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Walking load model for single footfall trace in three dimensions based on gait experiment

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Abstract. This paper investigates the load model for single footfall trace of human walking. A large amount of single person walking load tests were conducted using the three-dimensional gait analysis system. Based on the experimental data, Fourier series functions were adopted to model single footfall trace in three directions, i.e. along walking direction, direction perpendicular to the walking path and vertical direction. Function parameters such as trace duration time, number of Fourier series orders, dynamic load factors (DLFs) and phase angles were determined from the experimental records. Stochastic models were then suggested by treating walking rates, duration time and DLFs as independent random variables, whose probability density functions were obtained from experimental data. Simulation procedures using the stochastic models are presented with examples. The simulated single footfall traces are similar to the experimental records.

Keywords: human walking load; footfall load model; three dimensional gait analysis technique; stochastic model

1. Introduction

In the past few decades, structures have become lighter and flexible due to increasing aesthetical and multifunction architectural requirements for designing long span structures. Examples include pedestrian bridges, long-span floors for structures like stadiums and exhibitions halls with a column-free expanse, and cantilevered floors such as found in ballrooms and entertainment venues. As a result, human-induced structural vibration problems become increasingly prominent for these structures. Excessive structural vibrations induced by human activities such as walking, jumping, bobbing and running can seriously threaten the serviceability of structures if safe implementation is not properly considered at the structural design stage. Walking is the most common activity of occupants in a building. The walking load is a kind of

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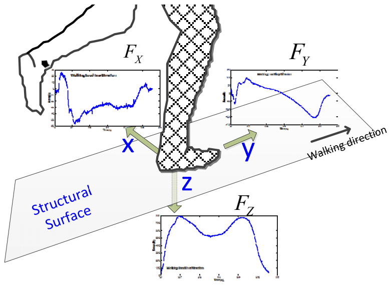


Fig. 1 Three components of walking load

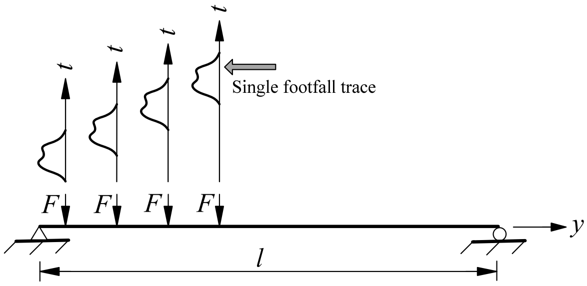


Fig. 2 Application method suggested by Ellingwood and Tallin (1984)

dynamic excitation that can cause structural vibration (Racic 2009). It can also cause dysfunction of vibration sensitive devices in high-tech factories, labs or hospitals (Ebrahimpour and Sack 2005, Pavic and Reynolds 2002). The footbridge 'T-bridge' in Japan and the London Millennium Footbridge in UK are two well-known examples demonstrating the significance of human-induced vibration serviceability problem (Fujino 1993, Dallard 2001).

At the design stage, vibration serviceability problems are usually addressed in two ways. One is to adjust the structure's fundamental frequency to be higher than a threshold value, and the other is to ensure the structure's vibration amplitude, due to human activities, to be lower than a limit. The second way is proved to be more accurate especially for massive structure like long-span concrete floor. When a person walks on the flat surface of a structure, each single footfall trace (SFT) can be divided into three components as shown in Fig. 1, which are 1) F_z that is perpendicular to the surface, 2) F_x that is in the walking plane and is perpendicular to the walking path, and 3) F_y that is in the walking plane and along the walking direction. Hereafter, F_x, F_y and F_z are termed as horizontal, longitudinal and vertical component respectively.

Because walking load is a moving load that varies in both time and locations, it is not an easy task to integrate the walking load into a finite element model (FEM) of a structure to calculate its dynamic responses. Besides, there are double-support phase during human walking which means both feet can be on the ground at the same time but at different locations. To overcome the problem, Ellingwood and Tallin (1984) proposed a simplified method that treated the walking load as a moving point-load. In this method, recorded or simulated SFT is applied to one node in the FEM of the structure at each walking step and then moved to successive nodes according to the walking speed, as demonstrated in Fig. 2. This method is easy for computer programing and can be directly applied to calculate group- or crowd-induced structural vibrations. Therefore, this method has already been adopted by several commercial structural analysis software. The accuracy of this

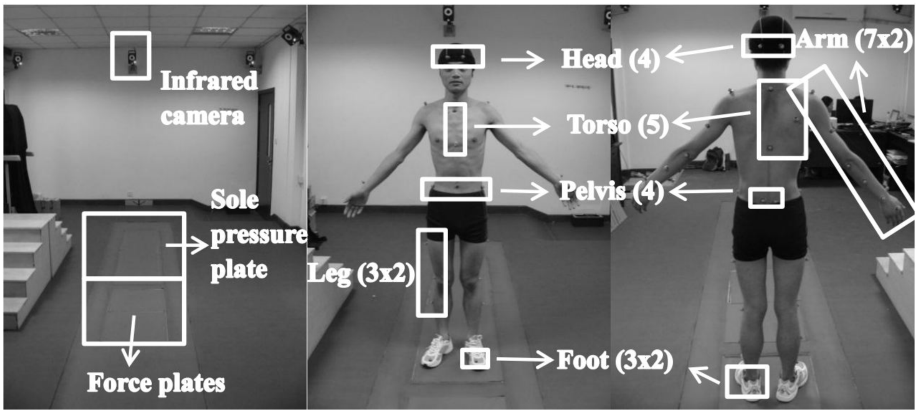


Fig. 3 Photo of the lab and a test subject with markers

simplified method, as shown by (Ellingwood and Tallin 1984, Chen *et al*. 2013), depends highly on the single footfall force model selected. In the past decades, many researchers have conducted experiments to investigate dynamic properties of human walking loads and to develop walking load models, such as Ohlsson (1982), Rainer *et al.* (1988), Kerr (1996), Kerr and Bishop (2001), Sahnaci and Kasperski (2005), Racic and Brownjohn (2011), Chen *et al.* (2013). Zivanovic *et al.* (2005), Racic *et al*. (2009, 2013) summarized and reviewed the development of experimental techniques and numerical models for walking loads. Most of these studies, however, focus on continuous walking load model, and also focus mainly on vertical component since it has the largest amplitude among all three components. Models for the three components in single footfall sense are very rare. Recently, Li *et al*. (2013) proposed a procedure to extract SFT from continuous walking load model by inverse analysis. A lots of unknown coefficients need to be determined which may introduce new uncertainties. Reliable models for SFT are still absent.

