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A novel harmony search based optimization of reinforced concrete biaxially loaded columns

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Abstract. A novel optimization approach for reinforced concrete (RC) biaxially loaded columns is proposed. Since there are several design constraints and influences, a new computation methodology using iterative analyses for several stages is proposed. In the proposed methodology random iterations are combined with music inspired metaheuristic algorithm called harmony search by modifying the classical rules of the employed algorithm for the problem. Differently from previous approaches, a detailed and practical optimum reinforcement design is done in addition to optimization of dimensions. The main objective of the optimization is the total material cost and the optimization is important for RC members since steel and concrete are very different materials in cost and properties. The methodology was applied for 12 cases of flexural moment combinations. Also, the optimum results are found by using 3 different axial forces for all cases. According to the results, the proposed method is effective to find a detailed optimum result with different number of bars and various sizes which can be only found by 2000 trial of an engineer. Thus, the cost economy is provided by using optimum bars with different sizes.

Keywords: reinforced concrete; biaxially loaded columns; optimization; metaheuristic algorithms; harmony search algorithm

1. Introduction

In design of reinforced concrete (RC) members, design of cross-sections and reinforcement are determined according to the knowledge of design engineer by assuming and checking the requirements defined in several design codes used in the region of the construction. Optimization of cross-sections and reinforcements can sustain a great economy in construction. Until now, several computational approaches have been proposed in design of RC members, but generally, these optimum designs were chosen from several preselected patterns of reinforcements. For that reason, the subject is already an active research area, especially for RC members subjected to bending in two directions in addition to axial and shear forces.

Metaheuristic algorithms have been proposed for optimization of RC members. Genetic

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algorithm (GA) inspired from natural selection is a widely employed algorithm in optimization of RC members such as beams (Govindaraj and Ramasamy 2005, Fedghouche and Tiliouine 2012), columns (Camp *et al.* 2003) and frames (Lee and Ahn 2003, Govindaraj and Ramasamy 2007). Two metaheuristic algorithms such as GA and simulated annealing (SA) are combined in the optimum design methodology for RC continuous beams by Leps and Sejnoha (2003). A hybrid optimization algorithm combining GA and discretized form of the Hook and Jeeves method was proposed for optimization of RC flat slab buildings by Sahab *et al*. (2005). Simulated annealing was employed in a multi-objective optimization approach for RC frames (Paya *et al.* 2008). A hybrid simulated annealing method was developed for optimum design of RC buildings by Li *et al*. (2010). Two heuristics (random walk and the descent local search) and metaheuristics (the threshold accepting and the simulated annealing) were used together for the optimum design of RC frames of bridges (Perea *et al.* 2008). Also, RC frames are optimized for minimization of embedded CO_2 emissions by employing SA (Paya-Zaforteza *et al.* 2009) and big bang-big crunch (BB-BC) (Camp and Huq 2013). RC retaining walls were optimized by employing several metaheuristics such as SA (Yepes *et al.* 2008), BB-BC (Camp and Akin 2012), harmony search (HS) (Kaveh and Abadi 2011) and charged system search (Talatahari *et al.* 2012). Tall piers for railway bridge viaducts were optimized by using an ant colony optimization based method by Martínez-Martín *et al*. (2013).

Harmony search algorithm inspired by musical performances were employed in the optimum design approaches for continuous beams (Akin and Saka 2010), RC frames (Akin and Saka 2012), T-shaped RC beams (Bekdaş and Nigdeli 2012) and RC columns (Bekdaş and Nigdeli 2014). Several metaheuristic algorithms were combined for optimum design of RC frames by Kaveh and Sabzi (2011). BB-BC algorithm was employed in the optimization methodology proposed by Kaveh and Sabzi (2012) for RC beams.

In this paper, a novel optimization process is proposed for cost optimization of RC biaxially loaded columns. The methodology covers the design constraints given in ACI-318 (building code requirements for structural concrete) (2005) and includes several stages employing random search iterations and harmony search algorithm. The proposed method uses different stages in control of design constraint related with different types of analyses. In these stages, optimum suitable values are iteratively searched by considering design constraints before ending the progress of all stages. Thus, the iteration process is shortened. Differently from other optimization approaches used for RC members, a detailed optimum design of reinforcement bars are done without using a defined number or size. Each bar may have a different size and the optimum number of bars is optimally changeably.

