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Use of design optimization techniques in solving typical structural engineering related design optimization problems

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Abstract. High powered computers and engineering computer systems allow designers to routinely simulate complex physical phenomena. The presented work deals with the analysis of two finite element method optimization techniques (First Order Method-FOM and Subproblem Approximation Method-SAM) implemented in the individual Design Optimization module in the Ansys software to analyze the behavior of real problems. A design optimization is a difficult mathematical process, intended to find the minimum or maximum of an objective function, which is mostly based on iterative procedure. Using optimization techniques in engineering designs requires detailed knowledge of the analyzed problem but also an ability to select the appropriate optimization method. The methods embedded in advanced computer software are based on different optimization techniques and their efficiency is significantly influenced by the specific character of a problem. The efficiency, robustness and accuracy of the methods are studied through strictly convex two-dimensional optimization problem, which is represented by volume minimization of two bars' plane frame structure subjected to maximal vertical displacement limit. Advantages and disadvantages of the methods are described and some practical tips provided which could be beneficial in any efficient engineering design by using an optimization method.

Keywords: design optimization; First Order Method; Subproblem Approximation Method; feasible/infeasible design space; robustness; accuracy

1. Introduction

Most of the structural engineering tasks are represented by complex physical and mathematical problems that are difficult to be solved manually (Su *et al*. 2014, Balek *et al.* 2012, Barnat *et al.* 2012, Bajer and Barnat 2012, Kala *et al.* 2011). Hence, if the aim of a civil or mechanical engineering project is to find an efficient design using any of the available optimization techniques manually, it is necessary to create as simple a mathematical model as possible. This leads a

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designer to use general coefficients to guarantee the safety of a design and the design ends up moving away from reality.

For those reasons, specialists for operating research and systems of information technologies deal with the implementation of optimization techniques in mechanical and civil engineering software (Cui and Cui 2015). There, the designers have the opportunity to apply complex mathematical algorithms for the creation of an efficient design models (Deshbhratar and Suple 2012, Fedorik 2013).

In case of solving complex structural or mechanical problems defined by multi-variable design space number of performed iterations might markedly influence computing time (Ródenas *et al.* 2011). It is recommended to perform for instance 3 times 20 random iterations initiated in different point with varying design space limits rather than 100 random iterations based on the same conditions. This improves the accuracy, robustness and time cost of the solution.

One way of using optimization methods in structural designs is to implement them in FEM-based (Finite Element Method) software (Morin *et al.* 2012). FEM is currently one of the most widely used methods in mechanical and civil engineering design for simulations of real systems (Kala and Kala 2011, Holomek and Bajer 2012, Kala *et al.* 2012, Karasek *et al.* 2012).

The algorithms of the presented optimization methods (FOM and SAM) are based on traditional operating research techniques (Rao 1996, Jividinejad 2012, Khoei 2015) with certain modifications for their ability to solve multipurpose problems (Shayanfar *et al.* 2013, Şahin 2014). The efficiency, robustness, and design space exploration of optimization methods can be improved by optimization tools.

One of the greatest challenges in the implementation of optimization techniques into engineering systems is their compatibility and ability to solve a wide range of systematic problems (Awad 2013, Bathe 2007, Himeur *et al.* 2014, Mallika and Rao, 2010). We present here a general framework for solving structural engineering-related optimization problems by applying mathematical methods established to find the minimal or maximal values of a function representing the structural design. The guideline helps to understand the operation of most commonly used optimization tools and to reach the optimal model of a system with commercial design tools without high requirements for designer skills in mathematics.

2. General procedure of design optimization

The Design Optimization module is an individual module intended for solving technical optimization problems within the Ansys program that applies the finite element method (Kala 2012, Kala and Kala 2012). A finite element model which is subjected to an optimization procedure uses the main components of the Ansys program for model creation (model creation pre-processor), solution (solution processor) and evaluation of obtained outcomes (database results post-processor).

Data flow during an optimization process performed by the Ansys program can be expressed by the following scheme (Fig. 1).

From the point of view of a designer, the analysis file is the most important element of the modelling procedure. It contains the parametrical expression of a model, parameterization of evaluated data from an initial design and an objective function. The parametrical model includes geometrical features of the model, which are used in the following example as design variables (DVs). The evaluated data parameterization presents state variables (SVs) and objective function

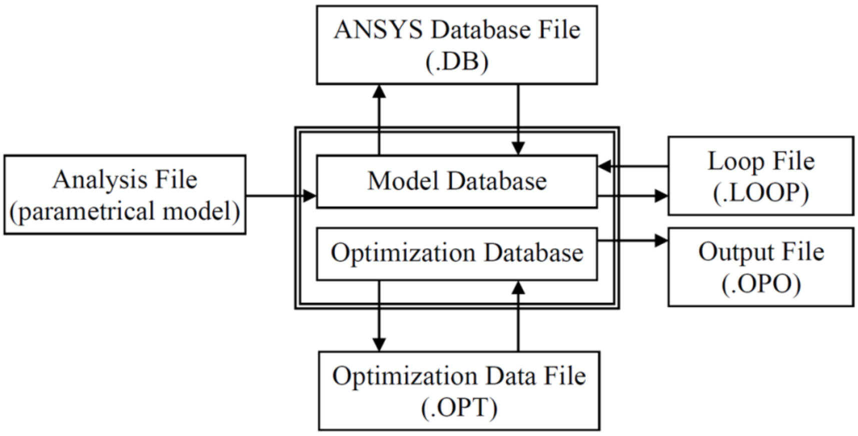

Fig. 1 Data flow during optimization procedure in ANSYS program

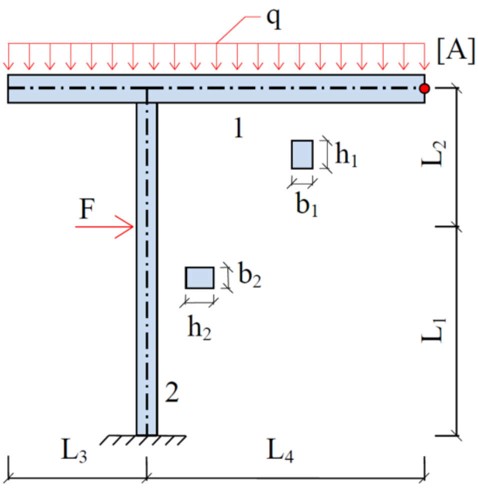

Fig. 2 Three bars' plane frame structure

(Obj). The analysis file is a key component of an optimization procedure because its content is used to create each consecutive iteration cycle in the optimization process until sufficient convergence is achieved. It includes one complete analysis from the beginning to the evaluation of the outcomes and their follow-up parameterization.

