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Stress intensity factor calculation for semi-elliptical cracks on functionally graded material coated cylinders

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Abstract. In this paper, the effect of functionally graded material (FGM) coatings on the fracture behavior of semi-elliptical cracks in cylinders is assessed. The objective is to calculate the stress intensity factor (SIF) of a longitudinal semi-elliptical crack on the wall of an aluminum cylinder with FGM coating. A three-dimensional finite element method (FEM) is used for constructing the mechanical models and analyzing the SIFs of cracks. The effect of many geometrical parameters such as relative depth, crack aspect ratio, FG coating thickness to liner thickness as well as the mechanical properties of the FG coating on the SIF of the cracks is discussed. For a special case, the validity of the FE model is examined. The results indicated that there is a particular crack aspect ratio in which the maximum value of SIFs changes from the deepest point to the surface point of the crack. Moreover, it was found that the SIFs decrease by increasing the thickness ratio of the cylinder. But, the cylinder length has no effect on the crack SIFs.

Keywords: fracture, functionally gradient materials (FGMs); cylinder; stress intensity factor (SIF); semi-elliptical crack; finite element method (FEM)

1. Introduction

Because of their good resistance against high temperatures, oxidation and wear, functionally graded materials (FGMs) are used as thermal coatings in engineering applications such as aerospace structures, jet engines, nozzles and gas turbines (Koizumi 1997). Functional gradation opens new possibilities for optimizing of mechanical structures to achieve a higher performance and materials' efficiency. In fact, the concept of the FGMs is to be used as an alternative to thermal barrier coatings in high temperature applications. The composition of FGMs changes from a pure material to another pure material along their thickness direction. Hence, the mechanical and physical properties of the FGMs vary along their thickness direction (Koizumi 1997, Ghannad *et al*. 2012).

In last two decades, development and analysis of FGMs were attracted the attentions of

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scientists and engineers. The fracture analysis was an important research topic which was focused. Initial studies analyzed stress field of crack tips in FGMs. Delale and Erdogan (1983) showed that the Poisson's ratio has a negligible effect on the mode I SIFs of cracks in a non-homogeneous material infinite plane. Eischen (1987) analyzed the stress fields at near the crack tip and found that the stress field functions in the crack tip are similar for the FGMs and the homogeneous materials. Further works revealed that the singularity functions around the crack tip stress field are same for the FGMs and homogenous elastic materials (Jin and Noda 1994, Jin and Batra 1996). The surface crack problem for a finite-thickness strip subjected to different loadings such as uniform tensile, bending and grip loads was studied by Erdogan and Wu (1997). With an introduction to the FGM concept, extensive researches on various aspects of the fracture of isotropic FGMs under mechanical loads were conducted by Marur and Tippur (2000).

Mode I (Anlas *et al*. 2000, Chen *et al*. 2000) and mixed-mode (Kim and Paulino 2002) crack problems were investigated by means of the finite element method (FEM) and the path independent *J*- integral for FGM materilas. Ozturk and Erdogan (1997, 1999) used integral equations to investigate Mode I and mixed-mode crack problems in an infinite non-homogeneous orthotropic medium with a crack aligned with one of the material directions considering constant Poisson's ratio. FEM was used by Kim and Paulino (2002) to study the mixed mode fracture problem in orthotropic FGMs. Dolbow and Gosz (2002) calculated the mixed-mode SIFs for two-dimensional (2D) cracks in FGMs using interaction energy integral. Parameswaran and Shukla (2002) studied the asymptotic stress field for cracks in FGMs. They assumed that the cracks are aligned along the direction of property gradation and the elastic modulus in FGMs is varied exponentially. Also, they computed the expansion of the crack deformation for shear and opening modes.

