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A simple shear deformation theory based on neutral surface position for functionally graded plates resting on Pasternak elastic foundations

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Abstract. In this work, a novel simple first-order shear deformation plate theory based on neutral surface position is developed for bending and free vibration analysis of functionally graded plates and supported by either Winkler or Pasternak elastic foundations. By dividing the transverse displacement into bending and shear parts, the number of unknowns and governing equations of the present theory is reduced, and hence, makes it simple to use. The governing equations are derived by employing the Hamilton's principle and the physical neutral surface concept. There is no stretching–bending coupling effect in the neutral surface-based formulation, and consequently, the governing equations and boundary conditions of functionally graded plates based on neutral surface have the simple forms as those of isotropic plates. Numerical results of present theory are compared with results of the traditional first-order and the other higher-order theories reported in the literature. It can be concluded that the proposed theory is accurate and simple in solving the static bending and free vibration behaviors of functionally graded plates.

Keywords: vibration; bending; FGM; plate theory; elastic foundation; neutral surface position

1. Introduction

Functionally graded materials (FGMs) are a new type of inhomogeneous materials (Yamanouchi *et al*. 1990, Koizumi 1993, 1997) whose macroscopic properties exhibit gradient change in space. Therefore, FGMs can be tailored to satisfy different requirements for material service performance at different parts or locations in a structure. Now FGMs have been used in many structural applications such as mechanical, aerospace, nuclear, and civil engineering.

Many papers, dealing with static and dynamic behaviour of functionally graded materials (FGMs), have been published recently. An interesting literature review of above mentioned work may be found in the paper of Birman and Byrd (2007). Reddy (2000) presented Navier's solutions,

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and finite element models including geometric non-linearity based on the third-order shear deformation theory for the analysis of functionally graded (FG) plates. Cheng and Batra (2000) derived the field equations for a functionally graded plate by utilising the first-order shear deformation theory or the third-order shear deformation theory and simplified them for a simply supported polygonal plate. An exact relationship was established between the deflection of the functionally graded plate and that of an equivalent homogeneous Kirchhoff plate. Vel and Batra (2002, 2004) developed a three-dimensional analysis of the transient thermal stresses, and the free and forced vibration of simple supported FG rectangular plates. Qian *et al*. (2004) conducted an investigation on free and forced vibrations and static deformations of an FG thick simply-supported square plate by using a higher-order shear and normal deformable plate theory and a meshless local Petrov-Galerkin method. Zhong and Yu (2006) used a state-space approach to analyze free and forced vibrations of an FG piezoelectric rectangular thick plate simply-supported at its edges. Free vibration analysis of FG simply-supported square plates was carried out by Pradyumna and Bandyopadhyay (2008) using a higher order finite element formulation, as a small part of their study work. Matsunaga (2008) studied natural frequencies and buckling stresses of FG simply-supported rectangular plates based on 2D higher-order approximate plate theory (2D HAPT). Lü *et al*. (2009) developed 3D exact solutions for free vibration of FG thick plates on Pasternak foundation. Some approximate 3D analyses for free vibration response of multilayered composite and FG plates have also been presented using the RMVT- and PVD-based finite layer methods (Wu and Li 2010a) and RMVT-based meshless collocation and element-free Galerkin methods (Wu and Chiu 2011). Benachour *et al*. (2011) investigated the free vibration of FG plate by the use of four variable refined plate theory. El Meiche *et al*. (2011) developed a novel hyperbolic shear deformation theory which takes into account transverse shear deformation effects for the buckling and free vibration analysis of thick FG sandwich plates. Brischetto (2013) developed an exact three-dimensional elastic model for the free vibration analysis of FG one-layered and sandwich simply-supported plates and shells. Bachir Bouiadjra *et al.* (2013) investigated the nonlinear thermal buckling behavior of FG plates using an efficient sinusoidal shear deformation theory. Houari *et al.* (2013) studied the thermoelastic bending response of FG sandwich plates using a new higher order shear and normal deformation theory. Belabed *et al*. (2014) developed an efficient and simple higher order shear and normal deformation theory for FG plates. Ait Amar Meziane *et al*. (2014) presented an efficient and simple refined shear deformation theory for the vibration and buckling of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions. Hebali *et al*. (2014) proposed a new quasi-three-dimensional (3D) hyperbolic shear deformation theory for the bending and free vibration analysis of FGM plate. Hamidi *et al.* (2015) presented a sinusoidal plate theory with 5-unknowns and stretching effect for thermo-mechanical bending of FG sandwich plates. Chakraverty and Pradhan (2014) studied the free vibration behavior of exponential functionally graded rectangular plates in thermal environment with general boundary conditions. Ait Yahia *et al.* (2014) investigated the wave propagation in FG plates with porosities using various higher-order shear deformation plate theories. Studies of a series of buckling, bending and vibration behavior of FG plate/beam and laminated plate can be found in recent references such as (Attia *et al*. 2015, Draiche *et al*. 2014, Khalfi *et al*. 2014, Nedri *et al*. 2014, Mahi *et al*. 2015).

Recently, investigations of FG plates resting on elastic foundations are identified as an interesting field. Although a few studies on the vibration and buckling analysis of isotropic homogeneous rectangular plates resting on elastic foundation have been carried out (see for example, Xiang *et al*. (1994), Xiang (2003), Lam *et al*. (2000), Zhou *et al*. (2004)) and their cited references), research studies on the dynamic and buckling behavior of their corresponding FG plates have received very little attention. Cheng and Kitipornchai (1999) proposed a membrane analogy to derive an exact explicit eigenvalue for compression buckling, hydrothermal buckling, and vibration of FG plates on a Winkler-Pasternak foundation based on the FSDT. Yang and Shen (2001) studied both free vibration and transient response of initially stressed FG rectangular thin plates subjected to impulsive lateral loads, resting on Pasternak elastic foundation, based on the CPT. The second-order statistics of the buckling of clamped FG rectangular plates that are resting on Pasternak elastic foundations and subjected to uniform edge compression was studied by Yang *et al*. (2005) in the framework of the FSDT. Ying *et al*. (2008) treated 2D elasticity solutions for bending and free vibration of FG beams resting on Winkler-Pasternak elastic foundations. Huang *et al*. (2008) used a benchmark 3D elasticity solution to study the bending behavior of FG thick simply-supported square plates on a Winkler-Pasternak foundation. Yaghoobi and Yaghoobi (2013) studied the buckling behavior of symmetric sandwich plates with FG face sheets resting on an elastic foundation using the first-order shear deformation plate theory and subjected to mechanical, thermal and thermo-mechanical loads. Bouderba *et al*. (2013) analysed the thermomechanical bending response of functionally graded plates resting on Winkler-Pasternak elastic foundations. Based on a refined trigonometric shear deformation theory, Tounsi *et al*. (2013a) analyzed the thermoelastic bending behavior of FG sandwich plates. Zidi *et al*. (2014) studied the bending response of FG plates under hygro-thermo-mechanical loading using a four variable refined plate theory.

Due to its high efficiency and simplicity, first-order shear deformation theory (FSDT) was used for analyzing moderately thick plates. First-order shear deformation theory (FSDT) proposed by Reissner (1950) and another one proposed by Mindlin (1951) are considered to be pioneering theories which take into account shear effects. Praveen and Reddy (1998) examined the nonlinear static and dynamic responses of functionally graded ceramic-metal plates using the first-order shear deformation theory (FSDT) and the von Karman strain. Croce and Venini (2004) formulated a hierarchic family of finite elements according to the Reissner-Mindlin theory. Zenkour *et al*. (2011) presented a mixed first-order transverse shear deformation plate theory (MFPT) for the bending response of an orthotropic rectangular plate resting on two-parameter elastic foundations. Then, Zenkour and Radwan (2013) extended this theory to the bending behaviour of FG plates. It is noted that the mixed first-order transverse shear deformation plate theory (Zenkour *et al*. 2011, Zenkour and Radwan 2013) is a modification of the conventional FSDT. In the MFPT, both the displacements and stresses must be considered arbitrary. Recently, Thai and Choi (2013a, b) developed a simple FSDT involving only four unknowns for FG plates and laminated composite plates. Furthermore, the benefits and disadvantages for the implementation of the simple FSDT-based numerical models are discussed by Yin *et al.* (2014). Thus, it can be noted that the models based on the first-order shear deformation theory (FSDT) are very often used owing to their simplicity in analysis and programming. Recently, Sadoune *et al.* (2014) developed a novel first-order shear deformation theory for laminated composite plates.

