Structural Engineering and Mechanics, *Vol. 54, No. 2 (2015) 221-237*
DOI: http://dx.doi.org/10.12989/sem.2015.54.2.221

Nonlinear damage detection using higher statistical moments of structural responses

Ling Yu*[1,2,3] and Jun-Hua Zhu[2,3]

[1]College of Civil Engineering and Architecture, China Three Gorges University, Yichang 443002, China
[2]MOE Key Lab of Disaster Forecast and Control in Engineering, Jinan University, Guangzhou 510632, China
[3]Science and Technology on Reliability Physics and Application Technology of Electronic Component Laboratory, Guangzhou 510610, China

(Received October 23, 2014, Revised February 14, 2015, Accepted March 8, 2015)

Abstract. An integrated method is proposed for structural nonlinear damage detection based on time series analysis and the higher statistical moments of structural responses in this study. It combines the time series analysis, the higher statistical moments of AR model residual errors and the fuzzy c-means (FCM) clustering techniques. A few comprehensive damage indexes are developed in the arithmetic and geometric mean of the higher statistical moments, and are classified by using the FCM clustering method to achieve nonlinear damage detection. A series of the measured response data, downloaded from the web site of the Los Alamos National Laboratory (LANL) USA, from a three-storey building structure considering the environmental variety as well as different nonlinear damage cases, are analyzed and used to assess the performance of the new nonlinear damage detection method. The effectiveness and robustness of the new proposed method are finally analyzed and concluded.

Keywords: structural damage detection; nonlinear damage detection; time series analysis; higher statistical moments

1. Introduction

Structural damage detection (SDD) plays the most pivotal role in the process of structural health monitoring (SHM) (Gul and Catbas 2011). Currently, the vibration-based structural damage detection (VBSDD) technique has been recognized and intensively studied as a promising tool for monitoring structural conditions and detecting structural damages (Doebling *et al.* 1998, Sohn *et al.* 2004, Carden and Fanning 2004, Yan *et al.* 2007, Zhou *et al.* 2013, Yu *et al.* 2013, Li and Chen 2013, Lei *et al.* 2015). Identifying the presence of the damage might be considered as the first step to take preventive actions and to start the process towards understanding the root causes of the problem (Gul and Catbas 2009). Most of the VBSDD methods can be classified into two groups: model based and feature based (Zhang 2007). For the latter, especially for those based on the time series statistical analysis in constructing time-series signature for direct damage diagnosis have gained considerable attention recently since their implementation for an automated SHM system is

*Corresponding author, Professor, E-mail: lyu1997@163.com

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

relatively more feasible compared to the model based methods (Lu and Gao 2005, Lee and Yun 2013, Yao and Pakzad 2014, Dorvash *et al*. 2014).

Most of the time series analysis based methodologies aim to fit time series models to the vibration data and then try to detect the damage by extracting damage features from these time series models. As one of key steps in the SHM, the ideal approach for features extraction is to choose features that are sensitive to damage, but are not sensitive to operational and environmental variations. However, such an approach is not always possible in real-world structures, and intelligent feature extraction procedures are usually required (Farrar and Worden 2007). Some of them directly compare the time series models whereas others use the residual errors when the new data is used with the previously created model. An AR (Auto-Regressive) model is fitted to the measured responses data from a healthy structure by Fugate *et al*. (2001), the corresponding residual errors are defined as the damage-sensitive features, and the X-bar and S control charts are employed to monitor the mean and variance of the selected features for SDD. Based on AR modeling of the signals from a laboratory tower, Zugasti *et al*. (2012) presented two SDD methods and reported that two methods were able to correctly detect damage in the structure that was simulated by loosening some of the bolts in the joints. Yao and Pakzad (2012) proposed two time series-based SDD algorithms by using statistical pattern recognition. They defined the Ljung-Box statistic of AR model residual sequence and the Cosh spectral distance of the estimated AR model spectrum as the damage indexes respectively. Compared with existing AR model based algorithms, the subsequent simulation and lab experiments showed that the Ljung-Box statistic is a more sensitive feature while the Cosh spectral distance tends to be more stable than Mahalanobis distance of coefficients. Further, Yao and Pakzad (2014) derived the sensitivity expressions of two damage features, namely the Mahalanobis distance of AR coefficients and the Cosh distance of AR spectra. The effectiveness of the proposed methods was illustrated in a numerical case study on a 10-DOF system. Sohn and Farrar (2001) proposed a two-step AR-ARX (auto-regressive and auto-regressive with eXogenous) model and subsequently used the standard deviation (STD) ratio of the residual error to indicate the damage. Lu and Gao (2005) also used the STD of the residual error as a damage feature. Gul and Catbas (2011) extracted different damage features from ARX models from the different clusters. Although the proposed methodology showed great success for the examples, they also acknowledged that the methodology should be verified with more experiments and be improved for damage detection with ambient vibration data as well.

In another study, Nair *et al*. (2006) employed an ARMA (Auto-Regressive Moving Average) model and the first three AR components as the damage sensitive feature. Omenzetter and Brownjohn (2006) used Auto-Regressive Integrated Moving Average (ARIMA) models to analyze the static strain data from a bridge during construction and operation, however, the authors reported that the damage location and their severity could not be identified although the method can detect structural changes. Carden and Brownjohn (2008) presented a statistical classification algorithm based on analysis of the time-series responses with ARMA models where the ARMA coefficients are fed to the classifier. The method is verified with experimental data respectively from the IASC-ASCE benchmark four-storey frame structure, the Z24 bridge and the Malaysia-Singapore Second Link bridge. The classifier is found to be capable of forming distinct classes corresponding to different structural states in most cases. However, the method may not be the most suitable SHM paradigm for structures with ambient excitation only. The sensitivity of ARMA models of static response data to typical infrastructural damage is unproven yet.