Most of the initial works on the fracture analyses in FGMs were done based on 2D assumptions. Nowadays, 3D crack problems in FGMs are presented. Walters *et al*. (2004) studied the fracture analysis of semielliptical surface cracks in FGM structures. Yildirim *et al*. (2005) investigated the fracture problem in a homogeneous substrate with FGM coating contains a semielliptical surface crack. In their research, the displacement correlation technique was used to analyze the structure which was subjected to mechanical or transient thermal loadings. Walters *et al*. (2006) computed the mixed-mode SIFs for semi-elliptical cracks in FGMs. Ayhan (2007) used a 3D FEM to calculate the SIF for semielliptical cracks in FGMs. He showed that the gradation of FGM structure can be modeled by dividing it to some layers. In this technique the mechanical properties of each layer is assumed to be constant but different from another layer. Also, Shahani and Kheirikhah (2007) presented mode I stress intensity factor for a circumferential semi-elliptical crack on the inner surface of a hoop-wrapped steel-lined CNG cylinder. They used 3D finite element method for modeling and analysis of the cracked cylinders.

The interfacial 2D crack problem between two different FGM plates was analyzed by Li *et al*. (2006). Recently, Kheirikhah and Khalili (2011) studied the influence of the discontinuity of the mechanical properties of FGMs on the stress intensity factors of semi-elliptical cracks through a 3D FEM.

Thermo-mechanical analysis of FGM cylinders and cylinders with FGM coating were also done by many researchers (Zimmerman and Lutz 1999, Liew *et al*. 2003, Pan and Roy 2006, Tutunc 2007, Hosseini *et al*. 2008, Li and Peng 2009). But, in the field of fracture analysis, only a few works can be found. Newman and Raju (1980, 1982) calculated SIFs for internal and external surface cracks in isotropic homogenous cylindrical pressure vessels. Li and Zou (1998) and Li *et al*. (1999) performed axisymmetric finite-element analyses of circumferentially cracked FGM cylinders. Mode-I SIFs were computed using a displacement correlation technique which links computed crack-face displacements with SIF expressions.

It seems the fracture analysis of the cylindrical structures with FGM thermal coating is important and necessary. Literature reviews indicate that there are no published results about SIF calculation of cracks on the inner surface of an FGM-coated cylinder. Therefore, the purpose of this paper is to introduce a FEM approach for assessment of the SIFs of the surface cracks on the inner surface of an aluminum cylinder with FGM coating. The effects of variety of geometrical parameters such as relative depth, crack aspect ratio (the ratio of crack length to crack depth), ratio of FGM thickness to liner thickness on the SIFs is investigated. Moreover, the effects of variety of the mechanical properties on the SIFs of the cracks are discussed.

2. Mechanical model

The cross-section view of a FGM coated aluminum cylinder with the corresponding geometrical parameters is shown in Fig. 1. As illustrated, a cylinder with a length of 2w, liner thickness of t, FGM thickness of t_{fg} and the inner liner radius of R_{in} is assumed. Because the longitudinal surface cracks in the cylinders are more critical than the circumferential ones (Newman and Raju 1980), an internal longitudinal semi-elliptical crack with a length of $2c$ and depth of a is assumed in the inner surface of the cylinder. Semielliptical cracks are described with two dimensionless parameters, defined as (Shahani and Kheirikhah 2007): the aspect ratio (a/c) and the relative depth (a/t). As presented in Fig. 1, point A is called the surface point (φ=0) and point B is called the deepest point (φ=π/2), where φ is the crack angle (Kheirikhah and Khalili 2011).

As mentioned, the material properties of a FGM coating changes along its thickness direction, gradually. The material properties graduate from the liner's property to the external surface property. To model the FGM gradation along the thickness of the coating, an exponential function is assumed for the gradation of mechanical properties (Ayhan 2007, Kheirikhah and Khalili 2011).

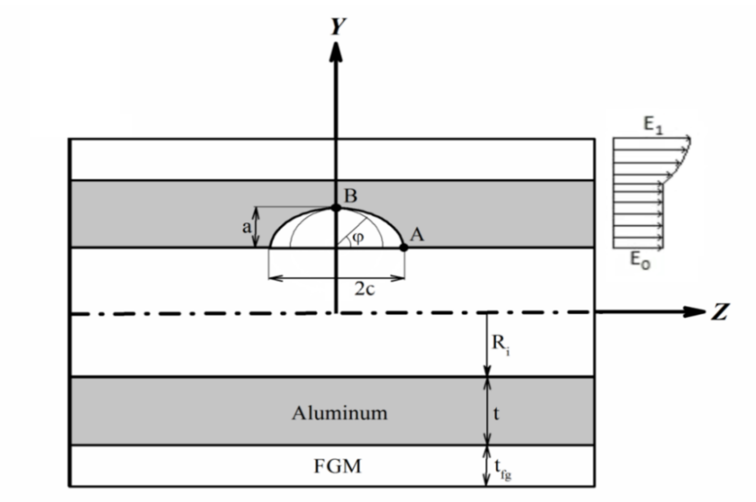

Fig. 1 Typical cross section of a cylinder with FGM coating and an internal semi-elliptical longitudinal crack