In this paper, a new first-order shear deformation theory (NFSDT) for the bending and free vibration analysis of functionally graded beams is developed including plate-foundation interaction. Using the same methodology presented by Thai and Choi (2013a, b), this theory is based on assumption that the in-plane and transverse displacements consist of bending and shear components, in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. Unlike the conventional first-order shear deformation theory (FSDT) (Reissner 1950, Mindlin 1951), the

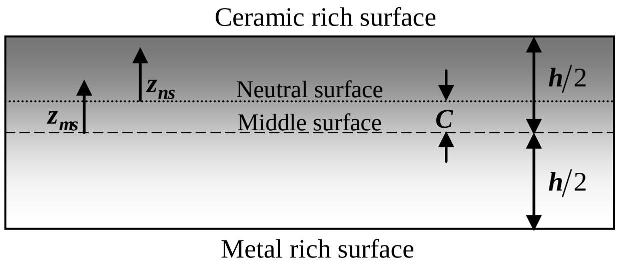

Fig. 1 The position of middle surface and neutral surface for a functionally graded plate

proposed first-order shear deformation theory contains two unknowns. Since, the material properties of FG plate vary through the thickness direction, the neutral plane of such plate may not coincide with its geometric middle plane (Yahoobi and Feraidoon 2010, Ould Larbi *et al*. 2013, Bouremana *et al*. 2013, Bousahla *et al*. 2014, Fekrar *et al*. 2014, Bourada *et al*. 2015). Indeed, Bouremana *et al*. (2013), Ould Larbi *et al*. (2013) show that the stretching-bending coupling in the constitutive equations of an FG beam does not exist when the coordinate system is located at the physical neutral surface of the plate. Therefore, the governing equations for the FG plate can be simplified. Based on the present theory and the exact position of neutral surface together with Hamilton's principle, the motion equations of the functionally graded plates resting on elastic foundation are obtained. Analytical solutions for bending and free vibration are obtained for a simply supported FG plate. Numerical examples are presented to show the validity and accuracy of the present NFSDT.

2. A new first-order shear deformation theory (NFSDT) for FG plates

Due to asymmetry of material properties of FG plates with respect to middle plane, the stretching and bending equations are coupled. But, if the origin of the coordinate system is suitably selected in the thickness direction of the FG plate so as to be the neutral surface, the properties of the FG plate being symmetric with respect to it. To specify the position of neutral surface of FG plates, two different planes are considered for the measurement of z, namely, z_{ms} and z_{ns} measured from the middle surface and the neutral surface of the plate, respectively, as depicted in Fig. 1.

2.1 Basic assumptions

The assumptions of the present theory are as follows:

• The origin of the Cartesian coordinate system is taken at the neutral surface of the FG plate.

• The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

• The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

• This theory assumes constant transverse shear stress and it needs a shear correction factor in order to satisfy the plate boundary conditions on the lower and upper surface.

• The transverse displacement w includes two components of bending w_b and shear w_s. These components are functions of coordinates x, y, and time t only.

2.2 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained as follows

$$u(x,y,z_{ns},t)=u_0(x,y,t)-z_{ns}\frac{\partial w_b}{\partial x} \tag{1a}$$

$$v(x,y,z_{ns},t)=v_0(x,y,t)-z_{ns}\frac{\partial w_b}{\partial y} \tag{1b}$$

$$w(x,y,z_{ns},t)=w_b(x,y,t)+w_s(x,y,t) \tag{1c}$$

where, u, v, w are displacements in the x, y, z directions, u_0, v_0, w_b and w_s are the neutral surface displacements.

The strains associated with the displacements in Eq. (1) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \tag{2}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_b}{\partial x^2}\\ -\dfrac{\partial^2 w_b}{\partial y^2}\\ -2\dfrac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial w_s}{\partial y}\\ \dfrac{\partial w_s}{\partial x}\end{Bmatrix} \tag{3}$$

2.3 Constitutive equations

The FG plate is made from a mixture of ceramic and metal and the properties are assumed to vary through the thickness of the plate. The volume-fraction of ceramic V_C is expressed based on z_{ms} and z_{ns} coordinates as

$$V_C=\left(\frac{z_{ms}}{h}+\frac{1}{2}\right)^p=\left(\frac{z_{ns}+C}{h}+\frac{1}{2}\right)^p \tag{4}$$

where p is the power law index which takes the value greater or equal to zero and C is the distance of neutral surface from the mid-surface. Using Eq. (4), the material non-homogeneous properties (P) of FG plate, as a function of thickness coordinate, become

$$P(z)=P_M+P_{CM}\left(\frac{z_{ns}+C}{h}+\frac{1}{2}\right)^p,\quad P_{CM}=P_C-P_M \tag{5}$$

where P represents the effective material property such as Young's modulus E and mass density ρ subscripts m and c represent the metallic and ceramic constituents, respectively. The value of the power law index p equal to zero represents a fully ceramic plate, whereas infinite p indicates a fully metallic plate.

The position of the neutral surface of the FG plate is determined to satisfy the first moment with respect to Young's modulus being zero as follows (Ould Larbi *et al*. 2013)

$$\int_{-h/2}^{h/2} E(z_{ms})(z_{ms} - C)dz_{ms} = 0 \tag{6}$$

Consequently, the position of neutral surface can be obtained as

$$C = \frac{\int_{-h/2}^{h/2} E(z_{ms}) z_{ms} dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms}) dz_{ms}} \tag{7}$$

It is clear that the parameter C is zero for homogeneous isotropic plates, as expected.

The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \text{ and } \begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} k_s Q_{44} & 0 \\ 0 & k_s Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{8}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. k_s is a shear correction factor which is analogous to shear correction factor proposed by Mindlin (1951). Using the material properties defined in Eq. (5), stiffness coefficients, Q_{ij}, can be expressed as

$$Q_{11} = Q_{22} = \frac{E(z_{ns})}{1-\nu^2}, \tag{9a}$$

$$Q_{12} = \frac{\nu\, E(z_{ns})}{1-\nu^2}, \tag{9b}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z_{ns})}{2(1+\nu)}, \tag{9c}$$

2.4 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Reddy 2002)

$$0 = \int_0^T (\delta U + \delta V - \delta K) dt \tag{10}$$

where δU is the variation of strain energy; δV is the variation of potential energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is calculated by

$$\delta U=\int_V\left(\sigma_x\delta\,\varepsilon_x+\sigma_y\delta\,\varepsilon_y+\tau_{xy}\delta\,\gamma_{xy}+\tau_{yz}\delta\,\gamma_{yz}+\tau_{xz}\delta\,\gamma_{xz}\right)dAdz_{ns}$$
$$=\int_A\left\{-M_x^b\frac{\partial^2\delta\,w_b}{\partial x^2}-M_y^b\frac{\partial^2\delta\,w_b}{\partial y^2}-2M_{xy}^b\frac{\partial^2\delta\,w_b}{\partial x\partial y}+Q_{yz}\frac{\partial\delta\,w_s}{\partial y}+Q_{xz}\frac{\partial\delta\,w_s}{\partial x}\right\}dA \tag{11}$$

where M, and Q are the stress resultants defined as

$$M_i^b=\int_{-\frac{h}{2}-C}^{\frac{h}{2}-C}z_{ns}\,\sigma_i dz_{ns}\,,\ (i=x,y,xy)\ \text{and}\ \left(Q_{xz},Q_{yz}\right)=\int_{-\frac{h}{2}-C}^{\frac{h}{2}-C}\left(\tau_{xz},\tau_{yz}\right)dz_{ns}. \tag{12}$$

The variation of potential energy of the applied loads can be expressed as

$$\delta V=-\int_A(q+f_e)\delta\,w\,dA \tag{13}$$

where q is the transverse applied load and f_e is the density of reaction force of foundation. For the Pasternak foundation model

$$f_e=K_W\,w-J_1\frac{\partial^2 w}{\partial x^2}-J_2\frac{\partial^2 w}{\partial y^2} \tag{14}$$

where K_W is the modulus of subgrade reaction (elastic coefficient of the foundation) and J_1 and J_2 are the shear moduli of the subgrade (shear layer foundation stiffness). If foundation is homogeneous and isotropic, we will get $J_1=J_2=J_0$. If the shear layer foundation stiffness is neglected, Pasternak foundation becomes a Winkler foundation.