However, all the above linear time series analysis based methods cannot deal with nonlinear damages effectively, such as the fatigue cracks that open and close upon dynamic loading (Chen and Yu 2013). Apart from the mean and standard deviation of the time histories, Mattson and Pandit (2006) adopted the higher-order moments of the residuals obtained from vector AR models to detect damage. They reported that the residual-based method is capable of identifying non-linear damage signatures that are too deeply buried in the system dynamics to be identified directly from the raw data, but they also found that only use of skewness and kurtosis as features for damage diagnosis is less reliable than the variance. Figueiredo *et al*. (2009) reported that skewness and kurtosis showed some differences in the damaged states when compared to the undamaged states. They can be used as features to detect damage that results in a linear system subsequently exhibiting nonlinear dynamic response.

In this study, an integrated method is proposed for nonlinear SDD based on time series analysis, the higher statistical moments of structural responses and the fuzzy c-means (FCM) clustering techniques. Six comprehensive damage indexes are developed in the arithmetic and geometric mean of the higher statistical moments. They are then classified by the FCM clustering method for achieving nonlinear damage detection. The background of theory of the integrated method is first presented in the section two. In order to assess the performance of the integrated nonlinear SDD method proposed here, the experimental data downloaded from the web site of the Los Alamos National Laboratory (LANL) USA on a three-storey building structure are adopted to conduct experimental verification in the following section. Some reasonable conclusions are made finally.

2. Background of theory

The theoretical background of the integrated method is developed for structural nonlinear damage detection based on time-series analysis in this section. Based on linear system theory, *AR* time series models are adopted to describe the acceleration time histories and used in the analysis of stationary time series processes, which obey probabilistic laws, in which the mean, variance and higher order moments are time invariant.

2.1 Data standardization

Supposing $x_i \in R^{n\times 1}$ (i=1,2...p), denotes amplitudes of measured acceleration response data with p data sample at all n measurement points. In order to eliminate the effects caused by environmental and operational variations from the measured acceleration responses, the data standardization is necessary as follows

$$\hat{x}_i = \frac{x_i - \bar{x}}{\sigma} \tag{1}$$

$$\bar{x} = \frac{1}{p}\sum_{i=1}^{p} x_i \tag{2}$$

$$\sigma^2 = \frac{1}{p-1}\sum_{i=1}^{p}(x_i - \bar{x})^2 \tag{3}$$

Where, $\bar{x}$, σ^2 and $\hat{x}_i$ are the mean, variance and standardized version of time series signal x_i respectively.

2.2 Traditional damage-sensitive index

AR models attempt to account for the correlations of the current observation in time series with its predecessors. A univariate *AR* model of order p at j-th measured acceleration signal, or *AR* (p), for the time series can be written as

$$A_j(q)x_j(k) = e_j(k) \tag{4}$$

$$A_j(q) = 1 + a_{1j}q^{-1} + a_{2j}q^{-2} + \cdots + a_{pj}q^{-p} \tag{5}$$

where $x_j(k)$(j=1,2,...m, k=1,2,...,n) are the current and previous values of the time series and $e_j(k)$ is *AR* model residual error. The *AR* coefficients a_{1j}, a_{2j},..., a_{pj} can be evaluated using a variety of methods. Here, the coefficients were calculated using the Yule-Walker equations (Box and Jenkins 1976). For the structural reference (health) state, the corresponding *AR* model can be made, the model parameter $A_j^{ref}(q)$ and residual error $e_j^{ref}(k)$ can be obtained. Similarly, for any unknown structural test sample $y_j(k)$, its residual error is

$$e_j^{test}(k) = A_j^{test}(q)y_j(k) \tag{6}$$

If the residual error is assumed as a Gaussian normal distribution with a zero mean, the traditional damage-sensitive index (DI) is defined as the standard deviation (STD) ratio of the unknown test state to the reference one as follows (Sohn and Farrar 2001)

$$\gamma^{std}(e_j) = \sigma(e_j^{test}) / \sigma(e_j^{ref}) \tag{7}$$

When the test samples come from the structural health state, *AR* model can effectively predict the sample, therefore the variance of the residual error is close to one of the reference sample, the STD ratio in Eq. (7) is approximately equal to one. When the test samples come from the structural damage state, the residual error will be increased, the STD ratio will larger than one, therefore, the STD ratio can be used to determine if the structures is damaged or not.

2.3 Order of AR model

The order of the *AR* model is an unknown value. A high-order model may perfectly match the data, but will not generalize to other data sets. On the other hand, a low-order model will not necessarily capture the underlying physical system response. In order to find out the optimum model order, several techniques are used in this study, such as Akaike's information criterion (*AIC*), partial autocorrelation function (*PAF*), final prediction error (*FPE*) etc. Finally, the *AIC* is chosen in this study, which is used to assess the general performance of linear models. In a simple way, this technique returns a value that is the sum of two terms as follows

$$AIC = -2L_m + 2m \tag{8}$$

Where L_m is the maximized log-likelihood of the residual error, and m is the number of adjustable parameters in the model. It assumes a tradeoff between the fit of the model and the model complexity. The first term is related to how well the model fits the data, i.e., if the model is too simple, the residual errors increase. On the other hand, the second term is a penalty factor related to the complexity of the model, which increases as the number of additional parameters grows (Box and Jenkins 1976).

2.4 Nonlinear damage-sensitive index

It should be noted that AR model is a kind of linear model and many classical statistical tests depend on the assumption of normality. This approach is based on the assumption that damage will introduce either linear deviation from the baseline condition or nonlinear effects in the signal and, therefore, the linear model developed with the baseline data will no longer accurately predict the response of the damaged system.

In order to establish the underlying distribution of the data, some higher statistical moments are used to estimate the probability density function (PDF) of the measured signals without normal distribution. Moreover, it is expected that the damage can introduce significant changes in the acceleration-time-history PDFs, as a consequence, the third and fourth statistical moments and PDFs are introduced as damage-sensitive features for effectively nonlinear damage detection in this study.