3. Problem description

Structural optimization problems are solved mathematically and the procedure is easily demonstrated with a simplified example. The aim of the presented optimization problem is to minimize the volume of a two-bar plane frame structure (Fig. 2) subjected to vertical displacement limit w of the structure in the point [A]. This can be acquired by varying the bars' cross-sectional heights defined by the parameters h_1 and h_2. The structure is fixed at the bottom of the column

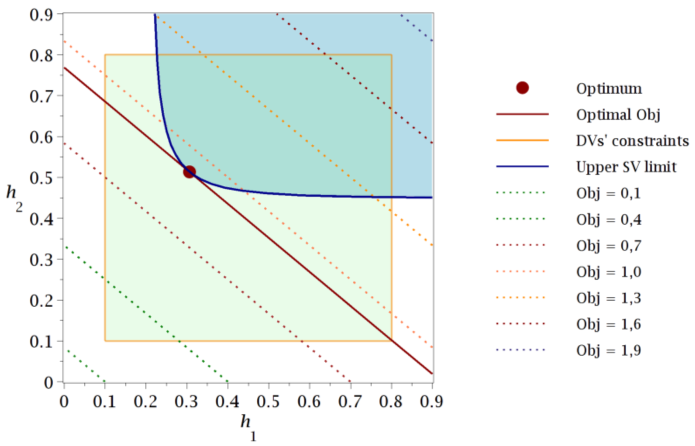

Fig. 3 Graphical expression of the optimal point of the design parameters h_1 and h_2

which is also loaded by the single cross force *F*. The horizontal bar is subjugated by the distributed load *q*. At first, the problem is analyzed by a graphical and manual solution to achieve an optimum, which is then used for the verification of the results acquired by First Order Method (FOM) and Subproblem Approximation Method (SAM).

The optimization problem is defined as

$$Find\ h = \begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix}, whichminimizef(\boldsymbol{h}) = h_1 + 1{,}2h_2, \tag{1}$$

subject to

$$\begin{gathered} 0{,}10 \leq h_1 \leq 0{,}80 \\ 0{,}10 \leq h_2 \leq 0{,}80 \end{gathered} \tag{2}$$

$$\left(w_{[A]} = \frac{1{,}7999.10^{-3}}{h_2^3} + \frac{1{,}9199.10^{-4}}{h_1^3}\right) \leq 0{,}02 \tag{3}$$

where $w_{[A]}$ is displacement in the point [*A*], and h_1 and h_2 are cross-sectional dimensions of the bars which represent the DVs of the problem.

3.1 Graphical expression and extreme localization

According to the definition (Eqs. (1) to (3)) a graphical expression of the presented problem can be obtained (Fig. 3). The axes of the graphical expression are represented by independent variables (DVs) and the feasible space is originated by an intersection of lower and upper limits of DVs h_1, h_2 and the upper limit of state variable (SV) $w_{[A]}$. The optimum of the optimization problem is located at the point where the objective function forms a tangent with the state variable function. The actual optimization problem represents a strictly convex optimization problem which leads to the existence of one, and only one, extreme within the frame of the defined design space.

The estimate obtained from the graphical expression of DV intervals is defined as follows:

$$h_1 = \langle 0{,}25 \ldots 0{,}35 \rangle,$$
$$h_2 = \langle 0{,}45 \ldots 0{,}55 \rangle$$

To find the optimum of the problem, a bisection method was used. Then the optimal point of the problem is defined by DVs' values:

$$h_1 = 3{,}0674.10^{-1}m,$$
$$h_2 = 5{,}1282.10^{-1}m^{\cdot}$$

Then the state variable and objective function (Obj) values are:

$$w_{[A]} = 0{,}02\,m,$$
$$f = 9{,}2215.10^{-1}m^3.$$

4. Solution by ANSYS: design optimization methods and tools

Before an optimization procedure by SAM and/or FOM is performed, a finite elements model and a parametrical model of the actual problem are created using the Ansys program. The FEM/FEA model is assembled from two-dimensional elements labelled as 'BEAM3'. Then the optimization variables (parameters) are defined as follows: the bars' cross-sections' heights h_1 and h_2 represent independent design variables; dependent variables are expressed by vertical displacement at the point [A] as a state variable, and the weight of the structure as the objective function (Obj).

4.1 Optimization tools

Other optimization tools that are available in the Design Optimization module attend to the exploration of the design space and the extreme values obtained by the optimization method (Fedorik 2013). They are represented by Random, Gradient, Sweep, Factorial and Single Loop Analysis Tool. The tools as presented in Ansys Release guide but their operation and characteristics are briefly introduced in the following subsections.

4.1.1 Random tool

Random Tool function has been developed to recognize the behavior and proportion of the objective function by defining random DV value for each iteration cycle. In practice, the DVs are generated randomly, not based on user's initial assumption, as shown in Eq. (4).

$$\mathbf{x} = \mathbf{x}^* = \textit{randomly generated vector} \tag{4}$$

The graphical expression of a model consisting of 50 random such design sets performed within the frame of the presented case is shown in Fig. 4. Advantages of the tool can be realized e.g. in the initial investigations of vast design spaces. This allows the avoidance of unexpectedly located extremes and smooth formatting of the approximated design space.