Hence, according to the defined coordinate system, the Young's modulus of the cylinder coating varies based on the following equation

$$E_{fg} = E_0 \; e^{\alpha y} \tag{1}$$

Where E_0 is Young's modulus at the liner of the cylinder and α is non-homogeneity parameter of the FGM coating which is defined as

$$\alpha = \frac{1}{t_{fg}} \; Log\left(\frac{E_1}{E_0}\right) \tag{2}$$

Where E_1 is Young's modulus of the external surface of the FGM coating. In this paper, the liner of the cylinder is assumed to be Aluminum with E_0=70 GPa and the surface layer of the FG coating is assumed to be an Alumina ceramic material with E_1=380 GPa (Kheirikhah and Khalili 2011). Schematic of Young's modulus gradation is also shown in Fig. 1.

It has been demonstrated that the variation of Poisson's ratio does not have a significant effect on the fracture behavior of FGMs (Ayhan 2007). Therefore, the Poisson's ratios of the liner and the coating are assumed to be constant (v=0.3) in the present study.

3. Finite element modeling

For the finite element analysis, a cylinder with an internal radius of R_{in}=0.15 m, liner thickness of t=0.01 m and total length of L=0.4 m has been selected. A 20-node brick element is employed for constructing the FE model of the problem in the ANSYS 12.0 standard code area (2004). The *YZ* and *XY* planes can be assumed as symmetry planes for the problem. Therefore, only a quarter of the cylinder is modeled. Also, only half of the each elliptical crack surfaces is constructed.

The stress field around the crack tip has square root singularity (Jin and Noda 1994, Jin and Batra 1996). Barsoum (1976) found that by moving the mid-node of a eight node isoparametric quadrilateral element to a quarter point position, the desired $1/\sqrt{r}$ variation for strains can be achieved along rays, within the element, the emanate from the node crack tip. this can be easily done by moving the mid-nodes to the quarter point. It cause to shape function in global coordinate becomes more similar to the $1/\sqrt{r}$ function.