The third statistical moment is a measure of the asymmetry of the PDF. The normalized third statistical moment is called as the skewness defined as follows

$$skewness(e_j) = E[e_j - m(e_j)]^3 / \sigma(e_j)^3 \tag{9}$$

Where, a positive skewness represents that the right tail is longer and that the area of the distribution is concentrated below the mean. On the other hand, a negative skewness means that the left tail is longer and that the area of the distribution is concentrated above the mean. The skewness of a standard normal distribution is zero.

The fourth statistical moment is a measure of the relative amount of data located in the tails of a probability distribution. The normalized fourth statistical moment is named as the kurtosis defined as follows

$$kurtosis(e_j) = E[e_j - m(e_j)]^4 / \sigma(e_j)^4 \tag{10}$$

Where, a kurtosis greater than three indicates a “peaked” distribution that has longer tails than a standard normal distribution. This means that there are more cases far from the mean. Kurtosis less than three indicates a “flat” distribution with shorter tails than a standard normal distribution. This property implies that fewer realizations of the random variable occur in the tails would be expected in a normal distribution. The kurtosis of a standard normal distribution is three.

Similar to Eq. (7), two damage-sensitive indexes are defined as the Skewness ratio and Kurtosis ratio of structural unknown test state to its reference state respectively as follows,

$$\gamma^{skew}(e_j) = \left| skewness(e_j^{test}) / skewness(e_j^{ref}) \right| \tag{11}$$

$$\gamma^{kur}(e_j) = kurtosis(e_j^{test}) / kurtosis(e_j^{ref}) \tag{12}$$

When the structure is in a healthy state, the skewness of the *AR* model residual error is close to zero, its kurtosis approaches to three. When the structure is damaged, the skewness will be positive or negative, the kurtosis will increase. When both the test and reference data samples come from same states, the corresponding skewness and kurtosis ratios will be identical and equal to one under same states respectively, or else they will be more or less than one, which can be used to detect the nonlinear damage of structures. However, the skewness and kurtosis estimates are extremely sensitive to values that fall in the tails of the distribution because they are magnified by a power of three or four, respectively. For this reason, they should be used very carefully.

2.5 Integrated damage-sensitive indexes

The STD ratio in Eq. (7) is a traditional damage-sensitive index (DI) suitable for linear model, while DIs defined in Eqs. (11)-(12) can partially represent the nonlinear damage of structures because the residual-based method is capable of identifying non-linear damage signatures that are too deeply buried in the system dynamics to be identified directly from the raw data, but only use of skewness and kurtosis as features for damage diagnosis is less reliable than the variance (Mattson and Pandit 2006). In order to integrate their functions of linear-nonlinear characteristics as a tradeoff at the same time, six DIs are defines in terms of arithmetic and geometric mean as follows

$$DI_1 = (\gamma^{std} + \gamma^{skew})/2 \tag{13}$$

$$DI_2 = (\gamma^{std} + \gamma^{kur})/2 \tag{14}$$

$$DI_3 = (\gamma^{std} + \gamma^{skew} + \gamma^{kur})/3 \tag{15}$$

$$DI_4 = \sqrt{\gamma^{std} \times \gamma^{skew}} \tag{16}$$

$$DI_5 = \sqrt{\gamma^{std} \times \gamma^{kur}} \tag{17}$$

$$DI_6 = \sqrt[3]{\gamma^{std} \times \gamma^{skew} \times \gamma^{kur}} \tag{18}$$

Here, the arithmetic mean is used to report central tendencies, but it is not a robust statistic because it is greatly influenced by outliers. Therefore, the geometric mean is used for comparing different items – finding a single “figure of merit” for these items – when each item has multiple properties that have different numeric ranges.

2.6 Structural damage detection (SDD)

In the previous section, damage indexes have been defined, but it is difficult to choose a threshold values that characterize damage. In order to perform the damage detection, fuzzy *c*-means clustering (*FCM*) algorithm, which was first presented by Bezdek (1981), and recently applied to SHM problems by da Silva *et al*. (2008), is employed to classify the features and to supply a fuzzy decision by using the membership of damage index in a cluster (Zhu *et al*. 2012). This algorithm is an unsupervised classification algorithm which uses a certain objective function, described in Eq. (19), for iteratively determining the local minima.

$$\min J(C,m) = \sum_{i=1}^{C}\sum_{j=1}^{N} u_{ij}^m d_{ij}^2 \tag{19}$$

$$center_i = \sum_{j=1}^{N} u_{ij}^m x_j \Big/ \sum_{j=1}^{N} u_{ij}^m \tag{20}$$

$$d_{ij}^2 = (x_j - center_i)^T (x_j - center_i) \tag{21}$$

$$u_{ij} = (d_{ij})^{\frac{-2}{m-1}} / \sum_{k=1}^{C} (d_{kj})^{\frac{-2}{m-1}} \tag{22}$$

where C is the total number of clusters and N is the total number of objects in calibration. u_{ij} is the membership function associated with the j-th object of the i-th cluster, which is updated by using Eq. (22) in each iteration step. The exponent m is a measurement of fuzzy partition. $center_i$ is the centroid of the i-th cluster, x_j is j-th object of data set to be clustered, which is set to be any of DIs here, d_{ij} denotes the distance between j-th object and the centroid of the i-th cluster, here, Euclidean distance is described as Eq. (21) (Matlab 2010).

3. Experimental verification

In order to assess the performance of the integrated SDD method proposed in this study, some experimental data from the three-storey building structure are adopted here, which is downloaded from the web site of the Los Alamos National Laboratory (LANL), USA (Figueiredo *et al*. 2009). The three-storey building structure as shown in Fig. 1 is used as a damage detection test bed, in which some detailed layout of the mass added at the base and nonlinearity source are shown in Fig. 2.