2. Methodology

A novel computation approach employing harmony search (HS) is developed for RC biaxially loaded columns (Fig. 1). In classical HS, the design variables are assigned with random numbers and analyses are conducted by iterative analyses. Then, the optimum solutions are found. In the proposed methodology, HS algorithm is combined with several random search stages because of two reasons. As the first reason, the design variables, which are not suitable for the constraints given in design codes, must be neglected after the design stages are conducted in classical HS. If a design variable is not suitable for a single constraint, all analyses are unnecessarily done. When additional iterative stages are employed in the methodology, this problem is prevented. As the

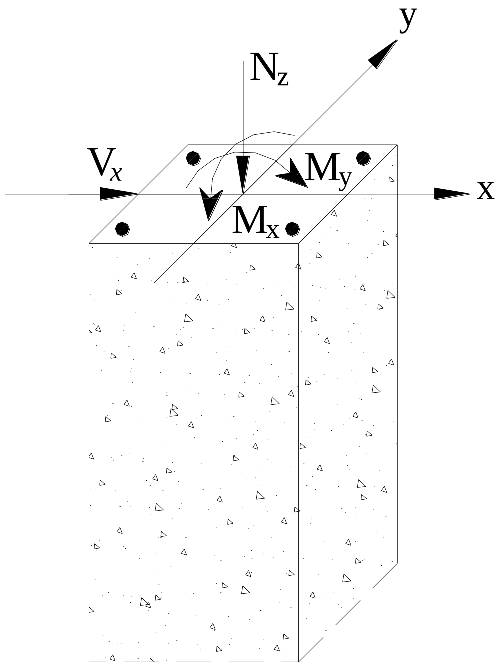

Fig. 1 RC biaxially loaded column

second reason, there are various design variables, which are related to each other and also the required internal forces are various in types such as axial force, flexural moments in two direction and shear force. Due to different criteria in design codes for axial forces, flexural capacity and shear capacity, the design must be optimum for all influences. By conducting random iteration stages during the generation of possible design variables, all design variables may be optimum and the best optimum solution can be searched according to the rules of HS. In classical HS algorithm, number of iterations may be too much to conduct or the possibility of finding optimum design variables may be not possible.

HS algorithm is an effective metaheuristic algorithm in optimization of engineering problems. HS algorithm was founded by imitating three possible choices of a musician such as playing any famous note from the performer's memory (usage of harmony memory), playing similar notes to known ones (usage of pitch adjusting) and composing new notes randomly (usage of randomization). The algorithm developed by Geem *et al.* (2001) has been also applied to problems in structural engineering such as cellular beams (Erdal *et al.* 2011), retaining walls (Kaveh and Abadi 2011), truss structures (Togan *et al.* 2011, Degertekin 2012, Gholipour *et al.* 2013) tuned mass dampers (Bekdaş and Nigdeli 2011), frame structures (Kaveh and Sabzi 2012, Martini 2011), modification of ground motions (Kayhan 2012) and RC members (Bekdaş and Nigdeli 2014, Bekdaş 2014, 2015).

The music inspired metaheuristic algorithm; HS uses a stochastic random search instead of a gradient search. Thus, it can be simply applied to engineering problems using discrete and continuous variables (Lee *et al.* 2005, Lee and Geem 2005). By using stochastic derivatives, the number of iterations is reduced and functions, which cannot be analytically derived, are solved (Geem 2008).

Table 1 Constraints on strength and dimensions of wall

Definition	Symbol
Flexural moment in both directions	M_{dx} and M_{dy}
Shear force	V_d
Axial force	N_d
Length of column	l
Strain corresponding ultimate stress of concrete	ε_c
Max. aggregate diameter	$D_{\max}$
Yield strength of steel	f_y
Compressive strength of concrete	f_c'
Elasticity modulus of steel	E_s
Specific gravity of steel	γ_s
Specific gravity of concrete	γ_c
Cost of the concrete per m^3	C_c
Cost of the steel per ton	C_s

The methodology of proposed method using random iteration stages and HS is briefly explained in this section. First, the design constants are defined. The constants are given in Table 1 with their symbols and definitions. Also, the compressive strength of concrete may be taken as a design variable. The possible strengths of concrete and the costs are depended to the location of the construction yard. In this paper, the compressive strength of concrete is taken as a design constant because the concrete with optimum strength may not be found in near region of the construction. In that situation, transportation cost may increase the total cost or the concrete may not be transported before the hardening begins.