4.1.2 Sweep tool

By applying the Sweep Tool, the designer monitors features of the objective function in the design space by a regular distribution of the design variable intervals. It means that an interval of each DV is divided into sections of equal length (as it is graphically shown in Fig. 5). The number of iterations is determined by designer, based on an assumption, i.e. how many sections have to be performed to obtain adequate information of the objective function features in the design space as

$$n_s = nN_s \tag{5}$$

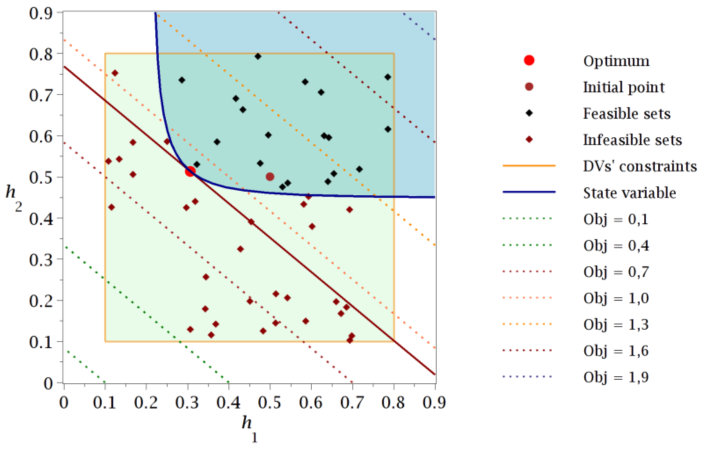

Fig. 4 Random tool showing 50 randomly selected DVs attempting to find the optimum

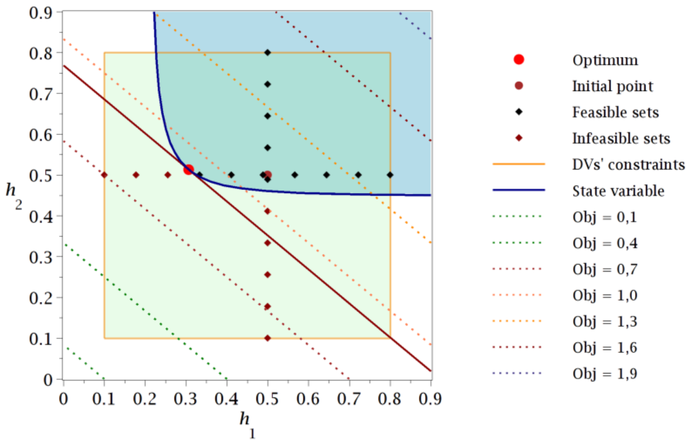

Fig. 5 Sweep tool functionality, dividing the constrained area with equally distributed DVs

where n is a number of design variables DVs and N_s is a number of sections for each design variable, where the computation will be performed. The Sweep Tool is suitable in cases where the design space is defined or otherwise known and investigation is required along defined DVs initiated in certain point in the design space.

4.1.3 Factorial tool

Factorial Tool is a statistical tool developed for acquiring information about the progress of the optimization procedure near the marginal points of a design space (Fig. 6). If the full factorial calculation is applied, with n design variables, total of n_{fa} of design sets are obtained, where

$$n_{fa} = 2^n \tag{6}$$

So, with the full factorial evaluation, all combinations of design variable limits are computed in n-dimensional design space. A decent investigation of the extreme points allows designer control

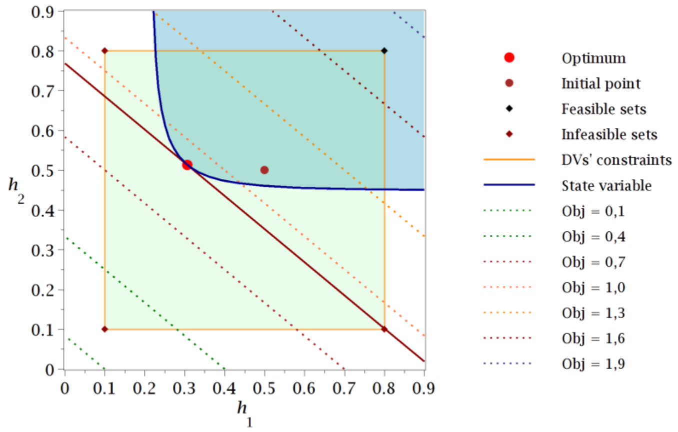

Fig. 6 Factorial tool looks optimum near the design space extremes

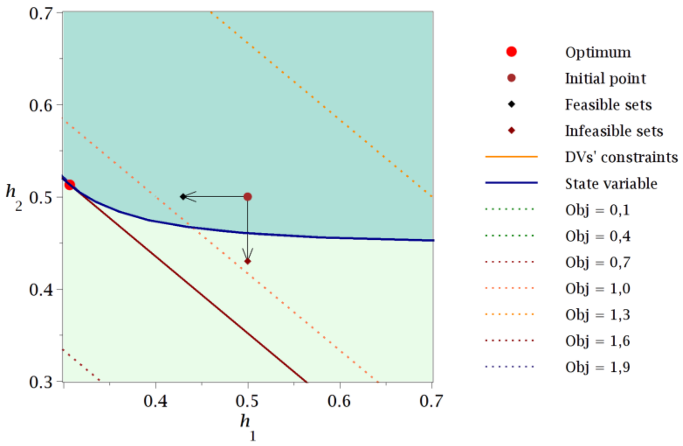

Fig. 7 Gradient tool investigates the proximity near a determined point

over variables' limits and defining the suitable size of the design space.

4.1.4 Gradient tool

Gradient Tool verifies the sensitivity of dependent variables (SVs and Obj). It computes gradients of the design variables based on the defined point in the design space (Fig. 7). The general expression of the objective function gradient is given as

$$\nabla fg = \left[\frac{\partial f_g}{\partial x_1}; \frac{\partial f_g}{\partial x_2}; \ldots\ldots\ldots; \frac{\partial f_g}{\partial x_n} \right] \tag{7}$$

Considering each design variable DV, the gradient of the objective function is expressed as follows

$$\frac{\partial f_g}{\partial x_i} = \frac{f_g(x + \Delta x_i \mathbf{e}) - f_g(x)}{\Delta x_i} \tag{8}$$

where **e** is a vector with 1 in *i*th article and 0 in all the others. Δx_i is expressed by

$$\Delta x_i = \frac{\Delta s}{100}\left(\overline{x}_i - \underline{x}_i\right) \tag{9}$$

where Δs is the difference of the step lengths. This tool is suitable for verification of the optimum point and its direct ambient proximity attained with the optimization procedure.

4.1.5 Single loop analysis tool

Single Loop Analysis Tool is a simple and direct tool which leads the designer to understand the design space of an optimization problem. It is a suitable tool for the evaluation of the state variable and the objective function values. Design variables are always determined by a designer explicitly. One iteration cycle with Single Loop Analysis Tool corresponds to one complete FEM analysis.