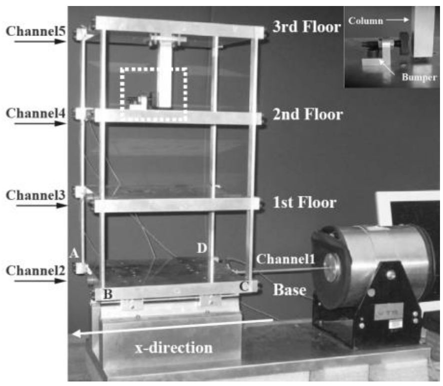
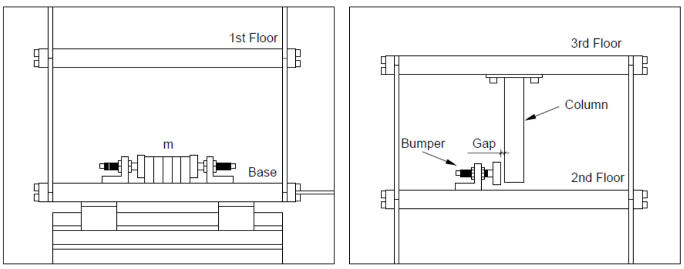


Fig. 1 Three-storey building model



(a) Mass, m, added at the base (b) Nonlinearity source

Fig. 2 Added mass and nonlinearity source

Table 1 Data labels of structural state conditions

Group	State	Case	State condition	Description
1	State #1	1-10	Undamaged	Baseline condition
	State #2	11-20	Undamaged	Mass = 1.2 kg at the base
	State #3	21-30	Undamaged	Mass = 1.2 kg on the 1st floor
	State #4	31-40	Undamaged	87.5% stiffness reduction in column 1BD
2	State #5	41-50	Undamaged	87.5% stiffness reduction in column 1AD and 1BD
	State #6	51-60	Undamaged	87.5% stiffness reduction in column 2BD
	State #7	61-70	Undamaged	87.5% stiffness reduction in column 2AD and 2BD
	State #8	71-80	Undamaged	87.5% stiffness reduction in column 3BD
	State #9	81-90	Undamaged	87.5% stiffness reduction in column 3AD and 3BD
	State #10	91-100	Damaged	Gap = 0.20 mm
	State #11	101-110	Damaged	Gap = 0.15 mm
3	State #12	111-120	Damaged	Gap = 0.13 mm
	State #13	121-130	Damaged	Gap = 0.10 mm
	State #14	131-140	Damaged	Gap = 0.05 mm
	State #15	141-150	Damaged	Gap = 0.20 mm and mass = 1.2 kg at the base
4	State #16	151-160	Damaged	Gap = 0.20 mm and mass = 1.2 kg on the 1st floor
	State #17	161-170	Damaged	Gap = 0.10 mm and mass = 1.2 kg on the 1st floor

3.1 Structural damage scenarios

The nonlinear damage was introduced through nonlinearities resulting from impacts with a bumper. When the structure is excited at the base, the suspended column hits the bumper. The level of nonlinearity depends on the amplitude of oscillation and the gap between the column and the bumper. The operational and environmental variety was simulated by adding mass and reducing stiffness at several different locations. Force and acceleration time series samples recorded for a variety of different structural state conditions were collected as shown in Table 1 together with information that describes the different states. Each state includes 10 observed cases, each case records 8192 consecutive data samples. For example, State#13-6-Test indicates the sixth observed data with case no of 126 in Table 1 for State #13 under unknown test condition. Therefore, there are 170 cases for 17 states in total, as listed in Table 1.

From Table 1, it can be found that the structural state conditions can be categorized into four main groups. The first group (State #1) is the baseline condition. The second group includes the states (States #2-#9) when the mass or stiffness of the structure are changed. Real-world structures have operational and environmental variability, which create difficulties in detecting and identifying structural damage. Such variability often manifests itself in linear changes in the stiffness or mass of a structure. In order to simulate such operational and environmental condition changes, tests are performed with different mass and stiffness conditions (States #2-#9). For example, the state condition labeled “State #4” described in Table 1 means that there is a 87.5% stiffness reduction in the columns located between the base and 1st floor at the intersection of plane B and D as illustrated in Fig. 2(b) by (Figueiredo *et al*. 2009, Chen and Yu 2013) (abbreviated as 1BD in Table 1, other abbreviations can be identified in the similar way). The stiffness reduction consists of replacing the corresponded column by another one with half the cross section thickness in the direction of shaking. The third group includes damaged state conditions (States #10-#14) simulated through the introduction of nonlinearities into the structure using a bumper and a suspended column, with different gaps between them. Finally, the fourth group includes the state conditions (States #15-#17) with nonlinear damage in addition to mass and stiffness changes used to simulate operational and environmental condition changes.

3.2 Effects of environmental conditions and structural damage

The dynamic characteristics of structures are easily affected by either structural damage or the operational environment conditions. How to determine whether it is due to the former or the latter, it is not easy. Sometimes the change in dynamic characteristics due to the latter is more significant than one by the former. Using the measured excitation force and acceleration responses, the frequency response function (FRF) can be obtained under the different conditions, as shown in Fig. 3. It can be seen from Fig. 3(a) that the structural frequencies have been shifted due to adding mass (State #3) or stiffness reduction (State #9) as compared with the FRF curve of baseline health condition of structure (State #1) at Channel 5 although the structure is all in undamaged conditions. If the structural damage conditions (States #14 & 17) are compared with the baseline health one, Fig. 3(b) shows that the second frequency of structure will be increased under the nonlinear damage of structure (State #14). Further, the third frequency of structure will be