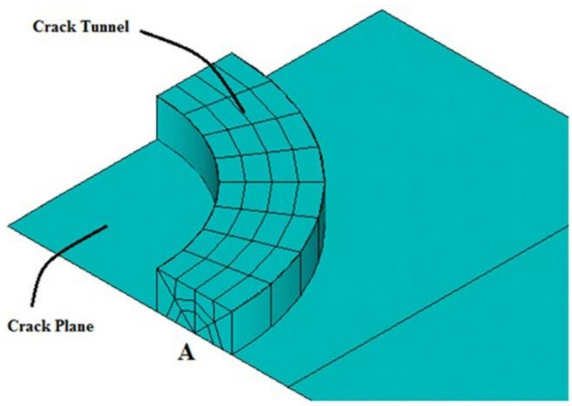

Fig. 2 Constructing 3D singular elements at the crack front

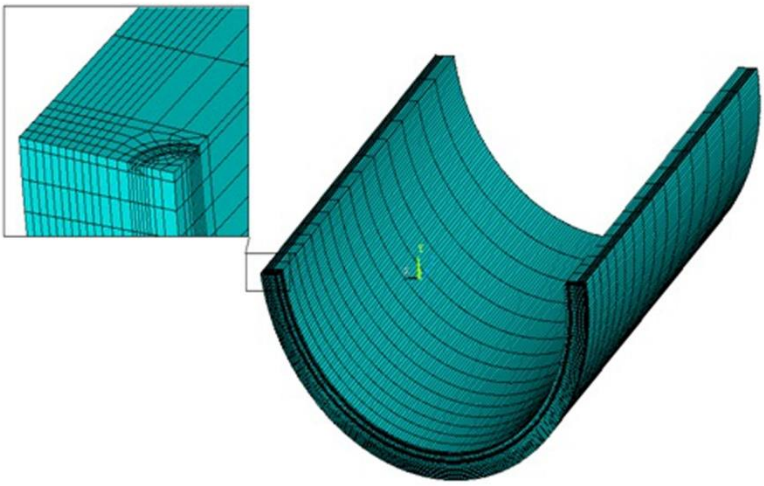

Fig. 3 Complete 3D FE model of the FGM-coated cylinder with an internal semi-elliptical longitudinal crack

For constructing the crack front in the ANSYS code, a temporary area is built and meshed at the surface point of the crack (point A). The mid side nodes of the first row elements around this point are shifted to the quarter point location. Then, by swiping the temporary area along the elliptical crack's front line, a meshed volume is created that may be named as the 'Crack Tunnel' (Kheirikhah and Khalili 2011, Shahani and Kheirikhah 2007). Fig. 2 shows the meshed crack tunnel. In this volume, only the first row elements around the crack's front are singular and the other elements are non-singular. Then, the remained parts of the liner are constructed using 20-node brick elements. The elastic module of all the liner elements is assumed to be E_0 during the modeling procedure.

In this research, the thickness of the FGM coating is divided into many layers to model its elastic modulus gradation, accurately. The elastic modulus of the elements in each layer is assumed to be constant and varies from another layer. Therefore, by increasing the number of layers, the accuracy of the elastic modulus gradation can be increased. In this study, the thickness of the FG coating is divided to ten layers. In Fig. 3, the complete quarter FE model of the FG cylinder with semielliptical longitudinal crack is shown.

In this problem, the *YZ* and *XY* planes are the symmetry planes. Therefore, the normal displacement component of these planes must be constrained. Thus, the boundary condition due to the symmetry conditions can be defined as

$$\text{For } XY \text{ plane:} \qquad w_{(z=0)} = 0 \tag{3}$$

$$\text{For } YZ \text{ plane:} \qquad u_{(x=0)} = 0 \tag{4}$$

Where u and w are the displacement along X and Z directions, respectively. Internal pressure is applied on all over the inner surfaces of liner. Because the crack is considered internally, the pressure is also applied on the crack surface.

Finally, the SIFs of the points along the crack's front are computed by ANSYS 12.0 standard code area.

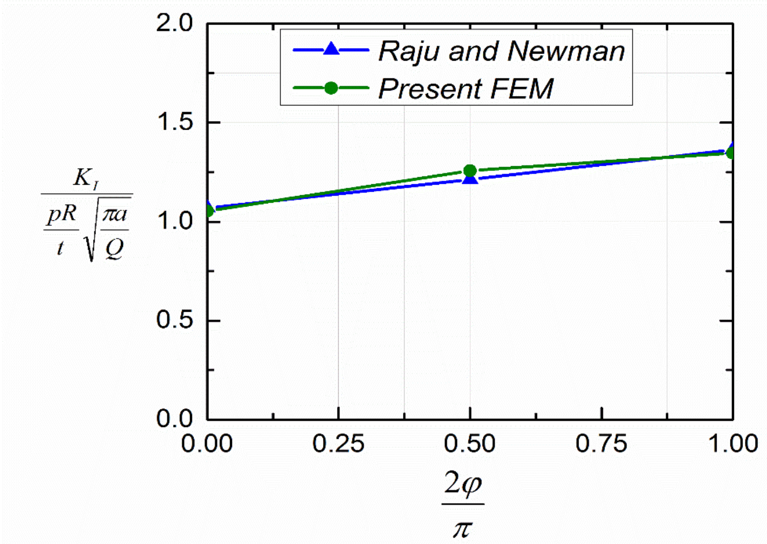

Fig. 4 SIFs for a semi-elliptical longitudinal crack in the internal surface of a cylinder