The design is done for the requirements of ACI-318 (building code requirements for structural concrete). The compressive stress block to neutral axis depth is taken as equivalent rectangular block as described in ACI-318. The elasticity modulus of concrete (E_c) is calculated according to the f_c' as seen in Eq. (1).

$$E_c = 4700\sqrt{f_c'} \tag{1}$$

The ranges of design variables are also defined. The design variables are cross-sectional dimensions of the column and size of the reinforcements such as longitudinal and shear. After the definition of design constants and ranges, the generation of initial harmony matrix (HM) is started. This matrix is generated by harmony vectors containing possible optimum values of design variables. A constant number (harmony memory size) of harmony vectors constructs the HM matrix. The set of design variables are given in Eq. (2). n represents the number of longitudinal reinforcements and it is also randomly chosen. If the minimum and maximum limits of the design variables are respectively defined with $x_{i,\min}$ and $x_{i,\max}$, a design variable is randomized as seen in Eq. (3) and randomly generated value is rounded to a physical value. In Eq. (3), rand (0, 1) is a randomly generated number between 0 and 1.

$$X = \begin{bmatrix} \mathrm{x}_1\text{: breadth of column (b)} \\ \mathrm{x}_2\text{: height of column (h)} \\ \mathrm{x}_3 - \mathrm{x}_{n+2}\text{: size of longitudinal reinforcements} \\ \mathrm{x}_{n+3}\text{: size of shear reinforcements} \\ \mathrm{x}_{n+4}\text{: distance between shear reinforcements} \end{bmatrix} \tag{2}$$

Table 2 Design constraints

Description	Constraints
Maximum shear force ($V_{n\max}$)	$g_1(X)$: $V_d \leq V_{n\max}=\min\{5.5bh;\ 0.2f_c'bh\}$
Maximum axial force ($N_{\max}$)	$g_2(X)$: $N_d \leq N_{\max}=\ 0.5f_c'bh$
Maximum steel bars spacing, $a_{\phi\max}$	$g_3(X)$: $a_\phi \leq a_{\phi\max}$=150 mm
Minimum steel bars spacing, $a_{\phi\min}$	$g_4(X)$: $a_\phi \geq a_{\phi\min}=\max\{1.5\phi$; 40 mm; $(4/3)D_{\max}\}$
Minimum steel area, $A_{s\min}$	$g_5(X)$: $A_s \geq A_{s\min}=0.01bh$
Maximum steel area, $A_{s\max}$	$g_6(X)$: $A_s \leq A_{s\max}=0.06bh$ (seismic design)
Flexural strength capacity, M_{dx} and M_{dy}	$g_7(X)$: $M_{dx} \geq M_{ux}$ and $M_{dy} \geq M_{uy}$
Concrete cover, c_c	$g_8(X)$: $c_c \geq 30$ mm
Axial force capacity, N_d	$g_9(X)$: $N_d \geq N_u$
Shear strength capacity, V_d	$g_{10}(X)$: $V_d \geq V_u$
Minimum shear reinforcement area, $A_{v\min}$	$g_{11}(X)$: $A_v \geq A_{v\min}=(bs/3f_y)$
Maximum shear reinforcement spacing, $s_{\max}$	$g_{12}(X)$: $s \leq s_{\max}=d/2$ or $d/4$ if $V_s \geq 0.33\sqrt{f_c'}bd$

$$\mathrm{x_i} = \mathrm{x}_{\mathrm{i},min} + (\mathrm{x}_{\mathrm{i},max} - \mathrm{x}_{\mathrm{i},min})\,\mathrm{rand}(0,1)\ . \tag{3}$$

In generation of harmony vectors, several random iteration stages are conducted. First, cross section dimensions (b and h) are randomized. For ductility conditions, the design constraints such as $g_1(X)$ and $g_2(X)$ must be checked. The cross sectional dimensions are iteratively randomized until these conditions are satisfied. All constraints are given in Table 2. The effective height of the column is defined with d. The symbol; ϕ is used as the diameter size of the related bar in the constraints.