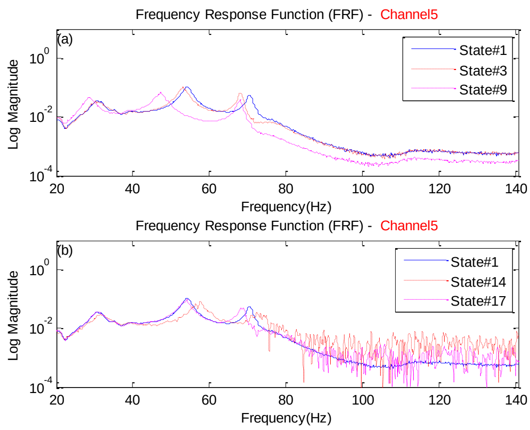


Fig. 3 Comparison on FRF curves

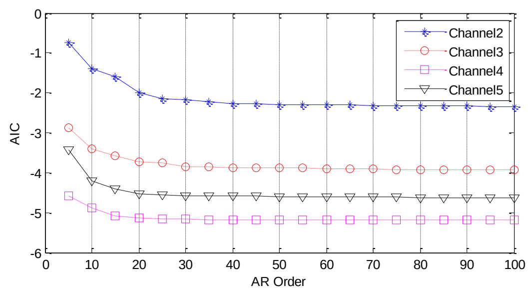


Fig. 4 Determination of AR model order

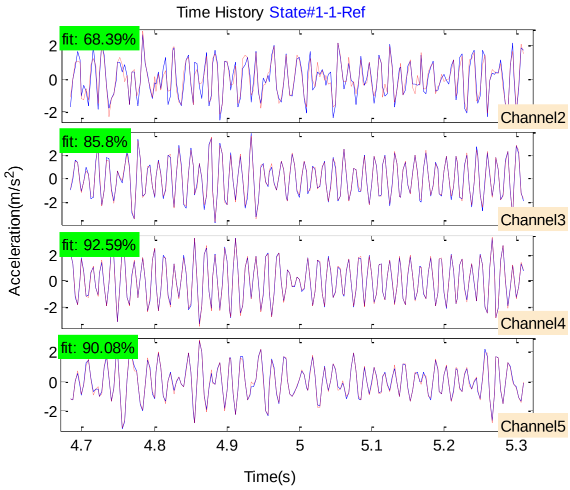


Fig. 5 Meansured and fitted data

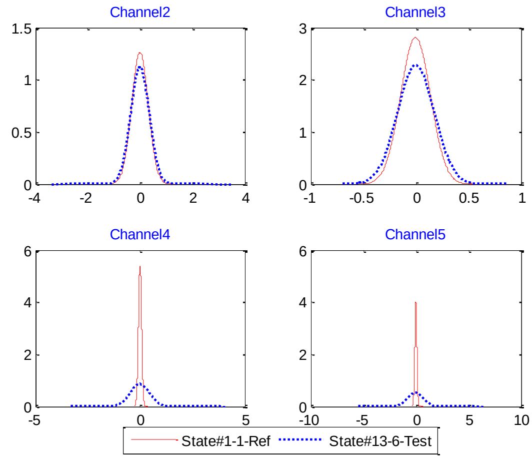


Fig. 6 PDFs of AR residuals

decreased if both the environmental condition and nonlinear damage are considered simultaneously (State #17). Therefore, it is very difficult to estimate the structural damage if the frequencies of structures are considered only.

3.3 Traditional damage-sensitive indexes

Fig. 4 shows the effects of *AR* model orders on the AIC of measured accelerations in baseline state (State #1). It can be seen that the changes in AIC curves are very small when the *AR* order is equal to or higher than 50, therefore, *AR* (50) is determined for prediction of the test samples in the following section. Fig. 5 compares the time histories of the measured acceleration responses with the fitted ones by using the AR (50) model. It can be found that the fitted ratio are reached to 68.3%, 85.8%, 92.59% and 90.08% for the data measured at Channels 2, 3, 4 and 5 respectively.

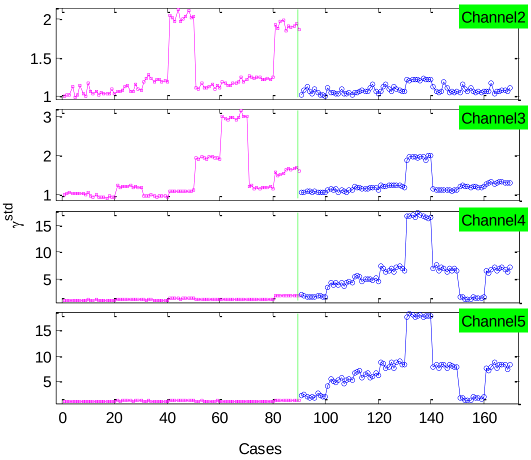


Fig. 7 STD ratios for all 170 cases

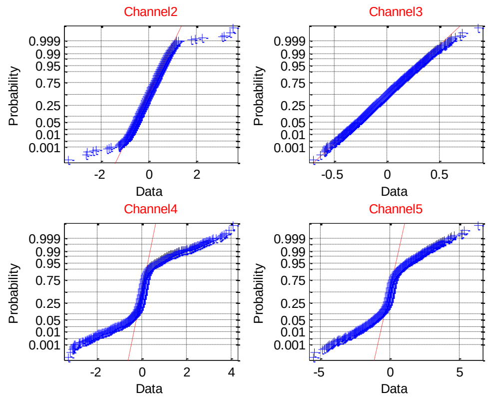


Fig. 8 Normal probability testing

Moreover, the residual errors are calculated, the estimated probability density functions (PDFs) of residual errors are compared in Fig. 6. They are corresponding to State #1 in red solid line and State #13 in blue dotted line respectively. As listed in Table 1, the State #1 indicates one undamaged state of structure, but the State #13 represents a damaged one, in which the gap between the column and the bumper is set to be 0.1 mm. It can be seen that the estimated PDFs of residual errors has changed obviously after the damage occurs in the State #13 condition, particular for that in the Channels 4 and 5 near to the gap. Therefore, the changes in the PDFs of residual errors can be used to identify the structural damage. The standard deviation (STD) ratios of the unknown test state to the reference one, as defined in Eq. (7), are shown in Fig. 7 for all 170 cases as listed in Table 1. The damaged and undamaged states can be easily identified.