4. Result and discussion

4.1 Verification

The validity of the model is examined in this section. As mentioned, no published results are available for the semi elliptical cracks on the inner surface of FGM cylinders. Hence, in order to demonstrate the accuracy of the present FE modeling and analysis, the SIFs of a longitudinal crack in the internal surface of a metallic cylinder is calculated by 3D FEM and compared with those of reported values (Newman and Raju 1982). Therefore, a cylinder with same geometrical parameters as the target reference paper is assumed and modeled. The modeled crack has a relative depth of a/t=0.5 and an aspect ratio of a/c=0.4. The internal pressure is assumed to be P=1 MPa. Fig. 4 compares the SIFs of the crack based on the present FE calculation and those of obtained result by Raju and Newman (1982). In this figure, K_I is the mode I SIFs of the semi-elliptical crack and Q is the shape factor for the elliptical crack which is defined as (Newman and Raju 1982)

$$Q = 1 + 1.464(\frac{a}{c})^{1.65} \tag{5}$$

The good agreement of the obtained results in special case with those of reported values confirms and guarantees the accuracy of the proposed FE solution in this paper.

4.2 Effect of the crack geometry

The variations of the mode I SIFs (K_I) of an internal crack against the crack depth ratio (a/t) for the point A located at the surface and point B located at the deepest of the crack is plotted and presented in Fig. 5. In this analysis, the constant geometry parameters are assumed to be a/c=1 and t_{fg}=t. As can be seen, the SIF increases by increasing the relative depth at those of defined points. The reason for this behavior can be addressed by referring to the fact that in a constant thickness

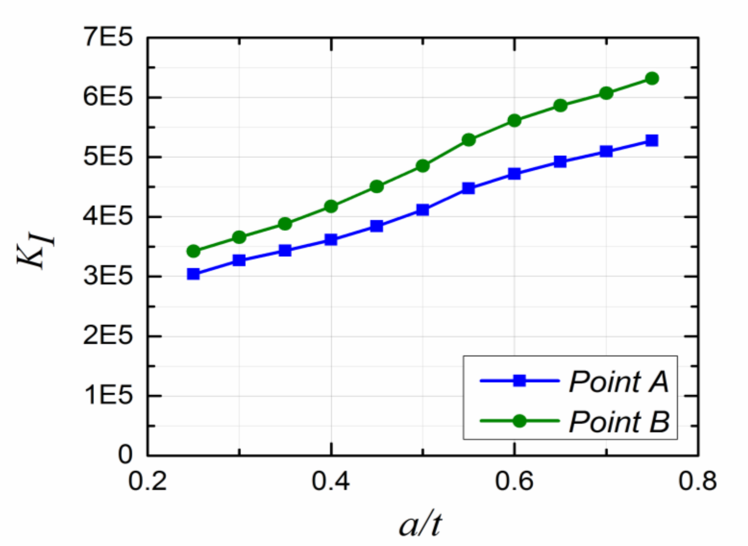

Fig. 5 Variation of the mode I SIF against the relative depth ratio

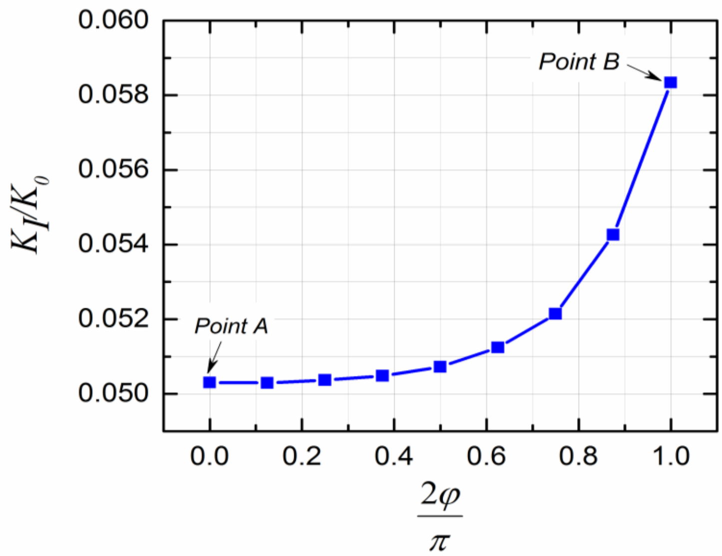

Fig. 6 Variation of the non-dimensional SIFs along the crack front

for the cylinder, an increase in the relative depth is due to the increment in of crack depth which leads to a greater value of SIFs (Kheirikhah and Khalili 2011). Moreover, it should be noted that at *a/t*=0.25, the SIF at point B is only slightly higher than point A (approximately 12.5%). But, the distance between the two corresponding lines starts increasing as the ratio of a/t increases. At the ending point (*a/t*=0.75), point B shows approximately 20 % higher SIF in compare to point A.

