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Random imperfection effect on reliability of space structures with different supports

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Abstract. The existence of initial imperfections in manufacturing or assembly of double-layer space structures having hundreds or thousands of members is inevitable. Many of the imperfections, such as the initial curvature of the members and residual stresses in members, are all random in nature. In this paper, the probabilistic effect of initial curvature imperfections in the load bearing capacity of double-layer grid space structures with different types of supports have been investigated. First, for the initial curvature imperfection of each member, a random number is generated from a gamma distribution. Then, by employing the same probabilistic model, the imperfections are randomly distributed amongst the members of the structure. Afterwards, the collapse behavior and the ultimate bearing capacity of the structure are determined by using nonlinear push down analysis and this procedure is frequently repeated. Ultimately, based on the maximum values of bearing capacity acquired from the analysis of different samples, structure's reliability is obtained by using Monte Carlo simulation method. The results show the sensitivity of the collapse behavior of double-layer grid space structures to the random distribution of initial imperfections and supports type.

Keywords: double-layer grids; random initial curvature imperfections; reliability; Monte Carlo simulation method; progressive collapse

1. Introduction

Double layer space trusses are one of the most frequently used forms of structure because of some major advantages such as high stiffness, relatively light weight, easy to erect and ability to cover large open areas (Thornton and Lew 1984). Although, double layer space trusses are highly redundant structures, but unfortunately collapse of such structures is not uncommon as highlighted by the collapse in 1978 of the double layer grid forming the roof of the Hartford coliseum (Smith and Epstein 1980).

Space trusses of double-layer grids generally have high degree of statical indeterminacy. Earlier work by Affan and Calladine (1989) estimated the number of redundant bars that could be removed without affecting truss stability, at 15% to 25% of the total number of truss members. This has led some designers to mistakenly believe that space trusses are highly reliable because the structure would still stand after removing a large number of redundant members (Affan 1987).

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Schmidt *et al*. (1980, 1982) studied different types of trusses, he showed that, when a truss member yields which losses its stiffness, but however the truss continues to carry more load. On the other hand, compression member buckling proved to be more critical. After buckling, a member loses most of its strength, thus shedding force to neighboring members, and frequently causing them to buckle as well. This would cause spreading member failure and progressive collapse which could takes only a few seconds to develop. The number of members that following their buckling truss changes to mechanism in less than a few seconds, is lower than the number reported by Affan.

Member's initial curvature, the most common form of member geometric imperfections, is known to be of common occurrence in practical trusses. In structures that typically contain hundreds or thousands of members, precise manufacturing throughout is almost impossible to achieve, and the existence of some imperfect members is always inevitable. Furthermore, description of a response of real-life structural systems is inevitably associated with various sources of uncertainties or random variables. Besides, basically the response of these structures itself has stochastic behavior. Therefore, in order to estimate, and furthermore, to insure the safety of a double-layer space structure, it is necessary to consider the effects of uncertainties in system parameters. In this regard, many investigations have been conducted by numbers of researchers, each including one or some of these random variables.

Wada and Wang (1992) studied the influence of random variation of member strength and construction errors on the mechanical behavior of double-layer space structures. They showed that construction errors, like assembly errors have enormous influence on the load carrying capacity of these structures. El-Sheikh (1991, 1995, 1997, 2002) presented a work on the study of the sensitivity of double-layer space structures to member geometric imperfections on overall strength and behavior and the location of truss critical areas at which imperfect members should be avoided. El-sheikh also has investigated the effect of member length imperfections on capacity and failure mechanism of triple-layer space structures.

Schenk and Schuëller (2005) assessed the stability of cylindrical shells with random imperfections. They employed the Monte Carlo simulation method and assumed the stress-strain relationship to be linear. Schenk and Schuëller (2007) investigated the effect of random imperfections on the critical load of isotropic, thin-walled, cylindrical shells under axial compression by means of static nonlinear finite element analyzes. They proposed the cumulative distribution functions of the limit load using the Monte Carlo simulation method. Broggi and Schuëller (2011) presented an efficient model of imperfections to analyze the buckling of composite cylindrical shells.