The variation of kinetic energy of the plate can be written as

$$\delta\,K=\int_V\left(\dot{u}\delta\,\dot{u}+\dot{v}\delta\,\dot{v}+\dot{w}\delta\,\dot{w}\right)\rho(z_{ns})dAdz_{ns}$$
$$=\int_A\left\{I_0\left[(\dot{w}_b+\dot{w}_s)\delta\,(\dot{w}_b+\dot{w}_s)\right]+I_2\left(\frac{\partial\,\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x}+\frac{\partial\,\dot{w}_b}{\partial y}\frac{\partial\delta\,\dot{w}_b}{\partial y}\right)\right\}dA \tag{15}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z_{ns})$ is the mass density; and (I_0, I_2) are mass inertias defined as

$$(I_0,I_2)=\int_{-\frac{h}{2}-C}^{\frac{h}{2}-C}(1,z_{ns}^2)\rho(z_{ns})dz_{ns} \tag{16}$$

Substituting the expressions for δU, δV, and δK from Eqs. (11), (13), and (15) into Eq. (10) and integrating by parts, and collecting the coefficients of δw_b, and δw_s, the following equations of motion of the plate are obtained

$$\delta u_0 : \; \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} \tag{17a}$$

$$\delta v_0 : \; \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} \tag{17b}$$

$$\delta w_b : \; \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - f_e + q = I_0(\ddot{w}_b + \ddot{w}_s) + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_2 \nabla^2 \ddot{w}_b \tag{17c}$$

$$\delta w_s : \; \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} - f_e + q = I_0(\ddot{w}_b + \ddot{w}_s) \tag{17d}$$

By substituting Eq. (2) into Eq. (8) and the subsequent results into Eq. (12), the stress resultants are obtained as

$$\begin{Bmatrix} N \\ M^b \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \end{Bmatrix}, \quad Q = A^s \gamma, \tag{18}$$

where

$$N = \{N_x, N_y, N_{xy}\}^t, \; M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \tag{19a}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \; D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{19b}$$

$$Q = \{Q_{xz}, Q_{yz}\}^t, \quad \gamma = \{\gamma_{xz}, \gamma_{yz}\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{19c}$$

where A_{ij}, D_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & D_{11} \\ A_{12} & D_{12} \\ A_{66} & D_{66} \end{Bmatrix} = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} Q_{11}\left(1, z_{ns}^2\right) \begin{Bmatrix} 1 \\ \nu \\ \frac{1-\nu}{2} \end{Bmatrix} dz_{ns}, \tag{20a}$$

and

$$\left(A_{22}, D_{22}\right) = \left(A_{11}, D_{11}\right), \tag{20b}$$

$$A_{44}^s = A_{55}^s = k_s \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \frac{E(z_{ns})}{2(1+\nu)} dz_{ns} \tag{20c}$$

By substituting Eq. (18) into Eq. (17), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_b, w_s) as

$$A_{11}d_{11}u_0 + A_{66}d_{22}u_0 + \left(A_{12} + A_{66}\right)d_{12}v_0 = I_0\ddot{u} - I_1\frac{\partial \ddot{w}_b}{\partial x} \tag{21a}$$

$$A_{22}d_{22}v_0 + A_{66}d_{11}v_0 + \left(A_{12} + A_{66}\right)d_{12}u_0 = I_0\ddot{u} - I_1\frac{\partial \ddot{w}_b}{\partial y} \tag{21b}$$

$$\begin{aligned} -D_{11}d_{1111}w_b - 2\left(D_{12} + 2D_{66}\right)d_{1122}w_b - D_{22}d_{2222}w_b - f_e + q = I_0\left(\ddot{w}_b + \ddot{w}_s\right) \\ + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_b \end{aligned} \tag{21c}$$

$$A_{55}^s d_{11}w_s + A_{44}^s d_{22}w_s - f_e + q = I_0\left(\ddot{w}_b + \ddot{w}_s\right) \tag{21d}$$

where d_{ij}, and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \ (i, j, l, m = 1{,}2). \tag{22}$$

Clearly, when the effect of transverse shear deformation is neglected (w_s=0), Eq. (21) yields the equations of motion of FG plate based on the classical plate theory and physical neutral surface concept.

3. Exact solution for FG plates

Rectangular plates are generally classified in accordance with the type of support used. We are here concerned with the exact solution of Eqs. (21a-d) for a simply supported FG plate. The following boundary conditions are imposed at the side edges for NFSDT

$$v_0 = w_b = w_s = N_x = M_x^b = 0 \ \text{at} \ \ x = 0, \ a \tag{23a}$$

$$u_0 = w_b = w_s = N_y = M_y^b = 0 \ \text{at} \ \ y = 0, \ b \tag{23b}$$

Based on the Navier approach, the following expansions of displacements are chosen to automatically satisfy the simply supported boundary conditions of plate

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\lambda\, x)\sin(\mu\, y)\, e^{i\omega t} \\ V_{mn}\sin(\lambda\, x)\cos(\mu\, y)\, e^{i\omega t} \\ W_{bmn}\sin(\lambda\, x)\sin(\mu\, y)\, e^{i\omega t} \\ W_{smn}\sin(\lambda\, x)\sin(\mu\, y)\, e^{i\omega t} \end{Bmatrix} \tag{24}$$

where U_{mn}, V_{mn}, W_{bmn}, and W_{smn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with (m, n) th eigenmode, and $\lambda = m\pi/a$, $\mu = n\pi/b$ and $i = \sqrt{-1}$.

For the case of a sinusoidally distributed load, we have

$$q = q_0 \sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{b}y\right) \tag{25}$$

where q_0 represents the intensity of the load at the plate center.

Substituting Eq. (24) into Eq. (21), the closed-form solutions can be obtained from

$$\left([K]-\omega^2[M]\right)\{\Delta\} = \{P\} \tag{26}$$

where $\{\Delta\}=\{U_{mn}, V_{mn}, W_{bmn}, W_{smn}\}^t$ and $[K]$ and $[M]$ are the symmetric matrixes given by

$$[K]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix},\ [M]=\begin{bmatrix} m_{11} & 0 & m_{13} & 0 \\ 0 & m_{22} & m_{23} & 0 \\ m_{13} & m_{23} & m_{33} & m_{34} \\ 0 & 0 & m_{34} & m_{44} \end{bmatrix} \tag{27}$$

in which

$$\begin{gathered} a_{11} = A_{11}\lambda^2 + A_{66}\mu^2 \\ a_{12} = \lambda\,\mu\left(A_{12} + A_{66}\right) \\ a_{13} = 0 \\ a_{14} = 0 \\ a_{22} = A_{66}\lambda^2 + A_{22}\mu^2 \\ a_{23} = 0 \\ a_{24} = 0 \\ a_{33} = D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4 + K_W + J_1\lambda^2 + J_2\mu^2 \\ a_{34} = K_W + J_1\lambda^2 + J_2\mu^2 \\ a_{44} = A_{55}^s\lambda^2 + A_{44}^s\mu^2 + K_W + J_1\lambda^2 + J_2\mu^2 \\ m_{11} = I_0 \\ m_{22} = I_0 \\ m_{13} = -\lambda\, I_1 \\ m_{23} = -\mu\, I_1 \\ m_{33} = I_0 + I_2\left(\lambda^2 + \mu^2\right) \\ m_{34} = I_0 \\ m_{44} = I_0 \end{gathered} \tag{28}$$

The components of the generalized force vector $\{P\}=\{P_1, P_2, P_3, P_4\}^t$ are given by

$$\begin{gathered} P_1=0 \\ P_2=0 \\ P_3=-q_0 \\ P_4=-q_0 \end{gathered} \tag{29}$$

Table 1 Material properties used in the FG plate

Properties	Metal		Ceramic	
	Aluminum (Al)	Titanium (Ti-6Al-4V)	Alumina (Al_2O_3)	Zirconia (ZrO_2)
E (GPa)	70	66.2	380	117.0
ρ (kg/m^3)	2702	–	3800	–

4. Results and discussion

In this section, various numerical examples are presented and discussed to verify the accuracy of present theory in predicting the bending, and vibration responses of simply supported FG plates. Two types of FG plates of Al/Al_2O_3 and Ti-6Al-4V/ ZrO_2 are used in this study, in which their material properties are listed in Table 1. For verification purpose, the obtained results are compared with those reported in the literature. In all examples, a shear correction factor of 5/6 is used for the present NFSDT. The Poisson's ratio of the plate is assumed to be constant through the thickness and equal to 0.3.