3.4 Extraction of integrated damage-sensitive indexes

Normal probability testing of *AR* residual errors in State#13-6 is shown in Fig. 8. It can be

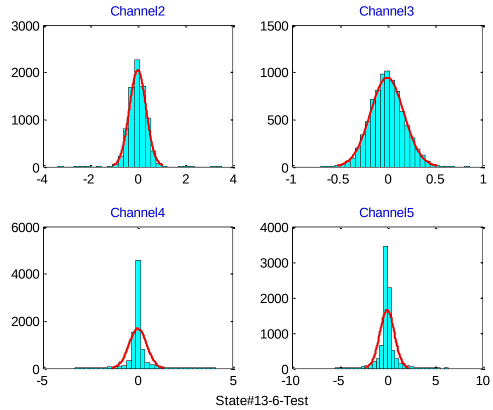


Fig. 9 Histogram with normal fitting

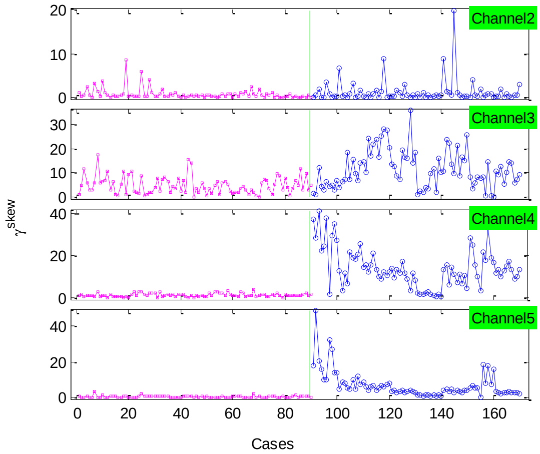


Fig. 10 Skewness ratios for cases

found that the unknown test State #13-6 is a damaged state because the *AR* residual errors are deviate from the normal distribution, particular for ones in Channels 4 and 5. It will not completely reflect the statistical distribution if the standard deviation (STD) of *AR* residual errors are used only. The histogram of *AR* residual errors in State #13-6 is compared with the normal fitting PSD curves in solid red line in Fig. 9, which indicates that the distribution of residual errors is obviously different from the normal fitting PSD curves, particularly in Channels 4 and 5. By the definition of kurtosis, the kurtosis of *AR* residual errors will greatly higher than ones of the normal fitting PSD.

For all 170 cases as listed in Table 1, the skewness and kurtosis ratios, as defined in Eqs. (11) and (12), are calculated and shown in Figs. 10 and 11, respectively. If they are compared with ones in Fig. 7, it can be found that after the structure is damaged, the states with lower STD ratios at Channels 4 and 5 in Fig. 7, i.e., states #10 and #16, correspond to ones with higher skewness and kurtosis ratios at Channels 4 and 5 in Figs. 10 and 11. This shows that the skewness and kurtosis ratios are the complementary to the STD ratio.

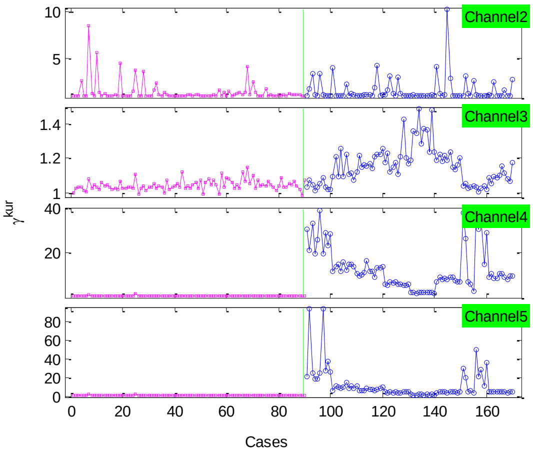


Fig. 11 Kurtosis ratios for cases

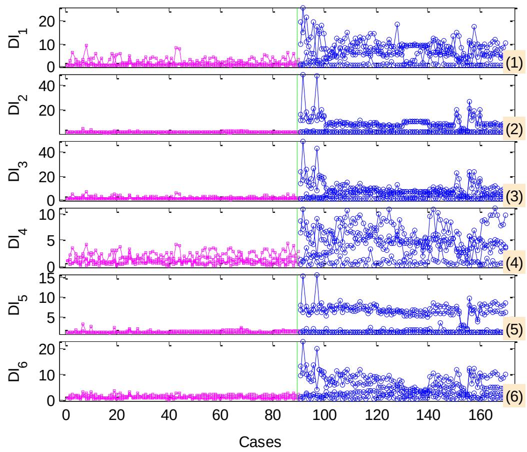


Fig. 12 Integrated damaged-sensitive indexes

Further, six integrated damage-sensitive indexes are calculated and shown in Fig. 12. In comparison to the STD ratios in Fig. 7, it can be seen that the distribution of damaged indexes are more reasonable, the damaged and undamaged states of the structure can be easily identified.

3.5 Structural damage detection

In order to perform the SDD, the fuzzy c-means clustering (*FCM*) algorithm is employed to clarify the damage-sensitive features and used to supply a fuzzy decision by using the membership of damage index in a cluster as defined in Eq. (19). Here, the computation parameters C=2 and m=2 respectively. The analytical results of membership for the traditional STD ratio is shown in Fig. 13. It can be found that there are no damage in both states #10 and #16. In fact, the both SDD results are not correct because both states #10 and #16 are in damaged states. In comparison to