This study aims to develop mathematical models for SFT in three dimensions. Section 2 discusses experiments on single person walking load using motion capture technology and force plates. Deterministic models are then proposed in Section 3. Model parameters are determined by analyzing experimental data. Section 4 further suggests stochastic models for SFT. Simulation procedure using the proposed models and examples are presented in Section 5, followed by summary and conclusions of this study in Section 6.

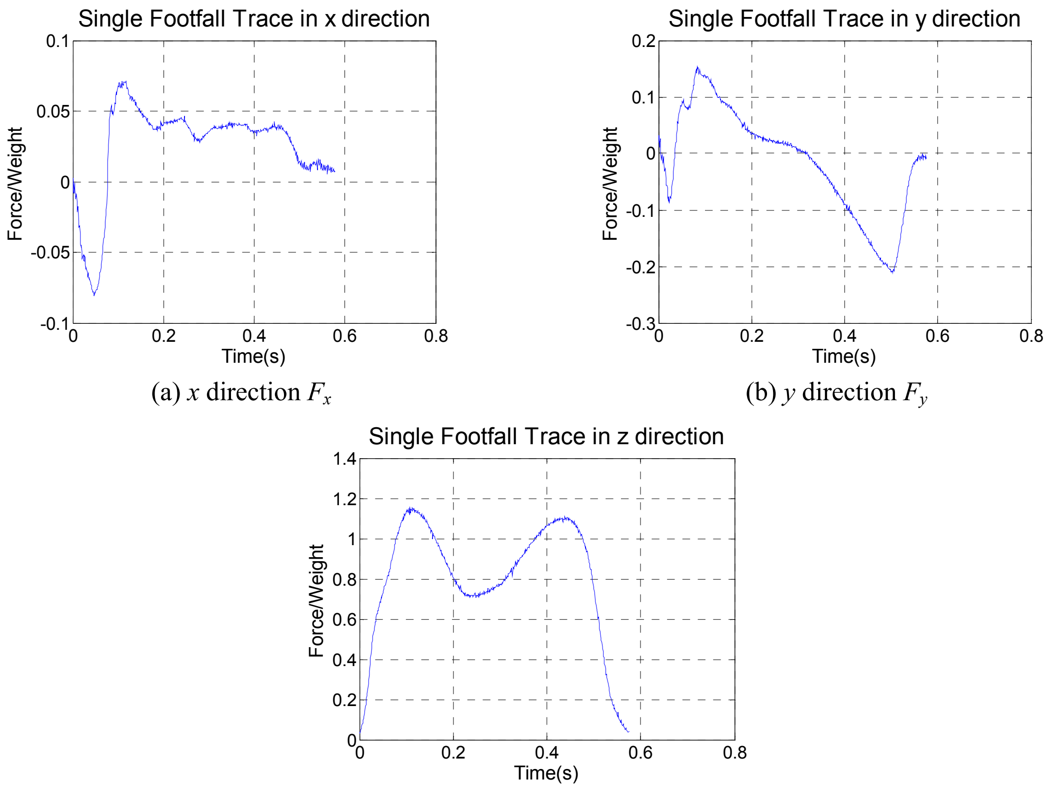
2. Experiments on walking load using 3D gait analysis technology

2.1 Test facilities

A series of experiments on human walking loads have been conducted with an experimental system combining two force plates and the three-dimensional (3D) gait analysis system. The 3D gait analysis system is capable of synchronously recording human motion and three-direction walking footfall traces. Since the duration of SFT is very short (typically less than one second) the usage of gait analysis system can significantly increase the measurement accuracy compared to traditional way using force plates only. The experiments were carried out in the Gait Lab at the Shanghai Institute of Traumatology and Orthopedics, Ruijin Hospital. The lab has a 9.0 m×5.0 m

Table 1 Statistics of age, body weight and height of all test subjects

Gender	Number	Age (y)		Weight (kg)		Height (mm)	
		Mean	Std	Mean	Std.	Mean	Std.
Male	59	23.4	2.34	65.1	4.13	1714.2	74.17
Female	14	22.8	1.15	51.2	8.77	1615.4	25.38



(a) x direction F_x (b) y direction F_y (c) z direction F_z

Fig. 4 Typical single footfall traces in three directions (male subject: height=1790 mm, body weight=706 N)

test area, where two force plates (AMTI OR6-7, USA) were flush-mounted on the ground, and a 2 m long sole pressure plate was installed on the ground adjacent to the force plates. The force plate is designed to measure footfall traces in three directions (Fig. 1). Moreover, ten infrared cameras were installed on the surrounding walls to capture the subject's motion by recording trajectories of reflective markers attached to test subjects' bony land-markers. Fig. 3 displays a picture of the test lab and a test subject with 39 markers on. In particular, three markers were placed on each foot in order to precisely monitor the movement of the foot, which enabled recording of high-precision gait parameters such as stride length, walking velocity, duration of each step and so on. The Vicon Motion Capture System was used in the test to measure the movement (reflective markers on subject) of the test subjects. Velocity and acceleration of each marker were calculated on-line by the motion capture analysis system. The motion capture system was integrated with analog data acquisition systems of the force plates to enable simultaneous measurements.

2.2 Test procedure

For this study, 73 subjects (59 male and 14 female) participated in the experiment. All subjects were volunteers and students of Tongji University. They were all healthy adults and were aware of the test purpose and test procedure. Statistics of the age, body weight and height of all the test subjects are listed in Table 1.