Then, the sizes of longitudinal reinforcements are randomized. Also, the number of longitudinal reinforcements is defined with a random number. This random number is between 2 and the maximum allowed number of reinforcements that satisfy the placement conditions given in $g_3(X)$. A symmetric design is done for both upper and lower faces of column. In optimization process, reinforcement bars can be placed in one or two lines. The clear distance (a_ϕ) between the reinforcement bars are checked for conditions such as $g_3(X)$ and $g_4(X)$. The randomization is iteratively conducted until orientations of bars are suitable for ACI-318.

First, the randomizations of bars are done for the direction in which value of the flexural moment is maximum. Then, additional reinforcements in the other direction are randomized if the constraint; $g_3(X)$ is not satisfied for the other direction. The reinforcement design ensuring the placement conditions are also checked for minimum and maximum reinforcement areas given as $g_5(X)$ and $g_6(X)$, respectively. Design of reinforcement is iteratively conducted until these criteria are suitable for ACI-318.

The required flexural moment is compared with the flexural moment capacity ($g_7(X)$) for the randomly assigned design variables. In the analyses, the effective height (d) of the column is calculated according to the size of reinforcements and clear cover of the column ($g_8(X)$). In this stage, an iterative analysis is done by searching the value of the distance from extreme compressive fiber to neutral axis (c) for axial forces and flexural moment in two directions. The deformation of the column is calculated for both directions and the results are superposed. Then location of the neutral axis is found. Then, the exact axial force and flexural moment capacities are found and constraints; $g_9(X)$ and $g_7(X)$ are checked. If the conditions are not met, the optimum cost of the design is penalized with a big value (10^6 \$). When the design is suitable for ACI-318, shear reinforcements can be randomized and shear force capacity ($g_{10}(X)$), minimum shear reinforcement area ($g_{11}(X)$) and minimum shear reinforcement spacing ($g_{12}(X)$) are checked. In $g_{12}(X)$, V_s is the shear force capacity provided by shear reinforcement.

After design variables are assigned with suitable values according to ACI-318, the optimum material cost of the column is calculated (Eq. (4)). The function given in Eq. (4) is the objective optimization, which are needs to be minimized. In this objective function, u_{st} is length of shear reinforcement.

$$f(X) = (bh - \sum A_s) l C_c + (\sum A_s + \frac{A_v}{s} u_{st}) l \gamma_s C_s \tag{4}$$

The initial harmony memory matrix (HMM) is constructed by merging initial harmony vectors and initial HMM is modified according to rules of HS algorithm in order to provide convergence to the optimum solution. A new vector is generated by using the same procedure as described before. In classical HS, with a possibility defined as harmony memory considering rate (HMCR), a new vector is generated around an existing vector in HM matrix. In this study, this rule is modified because the design variables are randomized with 50 mm differences for cross sectional dimensions and the size of reinforcements are randomized with 2 mm differences for practical production in construction yard. Also, the design constraints may not be provided if the randomizations are done around an existing vector. In replacement to this property, the ranges of design variables are updated according to the best design. The possibility of this modification is HMCR in the current study. If solution ranges are updated, lower bounds of the design variables are taken as the current best solutions with 50% possibility. Otherwise, the upper bound of solution ranges is updated and generation of design variables are limited with current best solution. Thus, the convergence to the optimum value is provided. If the cost of the newly generated harmony vector is lower than the existing worst one, the worst one is replaced with the newly generated vector. The iteration of generation a new vector (modification of HM matrix) continues for a constant iteration number. At the end of the iterative stage, the optimum cost and design variables are found. The flowchart of the method is given in Fig. 2.

3. Numerical examples

The optimization is conducted for different internal forces. Three different axial forces were investigated for 12 cases of biaxial flexural moment given in Table 3. For axial forces 1000 kN, 1500 kN and 2000 kN were taken in the analyses. The shear force value is taken as 100 kN for all cases. Design constants and ranges of design variables are shown in Table 4. The cost of material may vary according to the region of the construction. As a numerical example, the cost of steel for 1 ton is taken as 10 times of concrete per unit m^3.