Zhao *et al*. (2014) investigated the effects of random geometrical imperfections on concentrically braced frames and they showed that these imperfections have a substantial effect on design forces. They calculated the forces in the braces and their probabilistic distribution by using the Monte Carlo simulation method. De Paor *et al*. (2012) investigated the effects of random geometrical imperfections in shell cylinders and based on several experimental and numerical samples concluded that the random distribution of imperfections has a good concurrence with the normal distribution. Vryzidis *et al*. (2013) investigated the effect of random initial imperfection of steel pipes in their buckling capacity. Numerical analyses and experimental results have shown that not only initial imperfection has noticeable effects on the system's buckling capacity, but it affects the failure mode of the pipes as well. Kala (2013) investigated the effects of random distribution of imperfections on the lateral-torsional buckling of the **I** shaped rolled beams with simple supports. He concluded that the cross-section imperfection causes a decrease in the bearing capacity.

In this paper the effect of initial curvature imperfections in the bearing capacity of double-layer grids has been probabilistically investigated. Curvature imperfection distribution among the members has been considered probabilistically and the structure's reliability for different supports has been studied using the Monte Carlo simulation method. Considering the nonlinear behaviors and the existence of hundreds of random variables, calculating the structural reliability is very costly and time consuming. Therefore, in this paper, the structure's reliability has been obtained through a direct and simpler approach. All of the analyses have been carried out using the finite element software OpenSees (Mckenna *et al*. 2000).

2. Monte Carlo simulation method

Monte Carlo Method is computational algorithm that uses random sampling for computing and it is usually used for simulating complex engineering systems. Due to its reliance on repetitive calculations and random numbers, the Monte Carlo method will be set in such a way so that it can be run by computer. There are different methods such as first or second order approximation to determine the reliability which are employed in simple engineering problems (Melchers 1999).

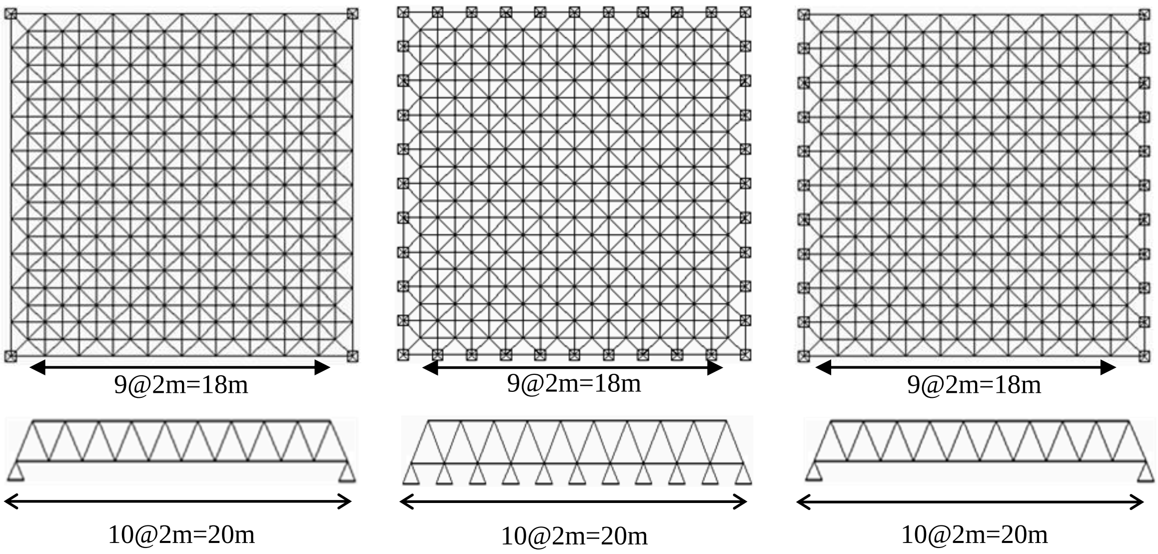
In many situations it is impossible to mathematically describe the response of some structural systems because of random nature and complex formulations of the problem. Even when we can find a mathematical model for predicting behavior of the system, there is no closed form solution for that equation. In such situations the simulation is one of the most common techniques to gain information about complicated problems. In fact, simulation is a special technique to approximate quantities that are difficult to obtain analytically. Among many of simulation procedures the Monte Carlo simulation method is one of the most famous and common procedures in solving complicated engineering problems.