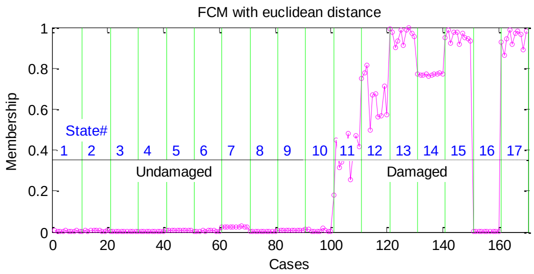


Fig. 13 Membership due to STD ratio

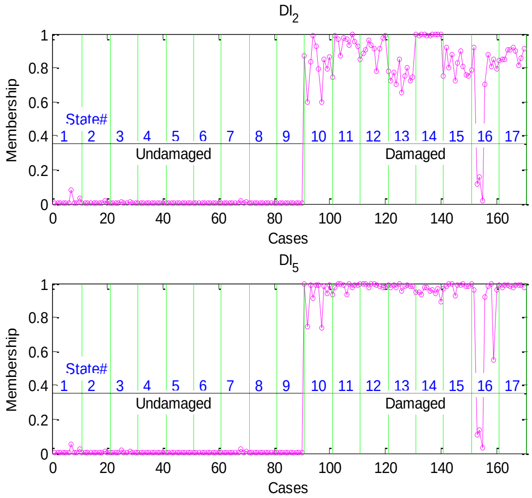


Fig. 14 Membership due to DI_2 and DI_5

other states, there are the largest gap between the column and the bumper, i.e., 0.2 mm, in both states #10 and #16. There are fewer opportunity to hit each other when the structure is excited, so the nonlinear damage severity is lower as well. However, the SDD result is correct in state #15 although the gap and the added mass are the same as ones in states #16. Only one difference between them is the different locations of added mass. It is at the base in state #15 but on the first floor in state #16. This affects the nonlinear interaction between the column and the bumper when the structure is excited at the base. It is also shown that the traditional STD ratio is easily affected by the environmental variability.

All the membership results from six integrated DIs are listed in Table 2, in which, abbreviated capital character 'FP', i.e., false positive, represents that the healthy state of structure is deemed as the damage state. While 'FN', i.e., false negative, means that the damage state of structure is deemed as the healthy state. Moreover, sign 'xx/yy' indicates that there are xx misdiagnoses out of yy data sample cases. The criterion of diagnosis decision is accepted as the follows: the structure is deemed as a healthy state if the membership value is lower than 0.5, otherwise, it is as a damage state.

It can be seen from Table 2 that all membership results from six DIs, i.e., from DI_1 through DI_6, are better than ones due to the traditional STD ratio (γ^{std}). The best result is from the DI_3 because there are only two misdiagnosis out of 169 data sample cases. DI_3 is the arithmetic mean value of

Table 2 Membership results for integrated damage-sensitive indexes

State	γ^{std}	DI_1	DI_2	DI_3	DI_4	DI_5	DI_6
FP	0/89	0/89	0/89	0/89	0/89	0/89	0/89
FN	**31/80**	**3/80**	**3/80**	**2/80**	**5/80**	**3/80**	**13/80**
1	0/9	0/9	0/9	0/9	0/9	0/9	0/9
2	0/10	0/10	0/10	0/10	0/10	0/10	0/10
3	0/10	0/10	0/10	0/10	0/10	0/10	0/10
4	0/10	0/10	0/10	0/10	0/10	0/10	0/10
5	0/10	0/10	0/10	0/10	0/10	0/10	0/10
6	0/10	0/10	0/10	0/10	0/10	0/10	0/10
7	0/10	0/10	0/10	0/10	0/10	0/10	0/10
8	0/10	0/10	0/10	0/10	0/10	0/10	0/10
9	0/10	0/10	0/10	0/10	0/10	0/10	0/10
10	**10/10**	0/10	0/10	0/10	0/10	0/10	0/10
11	**10/10**	**1/10**	0/10	0/10	0/10	0/10	0/10
12	**1/10**	0/10	0/10	0/10	0/10	0/10	0/10
13	0/10	0/10	0/10	0/10	0/10	0/10	0/10
14	0/10	0/10	0/10	0/10	**2/10**	0/10	**10/10**
15	0/10	0/10	0/10	0/10	0/10	0/10	0/10
16	**10/10**	**2/10**	**3/10**	**2/10**	**3/10**	**3/10**	**3/10**
17	0/10	0/10	0/10	0/10	0/10	0/10	0/10

STD, skewness and kurtosis ratios of AR residual errors. Therefore, the skewness and kurtosis indexes can provide a benefic complement to the traditional STD ratios. This also indicates that the complementary among STD, skewness and kurtosis ratios has been verified.

Moreover, it can be also found that the results from DI_2 and DI_5, both due to STD and kurtosis ratios in the arithmetic or geometric way, respectively, are better than ones due to both DI_1 and DI_4. The membership results due to DI_2 and DI_5 are shown in Fig. 14. Further, the DI_5 results are a little bit better than the DI_2 results because the distribution of DI_5 membership results are closer to the damage membership value of 100% as a whole when the structure is under all the nonlinear damaged states #10-#17, which indicates that all the nonlinear damaged states can be effectively identified.

4. Conclusions

In this study, an integrated method is proposed for structural nonlinear damage detection based on time series analysis, the higher statistical moments of structural responses and the fuzzy *c*-means (FCM) clustering techniques. Six comprehensive damage-sensitive indexes (DIs) are developed in the arithmetic and geometric manner of the higher statistical moments, and are classified by using the FCM clustering method to achieve nonlinear damage detection. Some experimental data downloaded from the web site of the Los Alamos National Laboratory (LANL) USA on a three-storey building structure are adopted to assess the effectiveness and robustness of the new nonlinear structural damage detection (SDD) method proposed in this study. The illustrated results show that: (1) The proposed integrated method is an effective tool for structural nonlinear damage detection, the damaged and undamaged states of the structure can be easily identified based on the newly proposed method. (2) The traditional standard deviation (STD) ratio of the residual errors is easily affected by the environmental variability. The skewness and kurtosis indexes can provide a benefic complement to the traditional STD ratio. (3) All membership results from six integrated DIs, i.e., from DI_1 through DI_6, are better than ones due to the traditional STD ratio. The distribution of six integrated DIs are more reasonable. The best result is from the DI_3, i.e. the arithmetic mean value of STD, skewness and kurtosis ratios of AR residual errors. DI_2 and DI_5, both due to combination of STD and kurtosis ratios in the arithmetic and the geometric way, respectively, are better than ones due to both DI_1 and DI_4. (4) Although the proposed integrated methodology showed great success for the examples under investigation, the authors also acknowledged that the methodology should be verified with more laboratory experiments using different types of structures in the field of structural engineering.

Acknowledgements

The project is jointly supported by the National Natural Science Foundation of China (50978123, 51278226 and11032005) and the Project of Science and Technology on Reliability Physics and Application Technology of Electronic Component Laboratory (9140C030605140C03015). The authors would like to acknowledge the Los Alamos National Laboratory (LANL) USA for freely downloading the experimental data of the three-storey building structure used in this study.