4.1 Bending problem

In this section, various numerical examples are described and discussed for verifying the accuracy of the present first-order shear deformation theory (NFSDT) in predicting the nondimensional deflections and stresses of FG plates subjected to sinusoidally distributed load. The various nondimensional parameters used are

$$\bar{w}=\frac{10E_c h^3}{q_0 a^4}w\left(\frac{a}{2},\frac{b}{2}\right),\quad \hat{w}=\frac{10^2 D}{a^4 q_0}w\left(\frac{a}{2},\frac{b}{2}\right),\quad \bar{\sigma}_x=\frac{h}{q_0 a}\sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{3}-C\right),$$

$$\hat{\sigma}_x=\frac{1}{10^2 q_0}\sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right),\quad \bar{\tau}_{xy}=\frac{h}{q_0 a}\tau_{xy}\left(0,0,-\frac{h}{3}-C\right),\quad \hat{\tau}_{xy}=\frac{1}{10q_0}\tau_{xy}\left(0,0,\frac{-h}{3}\right),$$

$$\bar{\tau}_{xz}=\frac{h}{q_0 a}\tau_{xz}\left(\frac{a}{2},0,\frac{h}{6}-C\right),$$

$$K_0=\frac{a^4 K_W}{D},\quad J_0=\frac{a^2 J_1}{D}=\frac{b^2 J_2}{D},\quad D=\frac{h^3 E_C}{12\left(1-\nu^2\right)},\quad \bar{z}=\frac{z}{h}$$

For the verification purpose, the results obtained by NFSDT are compared with the existing data in the literature. It should be noted that the results reported by Wu and Li (2010b) employed a RMVT-based third order shear deformation theory (TSDT). However, the results reported by Benyoucef *et al*. (2010) were based on the hyperbolic shear deformation theory. Wu *et al*. (2011) employed RMVT-based meshless collocation and element-free Galerkin methods for the quasi-3D analysis of multilayered composite and FGM plates. In the case of FG plates with elastic foundation, Bouderba *et al*. (2013) used a refined trigonometric shear deformation theory. Table 2 shows the comparison of nondimensional deflections and stresses of square plate without elastic foundation subjected to sinusoidally distributed load (*a/h*=10). The obtained results are compared with the work of above mentioned authors, and it can be concluded that, in general, the results are in good agreement with all the theories compared in this section, particularly with the results provided by Benyoucef *et al*. (2010) for all considered values of power law index *p*. In addition, it can be seen that the proposed theory (NFSDT) and conventional FSDT give identical results of deflections as well as stresses for all values of power law index *p*. It should be noted that the unknown function in present theory is four, while the unknown function in both FSDT and other shear deformation theories is five.

Table 3 presents the comparison of nondimensional displacements and stresses of FG rectangular plate supported by either Winkler or Pasternak elastic foundations and subjected to a mechanical load. The FG plate is taken to be made of Titanium and Zirconia as is described by Bouderba *et al*. (2013). The obtained results are compared with those of Bouderba *et al*. (2013) and a good agreement is observed for all considered values of power law index *p* and the elastic foundation parameters. It can be shown that the deflection and stresses are decreasing with the existence of the elastic foundations. The inclusion of the Winkler foundation parameter gives results more than those with the inclusion of Pasternak foundation parameters. As the volume fraction exponent increases for FG plates, the deflection will increase. The stresses are also sensitive to the variation of *p*.

Table 2 Comparison of nondimensional deflection and stresses of square Aluminum/alumina plate under sinusoidally distributed load (*a/h*=10)

p	Theories	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
1	Present (four unknowns)	0.5891	1.4968	0.6125	0.2348
	FSDT (five unknowns)	0.5891	1.4968	0.6125	0.2348
	Benyoucef et al. (2010)	0.5889	1.4894	0.6110	0.2622
	RMVT-based TSDT[a]	0.5890	1.4898	0.6111	0.2506
	RMVT-based collocation[b]	0.5876	1.5062	0.6112	0.2509
	RMVT-based Galerkin[b]	0.5876	1.5061	0.6112	0.2511
2	Present (four unknowns)	0.7552	1.4057	0.5459	0.2289
	FSDT (five unknowns)	0.7552	1.4057	0.5459	0.2289
	Benyoucef et al. (2010)	0.7572	1.3954	0.5441	0.2763
	RMVT-based TSDT[a]	0.7573	1.3960	0.5442	0.2491
	RMVT-based collocation[b]	0.7572	1.4129	0.5437	0.2495
	RMVT-based Galerkin[b]	0.7571	1.4133	0.5436	0.2495
4	Present (four unknowns)	0.8736	1.1922	0.5693	0.1899
	FSDT (five unknowns)	0.8736	1.1922	0.5693	0.1899
	Benyoucef et al. (2010)	0.8810	1.1783	0.5667	0.2580
	RMVT-based TSDT[a]	0.8815	1.1794	0.5669	0.2360
	RMVT-based collocation[b]	0.8826	1.1935	0.5674	0.2360
	RMVT-based Galerkin[b]	0.8823	1.1941	0.5671	0.2362
8	Present (four unknowns)	0.9623	0.9608	0.5887	0.1501
	FSDT (five unknowns)	0.9623	0.9608	0.5887	0.1501
	Benyoucef et al. (2010)	0.9741	0.9466	0.5856	0.2121
	RMVT-based TSDT[a]	0.9747	0.9477	0.5858	0.2263
	RMVT-based collocation[b]	0.9727	0.9568	0.5886	0.2251
	RMVT-based Galerkin[b]	0.9739	0.9622	0.5883	0.2261

[a]Given by Wu and Li (2010b)
[b]Given by Wu *et al*. (2011)

Table 3 Effect of the volume fraction exponent and elastic foundation parameters on the dimensionless and stresses of an FG rectangular Titanium/Zirconia plate (a=10h, b=2a, q_0=100)

p	K_0	J_0	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$
Ceramic	0	0	Present	0.68135	0.42148	0.86459	-0.30558
			Bouderba *et al*. (2013)	0.68131	0.42424	0.86240	-0.39400
			FSDT[(a)]	0.68135	0.42148	0.86459	-0.30558
			CPT[(a)]	0.65704	0.42148	0.86459	–
	100	0	Present	0.40525	0.25070	0.51426	-0.18175
			Bouderba *et al*. (2013)	0.40523	0.25233	0.51296	-0.23435
			FSDT[(a)]	0.40525	0.25070	0.51426	-0.18175
			CPT[(a)]	0.39652	0.25437	0.52183	–
	0	100	Present	0.083655	0.051750	0.10615	-0.037518
			Bouderba *et al*. (2013)	0.083654	0.052093	0.10589	-0.048377
			FSDT[(a)]	0.083655	0.051750	0.10615	-0.037518
			CPT[(a)]	0.08328	0.05342	0.10959	–
	100	100	Present	0.077198	0.047754	0.097959	-0.034622
			Bouderba *et al*. (2013)	0.077197	0.048071	0.097724	-0.044643
			FSDT[(a)]	0.077198	0.047754	0.097959	-0.034622
			CPT[(a)]	0.07688	0.04932	0.10116	–
0.5	100	100	Present	0.078732	0.045460	0.081870	-0.029835
			Bouderba *et al*. (2013)	0.078729	0.045788	0.081728	-0.038066
			FSDT[(a)]	0.078732	0.045460	0.081870	- 0.029835
			CPT[(a)]	0.078463	0.04693	0.08451	–
1	100	100	Present	0.079322	0.044575	0.073208	-0.027163
			Bouderba *et al*. (2013)	0.079321	0.044892	0.073054	-0.035023
			FSDT[(a)]	0.079322	0.044575	0.073208	-0.027163
			CPT[(a)]	0.07907	0.04604	0.07561	–
2	100	100	Present	0.079753	0.044297	0.067395	-0.024345
			Bouderba *et al*. (2013)	0.079758	0.044595	0.067185	-0.032215
			FSDT[(a)]	0.079753	0.044297	0.067395	-0.024345
			CPT[(a)]	0.07950	0.04581	0.06969	–
5	100	100	Present	0.080141	0.045462	0.064399	-0.022053
			Bouderba *et al*. (2013)	0.080150	0.045736	0.064125	-0.029922
			FSDT[(a)]	0.080141	0.045462	0.064399	-0.022053
			CPT[(a)]	0.07989	0.04710	0.06672	–
Metal	100	100	Present	0.081191	0.050227	0.058294	-0.020603
			Bouderba *et al*. (2013)	0.081190	0.050559	0.058148	-0.026565
			FSDT[(a)]	0.081191	0.050227	0.058294	-0.020603
			CPT[(a)]	0.08099	0.05196	0.06030	–

[(a)]Given by Bouderba et al. (2013)

It can be concluded from Tables 2 and 3 that in In general, a good agreement between the results is obtained, except for the case of the transverse shear stresses $\bar{\tau}_{xz}$ where a difference between the present theory and RMVT-based models is seen. The discrepancy between the present