These random numbers usually are generated using computer programs and some common algorithms of random number generation. Many procedures and subroutines for creating random numbers with arbitrary distributions are available. Details of such procedures are beyond the scope of this paper, but it should be mentioned that currently a simple algorithm based on a standard C++ library function is implemented in OpenSees software (Mckenna *et al*. 2000).

For every set of random numbers, the limit state function of system is obtained and ultimately based on the number of failures to the total number of scenarios, reliability is calculated. In practice, the possibility of failure is determined by generating a limited number of random variables. Thus the calculated failure probability is only an estimation of the systems actual failure probability. The minimum number of samples to acquire satisfactory results has been taken into consideration in different references (Melchers 1999).Although this method is very costly and time consuming, but with computers and analysis algorithms constantly getting modified and more powerful, it is used in a wide range of engineering problems (Nowak and Collins 2000).

3. Analytical models

In this paper, a 10×10 bay offset double-layer grid structure has been studied. The plan dimensions of the lower layer and the upper layer are 20×20 and 18×18 square meters respectively and the grid depth has been considered $\sqrt{3}\approx1.73$ meters. Each structure is composed of 1152



(a) GC (b) GS (c) GE

Fig. 1 Configuration of the selected double layer grids

Table 1 Member size of model grid structures

Kind of Support	Members length (m)	Sections (mm)
GC	2	CHS 114.3× 10
GE	2	CHS 101.6× 8
GS	2	CHS 88.9× 6.3

members. The supports have been considered in three different positions: Corner supports (GC), Edge supports (GE) and surrounding supports (GS). All supports are hinged as shown in Fig. 1. All members are pipes and the material's yield stress and elasticity modulus have been considered 360 Mpa and 210000 Mpa respectively. As an exception, the four diagonal members over the support nodes are considered to be solid bars so that the local collapse of these members and the instability of the entire structure will be prevented.

The design of the structure has been done under two different types of loads. The dead load which is equal to 0.5 kN/m^2, consists of the load of the covering and the joints. And the snow load which is equal to 2.0 kN/m^2. In this model, the dead loads and the snow load have been exerted on the joints of the upper layer as concentrated loads, proportionate to the load bearing area of each joint.

The structure has been designed using AISC allowable stress design method (AISC 2010), and to achieve minimum structural weight it was observed that compression is the dominant response of the system. All members were assumed to be made of the same circular steel tube with the length of 2000 mm, hence, three sections with specified characteristics in Table 1, are used.

4. Random variable modeling

Behavior of compression members has significant effect on failure mechanism of double-layer space structures. Almost all members of double-layer grid structures carry mostly axial forces. When a tension member reaches its yield point, its rigidity reduces to zero and the state continues until strain hardening occurs. But when a compression member buckles under compression forces, then it may not be able to carry additional loads. When the member continues to bowing, it is necessary that axial force also be reduced for keeping the balance situation.

In other words, a compression member represents softening behavior after buckling. When applied load exceeds the elastic limit point of the structure, then buckling of some compression members leads to sudden reduction in load carrying capacity of the whole structure and redistribution of internal forces. If the structure can tolerate this redistribution, then it might be able to convey few additional load, otherwise some other members will fail and progressive collapse of the system will be possible.