4.2 Subproblem approximation method

The Subproblem Approximation Method (SAM) is an iterative method based on an approximated function. At first, the approximation of the dependent variables (Obj - objective function and SVs - state variables) by least squares fitting is performed, and then the approximated objective function is minimized or maximized. Thus, the aim of the process is minimizing or maximizing an approximated function instead of the true function. More efficient methods for finding the extreme of a function are unconstrained optimization methods. For this reason, the defined constrained optimization problem is converted to an unconstrained optimization problem (Güler, 2010). The general approximated objective function is defined in a fully quadratic form with cross terms as (Ansys Release 11.1)

$$\hat{f} = a + \sum_{i=1}^{n} b_i \mathbf{x}_i + \sum_{i=1}^{n} c_i x_i^{\,2} + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d_{i\,j} \mathbf{x}_i \mathbf{x}_j \tag{10}$$

where ˆ indicates that the function is approximated, n represents number of iterations, **x** is the vector of design variables, a, b, c and d are coefficients determined by the weighted least square technique, i represents the number of variables and j is the quantity of performed loops. The transformation is performed by the penalty function method which is applied to the objective function. The penalty function replaces the previously defined constraints in limits of DVs and SVs. Then the minimizing problem with use of the SAM is expressed by

$$F(\mathbf{x}, q^{(j)}) = \hat{f} + f_0 q^{(j)} \left(\sum_{i=1}^{n} X(\mathbf{x}_i) + \sum_{i=1}^{m_1} G(\hat{g}_i) + \sum_{i=1}^{m_2} H(\hat{h}_i) + \sum_{i=1}^{m_3} W(\hat{w}_i) \right) \tag{11}$$

where X is the penalty function, used to express the constraints of design variables, G, H and W are penalty functions which substitute constraints of state variables, $F(\mathbf{x},q^{(j)})$ is the unconstraint objective function, f_0 is the reference objective function and $q^{(j)}$ is the penalty parameter. The following table (Table 1) describes the essential features of the SAM method applied in the presented analysis.

Within the frame of the presented optimization problem, the SAM is analyzed according to a different approximation of dependent variables SV and Obj and a different pointing of weighting

Table 1 SAM properties, methods of variables' approximations and weighting factors' definitions

Setting	Superscript	Conditions
Obj (Objective function fitting)	0	Quadratic + cross-term curve
	1	Linear curve
	2	Quadratic curve
SVs (State variables fitting)	0	Quadratic curve
	1	Linear curve
	3	Quadratic + cross-term curve
W (Weighting factors)	0	Design space, Obj and feasibility of solution
	1	All are unity
	2	Distance in design space
	3	Obj (Objective function)
	4	Feasibility/infeasibility of solution

Table 2 SAM results initiated by 5 random loops

Category	Group	Settings	Obj.10^{-1} [m^3]	SV.10^{-2} [m]	$DV_1.10^{-1}$ [m]	$DV_2.10^{-1}$ [m]	Iterations [-]
1	1	($Obj^{0,1,2}$,SVs^0,W^0)	9,2274	1,9988	3,0672	5,1335	147 (1219)
	2	($Obj^{0,1,2}$,SVs^0,W^1)	9,2273	1,9990	3,0620	5,1377	398 (1374)
	3	($Obj^{0,1,2}$,SVs^0,W^2)	9,2278	1,9986	3,0719	5,1299	381 (1257)
	4	($Obj^{0,1,2}$,SVs^0,W^3)	9,6820	1,9892	2,5796	5,9187	27 (1176)
	5	($Obj^{0,1,2}$,SVs^0,W^4)	9,2291	1,9978	3,0670	5,1351	150 (1177)
2	6	($Obj^{0,1,2}$,SVs^1,$W^{0,4}$)	10,0170	1,8130	2,6488	6,1404	10 (1168)
	7	($Obj^{0,1,2}$,SVs^1,W^1)	9,7336	1,7053	3,1723	5,4678	8 (1166)
	8	($Obj^{0,1,2}$,SVs^1,W^2)	9,3746	1,9658	2,8143	5,4669	14 (1228)
	9	($Obj^{0,1,2}$,SVs^1,W^3)	10,2580	1,8406	2,5490	6,4241	18 (1183)
3	10	($Obj^{0,1,2}$,SVs^3,W^0)	9,2278	1,9986	3,0702	5,1313	135 (1217)
	11	($Obj^{0,1,2}$,SVs^3,W^1)	9,2290	1,9979	3,0595	5,1413	337 (1219)
	12	($Obj^{0,1,2}$,SVs^3,W^2)	9,2300	1,9986	3,0224	5,1730	158 (1189)
	13	($Obj^{0,1,2}$,SVs^3,W^3)	9,2607	1,9783	3,0424	5,1820	134 (1181)
	14	($Obj^{0,1,2}$,SVs^3,W^4)	9,2281	1,9984	3,0774	5,1256	476 (1458)

factor (Table 1). The objective function in this problem is represented by the linear Eq. (1) which leads to its changeless form due to the different approximation proceeding. Furthermore, the SAM is depended on a different number of random iterations which are evaluated before the first approximation is formed. The design space was explored by 5, 50, and 100 loops where the Random Tool was applied.

The following table (Table 2) represents the summarized variables' values in the best design sets obtained in the analysis where the SAM proceeding was initiated by 5 random loops. The presented results are divided into three categories according to the approximation type of dependent variable SV (Table 1).