After sufficient warm-up and rehearsal, each test subject was required to perform seven test cases. In three cases, participants were allowed to walk at self-selected (without sound instruction) slow, normal and fast rates. In other four cases, participants were asked to walk at fixed 1.5, 1.75, 2.0 and 2.25 Hz that were guided by an electronic metronome. Every test case was repeated 6 or 7 times. Each test subject's walking procedure was visually monitored to make sure each foot fully stepped on the surface of one force plate while not having the other foot in contact with the same plate. Thus, each force plate successfully recorded the SFTs of one foot in three dimensions, i.e., F_x, F_y and F_z. The sampling frequency of the force plate was 1000 Hz. The Vicon System monitored spatial locations of all the markers with a sampling frequency of 200 Hz (200 frames per second). Typical SFTs measured by one force plate are shown in Fig. 4. A more comprehensive description of the experiment can be found in Chen *et al*. (2011).

3. Deterministic model of single footfall trace

3.1 Fundamental equations

Assume SFT $f(t)$ of each step is identical, it can then be expressed as Fourier series in Eq. (1).

$$f(t) = G \cdot \left(a_0 + \sum_{i=1}^{n} \alpha_n \sin\left(\frac{2\pi i}{T} t + \varphi_n \right) \right) \tag{1}$$

where

$$\alpha_n = \sqrt{a_n^2 + b_n^2} \tag{2}$$

$$\varphi_n = \arctan\left(\frac{a_n}{b_n} \right) \tag{3}$$

$$a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} f(t)\, dt \tag{4}$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos n\omega t\, dt \tag{5}$$

$$b_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \sin n\omega t\, dt \tag{6}$$

In Eq. (1), α_n is termed dynamic load factors (DLFS) that can be calculated through Eq. (2) by Fourier coefficients a_n and b_n that are defined in Eqs. (5)-(6). φ_n is the phase angle that defined in Eq. (3). The constant coefficient a_0 is determined by Eq. (4). In all above equations, T is the duration of the single footfall trace, n is the model's order and G is the human body weight. Eqs.

(1)-(6) form the foundation equations for single footfall traces in three dimensions.

3.2 Duration time T of the single footfall trace

The statistical results of the duration time T of single footfall traces are listed in Table 2. Note that the duration time decreased as the walking speed or step frequency increased. A quadratic function, Eq. (7), is found by least square regression to present the relationship between the walking rate f_p and the duration time T of SFT (Fig. 5). For simulation applications, the duration of the simulated SFT can be calculated by Eq. (7) once the walking frequency f_p is given.

$$T\left(f_p\right) = 0.1315 f_p^2 - 0.8716 f_p + 1.8244 \tag{7}$$

3.3 Model order n

An appropriate truncated order n is necessary in the single footfall load model, Eq. (1), to reveal the real footfall load characteristics in both time and frequency domain. To determine the number of order, we compared the measured single footfall traces and their reconstructed counterpart by Eq. (1) with different numbers of order ranging from 1 to 6. As an example, Fig. 6

Table 2 Statistical results of the duration time

Test cases		Duration time(unit: second)		
		Mean	Standard variation	Range
Free walk	Slow	0.676	0.085	0.549~0.932
	Normal	0.580	0.091	0.394~0.715
	Fast	0.514	0.073	0.371~0.885
Sound guided walk	1.5Hz	0.788	0.095	0.657~0.875
	1.75Hz	0.681	0.081	0.589~0.716
	2.0Hz	0.598	0.043	0.514~0.862
	2.25Hz	0.559	0.073	0.470~0.751
All conditions		0.631	0.171	0.371~0.932

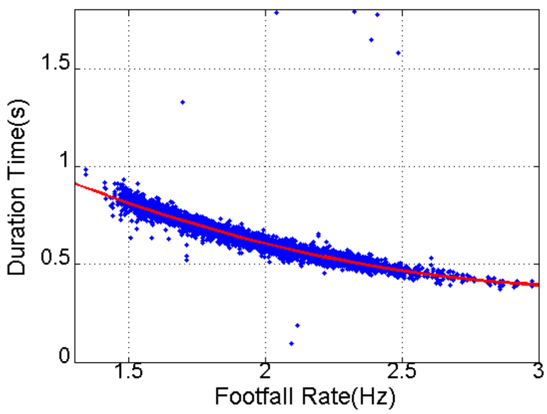
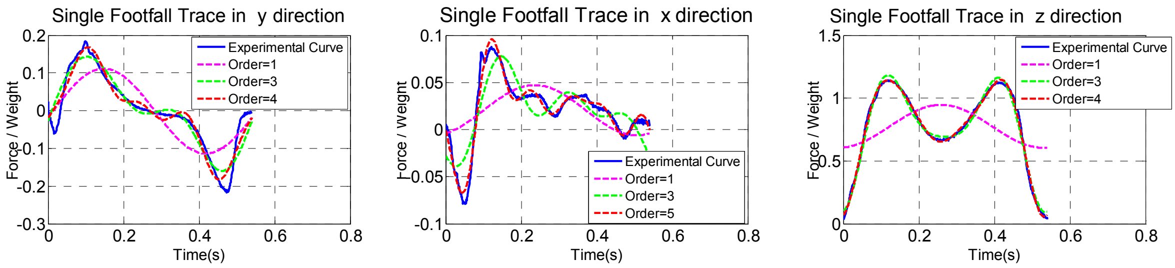
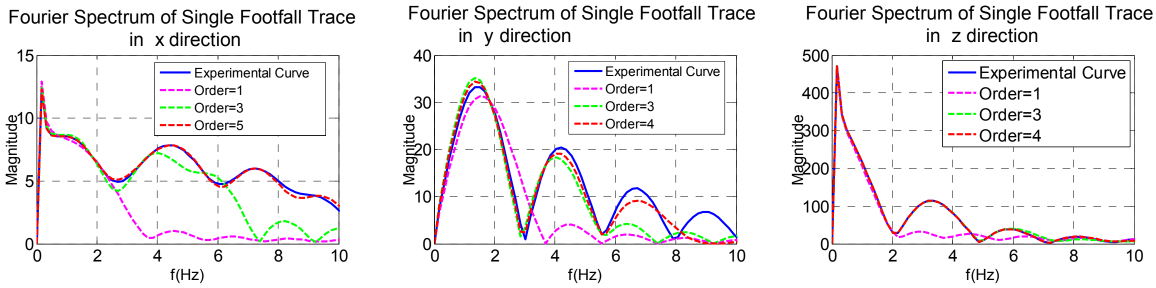