Behavior of a simple pin-ended compression member is a function of three basic factors namely: the slenderness ratio, the yield stress of the material and the initial imperfection of the member. In this paper one pin-ended truss type member with initial curvature imperfection was considered to model random characteristic of buckling behavior of the member (Fig. 2). It was assumed that the geometric imperfection was in the shape of a half-sine wave along the length. The maximum initial deviation of the member in its mid-point is denoted e. This member was created in OpenSees with twenty Elastic-Perfectly Plastic non-linear beam-column elements with equal length, integrated at 4 points along the element. The integration is based on the Gauss-Legendre quadrature rule which enforces Bernoulli beam assumptions. This section was captured with Fiber Section element class in OpenSees, so that the whole section was divided to 16 subdivisions (fibers) in circumferential direction and to 4 subdivisions in radial direction as shown in Fig. 2. Finally, the axial force-displacement relationship of the imperfect member was obtained through displacement control analysis using Newton-Raphson algorithm with considering both geometric and material non-linearity (Sheidaii and Abedi 2001)

For the sake of simulation of random nature of the initial geometric imperfection it was assumed that the amount of the maximum imperfection in the mid-point of the column (i.e. e), has Gamma probability distribution (Schuëller 1987). The parameters of the Gamma distribution are chosen in such a way so that the mean and maximum values of the imperfection are equal to 0.05 and 0.1 percent of the member’s length, respectively (i.e., $0.0005L=0.0005\times2000=1$ mm and $0.001L=0.001\times2000=2$ mm). Because application of members with rather large imperfection can easily be avoided. In this manner, the probability density function of the imperfection was