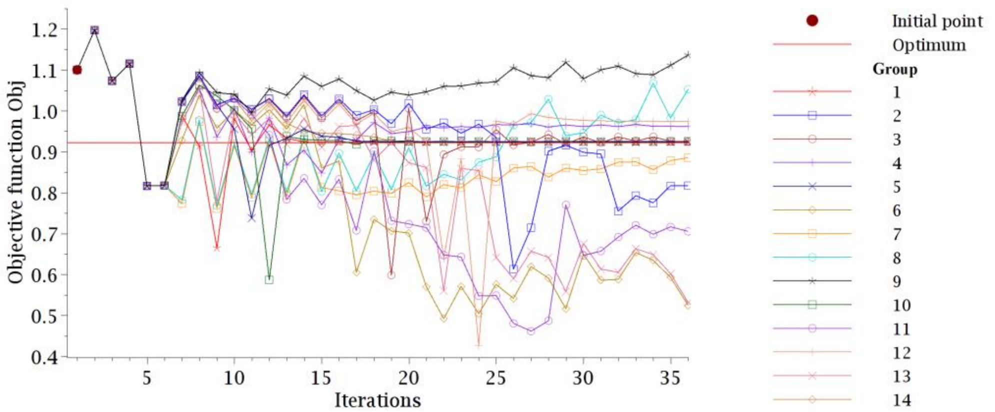

Fig. 8 Obj proceeding by SAM showing the optimization procedure converging towards the optimum

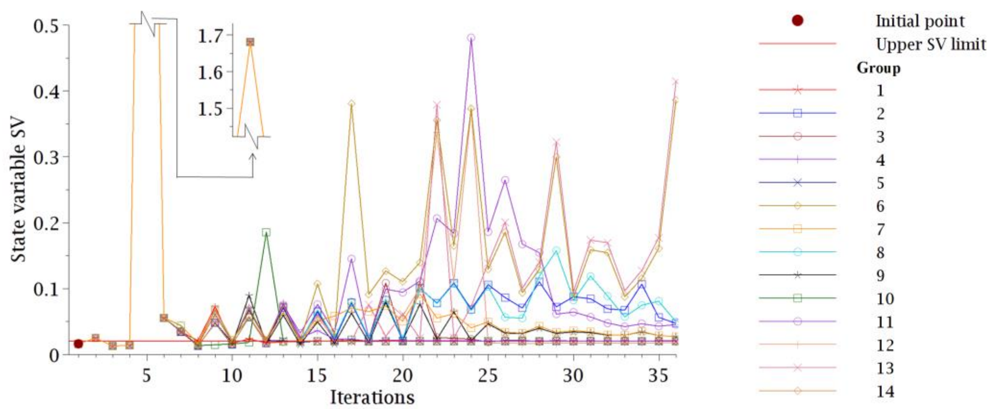

Fig. 9 SV proceeding by SAM showing the optimization procedure converging towards the optimum

In the case where the SAM process is initiated by 5 random loops, the SV approximation performed by a linear fitting prevents the accuracy of the solution. Also, quadratic SV approximation with the combination of weight factors directed into the objective function values converges at a distant location from the actual optimum. The remaining cases where the quadratic and quadratic plus cross-term fitting approximate the SV achieve ambient of the optimum. The progress of dependent variables (Obj and SV) due to the first 30 SAM iterations initiated by the 5 random loops is pictured in the following figures (Figs. 8 and 9).

The features of the best design sets and numbers of iterations by SAM performed initiated by 50 and 100 loops are summarized in the following tables (Tables 3 and 4). In multi-variable design spaces, the number of performed iterations might markedly influence computing time, improve accuracy, robustness and time cost of the solution. Unwarranted iterations are also wasting the resources but the current low cost of computing time thorough model verification schemes are generally warranted.

Exploration of the design space by 50 random loops causes more reliable convergence to the optimum at the expense of accuracy. Except in linear approximation of the SV, the ambient of the actual optimum was found.

Table 3 SAM results initiated by 50 random loops

Category	Group	Settings	Obj.10^{-1} [m^3]	SV.10^{-2} [m]	DV1.10^{-1} [m]	DV2.10^{-1} [m]	Iterations [-]
1	1	$(Obj^{0,1,2},SVs^0,W^0)$	9,2286	1,9981	3,0771	5,1262	177 (1211)
	2	$(Obj^{0,1,2},SVs^0,W^1)$	9,3479	1,9959	2,7764	5,4763	176 (1256)
	3	$(Obj^{0,1,2},SVs^0,W^2)$	9,2556	1,9900	2,9584	5,2477	169 (1366)
	4	$(Obj^{0,1,2},SVs^0,W^3)$	9,2340	1,9946	3,0631	5,1424	158 (1242)
	5	$(Obj^{0,1,2},SVs^0,W^4)$	9,3712	1,9953	2,7542	5,5142	57 (1255)
2	6	$(Obj^{0,1,2},SVs^1,W^0)$	9,5349	1,8691	2,8606	5,5619	152 (1229)
	7	$(Obj^{0,1,2},SVs^1,W^1)$	9,5854	1,7841	3,2265	5,2991	19 (1311)
	8	$(Obj^{0,1,2},SVs^1,W^2)$	9,4194	1,9669	2,7642	5,5459	53 (1211)
	9	$(Obj^{0,1,2},SVs^1,W^3)$	9,5854	1,7841	3,2265	5,2991	19 (1216)
	10	$(Obj^{0,1,2},SVs^1,W^4)$	9,4098	1,9723	2,7626	5,5393	137 (1216)
3	11	$(Obj^{0,1,2},SVs^3,W^0)$	9,2326	1,9988	3,1436	5,0742	143 (1221)
	12	$(Obj^{0,1,2},SVs^3,W^1)$	9,3242	1,9977	2,7993	5,4374	252 (1221)
	13	$(Obj^{0,1,2},SVs^3,W^2)$	9,2370	1,9974	2,9866	5,2086	158 (1234)
	14	$(Obj^{0,1,2},SVs^3,W^3)$	9,2362	1,9988	2,9787	5,2146	346 (1259)
	15	$(Obj^{0,1,2},SVs^3,W^4)$	9,3593	1,9935	2,7696	5,4914	194 (1243)