The optimum results for 1000 kN axial force and all biaxial moment cases (numerical example 1) are illustrated in Table 5. For case 1, additional reinforcements are not needed but web reinforcements are found when the flexural moment is increased. For cases 2-10, the web reinforcements are generally assigned with minimum diameter sizes. For that cases, the flexural moment in the direction which is not critical, is low. Only web reinforcement is needed in order to ensure design constraint given as $g_3(X)$, in order to maintain the maximum distance between the

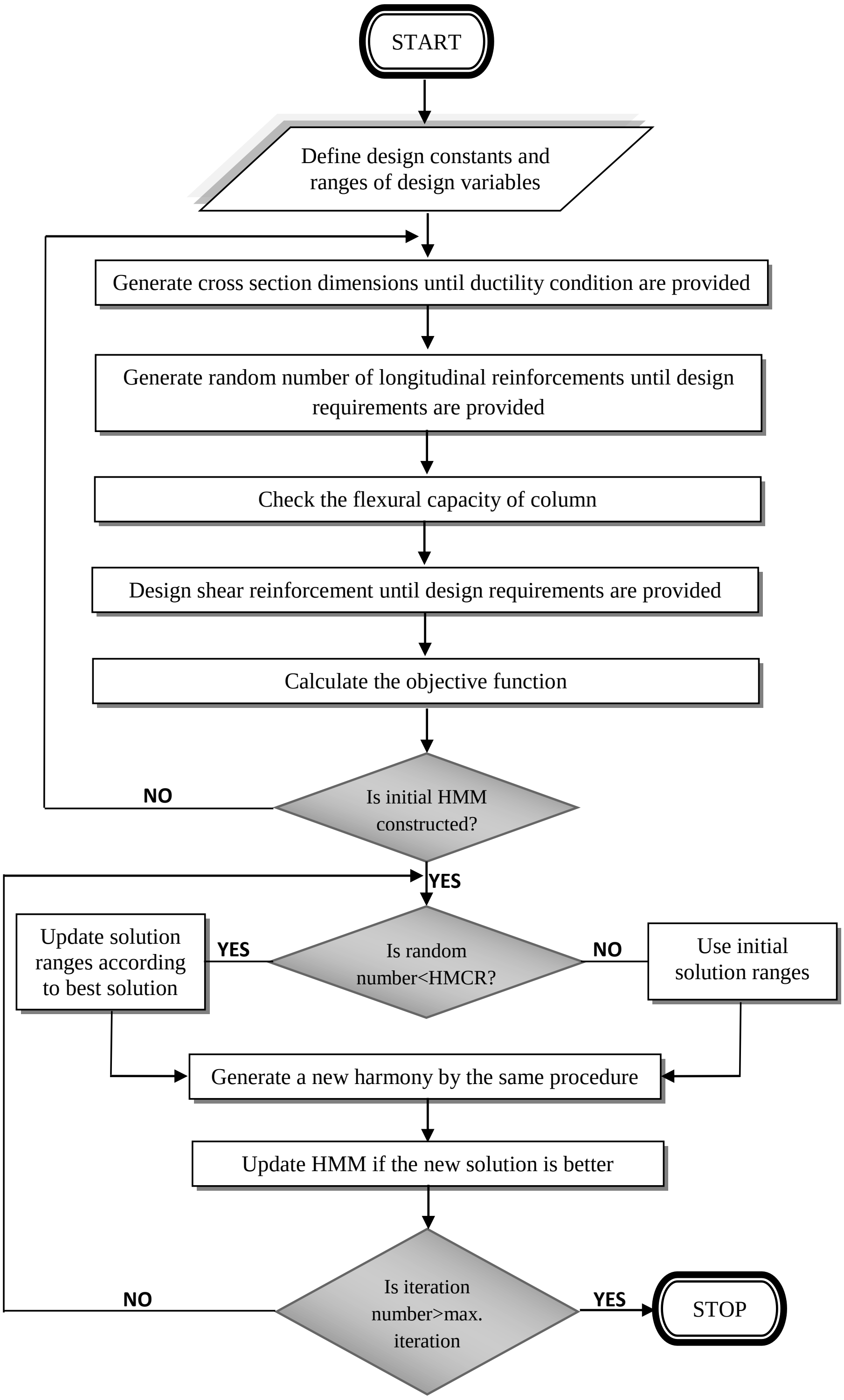

Fig. 2 Flowchart of the methodology

Table 3 Biaxial flexural moments for different cases

Case	M_y (kNm)	M_x (kNm)
1	100	100
2	200	100
3	300	100
4	400	100
5	500	100
6	600	100
7	700	100
8	800	100
9	900	100
10	1000	100
11	500	500
12	800	800