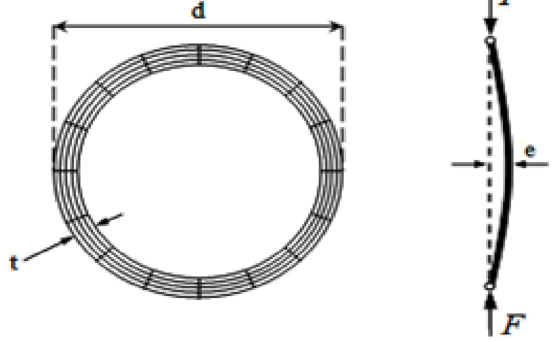


Fig. 2 Geometrical and meshing specifications of the compression member model

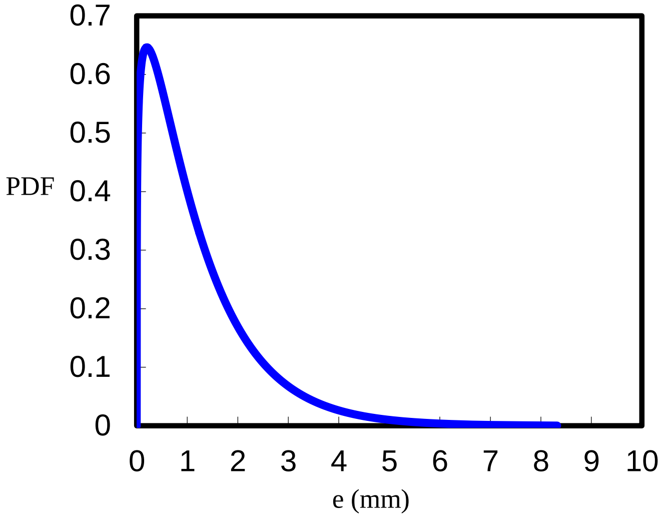


Fig. 3 Probability density function with gamma distribution

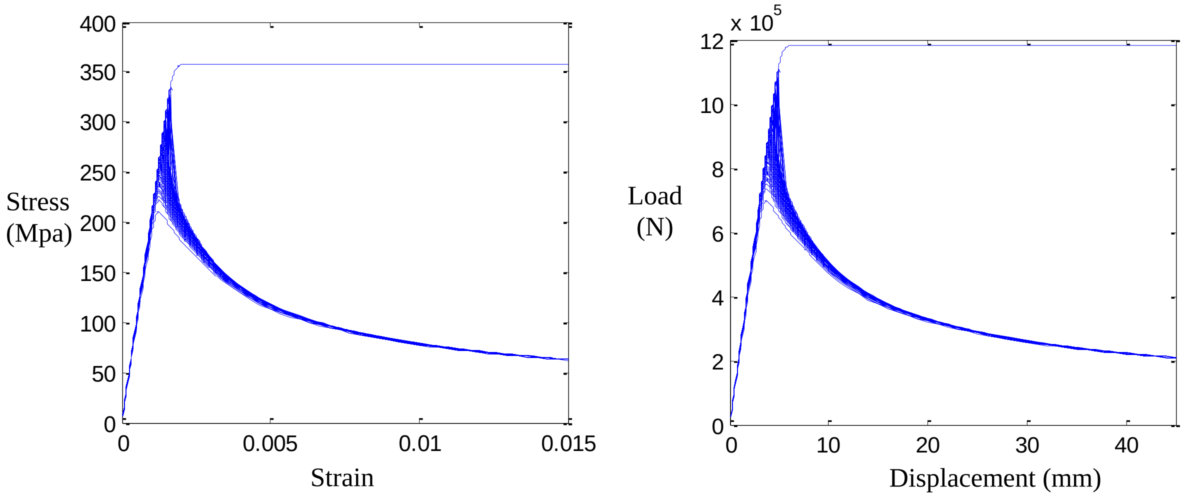
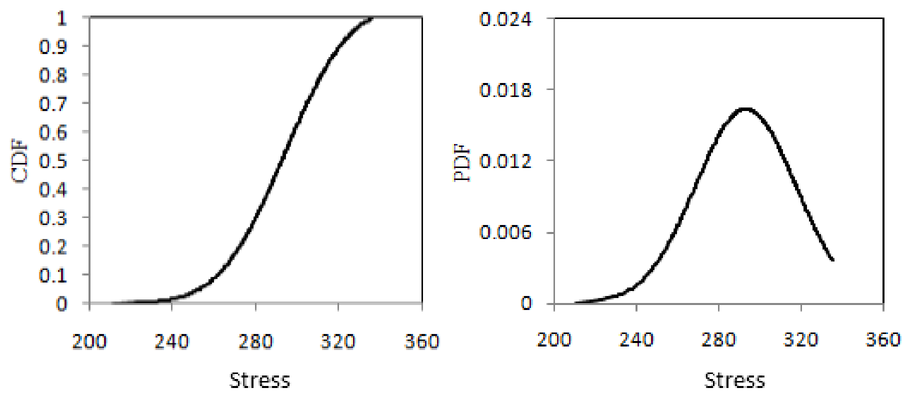


Fig. 4 Relevant axial force-displacement and stress-strain for compressive member with geometrical imperfections for section CHS 114.3×10

obtained by generating 1152 random numbers in Gamma distribution with above characteristics (Fig. 3). Then for each of these imperfections the analysis performed and the maximum load that the member can convey calculated for each case.

Fig. 4 shows the relevant axial force-displacement relationship of each imperfection for section CHS 114.3×10. Note that the figure has been drawn according to the corresponding stress values. The probability distribution of the buckling stress and its relevant statistical parameters can be determined by statistical calculations. In this case the probability density function and cumulative distribution function of bucking load of the column in terms of relevant stresses were obtained as shown in Fig. 5. It can be seen that the post-bucking negative rigidity of the compression member



(a) (b)

Fig. 5 Functions (a) cumulative distribution and (b) probability density for compressive member buckling stress for section CHS 114.3×10