Table 4 SAM results initiated by 100 random loops

Category	Group	Settings	Obj.10^{-1} [m^3]	SV.10^{-2} [m]	DV1.10^{-1} [m]	DV2.10^{-1} [m]	Iterations [-]
1	1	$(Obj^{0,1,2},SVs^0,W^0)$	9,2291	1,9978	3,0670	5,1351	243 (1263)
	2	$(Obj^{0,1,2},SVs^0,W^1)$	9,2346	1,9944	3,0527	5,1516	249 (1333)
	3	$(Obj^{0,1,2},SVs^0,W^2)$	9,2791	1,9685	3,0188	5,2169	194 (1319)
	4	$(Obj^{0,1,2},SVs^0,W^3)$	9,2313	1,9964	3,0607	5,1421	197 (1271)
	5	$(Obj^{0,1,2},SVs^0,W^4)$	9,4314	1,9984	2,6992	5,6102	839 (1278)
2	6	$(Obj^{0,1,2},SVs^1,W^0)$	9,5152	1,8259	3,2377	5,2312	84 (1259)
	7	$(Obj^{0,1,2},SVs^1,W^1)$	9,5152	1,8259	3,2377	5,2312	84 (1302)
	8	$(Obj^{0,1,2},SVs^1,W^2)$	9,4516	1,9916	2,6955	5,6302	243 (1326)
	9	$(Obj^{0,1,2},SVs^1,W^3)$	9,5152	1,8259	3,2377	5,2312	84 (1295)
	10	$(Obj^{0,1,2},SVs^1,W^4)$	9,5152	1,8259	3,2377	5,2312	84 (1259)
3	11	$(Obj^{0,1,2},SVs^3,W^0)$	9,2292	1,9977	3,0678	5,1345	379 (1334)
	12	$(Obj^{0,1,2},SVs^3,W^1)$	9,3157	1,9980	2,8093	5,4220	247 (1276)
	13	$(Obj^{0,1,2},SVs^3,W^2)$	9,2391	1,9951	2,9960	5,2026	255 (1429)
	14	$(Obj^{0,1,2},SVs^3,W^3)$	9,2333	1,9987	2,9954	5,1983	246 (1309)
	15	$(Obj^{0,1,2},SVs^3,W^4)$	9,3926	1,9854	2,7532	5,5329	987 (1298)

If the solution is initiated by 100 random loops, the SAM method requires a great number of iterations to achieve convergence especially if the weighting factor is directed to

Table 5 Initial design sets

Obj [m^3]	SV [m]	DV_1 [m]	DV_2 [m]	Status
0,2200	1,9922	0,1	0,1	infeasible
1,1000	$1,5966.10^{-2}$	0,5	0,5	feasible
1,7600	$3,9094.10^{-3}$	0,8	0,8	feasible

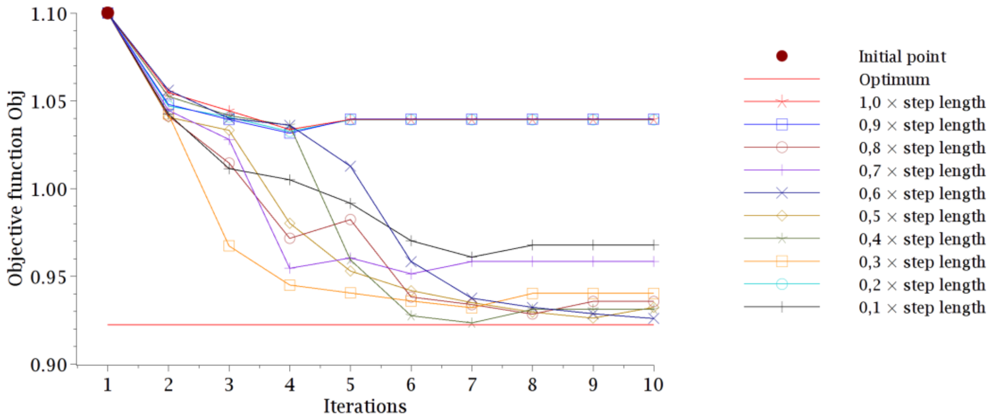

Fig. 10 Obj proceeding by FOM showing the optimization procedure converging towards the optimum

feasibility/infeasibility of the obtained design sets. In the case that the SV variable is approximated by a linear fitting the SAM is not able to improve features of the best achieved random design set (no. 84). The actual optimum ambient is achieved in solutions where the SV approximation is performed by quadratic and quadratic plus cross-term fitting.

4.3 First order method

Unlike SAM, the First Order Method (FOM) uses a derivation of functions to solve an optimization problem. The objective function and the penalty functions of the state variable are derived, which leads to the problem of searching a certain direction in the design space. For each iteration, a browsing of the direction by the steepest descent method and the conjugate gradient method is performed. This means that several sub iterations are performed in each iteration computing both the direction and descent of the functions. The function which solves optimization problem by the first order method has the general form

$$F(x,q)=\frac{f}{f_0}+\sum_{i=1}^{n}X_x(\mathbf{x}_i)+q\left(\sum_{i=1}^{m1}W_g(g_i)\sum_{i=1}^{m2}W_h(h_i)\sum_{i=1}^{m3}W_w(w_i)\right) \tag{12}$$

where F is the unconstrained objective function. The term X_x is the penalty function, which compensates constraints of the design variables DVs and W_g, W_h and W_w are limit values of the state variables (SVs). f_0 then represents a reference objective function, which was achieved in the current group of the design sets. An appropriate penalty parameter q monitors how well the design constraints are being satisfied.

To analyze the efficiency of the FOM, different cases with varying step lengths' range of

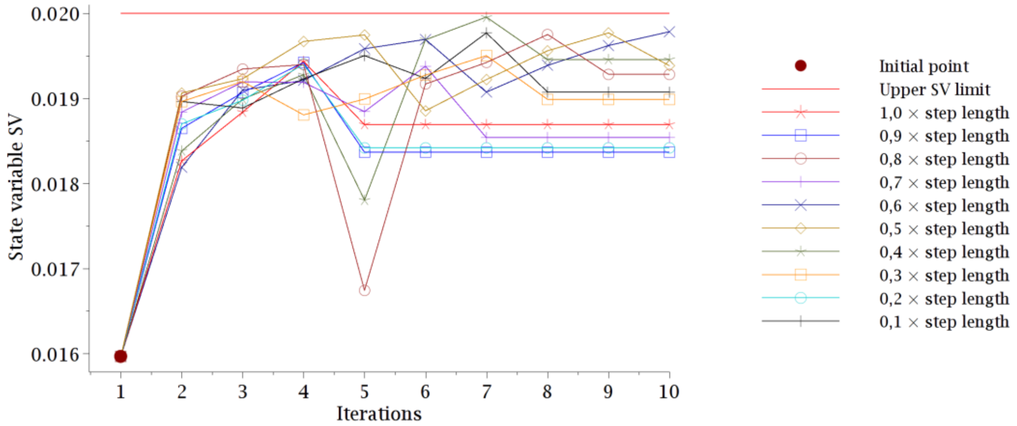

Fig. 11 SV proceeding by FOM showing the optimization procedure converging towards the optimum

gradients and different initial point locations are studied. The following table (Table 5) shows optimization variables' values at analyzed initial points, where the initial points DV_1=DV_2=0,5 and DV_1=DV_2=0,8 are located in the feasible design space and DV_1=DV_2=0,1 is in the infeasible design space because of exceeding the upper *SV* limit (3). The progress of the dependent variables (*Obj* and *SV*) in the first 10 loops by the FOM performed initiated by h_1=0,5 and h_2=0,5 are shown in the figures (Figs. 10 and 11) below.