References

Bezdek, J.C. (1981), *Pattern recognition with fuzzy objective function algorithm*, Plenum Press, NY.

Box, G. and Jenkins, G. (1976), *Time series analysis: forecasting and control*, Prentice Hall, Englewood Cliffs, New Jersey.

Carden, E.P. and Brownjohn, J.M. (2008), "ARMA modelled time-series classification for structural health monitoring of civil infrastructure", *Mech. Syst. Sig. Proc*., **22**(2), 295-314.

Carden, E.P. and Fanning, F. (2004), "Vibration based condition monitoring: a review", *Struct. Hlth. Monit.*, **3**(4), 355-377.

Chen, L.J. and Yu, L. (2013), "Structural nonlinear damage identification algorithm based on time series ARMA/GARCH model", *Adv. Struct. Eng.*, **16**(9), 1597-1609.

da Silva, S., Dias Júnior, M., Lopes Junior, V. and Brennan, M.J. (2008), "Structural damage detection by fuzzy clustering", *Mech. Syst. Sig. Proc*., **22**, 1636-1649.

Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), "A summary review of the vibration-based damage identification methods", *Shock Vib. Dig.*, **30**(2), 91-105.

Dorvash, S., Pakzad, S.N. and LaCrosse, E.L. (2014), "Statistics based localized damage detection using vibration response", *Smart Struct. Syst.*, **14**(2), 85-104.

Farrar, C.R. and Worden, K. (2007), "An introduction to structural health monitoring", *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.*, **365**(1851), 303-315.

Figueiredo, E., Park, G., Figueiras, J. Farrar, C. R. and Worden, K. (2009), "Structural health monitoring algorithm comparisons using standard data sets", Los Alamos National Laboratory Report, LA-14393.

Fugate, M.L., Sohn, H. and Farrar, C. R. (2001), "Vibration-based damage detection using statistical process control", *Mech. Syst. Sig. Proc*., **15**(4), 707-721.

Gul, M. and Catbas, F.N. (2009), "Statistical pattern recognition for structural health monitoring using time series modeling: theory and experimental verifications", *Mech. Syst. Sig. Proc*., **23**(7), 2192-2204.
Gul, M. and Catbas, F.N. (2011), "Structural health monitoring and damage assessment using a novel time series analysis methodology with sensor clustering", *J. Sound and Vib.*, **330**(6), 1196-1210.
Lee, S.G. and Yun, G.J. (2013), "A statistical reference-free damage identification for real-time monitoring of truss bridges using wavelet-based log likelihood ratios", *Smart Struct. Syst*., **12**(2), 181-207.
Lei, Y., Chen, F. and Zhou, H. （2015), "A two-stage and two-step algorithm for the identification of structural damage and unknown excitations: numerical and experimental studies", *Smart Struct. Syst*., **15**(1), 57-80.
Li, Y.Y. and Chen, Y. (2013), "A review on recent development of vibration-based structural robust damage detection", *Struct. Eng. Mech.*, **45**(2), 159-168.
Lu, Y. and Gao, F. (2005), "A novel time-domain auto-regressive model for structural damage diagnosis", *J. Sound Vib.*, **283**(3-5), 1031-1049.
MATLAB (2010), Statistics Toolbox, User's Guide, http://www.mathworks.com, The MathWorks, Inc.
Mattson, S. and Pandit, S. (2006), "Statistical moments of autoregressive model residuals for damage localisation", *Mech. Syst. Sig. Proc.*, **20**, 627-645.
Nair, K.K., Kiremidjian, A.S. and Law, K.H. (2006), "Time series-based damage detection and localization algorithm with application to the ASCE benchmark structure", *J. Sound Vib.*, **291**(1-2), 349-368.
Omenzetter, P. and Brownjohn, J.M. (2006), "Application of time series analysis for bridge monitoring", *Smart Mater. Struct.*, **15**, 129-138.
Sohn, H. and Farrar, C.R. (2001), "Damage diagnosis using time series analysis of vibration signals", *Smart Mater. Struct.*, **10**, 1-6.
Sohn, H., Farrar, C.R., Hemez, F.M., Shunk, D.D., Stinemates, D.W. and Nadler, B.R. (2004), "A review of structural health monitoring literature: 1996-2001", Los Alamos National Laboratory Report.
Yan, Y.J., Cheng, L., Wu, Z.Y. and Yam, L.H. (2007), "Development in vibration-based structural damage detection technique", *Mech. Syst. Sig. Proc.*, **21**, 2198-2211.
Yao, R. and Pakzad, S.N. (2014), "Damage and noise sensitivity evaluation of autoregressive features extracted from structure vibration", *Smart Mater. Struct.*, **23**, 025007.
Yao, R. and Pakzad, S.N. (2012). "Autoregressive statistical pattern recognition algorithms for damage detection in civil structures", *Mech. Syst. Sig. Proc.*, **31**, 355-368.
Yu, L., Zhu, J.H. and Yu, L.L. (2013), "Structural damage detection in a truss bridge model using fuzzy clustering and measured FRF data reduced by principal component projection", *Adv. Struct. Eng.*, **16**(1), 207-217.
Zhang, Q.W. (2007), "Statistical damage identification for bridges using ambient vibration data", *Comput. Struct.*, **85**, 476-485.
Zhou, L.R., Yan, G.R., Wang, L. and Ou, J.P. (2013), "Review of benchmark studies and guidelines for structural health monitoring", *Adv. Struct. Eng.*, **16**(7), 1187-1206.
Zhu, J.H., Yu, L. and Yu, L.L. (2012), "An eigenspace projection clustering method for structural damage detection", *Struct. Eng. Mech.*, **44**(2), 179-196.
Zugasti, E., Gómez González, A., Anduaga, J., Arregui, M.A. and Martínez, F. (2012), "NullSpace and AutoRegressive damage detection: a comparative study", *Smart Mater. Struct.*, **21**, 085010.