Fig. 6 shows the variations of the non-dimensional mode I SIFs (K_I /K_0) along crack front, from the point *A* (surface point) to the point *B* (deepest point). In all calculations in this paper, the parameter K_0 is defined as

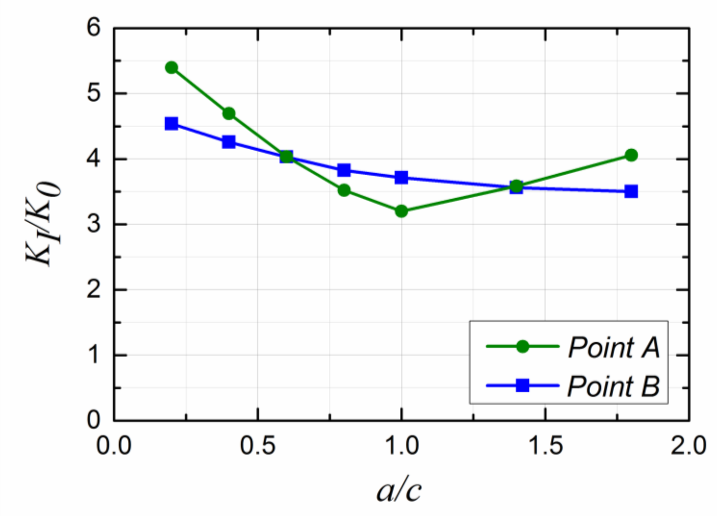

Fig. 7 Variations of the SIFs against the crack aspect ratio

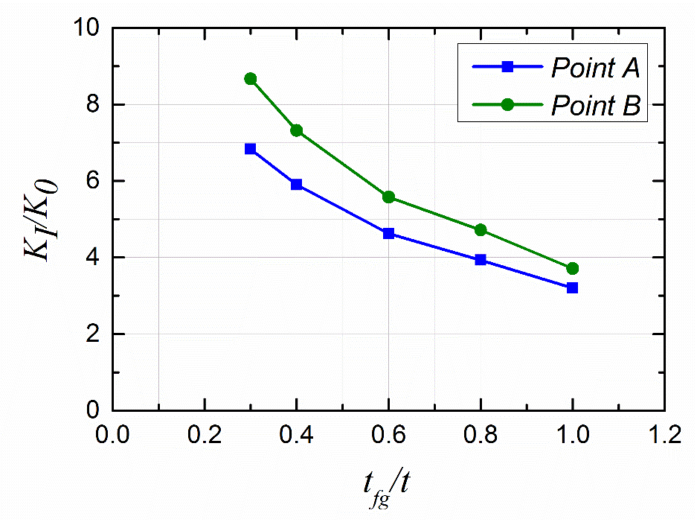

Fig. 8 Variations of the SIFs against the cylinder thickness ratio.

$$K_0 = P\sqrt{\pi a} \tag{6}$$

In this analysis, the constant geometry parameters are assumed to be a/c=1, $t_{fg}=t$ and a/t=0.5. It became evident that for the semi-circular cracks (a/c=1), the non-dimensional SIFs increases along crack front from the surface point to the deepest point. The ratio of K_I/K_0 starts from 0.0503 at point A and reaches an approximately 16 percent higher value of 0.0583 at point B.

Fig. 7 shows the variation of non-dimensional SIFs (K_I/K_0) with respect to the crack aspect ratio for the surface and the deepest points. In this analysis, the constant geometry parameters are assumed to be a/t=0.5 and $t_{fg}=t$.

