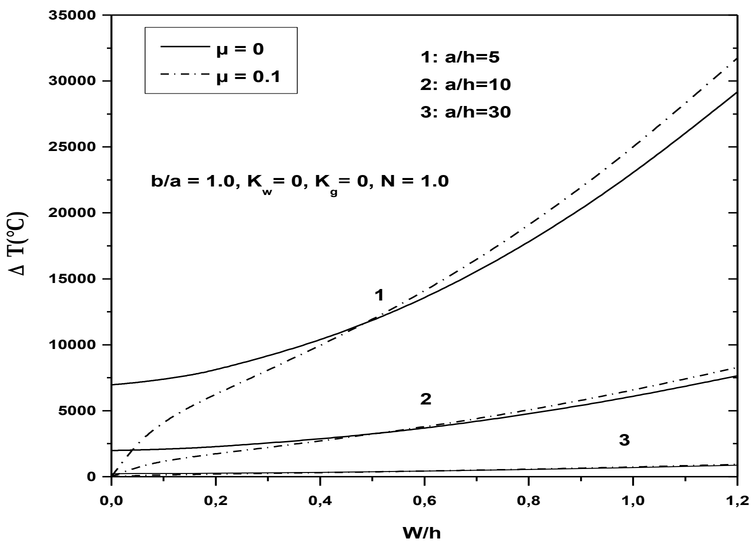

Fig. 10 Effects of the thickness ratios on the post-buckling of FG plates under temperature gradient (all IM edges)

that the increase of thickness ratios makes the capability of temperature resistance of the plates to be decreased.

The thermo-mechanical post-buckling behaviour of FG plates with different values of thickness ratios (K_w) and subjected to uniaxial compression is demonstrated in Fig. 11. It can be seen again that the capacity of mechanical load bearing of the FG plates is more reduced with the increase of the thickness ratios.

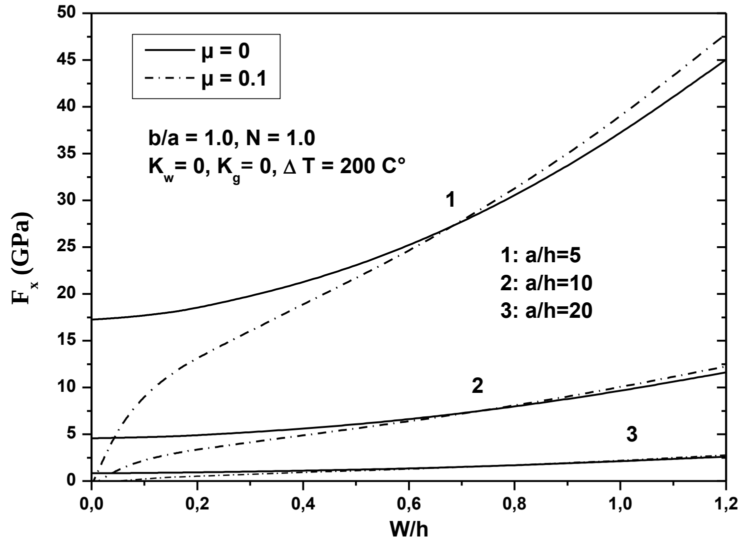

Fig. 11 Effects of the thickness ratios on the post-buckling of FG plates under uniaxial compression (immovable on ***y***=0, ***b***)