The termination of the procedure consists of achieving the convergence criteria which, in this case, are defined as the differences of the objective function values in the two consecutive design sets. The variables' values in the best sets obtained and the total number of iterations by the FOM method performed are summarized in Table 6.

In the case where the FOM method is initiated in the infeasible design space, the solution requires markedly more iterations to achieve the convergence criteria. On the other hand, the high number of performed loops allows finding the minimum of the objective function in the feasible design space. The FOM analyses initiated by the feasible design sets require a smaller number of loops to achieve the convergence criteria at the expense of accuracy.

To guarantee achieving the optimum of the problem, design space exploration is recommended. In the case where the presented problem is initiated by 5 random loops, the optimum is obtained in all presented cases.

5. Conclusions

The function and procedure to apply First Order Method and Subproblem Approximation Method, both given as Ansys tools, were presented and analyzed using a simple two-bar plane frame structure weight minimization problem. The optimization problem was expressed by two dependent (Obj and SV) and two independent variables (DVs). To control and verify efficiency and accuracy of analyzed methods' resultant design sets was manually evaluated and the graphical solutions provided for each method.

The FOM was analyzed depending on different initial point location and varying step lengths of gradient. The initial points were located at the lower and upper constraints of the design variables

Table 6 Initial design sets

Initial point	Step length [%]	Obj.10^{-1} [m^3]	SV.10^{-2} [m]	DV_1.10^{-1} [m]	DV_2.10^{-1} [m]	Iterations [-]
DV_1=0,1 DV_2=0,1	100	9,2369	1,9927	3,0688	5,1401	39 (100)
	90	9,2378	1,9921	3,0691	5,1406	43 (103)
	80	9,2316	1,9962	3,0631	5,1657	52 (107)
	70	9,2357	1,9942	3,1058	5,1083	16 (102)
	60	9,2301	1,9971	3,0665	5,1363	43 (104)
	50	9,2300	1,9972	3,0751	5,1291	46 (103)
	40	9,2307	1,9967	3,0646	5,1384	93 (129)
	30	9,2313	1,9967	3,0456	5,1547	29 (102)
	20	9,2315	1,9967	3,1000	5,1096	47 (101)
	10	9,2278	1,9986	3,0632	5,1372	99 (129)
DV_1=0,5 DV_2=0,5	100	10,3350	1,9435	4,7214	4,6784	4 (9)
	90	10,3160	1,9422	4,6972	4,6821	4 (9)
	80	9,2844	1,9750	3,2340	5,0420	8 (12)
	70	9,5122	1,9376	3,6409	4,8928	6 (10)
	60	9,2593	1,9783	3,0707	5,1572	10 (13)
	50	9,2613	1,9770	3,0829	5,1486	9 (12)
	40	9,2330	1,9954	3,0887	5,1203	7 (12)
	30	9,3188	1,9503	3,2277	5,0759	7 (12)
	20	10,3250	1,9403	4,7054	4,6829	4 (9)
	10	9,6089	1,9771	3,5651	4,7865	7 (12)
DV_1=0,8 DV_2=0,8	100	9,4574	1,9827	3,6431	4,8452	8 (13)
	90	10,6850	1,9491	5,1173	4,6394	4 (9)
	80	9,2676	1,9871	2,9349	5,2773	7 (12)
	70	9,3208	1,9496	2,9724	5,2903	10 (16)
	60	10,7110	1,9326	5,1280	4,6528	4 (10)
	50	9,4097	1,9442	3,4624	4,9561	7 (12)
	40	10,6520	1,9657	5,1005	4,6265	6 (11)
	30	9,4752	1,9846	3,6763	4,8325	7 (14)
	20	9,3090	1,9881	3,3653	4,9531	12 (15)
	10	10,2030	1,9932	4,6296	4,6446	10 (15)

and in the middle of their defined ranges. In the case where the initial point is defined in the infeasible design space (lower constraints of the DVs), the FOM method requires markedly more iterations to achieve convergence criteria, but with no effect on efficiency and accuracy of the solution. If the initial point is localized in the feasible design space, the convergence criteria were achieved already in max 10 iterations at the expense of accuracy. To improve the efficiency and accuracy of the FOM procedure, the defined design space can be explore further and adapt convergence criteria considering the features of the optimization problem at hand.

According to the optimization problem definition within the frame of the SAM, the dependent variable (SV) approximation techniques and the effect of weighting factor were observed. The SAM efficiency was analyzed depending on the number of random loops performed beforehand. The cases where the SV approximation was performed by quadratic and quadratic plus cross-term fitting achieved the optimum. The number of loops evaluated beforehand influences the dependent variables' function forms depending on their location within the design space. To improve stability and accuracy of the SAM procedure, it is recommended to explore the design space of the problem and regulate its constraints according to obtained features.

The presented optimization methods and tools are implemented in the Ansys Design Optimization module and allow the designer an efficient design of a wide range of problems. The robustness and accuracy of the solutions are not guaranteed, which is why these features can be improved by the application of one of the optimization tools or their combination for more detailed exploration of the design space before the optimization method is used and/or for controlling of the best design obtained by the First Order or Subproblem Approximation method.

It is recommended to use the SAM techniques in cases where the features of the design space are unknown. Suitable initial conditions before the optimization procedure is applied are determined with the Random Tools loops which allow a generalized investigation of the design space. In the case the estimated location of searched extreme is known, the FOM is more advisable.

Within the frame of the optimization problem defined by more variables and complex geometry, it is more suitable to use the SAM from the point of view of time consumption, although it demands more iterations to achieve required results. In the case the initial investigation is performed by Random Tool the FOM procedure starts its solution from the most convenient point obtained towards lower (minimization) or higher (maximization) location in the ambient of the point.