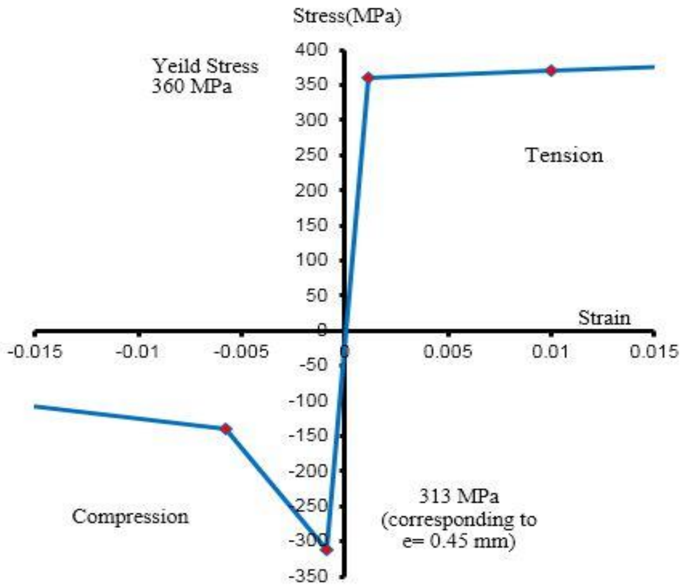
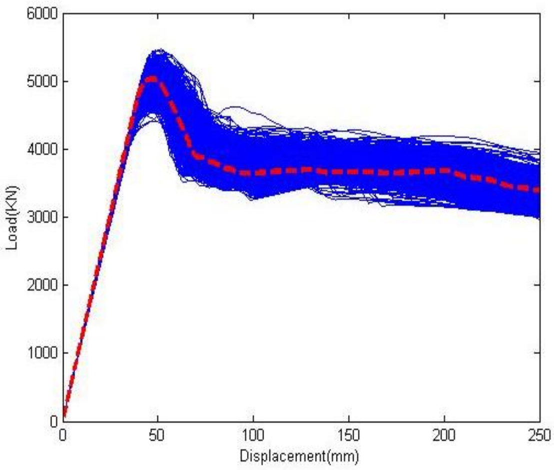
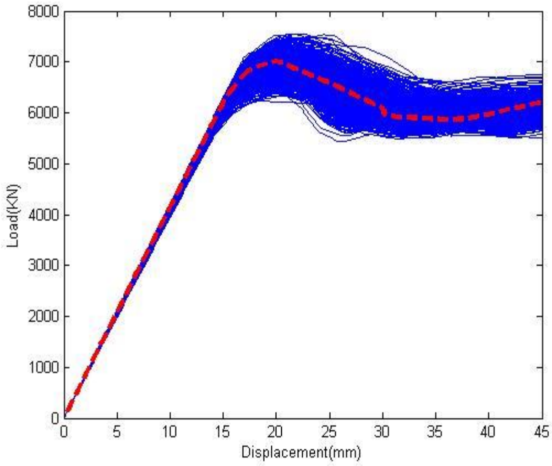


Fig. 6 Idealized axial stress-strain relationship for imperfect members in tension and compression for section CHS 114.3×10 and *e*=0.45 mm

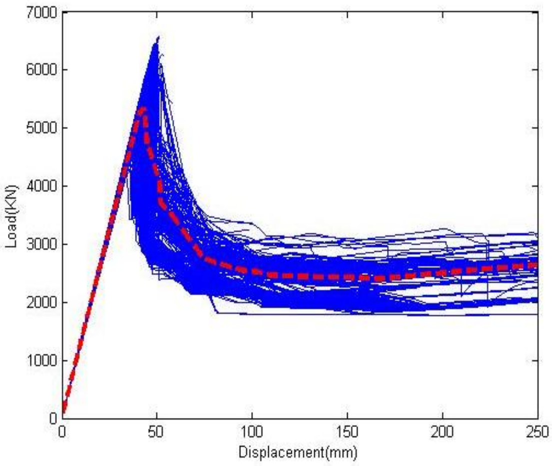
has severe sensitivity to initial imperfections. Also, the mean and the standard deviation of the buckling stress of the member were obtained as μ_s=313 N/mm^2 and σ_s=28 N/mm^2, respectively. So the idealized axial stress-strain relationship of the imperfect member and its characteristics in tension and compression were modeled as Fig. 6 in the following analyses. It should be noted that the yield stress and modulus of elasticity of imperfect member are assumed deterministic parameters, but buckling stress is assumed probabilistic value, and is a function of member initial curvature imperfection.