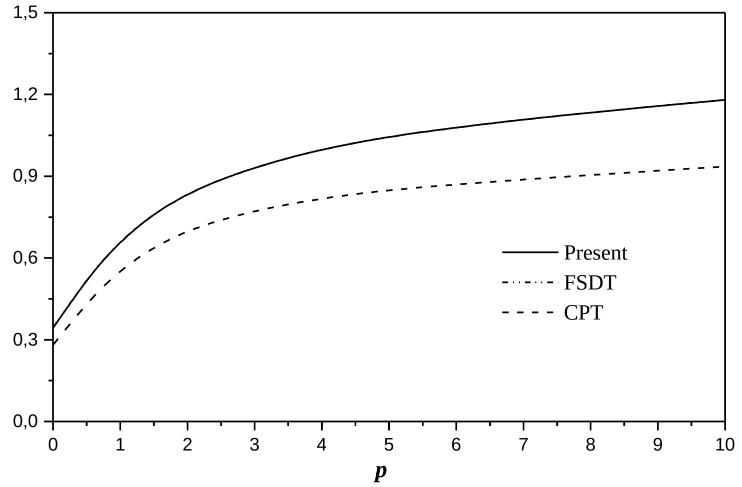

Fig. 2 Comparison of the variation of nondimensional deflection $\overline{w}$ of square Aluminum/ alumina plate under sinusoidally distributed load versus power law index p (a/h=5)

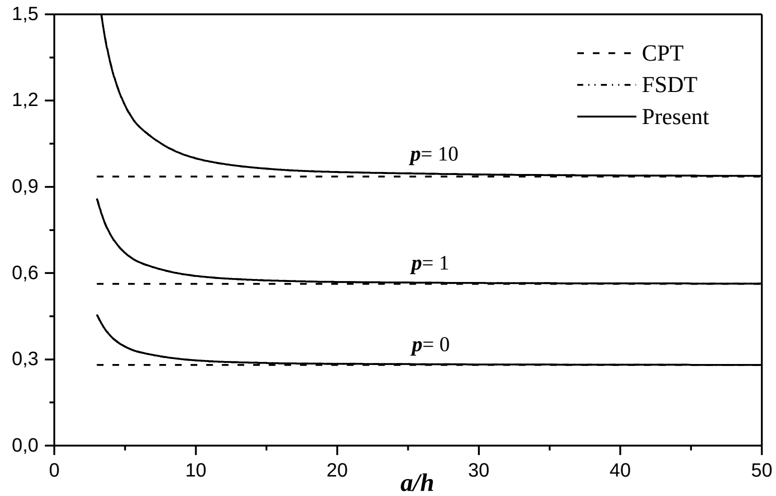

Fig. 3 Comparison of the variation of nondimensional deflection $\overline{w}$ of square Aluminum/ alumina plate under sinusoidally distributed load versus thickness ratio a/h

theory and the accurate solutions increases, when the power law index becomes greater. This is due to the fact that the present theory violates the stress-free boundary conditions on the plate surface and because of the use of a constant shear correction factor for all values of power law index p. To overcome this problem, the transverse shear stresses can be calculated by using the equilibrium equations, rather than using the constitutive equation.

To illustrate the accuracy of present theory for wide range of power law index p, thickness ratio a/h, and aspect ratio a/b, the variations of nondimensional deflection $\overline{w}$ with respect to power law index p, thickness ratio a/h, and aspect ratio a/b are illustrated in Fig. 2, Fig. 3 and Fig. 4, respectively, for FG plate subjected to sinusoidally distributed load. The obtained results are

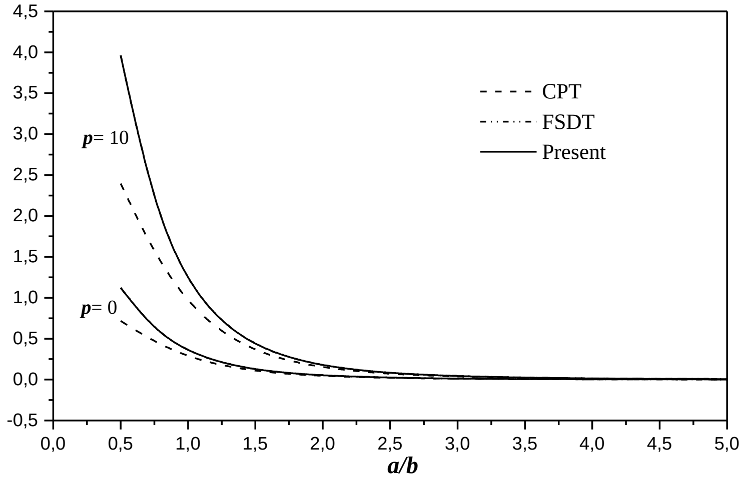

Fig. 4 Comparison of the variation of nondimensional deflection $\overline{w}$ of Aluminum/ alumina plate under sinusoidally distributed load versus aspect ratio *a/b* (*a/h*=5)

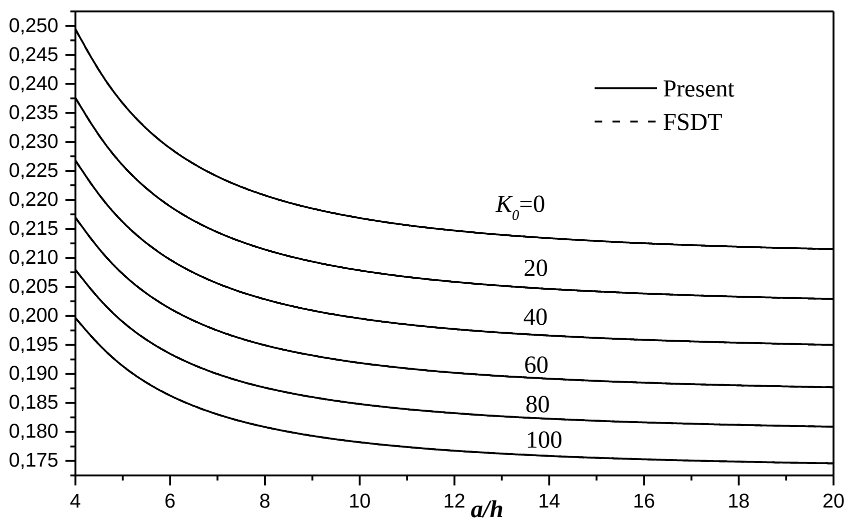

Fig. 5 Effect of Winkler modulus parameter on the dimensionless center deflection $\hat{w}$ of square Titanium/Zirconia plate (p=2) for different side-to-thickness ratio *a/h* with J_0=10 and q_0=100.

compared with those predicted by CPT and the conventional FSDT. It can be seen that the results of present theory and the conventional FSDT are identical, and the CPT underestimates the deflection of plate. Since the transverse shear deformation effects are not considered in CPT, the values of nondimensional deflection $\overline{w}$ predicted by CPT are independent of thickness ratio *a/h* (see Fig. 3). Fig. 4 shows the effects of the aspect ratio *a/b* on the dimensionless deflection $\overline{w}$ of FG plate. The deflections caused by applying different theories decreases as *a/b* increases. It should be noted that the proposed theory (NFSDT) involves four unknowns as against five in case of conventional FSDT.