Contrary to the FOM, SAM does not focus on the random extreme point only, but it searches the optimum of the approximated objective function all over the design space. Hence, the selection and proper usage of design tools can markedly reduce the time, cost, and accuracy of a structural model design. In the case of using an optimization method in efficient structural design, it is recommended to pay closer attention to the method selection. Some tools are proven to perform sub-optimally on computing certain engineering problems – this can easily be verified by changing the optimization methods and perceiving the software reach alternate solutions, both labelled "optimum". Both analyzed methods described here also have certain advantages and disadvantages in dependence on the characteristics of a problem. Matching the right tool with solving the structural optimization task surely makes a difference.

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References

Awad, Z.K. (2013), "Optimization of a sandwich beam design: analytical and numerical solutions", *Struct.*

Eng. Mech., **48**(1), 93-102.
Balek, V., Bydžovský, J., Dufka, A., Drochytka, R. and Beckman, I. N. (2012), "Use of emanation thermal analysis to characterize microstructure development during Portland cement hydration", *J. Therm. Anal. Calorim.*, **111**(1), 323-327.
Bajer, M. and Barnat, J. (2012) "The glue-concrete interface of bonded anchors", *Constr. Build. Mater.*, **34**(9), 267-274.
Barnat, J., Bajer, M. and Vyhnankova, M. (2012), "Bond Strength of Chemical Anchor in High-strength Concrete", *Proceedings of the 23rd Czech and Slovak Conference on Steel Structures and Bridges*, *Procedia Eng.*, **40**, 38-43.
Bathe, K.J. (2007), "On reliable finite element methods for extreme loading conditions", *Extreme Man-made and Natural Hazards in Dynamics of Structures*, Eds. A. Ibrahimbegovic and I. Kozar, Springer-Verlag, 71-102.
Cui, G. and Cui C. (2015), "Shape optimization based on ANSYS", *J. Inform. Comput. Sci. (JICS)*, **12**(11), 4291-4297.
Deshbhratar, R.J. and Suple, Y.R. (2012), "Analysis & optimization of crankshaft using fem", *Int. J. Mod. Eng. Res. (IJMER)*, **2**(5), 3086-3088.
Fedorik, F. (2013), "Analysis of gradient and approximation design optimization methods' robustness", *Proceedings of the 19th International Conference on Soft Computing-Mendel 2013*, Brno, Czech Republic.
Fedorik, F. (2013), "Efficient design of a truss beam by applying first order optimization method", *Proceedings of the ICNAAM 2013-11th International Conference of Numerical Analysis and Applied Mathematics*, Rhodes, Greece.
Güler, O. (2010), *Foundation of Optimization, Graduate Texts in Mathematics 258*, Springer Science+ Business Media, LLC 2010.
Holomek, J. and Bajer, M. (2012), "Experimental and numerical investigation of composite action of steel concrete slab", *Procedia Eng.*, **2012**(40), 143-147.
Himeur, M., Benmarce, A. and Guenfoud, M. (2014), "A new finite element based on the strain approach with transverse shear effect", *Struct. Eng. Mech.*, **49**(6), 793-810.
Jividinejad, A. (2012), "Theory of parametric design optimization approach via finite element analysis", *Adv. Theo. Appl. Mech.*, **5**(5), 217-224.
Kala, J., Hradil, P. and Salajka, V. (2011), "Measures which can be used to predict, prevent and resolve the problems of liveliness in footbridges", *Proceedings of the 17th Int. Conf. Engineering Mechanics 2011*, Svratka, Czech Republic.
Kala, Z. (2012), "Geometrically non-linear finite element reliability analysis of steel plane frames with initial imperfections", *J. Civil Eng. Manage.*, **18**(1), 81-90.
Kala, Z. and Kala, J. (2012), "Lateral-torsional buckling analysis of I-beams using shell finite elements and nonlinear computation methods", *International Conference of Numerical Analysis and Applied Mathematics (ICNAAM)*, Greece.
Kala, Z. and Kala, J. (2011), "Sensitivity analysis of stability problems of steel columns using shell finite elements and nonlinear computation methods", *Proceedings of the 17th International Conference Engineering Mechanics 2011*, Svratka, Czech Republic.
Kala, J., Salajka, V. and Hradil, P. (2012), "Sensitivity investigation of eigenvalue problem of water tower construction interacting with fluid", *J. Vib.*, **14**(3), 1151-1159.
Karasek, R., Bajer, M. and Jurdova, K. (2012), "Influence of column construction type on the bearing capacity of bonded anchors", *Proceedings of the 23rd Czech and Slovak Conference on Steel Structures and Bridges, Procedia Engineering*, **40**, 195-198.
Khoei, A.R. (2015), *Extended Finite Element Method-Theory and Practice*, Pondicherry, India.
Mallika, A. and Ramana Rao, N.V. (2010), "Thickness optimization of vibrating shells for minimum volume", *Int. J. Civil Struct. Eng.*, **1**(2), 211.
Morin, P., Nochetto, R.H., Pauletti, M.S. and Verani, M. (2012), "Adaptive finite element method for shape optimization", *ESAIM: Control, Optim. Calcul. Var.*, **18**(4), 1122-1149.

Rao, S.S. (1996), *Engineering Optimization-Theory and Practice*, Fourth Edition, School of Mechanical Engineering, Purdue University, John Wiley & Sons, Indiana.

Ródenas, J.J., Bugeda, G., Albelda, J. and Oňate, E. (2011), "On the need for the use of error-controlled finite element analyses in structural shape optimization processes", *Int. J. Numer. Meth. Eng.*, **87**(11), 1105-1126.

Şahin, A. and Bayraktar, A. (2014), "Computational finite element model updating tool for modal testing of structures", *Struct. Eng. Mech.*, **51**(2), 229-248.

Shayanfar, M.A., Ashoory, M., Bakhshpoori, T. and Farhadi, B. (2013), "Optimization of modal load pattern for pushover analysis of building structures", *Struct. Eng. Mech.*, **47**(1), 119-129.

Su, J., Chen, Z., Wang, Z. and Xi, G. (2014), "An adaptive finite element method for shape optimization in stationary incompressible flow with damping", *Int. J. Numer. Anal. Model.*, **5**(1-2), 79-96.

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