(a) GC



(b) GS



(c) GE

Fig. 7 Load-displacement diagrams of the double-layer space structures with different supports

5. Results of the push-down analyses with random imperfection

As mentioned earlier, in this case only the buckling stress of the column was considered as a random variable for modeling all types of initial imperfections. For this purpose, obtained probability distribution of the buckling stresses of the members was used as a base for the following analyses of the double-layer space structure. In other words, again by using Monte Carlo simulation, non-linear finite element push down analyses were performed for double-layer grid, and for each of them the maximum load carrying capacity of the system was calculated. Effects of both material and geometric non-linearity were considered in the analyses. It should be mentioned that for modeling the random nature of distribution of imperfections through the double-layer grid, since the structure totally contains 1152 pin-ended members, in each analysis 1152 buckling stress values were chosen among the above stresses and uniformly distributed among the all members.

This procedure should be repeated for many times so the results would be reliable. A sensitivity analysis showed that by performing 600 analyzes, the accuracy of the results will increase up to 99%. In this paper, for more certainty, 1000 simulations were performed and the results are given as force-displacement relationship for the node in the middle bottom layer of the double-layer grid as shown in Fig. 7. For every 1000 analyzes, approximately 5 hours of CPU time were spent. However, preparing the input file was quite time consuming.

According to the load-displacement diagrams of the GC, GS, GE grids and Table 2, it can be seen that the GS grid can carry loads 1.38 times and 1.24 times more than the GC and GE grids, respectively. Also, the GE grid's load carrying capacity is 1.11 times more than that of the GC grid. Also as it is illustrated in Fig. 7, red dashed line is the average of thousands load-displacement diagram demonstrator.

6. Reliability

Reliability of a structural system is defined as the probability that it will perform its intended function without failing. In the present context, the approach to calculate the reliability is based on the definition of the probability of failure. Reliability of a system is usually denoted by R and is defined as $R=1-P_f$, where P_f is the probability of failure of the system. In this study, the only considered random variable is the imperfection or the relevant buckling stress of the member. The reliability of the system R for the specified applied load F_s is equal to the probability of the system failure load F being greater than or equal to the load F_s. This can be expressed mathematically as Eq. (1):

$$R(Fs) = P(F \geq Fs) \tag{1}$$

By using this definition the reliability of double-layer space structure was calculated and the results were given in Fig. 8 and Table 3. Second, third and fourth columns show the amount of strength reduction in each case because of the imperfections. Note that the last three columns in Table 3, gives load factor for each case. It means, if a designer wants to become sure about the safety of an imperfect structure, he must multiply the design load of the system by corresponding load factor in the Table 3, afterward, the structure can be assumed perfect in analysis and design procedures.

It should be mentioned that the perfect structure theoretically is a structure without any imperfection. But in practice, construction of such an ideal structure is impossible, so in the present context, the structure that indicated maximum load carrying capacity among other structures with different imperfections was considered as a perfect or most nearest structure to the perfect structure.

Table 2 Statistical specifications of the collapse load

Grid's name	Failure load (KN)				
	Mean (χ) (KN)	Standard Deviation (σ) (KN)	Coefficient of Variation (σ/χ)	P_{MAX} (KN)	P_{MIN} (KN)
GC	5023.5	173.9	0.035	5469.5	4411.2
GE	5588.7	389.7	0.070	6577.3	4273.6
GS	6941.9	208.3	0.030	7553.2	6233.4