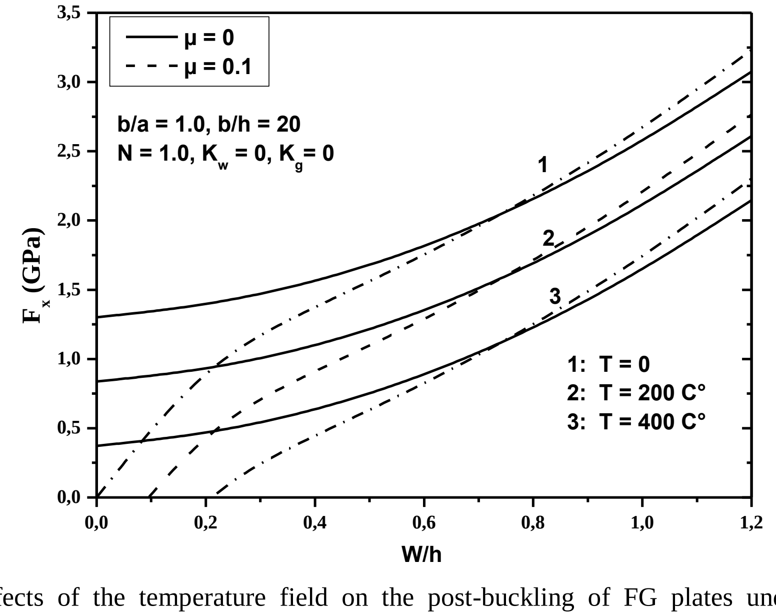

Fig. 12 Effects of the temperature field on the post-buckling of FG plates under uniaxial compression (immovable on ***y***=0, ***b***)

The thermo-mechanical post-buckling response of FG plates exposed to temperature field and subjected to uniaxial compression is illustrated in Fig. 12. As can be seen, the capacity of mechanical load bearing of the FG plates is more reduced because of the enhancement of pre-existent thermal load.

Finally, interactive influences of elastic foundations and temperature gradient on the post-

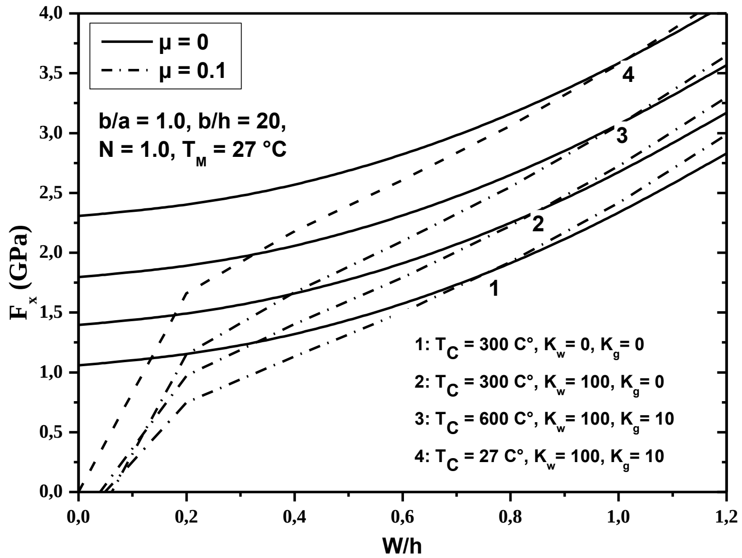

Fig. 13 Interactive effects of elastic foundation and nonlinear temperature distribution on the post-buckling of FG plates under uniaxial compression (immovable on ***y***=0, ***b***).

buckling of the FG plates subjected to uniaxial compressive loads are shown in Fig. 13. As can be remarked, in spite of the increase of ceramic-rich surface temperature, Pasternak type foundations have very beneficial effects on the improvement of thermo-mechanical loading capacity of the FG plates. It is also observed that the spring constant factors have significant impact on the stability of the plates, particularly when K_g is included.

4. Conclusions

In this work, an analytical formulation is proposed to study the mechanical, thermal and thermomechanical buckling and post-buckling responses of thick FG plates supported by elastic foundations. The developed approach is based on the Shi's higher order shear deformation theory to determine accurate predictions for buckling loads and postbuckling loading carrying capacity of thick plates. The obtained analytical expressions of load–deflection curves have practical contribution in analysis and design. The results prove that elastic foundations have a beneficial effect on the stability of FG plates. In addition, it is concluded also that the power law index, in-plane boundary restraint, imperfection and temperature conditions have significant influences on the response of the plates.

Acknowledgments

This research was supported by the Algerian National Thematic Agency of Research in Science and Technology (ATRST) and university of Sidi Bel Abbes (UDL SBA) in Algeria.

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