Fig. 5 Relationship between duration time and walking rate



(a) Comparison of time histories



(b) Comparison of Fourier amplitude spectrum

Fig. 6 Comparison of reconstructed single footfall traces with experimental records

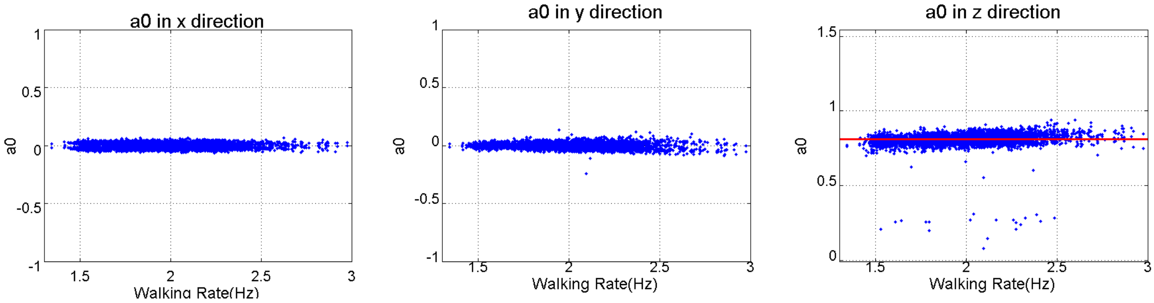


Fig. 7 Variation of constant term a_0 with walking rates in three dimensions

compares the reconstructed traces with the measured traces. It is seen from Fig. 6(a) that theconstructed traces where n=5, 4 and 4, in x, y and z direction agree well with the measured ones. Fig. 6(b) shows the comparison results in frequency domain. Similarly, in x direction, the single footfall load model where n=5 is able to represent the characteristics in frequency domain while n=4 could reach good agreement in both y and z directions. The observations of all the other cases are the same. Therefore, n=5, 4 and 4 in x, y and z direction are recommended to simulate a single footfall trace by Eq. (1).

3.4 The value of a_0

Based on the experimental data, the relationship between the constant term a_0 and the walking rate f_p was analysed and depicted in Fig. 7. Note that mean values of a_0 are almost zero in x and y direction, suggesting that we can neglect a_0 in the two directions. In direction z, a_0 is close to a