Table 4 Design constants and ranges of design variables

Symbol	Unit	Value
b	mm	300-600
h	mm	300-600
l	mm	3000
ε_c	-	0.003
ϕ	mm	16-30
ϕ_v	mm	8-14
D_{max}	mm	16
f_y	MPa	420
f_c'	MPa	25
E_s	MPa	200000
γ_s	t/m^3	7.86
γ_c	t/m^3	2.5
C_c	\$/m^3	40
C_s	\$/t	400

reinforcements bars. Since the flexural moments are same for the last two cases (11 and 12), additional reinforcements are needed as web reinforcements in order to carry both flexural moments. A typical drawing for the results of Case 2 and 1000 kN axial force is given in Fig. 3.

In Table 6, the optimum results are given for 1500 kN axial force and all biaxial moment cases in numerical example 2. The optimum cross sectional dimensions are generally larger than the results in Table 5 for 1500 kN because of the increase of axial force. The optimum costs for numerical example 2 are near to the costs of the numerical example 1 for several cases such as 2, 7 and 12. The increase of axial force has a little advantage on reduction of tensile stresses.

Table 5 Optimum design of columns for numerical example 1

Case no	*b* (mm)	*h* (mm)	Bars in each face (critical direction)	Web reinforcement in each face (the other direction)	Shear reinforcement diameter/distance (mm)	Total Cost ($)
1	350	350	1Φ18+1Φ16+1Φ20	-	Φ8/150	32.57
2	300	500	1Φ22+1Φ18	2Φ16	Φ8/220	40.85
3	300	600	1Φ18+1Φ16+1Φ22	1Φ16	Φ8/270	44.32
4	300	600	2Φ24+ 1Φ16+1Φ20	1Φ16	Φ8/270	55.20
5	400	600	1Φ16+ 2Φ18+1Φ22+1Φ20	1Φ18+1Φ16	Φ8/270	67.23
6	350	600	5Φ18+2Φ20+1Φ30	1Φ16	Φ8/270	81.11
7	500	600	3Φ20+1Φ16+1Φ18+3Φ22	1Φ16	Φ8/240	91.54
8	550	550	4Φ22+1Φ26+1Φ28+1Φ16+1Φ18	1Φ16+1Φ18	Φ8/210	107.81
9	500	600	5Φ22+1Φ24+2Φ18+1Φ20+3Φ16	1Φ16+1Φ18	Φ8/240	119.39
10	500	600	1Φ20+3Φ24+1Φ22 2Φ28+1Φ18+1Φ30	3Φ18+1Φ16	Φ8/240	137.53
11	550	600	1Φ20+1Φ30+1Φ18+1Φ26	1Φ22+1Φ18	Φ8/210	90.15
12	600	600	1Φ30+ 1Φ18+1Φ16+4Φ26+1Φ20	3Φ22+1Φ30	Φ8/200	150.34