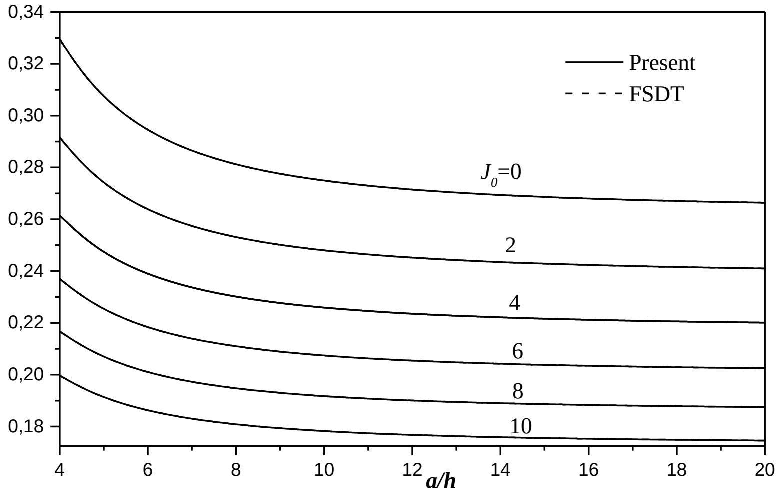

Fig. 6 Effect of Pasternak shear modulus parameter on the dimensionless center deflection $\hat{w}$ of square Titanium/Zirconia plate (p=2) for different side-to-thickness ratio a/h with K_0=10 and q_0=100

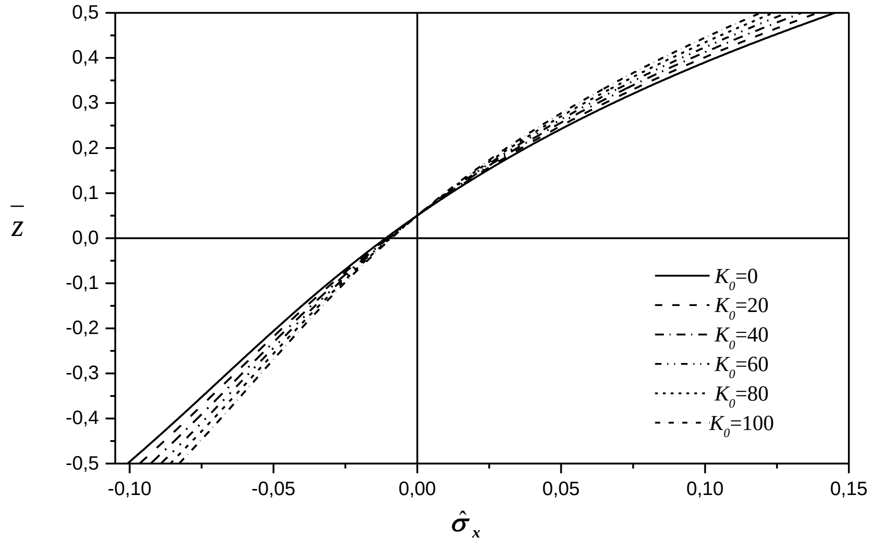

Fig. 7 Variation of dimensionless axial stress ($\hat{\sigma}_x$) through-the-thickness of a square Titanium/Zirconia plate (p=2) for different values of Winkler modulus parameter K_0 with J_0=10, q_0=100 and a/h=10

The effect of the elastic foundation parameters (K_0 and J_0) and side-to-thickness ratio a/h on the center deflection $\hat{w}$ of FG square plate (p=2) is explained in Figs. 5 and 6. As expected, the deflections decrease gradually as either the Winkler modulus parameter K_0 or the Pasternak shear modulus parameter J_0 increases. It can be also seen that the increase of side-to-thickness ratio a/h leads to a decrease of the center deflection of the FG plate.

The effect of the elastic foundation parameters (K_0 and J_0) on the axial stress $\hat{\sigma}_x$ of FG square plate (p=2) is shown in Figs. 7 and 8. It can be seen that the maximum compressive stresses occur

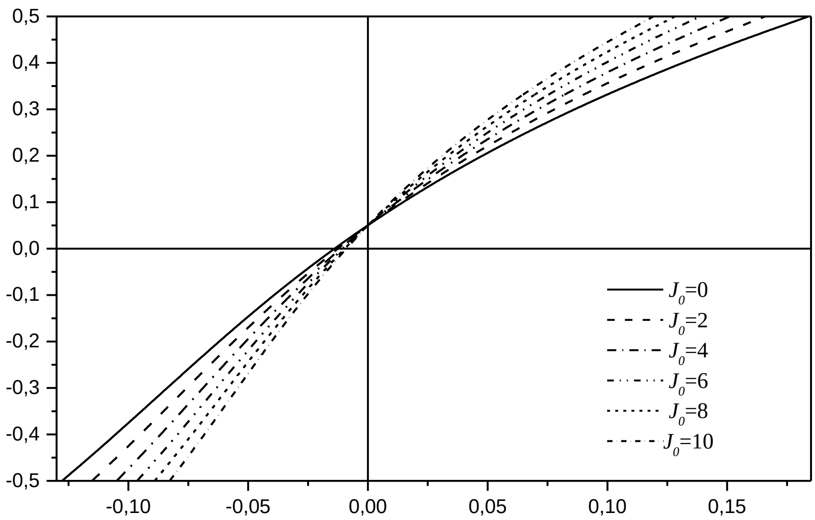

Fig. 8 Variation of dimensionless axial stress ($\hat{\sigma}_x$) through-the-thickness of a square Titanium/Zirconia plate (p=2) for different values of Pasternak shear modulus parameter J_0 with K_0=10, q_0=100 and a/h=10

Table 4 Comparison of nondimensional fundamental frequency ($\hat{\omega}$) of square Aluminum/alumina plate

a/h	Method	Power law index (*p*)				
		0	0.5	1	4	10
5	FSDT[(a)]	0.2112	0.1805	0.1631	0.1397	0.1324
	TSDT[(b)]	0.2113	0.1807	0.1631	0.1398	0.1301
	Present	0.2112	0.1805	0.1631	0.1397	0.1324
10	FSDT[(a)]	0.0577	0.0490	0.0442	0.0382	0.0366
	TSDT[(b)]	0.0577	0.0490	0.0442	0.0381	0.0364
	Present	0.0577	0.0490	0.0442	0.0382	0.0366
20	FSDT[(a)]	0.0148	0.0125	0.0113	0.0098	0.0094
	TSDT[(b)]	0.0148	0.0125	0.0113	0.0098	0.0094
	Present	0.0148	0.0125	0.0113	0.0098	0.0094

[(a)]Hosseini-Hashemi *et al.* (2011a)
[(b)]Hosseini-Hashemi *et al*. (2011b)

at a point near the bottom surface and the maximum tensile stresses occur, of course, at a point near the top surface of the FG plate. In addition, it can be observed from these figures that the elastic foundation has a significant effect on the maximum values of the axial stress. It is observed that normal stress decreases gradually with K_0 or J_0. However, the effect of Pasternak shears modulus parameter is more significant than Winkler modulus parameter.

4.2 Free vibration problem

For convenience, the following nondimensionalizations are used in presenting the numerical results in graphical and tabular form:

$$\hat{\omega} = \omega\, h\sqrt{\rho_c / E_c}\ ,\ \bar{\omega} = \omega \frac{a^2}{h}\sqrt{\rho_c / E_c}\ ,\ \Omega = \omega\, h\sqrt{\rho_m / E_m}\ ,\ K_0 = \frac{a^4 K_W}{D_m};$$

$$J_0 = \frac{a^2 J_1}{D_m} = \frac{b^2 J_2}{D_m},\ D = \frac{h^3 E_m}{12\left(1-\nu^2\right)}$$

Table 5 Comparison of the first four nondimensional frequency ($\bar{\omega}$) of rectangular Aluminum/alumina plate (b=2a)