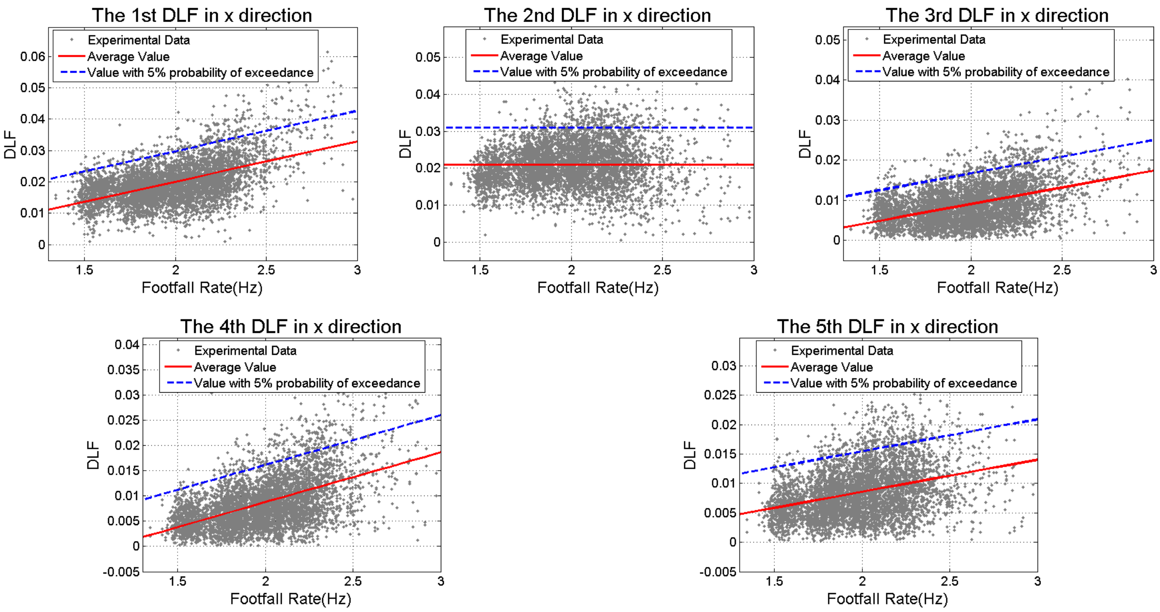


Fig. 8 Variation of the first five DLFs in x direction with footfall rate

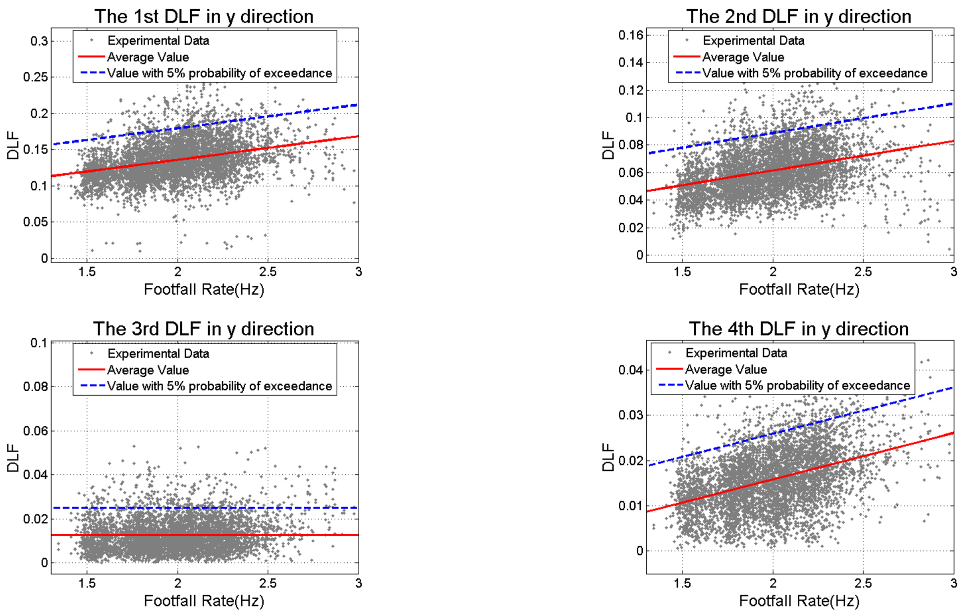


Fig. 9 Variation of the first four DLFs in y direction with footfall rate

constant value 0.81. The result is consistent with the physical mechanism of human walking that body weight is parallel to direction z and vertical to the horizontal directions x and y. For continuous walking load trace, the constant term a_0 is 1.0 because the mean value of the trace is equal to the person's body weight. However, it is a common mistake in some research to take a_0 as

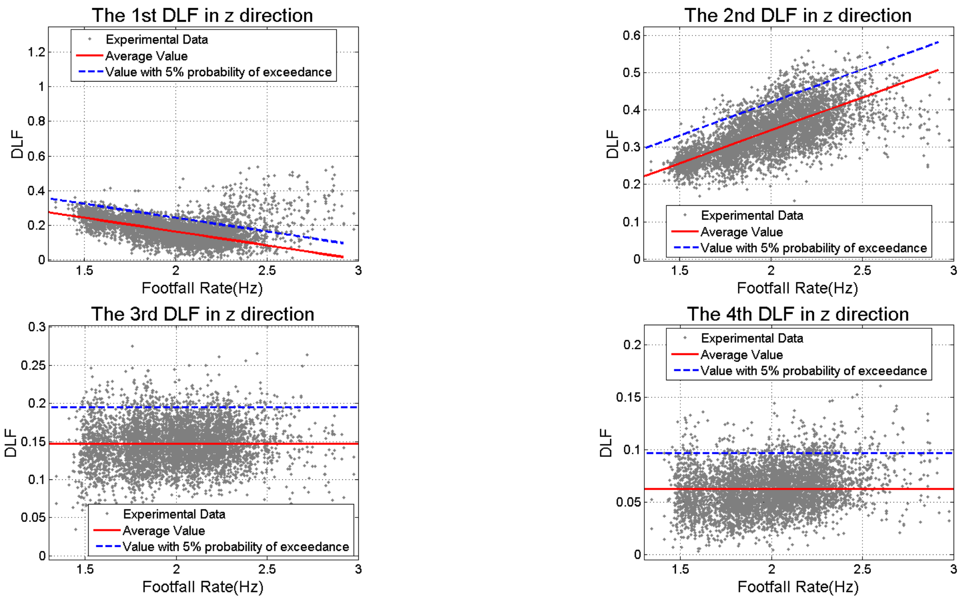


Fig. 10 Variation of the first four DLFs in z direction with footfall rate

Table 3 Mean values for DLFs in single footfall load model

Order	x direction	y direction	z direction
1	$0.0129f_p-0.0057$	$0.0324f_p+0.0712$	$-0.1598f_p+0.4828$
2	0.021	$0.0214f_p+0.0186$	$0.1771f_p-0.0100$
3	$0.0083f_p-0.0076$	0.013	0.147
4	$0.0099f_p-0.0110$	$0.0103f_p-0.0047$	0.062
5	$0.0055f_p-0.0023$	/	/

1.0 for SFT. The smaller constant a_0 here is because in SFT the double support phase in the human walking has not been directly accounted in SFT. The duration of a double support phase is about 20% of the gait duration, thus a value of a_0=0.81 is reasonable for SFT.

3.5 Dynamic load factors α_n

Figs. 8-10 depict the relationship between dynamic load factors α_n and footfall rate in the x, y and z direction, respectively. Least-square linear fits to the data were conducted, and the results are plotted in these figures. DLF values with 5% exceeding probability are also provided for design reference in the form of mean values plus 1.645 times standard deviation values.

It is seen from Figs. 8-10 that all the DLFs in x, y and z directions can be broadly treated as constant values or a linear relationship with the walking rate. All the fitted functions and constant values are summarized in Table 3.

3.6 Phase angle

The experimental results of phase angles are very scattered, and no clear relationship between

Table 4 Phase angles in single footfall load model

Order	x direction	y direction	z direction
1	$\pi/2$	$-\pi/20$	$-\pi/2$
2	$2\pi/5$	$-\pi/20$	$-\pi/2$
3	$\pi/3$	$-\pi/3$	$-\pi/2$
4	$-\pi/5$	$-\pi/10$	$-\pi/3$
5	$-\pi/3$	/	/

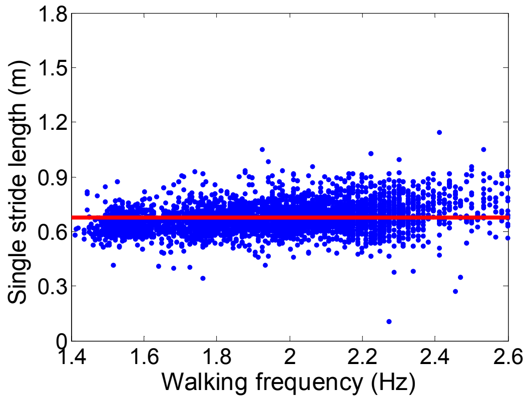


Fig. 11 Variation of the single stride length with footfall rate

the phase angle and walking rate can be observed. Therefore, the mean value of phases angles of each order are provided in Table 4 for the three directions.

3.7 Single stride length

Another important parameter of people walking is the single stride length, denoted by ΔL. When integrating the walking loading model into a structure's finite element model, the stride length is dominant in defining the mesh size. The experimental data shows that the stride length doesn't change much with the walking frequency, as demonstrated in Fig. 11. We also attempted to connect the stride length with the test subject's height but no clear relationship was observed in our experiments. Therefore, we regarded the single stride length as a constant value at 0.67 m.