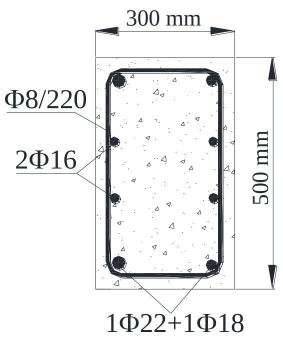

Fig. 3 Reinforcement orientations for Case 2 and 1000 kN axial force

Table 6 Optimum design of columns for numerical example 2

Case no	*b* (mm)	*h* (mm)	Bars in each face (critical direction)	Web reinforcement in each face (the other direction)	Shear reinforcement diameter/distance (mm)	Total Cost
1	450	300	3Φ18+1Φ16	-	Φ8/120	38.43
2	350	450	3Φ18+1Φ16	-	Φ8/190	40.66
3	400	500	3Φ18+ 1Φ16	1Φ16	Φ8/220	49.69
4	300	600	1Φ24+ 2Φ22	4Φ16	Φ8/270	62.58
5	500	550	1Φ22+ 1Φ24+1Φ30+1Φ20	1Φ18	Φ8/240	76.57
6	500	600	2Φ24+1Φ22+2Φ18	3Φ16	Φ8/240	85.16
7	550	600	2Φ16+3Φ22+1Φ20	2Φ16+1Φ18	Φ8/210	91.50
8	500	600	2Φ26+2Φ28+1Φ22+1Φ16+1Φ18	3Φ16	Φ8/240	110.03
9	550	600	10Φ18+6Φ16	2Φ16	Φ8/210	122.04
10	550	600	3Φ22+2Φ16+2Φ20 2Φ18+2Φ24+1Φ26+1Φ30	2Φ16	Φ8/210	141.99
11	600	600	1Φ20+ 1Φ22+1Φ26	1Φ24+1Φ28	Φ8/200	91.60
12	600	600	3Φ22+ 2Φ20+3Φ26+1Φ24+1Φ18	2Φ20+1Φ24+1Φ16	Φ8/200	148.53

Table 7 Optimum design of columns for numerical example 3

Case no	*b* (mm)	*h* (mm)	Bars in each face (critical direction)	Web reinforcement in each face (the other direction)	Shear reinforcement diameter/distance (mm)	Total Cost
1	400	400	2Φ20	1Φ18	Φ8/170	39.69
2	350	500	1Φ22+1Φ16	2Φ16	Φ8/220	43.07
3	400	500	3Φ18+ 1Φ16	1Φ16	Φ8/220	49.69
4	350	600	1Φ22+ 1Φ28+1Φ16	1Φ16	Φ8/270	54.84
5	450	600	3Φ18+ 1Φ22+1Φ16	2Φ18	Φ8/260	70.90
6	350	600	3Φ20+2Φ24+1Φ22+ 1Φ18	1Φ18+1Φ16	Φ8/270	83.51
7	500	600	1Φ28+3Φ20+1Φ26+ 1Φ16	2Φ18	Φ8/240	92.65
8	500	600	8Φ18+1Φ16+1Φ20	1Φ16	Φ8/240	103.48
9	450	600	1Φ30+4Φ20+1Φ22+ 2Φ26+1Φ24	2Φ16	Φ8/260	119.47
10	550	600	2Φ26+1Φ28+1Φ24+ 2Φ22+1Φ20+2Φ18	3Φ16+1Φ18	Φ8/210	129.82
11	550	600	1Φ28+ 1Φ18+1Φ30+1Φ26	1Φ20	Φ8/210	89.80
12	600	600	1Φ20+ 5Φ18+2Φ28+2Φ24	2Φ28+ 1Φ26+1Φ18	Φ8/200	162.86

The optimum results are presented for the third numerical example with 2000 kN axial force and all biaxial moment cases in Table 7. When the results are compared with the results of the numerical example 2, a significant reduction on the optimum total cost values are generally seen for case 3. In this numerical example axial compression force is effective to reduce tensile stresses and for that reason concrete sections which are useful for compressive stresses, are the biggest for numerical example 3. In case 12 of numerical example 3, a significant increase of the total cost is seen. In that case the limits of the cross section dimensions are found as the optimum ones. For that reason, reinforcement bars working on compressive are also needed.

4. Discussions and conclusions

A detailed reinforcement design is provided instead of an optimum reinforcement ratio. Thus, the optimum solutions are not theoretical. The solutions are practical optimums. Harmony search is a widespread algorithm for structural problems. It is easy to apply and adaptation of HS for the design problem is suitable. The problem uses discrete variables and the algorithms using the modification of all design variables is not possible since the number of bars is a random design variable in design. Also, all reinforcement bars as many as a random bar number are optimized with different sizes. Additionally, it is not possible to find a theoretical optimum solution for different numbers of bars or different bar sizes. Also, the cost optimization problem is non-linear since concrete and steel are different in price and behavior. In that situation, only cross-sections or reinforcements may have theoretical optimums.