a/h	Mode (m,n)	Method	Power law index (*p*)						
			0	0.5	1	2	5	8	10
5	1(1,1)	FSDT[(a)]	3.4409	2.9322	2.6473	2.4017	2.2528	2.1985	2.1677
		TSDT	3.4412	2.9347	2.6475	2.3949	2.2272	2.1697	2.1407
		Present	3.4409	2.9322	2.6473	2.4017	2.2528	2.1985	2.1677
	2(1,2)	FSDT[(a)]	5.2802	4.5122	4.0773	3.6953	3.4492	3.3587	3.3094
		TSDT	5.2813	4.5180	4.0781	3.6805	3.3938	3.2964	3.2514
		Present	5.2802	4.5122	4.0773	3.6953	3.4492	3.3587	3.3094
	3(1,3)	FSDT[(a)]	8.0710	6.9231	6.2636	5.6695	5.2579	5.1045	5.0253
		TSDT	8.0749	6.9366	6.2663	5.6390	5.1425	4.9758	4.9055
		Present	8.0710	6.9231	6.2636	5.6695	5.2579	5.1045	5.0253
	4(2,1)	FSDT[(a)]	9.7416	8.6926	7.8711	7.1189	6.5749	5.9062	5.7518
		TSDT	10.1164	8.7138	7.8762	7.0751	6.4074	6.1846	6.0954
		Present	10.1089	8.6926	7.8711	7.1189	6.5749	6.3707	6.2683
10	1(1,1)	FSDT[(a)]	3.6518	3.0983	2.7937	2.5386	2.3998	2.3504	2.3197
		TSDT	3.6518	3.0990	2.7937	2.5364	2.3916	2.3411	2.3110
		Present	3.6518	3.0983	2.7937	2.5386	2.3998	2.3504	2.3197
	2(1,2)	FSDT[(a)]	5.7693	4.8997	4.4192	4.0142	3.7881	3.7072	3.6580
		TSDT	5.7694	4.9014	4.4192	4.0090	3.7682	3.6846	3.6368
		Present	5.7693	4.8997	4.4192	4.0142	3.7881	3.7072	3.6580
	3(1,3)	FSDT[(a)]	9.1876	7.8145	7.0512	6.4015	6.0247	5.8887	5.8086
		TSDT	9.1880	7.8189	7.0515	6.3886	5.9765	5.8341	5.7576
		Present	9.1876	7.8145	7.0512	6.4015	6.0247	5.8887	5.8086
	4(2,1)	FSDT[(a)]	11.8310	10.0740	9.0928	8.2515	7.7505	7.5688	7.4639
		TSDT	11.8315	10.0810	9.0933	8.2309	7.6731	7.4813	7.3821
		Present	11.8307	10.0737	9.0928	8.2515	7.7505	7.5688	7.4639
20	1(1,1)	FSDT[(a)]	3.7123	3.1456	2.8352	2.5777	2.4425	2.3948	2.3642
		TSDT	3.7123	3.1458	2.8352	2.5771	2.4403	2.3923	2.3619
		Present	3.7123	3.1456	2.8352	2.5777	2.4425	2.3948	2.3642
	2(1,2)	FSDT[(a)]	5.9198	5.0175	4.5228	4.1115	3.8939	3.8170	3.7681
		TSDT	5.9199	5.0180	4.5228	4.1100	3.8884	3.8107	3.7622
		Present	5.9198	5.0175	4.5228	4.1115	3.8939	3.8170	3.7681
	3(1,3)	FSDT[(a)]	9.5668	8.1121	7.3132	6.6471	6.2903	6.1639	6.0843
		TSDT	9.5669	8.1133	7.3132	6.6433	6.2760	6.1476	6.0690
		Present	9.5668	8.1121	7.3132	6.6471	6.2903	6.1639	6.0843
	4(2,1)	FSDT[(a)]	12.4560	10.5660	9.5261	8.6572	8.1875	8.0207	7.9166
		TSDT	12.4562	10.5677	9.5261	8.6509	8.1636	7.9934	7.8909
		Present	12.4562	10.5657	9.5261	8.6572	8.1875	8.0207	7.9165

[(a)]Hosseini-Hashemi *et al*. (2011a)

Table 6 Comparison of nondimensional fundamental frequency (Ω) of FG square Aluminum/alumina plate resting on elastic foundation

K_0	J_0	*h/a*	Method	Power law index (*p*)				
				0	0.5	1	2	5
0	0	0.05	Present	0.0291	0.0246	0.0222	0.0202	0.0191
			FSDT	0.0291	0.0246	0.0222	0.0202	0.0191
			TSDT[a]	0.0291	0.0249	0.0227	0.0209	0.0197
		0.1	Present	0.1133	0.0963	0.0868	0.0789	0.0744
			FSDT	0.1133	0.0963	0.0868	0.0789	0.0744
			TSDT[a]	0.1134	0.0975	0.0891	0.0819	0.0767
		0.2	Present	0.4150	0.3546	0.3204	0.2904	0.2711
			FSDT	0.4150	0.3546	0.3204	0.2904	0.2711
			TSDT[a]	0.4154	0.3606	0.3299	0.3016	0.2765
0	100	0.05	Present	0.0406	0.0386	0.0378	0.0374	0.0377
			FSDT	0.0406	0.0386	0.0378	0.0374	0.0377
			TSDT[a]	0.0406	0.0389	0.0382	0.0380	0.0381
		0.1	Present	0.1597	0.1526	0.1494	0.1478	0.1489
			FSDT	0.1597	0.1526	0.1494	0.1478	0.1489
			TSDT[a]	0.1599	0.1540	0.1517	0.1508	0.1515
		0.2	Present	0.6074	0.5855	0.5752	0.5698	0.5734
			FSDT	0.6074	0.5855	0.5752	0.5698	0.5734
			TSDT[a]	0.6080	0.5932	0.5876	0.5861	0.5879
100	0	0.05	Present	0.0298	0.0255	0.0233	0.0214	0.0205
			FSDT	0.0298	0.0255	0.0233	0.0214	0.0205
			TSDT[a]	0.0298	0.0258	0.0238	0.0221	0.0210
		0.1	Present	0.1161	0.0999	0.0910	0.0837	0.0799
			FSDT	0.1161	0.0999	0.0910	0.0837	0.0799
			TSDT[a]	0.1162	0.1012	0.0933	0.0867	0.0821
		0.2	Present	0.4268	0.3698	0.3380	0.3107	0.2941
			FSDT	0.4268	0.3698	0.3380	0.3107	0.2941
			TSDT[a]	0.4273	0.3758	0.3476	0.3219	0.2999
100	100	0.05	Present	0.0411	0.0392	0.0384	0.0381	0.0384
			FSDT	0.0411	0.0392	0.0384	0.0381	0.0384
			TSDT[a]	0.0411	0.0395	0.0388	0.0386	0.0388
		0.1	Present	0.1617	0.1549	0.1519	0.1505	0.1517
			FSDT	0.1617	0.1549	0.1519	0.1505	0.1517
			TSDT[a]	0.1619	0.1563	0.1542	0.1535	0.1543
		0.2	Present	0.6156	0.5948	0.5852	0.5804	0.5845
			FSDT	0.6156	0.5948	0.5852	0.5804	0.5845
			TSDT[a]	0.6162	0.6026	0.5978	0.5978	0.5993

[a]Given by Baferani *et al*. (2011)

The nondimensional natural frequency $\hat{\omega}$ of square plate obtained from the proposed theory (NFSDT) is compared with those reported by Hosseini-Hashemi *et al*. (2011a) based on FSDT and

Hosseini-Hashemi *et al*. (2011b) based on third shear deformation theory (TSDT). The results are given in Table 4 for different values of thickness ratio *a/h* and power law index *p*. From Table 4, it can be observed that the present results are in excellent agreement with those acquired by the FSDT (Hosseini-Hashemi *et al*. 2011a), and TSDT (Hosseini-Hashemi *et al*. 2011b).

The four natural frequency parameters ($\overline{\varpi}$) of the rectangular FG plate (b=2a) for different values of the power law index p and thickness ratio *a/h* are compared with those given by Hosseini-Hashemi *et al*. (2011a) based on FSDT and with those obtained using TSDT (Reddy 2000) in Table 5. It can be seen that the results predicted by the new first-order shear deformation theory (NFSDT) and TSDT are almost identical for all modes of vibration of thin to thick plates. Also, the proposed theory with only four unknown functions gives more accurate prediction of natural frequency compared to the conventional FSDT which needs five unknown functions.
Fundamental frequency parameters Ω of the Al/Al_2O_3 square plate are listed in Table 6 for various values of thickness to length ratio (*h/a*=0.05, 0.1, and 0.2), power law index (p=0, 0.5, 1, 2, and 5), and foundation stiffness parameters (K_0, J_0). The present results are compared with those obtained by the conventional FSDT and by Baferani *et al*. (2011) using TSDT. Table 6 proves the fact that all results are in excellent agreement with each other.

The variations of nondimensional fundamental frequency $\overline{\varpi}$ of square plate with respect to power law index p and thickness ratio *a/h* are compared in Fig. 9 and Fig. 10, respectively. It is observed that the nondimensional frequencies $\overline{\varpi}$ predicted by the new first-order shear deformation theory (NFSDT) and the conventional FSDT are identical, and the CPT overestimates the frequency of thick plate. Fig. 11 shows the effects of the aspect ratio *a/b* on nondimensional fundamental frequency $\overline{\varpi}$ of FG plate. It is observed that the frequency parameter increases for plates with higher aspect ratio *a/b*. It is observed that the proposed theory and the conventional FSDT give identical results.