3.8 Deterministic model for SFT and its applicatin procedure

Based on all the above discussions, the deterministic model for SFT can be established as Eqs. (8), (9) and (10) for x, y and z direction, respectively.

$$f_x(t) = G \cdot \sum_{i=1}^{5} \alpha_{xi} \sin\left(\frac{2\pi i}{T} t + \varphi_{xi}\right) \tag{8}$$

$$f_y(t) = G \cdot \sum_{i=1}^{4} \alpha_{yi} \sin\left(\frac{2\pi i}{T} t + \varphi_{yi}\right) \tag{9}$$

$$f_z(t) = G \cdot \left(a_0 + 4\sum_{i=1}^{5} \alpha_{zi} \sin\left(\frac{2\pi i}{T} t + \varphi_{zi} \right) \right) \tag{10}$$

The application procedure of the above model to simulate SFT can be divided into the following four steps.

Step 1. Determine the walking frequency f_p and design value of body weight G according to certain design guidelines' recommendations.

Step 2. Calculate the duration of a single footfall trace T by Eq. (7).

Step 3. Generate a single footfall trace in x, y or z direction by Eqs. (8), (9) or (10). For each order i, the corresponding DLF and phase angle can be determined by Table 3 and Table 4.

Step 4. Integrate the single footfall trace into a finite element model, or other structural analysis model, through a step-by-step manner using a constant stride length of 0.67 m.

4. Stochastic model for single footfall trace

The above deterministic model gives identical single footfall trace at each step. Experimental records reveal that the walking load is actually a narrow-banded random process. The randomness of walking process comes from two main sources: intra-subject variability and inter-subject variability (Racic *et al*. 2009). The former means a person can't generate identical footfall trace at each step, the later means even at the same walking rates two different persons will generate different footfall traces. A simplified stochastic model can be developed by treating the parameters in Eqs. (8)-(10) as random variables, whose probability distribution property can be determined from the experimental results.

4.1 Statistical characteristics of model parameters

4.1.1 Walking rate

To obtain the probability distribution of the walking rate, data from free-walking test cases have been analysed statistically, and the results are shown in Fig. 12. The walking rate generally follows a normal distribution with a mean value of 2.10 Hz and standard deviation of 0.269 Hz.

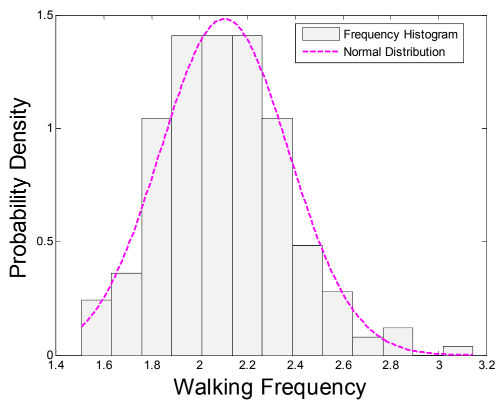


Fig. 12 Statistics of walking rate from the free walking cases

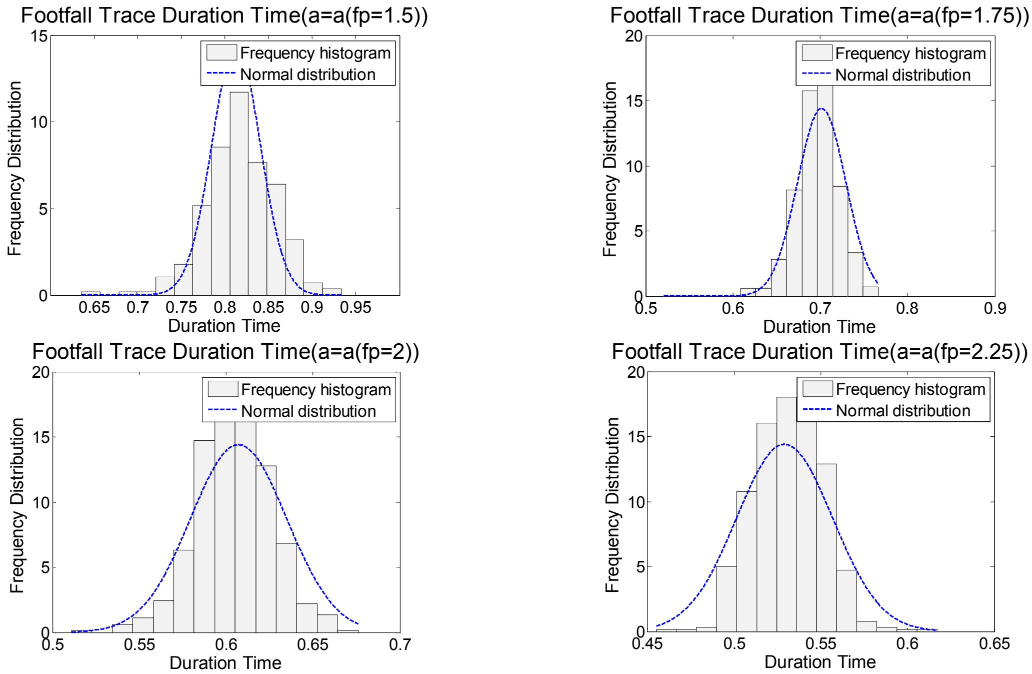


Fig. 13 Statistics of single footfall trace duration time

Table 5 Standard deviation of DLFs in single footfall load model

Order	*x* direction	*y* direction	*z* direction
1	0.0059	0.0266	0.0720
2	0.0061	0.0166	0.0494
3	0.0047	0.0076	0.0293
4	0.0045	0.0061	0.0208
5	0.0042	/	/