The optimum analyses results are done for 3 different axial forces and different cases of biaxial bending moments are investigated. In the first 10 cases, a minor bending moment is used in one direction while the moment in the other direction is increased between 100 kNm and 1000 kNm.

Table 8 Optimum objective function for various runs

Run	1	2	3	4	5	6	7	8	9	10	11	12
$f(X)$ ($)	87.16	86.23	85.25	85.16	85.16	86.39	85.51	85.16	85.16	85.16	85.16	86.89
iteration	579	1105	37	505	359	757	1186	169	934	309	128	1661

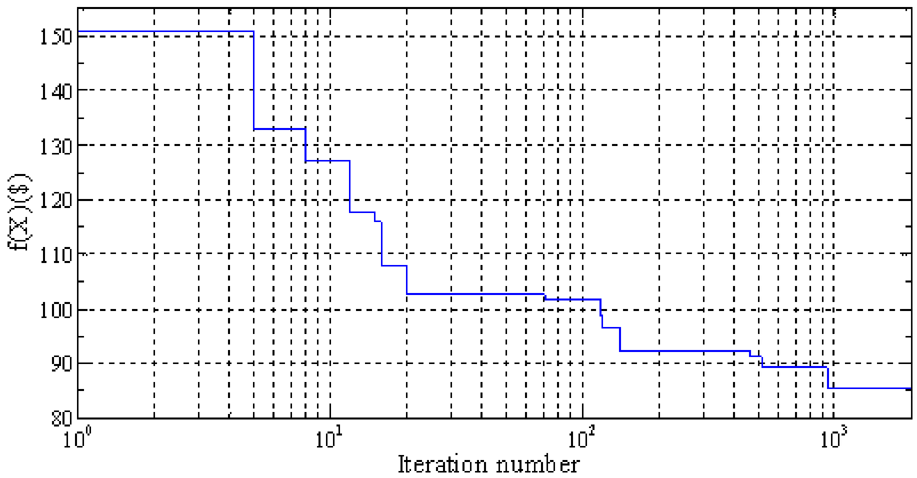

Fig. 4 Convergence to the optimum value

As seen from the results of these cases, the essential reinforcements are assigned in only critical direction and reinforcements with small diameters are placed for other direction in order to ensure the maximum clear distance between the reinforcement bars. Since the effect of biaxial bending is not clearly seen in these cases, additional two cases are also presented. In these cases, the flexural moment is equal for two directions and reinforcements are near to each other for both directions.

In several cases of numerical examples, the optimum cost is getting lower than other cases by the increase of the axial force. In that situation, compressive stresses are effective in more locations of the cross section and the concrete cross sectional size is enlarged. Due to reduction of tensile stresses, the areas of the reinforcements are getting lower than other cases. By increasing concrete volume and decreasing the steel tonnage, minimization of the cost is provided.

The optimum results of numerical examples were searched for 2000 iterations. By using the proposed methodology, 2000 possible designs of an engineer are checked for the minimum material cost. In Fig. 4, the convergence plot is given for the case 6 and 1500 kN axial force. As seen in the plot, the objective function is over 125 $ for the first 10 iterations. In technical practice of an engineer when designing a biaxially loaded column, it is not practical to conduct analyses more than 10. For that reason, the optimum value (85.16 $) is nearly 32% lower than engineering practices. The optimization of the same case is done for several times in order to test the sensitivity of the optimum results. The optimum objective function of various optimum trials are given in Table 8 and the iterations where the optimums are found are shown. According to the results, the standard derivative is 0.76 (normalized one is equal to 0.0088) for that case. The same optimum results are found for different run.

The duration of a generation is different because the proposed method uses additional random searches for different stages of optimization. The duration of a generation is between 2 and 6 second by using a Matlab code at a 3.4 GHz quad core personal computer with 32 GB RAM.

The optimum results show sensibility to internal forces by increasing the cross section dimension or reinforcement area. This situation proves that, the proposed methodology is effective to find the final optimum values. Due to economical difference of concrete and steel, cross-sectional dimensions (especially height for maximum flexural moment capacity) are getting bigger and approach to the limits of the optimization ranges. The method is feasible for RC members such as columns under biaxial bending in addition to compressive and shear stresses.

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