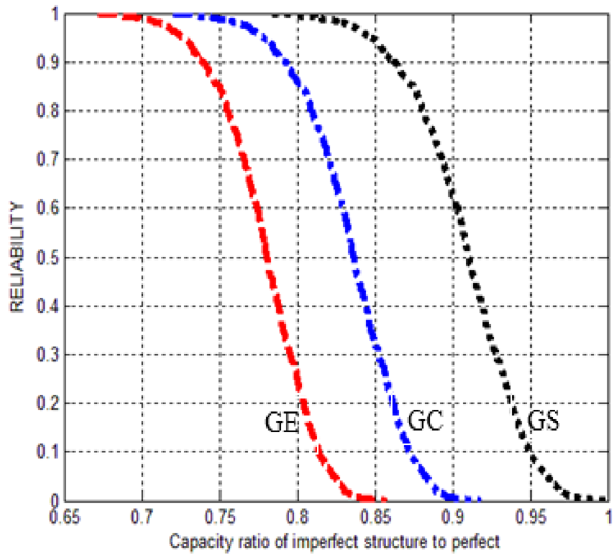


Fig. 8 Reliability for two layer grid with different supports

Table 3 Reliability for two layer grid with different supports

Reliability	capacity ratio of imperfect structure to perfect structure			Coefficient Load factor		
	GS	GE	GC	GS	GE	GC
1	0.846	0.683	0.721	1.15	1.32	1.28
0.99	0.863	0.723	0.768	1.14	1.28	1.23
0.98	0.870	0.737	0.779	1.13	1.26	1.22
0.97	0.875	0.743	0.786	1.13	1.26	1.21
0.96	0.878	0.756	0.792	1.12	1.24	1.21
0.95	0.881	0.763	0.796	1.12	1.24	1.20
0.94	0.883	0.768	0.800	1.12	1.23	1.20
0.93	0.885	0.773	0.804	1.12	1.23	1.20
0.92	0.887	0.777	0.807	1.11	1.22	1.19
0.91	0.889	0.780	0.810	1.11	1.22	1.19
0.9	0.890	0.784	0.812	1.11	1.22	1.19
0.85	0.897	0.799	0.822	1.10	1.20	1.18
0.8	0.902	0.810	0.831	1.10	1.19	1.17
0.75	0.906	0.820	0.838	1.09	1.18	1.16
0.7	0.910	0.829	0.844	1.09	1.17	1.16

7. Discussion

It can be seen from Fig. 8 and Table 3 that for designing a GC system with reliability equal to for example 0.99, it should be noted that the capacity of the imperfect structure will be equal to 76.8 percent of the capacity of the perfect system which is 23 percent less than that. In other words, for designing a system with reliability equal to 0.99, the design capacity must be considered 23 percent greater than the capacity of the perfect structure. Or for designing a system with reliability equal to for example 0.95, the capacity of the imperfect structure will be equal to 79.6 percent of the capacity of the perfect system which is 20 percent less than that, and so on.

The above information also, can be interpreted in the following way. Assume the goal is to design a GS double-layer space structure with consistent level of reliability without conducting detailed reliability analysis. In this case, if the target reliability of the system is for example 0.98, the design load of the system must be multiplied by corresponding load factor in the fifth column of Table 3 (i.e., 1.13).

In general, existence of initial imperfections in selected double-layer space structure reduced the capacity of the system from 9 percent through 32 percent. Hence, this stochasticity should be suitably considered in estimating realistic response of these structures.

8. Conclusions

The existence of some imperfections in double-layer space structures with hundreds of members and joints is almost inevitable. This paper examines the effect of these random imperfections on reliability and overall strength of double-layer space structures. Analyses of a selected off set double-layer grid structure by using Monte Carlo simulation method, indicate that these structures are highly sensitive to random imperfections. It is shown that by drawing out diagrams like Fig. 8 and applying them in structural designs one can easily determine the design load of a structure with desired safety. In fact, these diagrams help designers to create designs with consistent reliability level without conducting a detailed reliability analysis for each design.

As it is expected, by increasing the number of supports, the structure’s bearing capacity increases and structures with surrounding supports show far greater capacities than those that have corner and edge supports. And when one of the members collapses, there won’t be a significant drop in the bearing capacity of the GS structures because of the multiple alternative paths through which the load can be redistributed in the structure. As it is demonstrated in Fig. 7, GC and GS have flexible behavior while GE has brittle behavior.

As it is illustrated in Fig. 8, GS and GC have more reliability ratio in comparison with GE which it shows that the better behavior can not be taken with increasing the number of supports. Actually, behavior of flat double-layer space structures with corner or surrounding supports is similar to two-way slab behavior, but edge support’s behavior is similar to one-way slab. Thus, collapse behavior of flat double-layer space structures is significantly affected by number and situation of supports.

This investigation has been done on a double-layer grid space structure whose members are identical circular profiles. Therefore, the obtained results cannot be generalized to other types of space structures and further investigations have to be carried out in this regard.

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