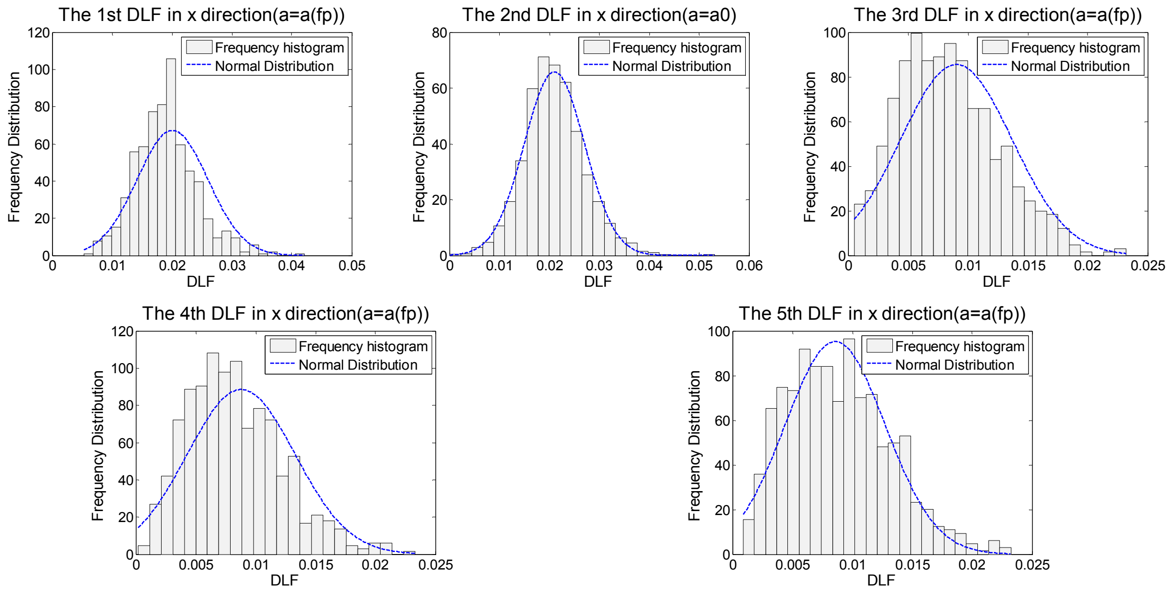
Matsumoto *et al*. (1972) conducted statistical research on walking rate with a sample size of 505 persons. They believed walking rate follows normal distribution with a mean of 2.0 Hz and standard deviation of 0.173 Hz. Zivonavic *et al*. (2007) reported a normal distribution with mean value at 1.87 Hz and standard deviation at 0.186 Hz. Moreover, the mean walking frequencies measured by Kerr and Bishop (2001), Pachi and Ji (2005) are 1.9 Hz and 1.8 Hz. All these measurements have confirmed that the walking rate follows normal distribution whose mean value and standard deviation might change for different population or different walking scenario.

4.1.2 Duration time

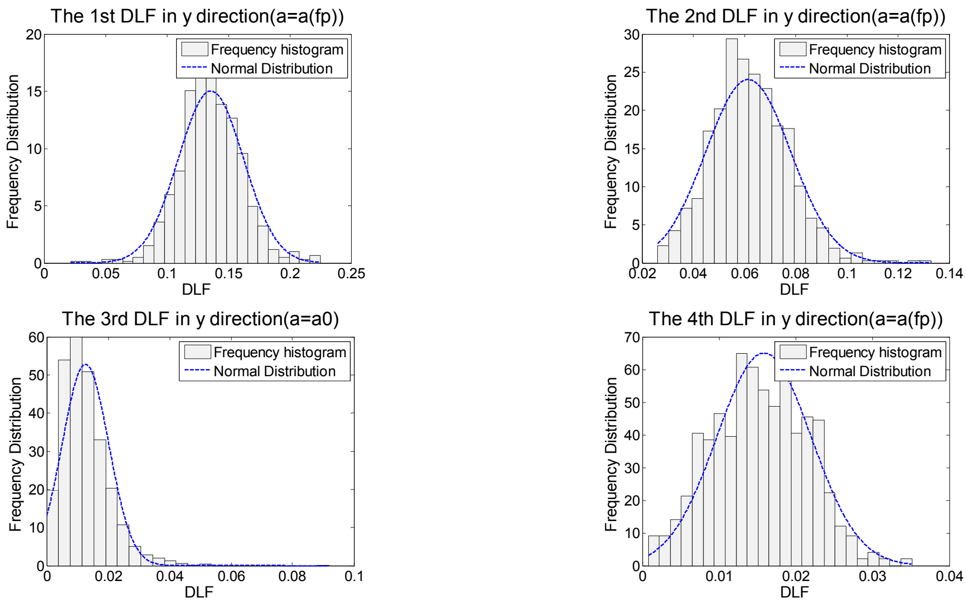
Statistical analysis result of duration time in four frequency ranges, which are $f_p \in [1.45, 1.55]$Hz, $f_p \in [1.70, 1.80]$Hz, $f_p \in [1.95, 2.05]$Hz and $f_p \in [2.20, 2.30]$Hz corresponding to four test cases, are shown in Fig. 13. We found that the the duration time of a single footfall trace is approximately normally distributed. Mean value of the distribution can be estimated by Eq. (7) and the standard deviation can be taken as constant value of σ=0.0277 second.

4.1.3 Dynamic load factors

Fig. 14 shows the frequency distribution of DLF data within $f_p \in [1.95, 2.05]$Hz in three directions with fitted normal distribution curves. DLS were found, based on the experimental data, to follow normal distribution, whose expected mean value follows Table 3 and standard deviation follows Table 5.