It was found that the SIFs of the deepest point decrease by increasing the crack aspect ratio, continuously. But, for the surface point, the lowest SIF is occurred for the semi-circular crack (a/c=1). Moreover, it can be seen that for the range of 0.6<a/c<1.4, the deepest point is more critical than the surface point and for out of the above range, the surface point is more critical than the deepest point.

4.3 Effect of the coating thickness

In this section, the effect of cylinder thickness ratio (t_{fg} /t) on the stress intensity factor of internal longitudinal cracks is investigated. Fig. 8 shows the variations of non-dimensional mode I SIF (K_I /K_0) against the cylinder thickness ratio for the surface and the deepest points. The analysis is performed for the different cylinder thickness ratios of t_{fg} /t=0.25, 0.4, 0.6, 0.8 and 1. The crack geometrical parameters and the liner thickness are assumed to be constant as a/c=1, a/t=0.5 and t=0.01. It can be seen that the non-dimensional SIFs at the defined points decrease by increasing the thickness of FGM coating. The reason behind this behavior can be attributed to reduction of the applied stress to the liner and hence reduction of the SIFs, as the thickness of the FGM coating increases.

4.4 Effect of the mechanical properties

This section of the paper is to investigate the change of SIFs where the mechanical properties ratios are altered. In this analysis, the constant geometry parameters are assumed to be a/c=1, a/t=0.5 and t_{fg}=t. Also, the Young's modulus of the liner is assumed to be constant (E_0=70 GPa). But the Young's modulus of the surface layer of the FG coating subjected to change. The analysis is performed for the different stiffness ratios of E_1/E_0=2, 3, 4 and 5. Fig. 9 presents the variation of non-dimensional mode I SIF (K_I /K_0) against the stiffness ratio for the surface and the deepest points. As shown, the SIFs of the both points have a tendency to decrease as the Young's modulus of the coating increases.

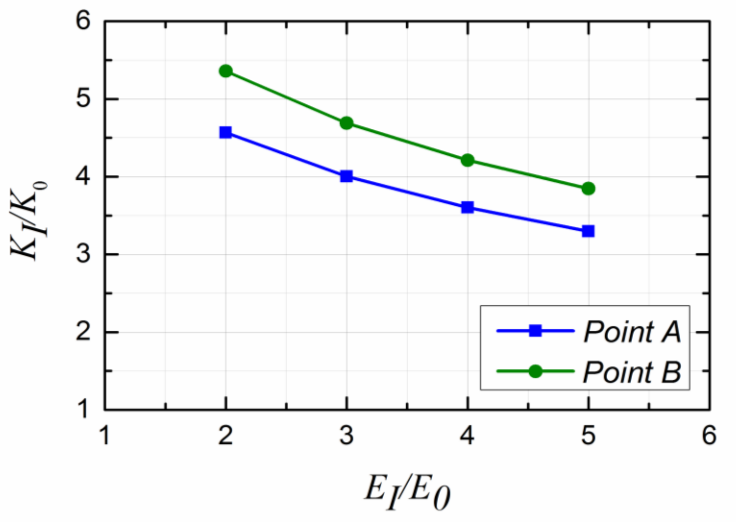

Fig. 9 Variations of the SIFs against the stiffness ratio

5. Conclusions

In this paper, the fracture analysis of an aluminum cylinder FGM-coated consisting of a longitudinal semi-elliptical crack on its wall was studied. A 3D FEM was used to calculate the SIFs of cracks. The effects of relative parameters such as crack depth to thickness, crack aspect ratio and thickness ratio of Aluminum to FGM coating as well as the changes in modulus of elasticity on the SIF of the cracks were addressed. Therefore, based on the mentioned analyzes, that the following conclusions can be drawn:

• The SIFs increase by increasing in the relative depth.

• For the semi-circular cracks (a/c=1), the non-dimensional SIFs increases along crack front from the surface point to the deepest point.

• The SIFs of the deepest point decrease with increase in crack aspect ratio, But for the surface point, the lowest SIF is occurred for the semi-circular crack (a/c=1).

• The SIFs decrease as the thickness ratio increases.

• The SIFs decrease as the Young's modulus of the coating increases.

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