The effect of foundation stiffness on the non-dimensional fundamental frequencies (Ω) of FG square plates is shown in Fig. 13 (*a/h*=10). The figure shows that frequencies of FG plates increase

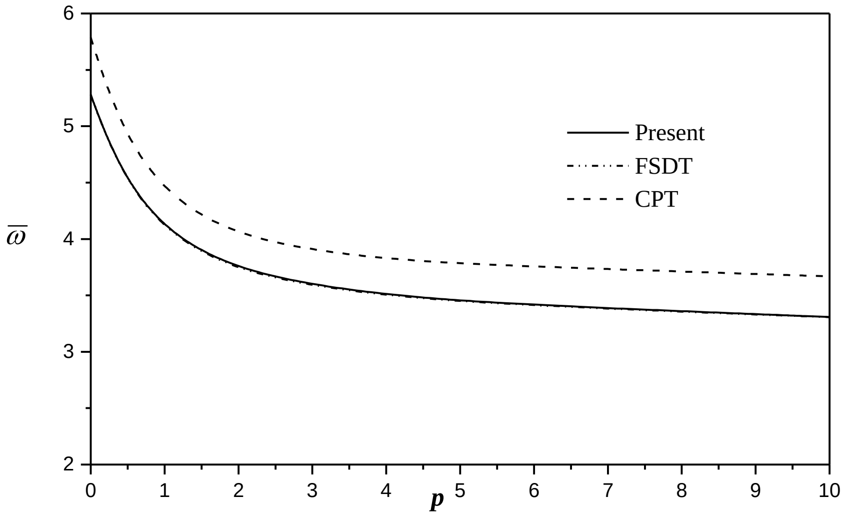

Fig. 9 Comparison of nondimensional fundamental frequency $\overline{\varpi}$ of square Aluminum/ alumina plate versus power law index p (*a/h*=5)

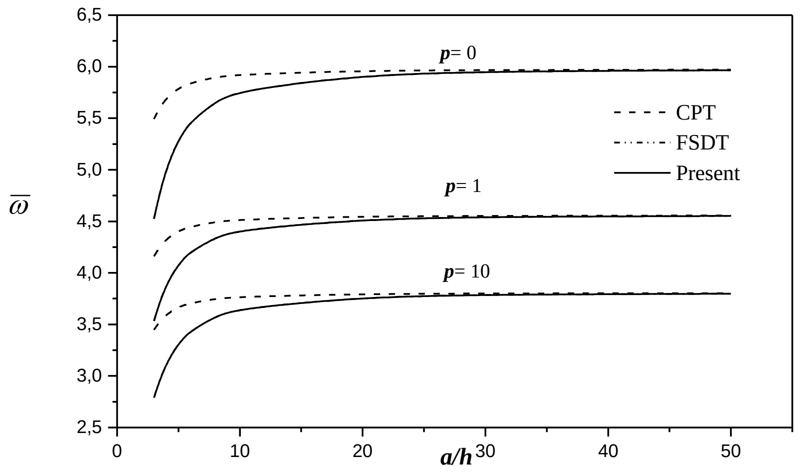

Fig. 10 Comparison of nondimensional fundamental frequency $\overline{\omega}$ of square Aluminum/ alumina plate versus thickness ratio *a/h*

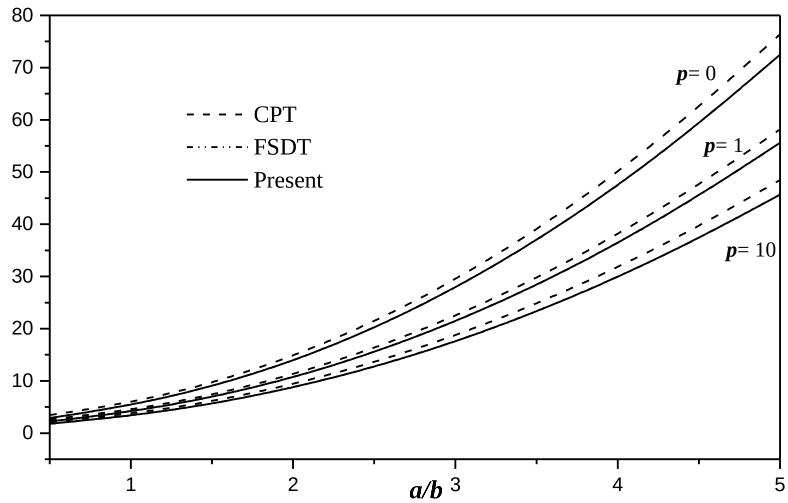

Fig. 11 Comparison of the variation of nondimensional fundamental frequency $\overline{\omega}$ of Aluminum/ alumina plate versus aspect ratio *a/b* (*a/h*=5).

when foundation parameters increase. It is also noted in this case that the effect of Pasternak shears modulus parameter is more significant than Winkler modulus parameter.

5. Conclusions

A new first-order shear deformation theory (NFSDT) was proposed to analyse static and dynamic behaviour of functionally graded plates resting on Winkler-Pasternak elastic foundations. The neutral surface position for such plates has been determined. The effectiveness of the theory is

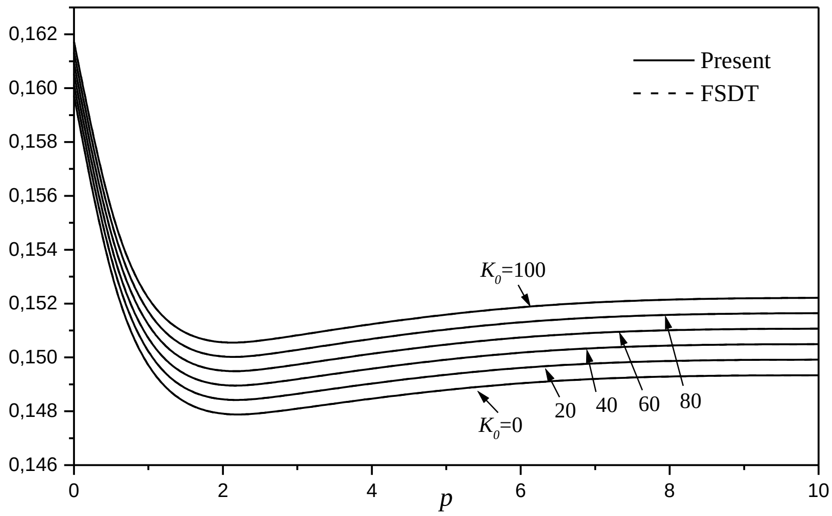

Fig. 12 Effect of Winkler modulus parameter on the dimensionless fundamental frequency Ω of square Aluminum/ alumina plate for different power law index *p* (a/h=10, J_0=100)

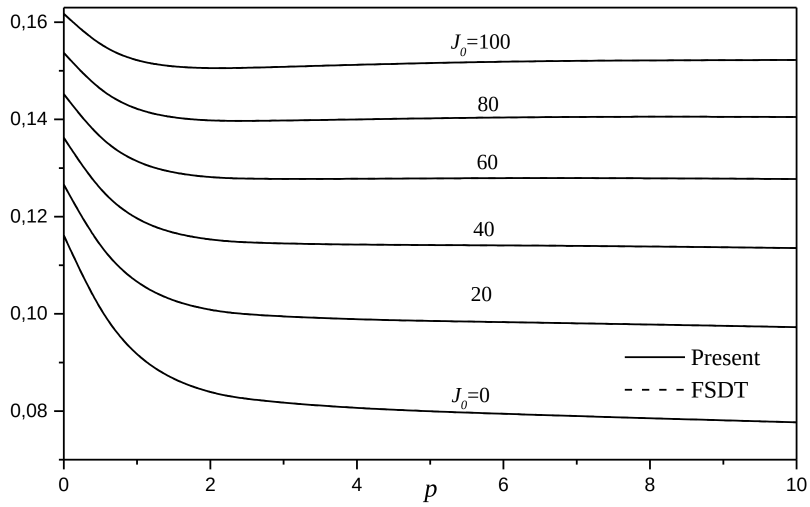

Fig. 13 Effect of Pasternak shear modulus parameter on the dimensionless fundamental frequency Ω of square Aluminum/alumina plate for different power law index *p* (a/h=10, K_0=100).

brought out by applying them for static as well as dynamic analysis. The results obtained using this new theory, are found to be in excellent agreement with previous studies. Unlike the conventional first shear deformation theory, the proposed first shear deformation theory contains only four unknowns rather than the usual five and eliminates the stretching – bending coupling effect, resulting in reduced computational expense and significantly facilitating engineering analysis. However, the disadvantages for the implementation of the present NFSDT-based numerical models is discussed and resolved recently by Yin *et al.* (2014). In conclusion, it can be said that the proposed theory NFSDT is not only accurate but also provides an elegant and easily implementable approach for simulating the static and dynamic behavior of functionally graded plates resting on elastic foundations. The formulation lends itself particularly well to finite element simulations (Curiel Sosa *et al*. 2012, Curiel Sosa *et al*. 2013), other numerical methods employing symbolic computation for plate bending problems (Rashidi *et al*. 2012) and also in analysing nanostructures (Heireche *et al*. 2008, Benzair *et al*. 2008, Tounsi *et al*. 2013b, c, Berrabah *et al*. 2013, Benguediab *et al*. 2014, Semmah *et al*. 2014), which will be considered in the near future.

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