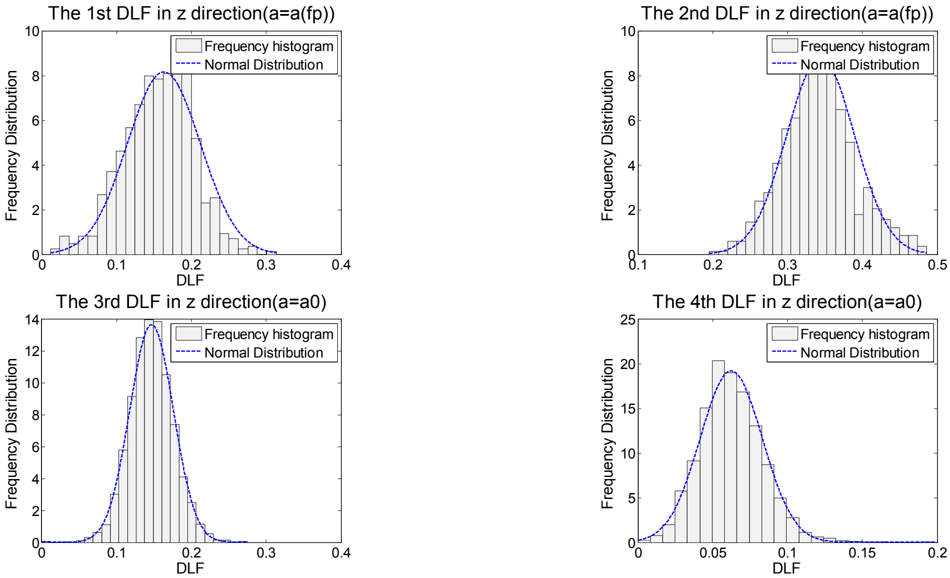


(a) Statistics of DLFs in direction x



(b) Statistics of DLFs in direction y

Fig. 14 Statistics of DLFs in three directions ($f_p \in [1.95, 2.05]$Hz)



(c) Statics of DLFs in direction *y*

Fig. 14 Continued

4.1.4 Other parameters

From current experimental data, the Fourier coefficient a_0 and single stride length ΔL can be treated as constant values. The phase angle of each order can be taken as constant following value in Table 4. The phase angle can also be treated as a random variable uniformly distributed in a 2pi range centered at the value given in Table 4.

4.2 Application procedure of the stochastic load model

A single footfall trace can be generated with the above stochastic model by the following steps.

(1) Determine the walking occupant's body weight G and walking rate f_p. The walking rate should be determined according to the requirement of structural analysis. If the walking occupant is assumed to walk in a natural way, the walking rate could be generated randomly, following normal distribution with mean value at 2.10 Hz and standard deviation at 0.269 Hz.

(2) Determine the probability density functions of DLFs after walking rate is known. All DLFs, i.e., α_{xi} (i=1,2,3,4,5), α_{yi} (i=1,2,3,4) and α_{zi} (i=1,2,3,4), follow normal distribution with mean values and standard deviation given in Table 3 and 5. Duration time T is calculated by Eq. (7). Phase angle φ is determined following Table 4.

(3) Generate three direction single footfall load time history by Eqs. (8)-(10).

5. Simulation examples

The above procedure is adopted to simulate single footfall trace. A typical example is given in Fig. 15. Solid lines in Fig. 15 represent measured and simulated single footfall load in three directions. The dashed lines are 10 simulated single footfall traces. The comparison shows that the

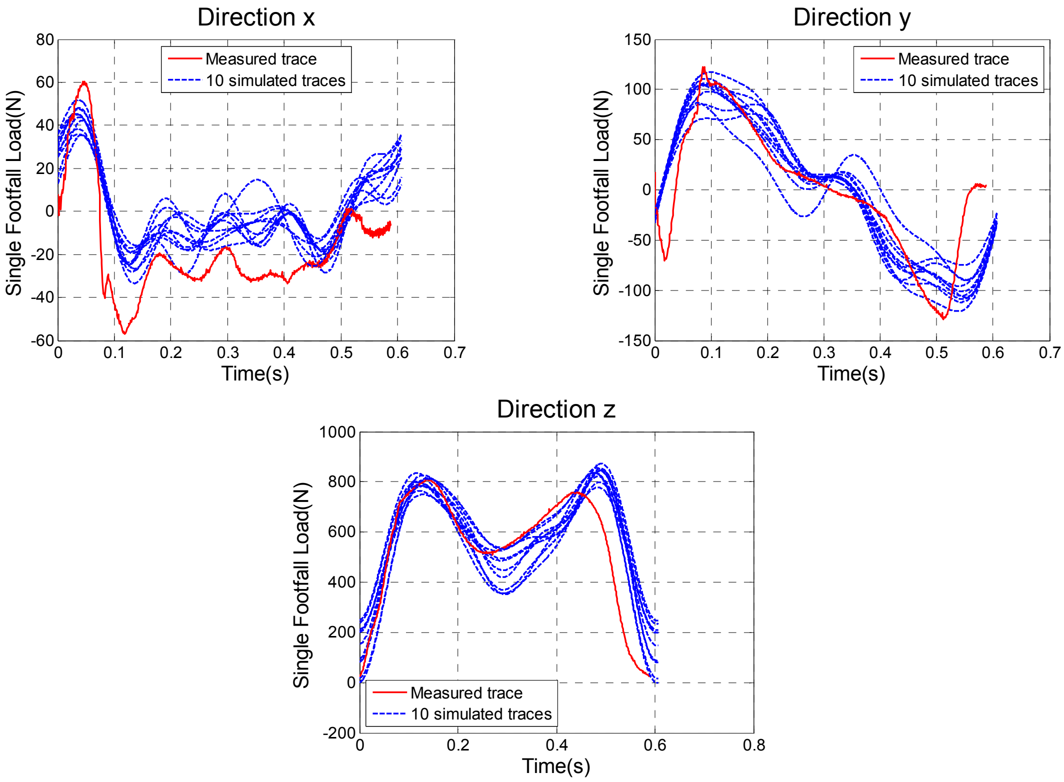


Fig. 15 Stochastic simulated single footfall load time histories

stochastic simulated traces provide good agreement in both peak value and variation trend with the measured time histories, which shows that the stochastic model and simulation method is reasonable and feasible.

6. Conclusions

This paper suggests a deterministic and a stochastic model for single footfall trace in three dimensions based on experimental measurements. The suggested model represents single footfall traces by Fourier series. Model parameters such as walking rate, dynamic load factors, phase angles and stride length are determined by analyzing a large amount of single person walking load experimental data. The truncated order of Fourier series for vertical, horizontal and longitudinal component are found as 4, 5 and 4 respectively. In deterministic model, model parameters are taken as constants whilst in stochastic model they are treated as random variables. Experimental measurements suggest that the walking rate, dynamic load factors and duration time generally follow normal distribution. Application procedure is provided to generate single footfall trace by the suggested single footfall trace model. Comparison with experimental data proves that the simulated traces can capture the main characteristics of a real single footfall trace. The simulated single footfall trace can be integrated into structure's finite element model to calculate its dynamic responses due to human walking.

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