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Vibration of sumberged functionally graded cylindrical shell based on first order shear deformation theory using wave propagation method

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Abstract. This paper focuses on vibration analysis of functionally graded cylindrical shell submerged in an incompressible fluid. The equation is established considering axial and lateral hydrostatic pressure based on first order shear deformation theory of shell motion using the wave propagation approach and classic Flügge shell equations. To study accuracy of the present analysis, a comparison carried out with a known data and the finite element package ABAQUS. With this method the effects of shell parameters, m, n, h/R, L/R, different boundary conditions and different power-law exponent of material of functionally graded cylindrical shells, on the frequencies are investigated. The results obtained from the present approach show good agreement with published results.

Keywords: vibration; submerged cylindrical shell; functionally graded material; first order shear deformation theory; wave propagation approach; Flügge shell equations

1. Introduction

Due to the increasing demands for high structural performances, the study of functionally graded materials in structures has received considerable attention in recent years (Farahani *et al*. 2014). Cylindrical shells in contact with fluid are the practical element of many types of engineering structures, such as pressure vessels, oil tankers, aero planes, ships, nuclear reactors and marine crafts and any types of engineering structures that are affected in high temperatures. Recently several researches have been carried out on analysis of FGM cylinders submerged in acoustic media. Loy *et al*. (1999) studied the vibration characteristics of cylindrical shells structured from functionally graded material. They deduced that the behavior of FGM cylindrical shells is similar to that of isotropic ones, but the two configurations of the shells affect their natural frequencies. The dynamic characteristics of a circular cylindrical shell in contact with a liquid are theoretically studied by Askaria and Jeongb (2010). In their paper the liquid is assumed to be incompressible and inviscid, the liquid motion be described as the velocity potentials written in

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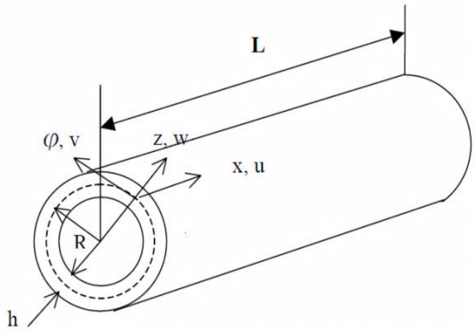

Fig. 1 Coordinate system of Functionally graded cylindrical shell

terms of the appropriate Bessel functions for both the inner and the outer liquid regions. Kenji Saijyou (2006) precented the relationship between the dominant mode of the submerged thin cylindrical shell and the flexural wave velocity and the natural frequencies corresponding to the vibration mode is obtained as the solution of characteristic equation of thin cylindrical shell. A submerged evacuated circular cylindrical shell subjected to a sequence of two external shock waves generated at the same source is considered by Iakovleva *et al*. (2013). They used classical methods of mathematical physics with the finite-difference methodology for employed to simulate the interaction. Sharma *et al*. (1998) have analyzed frequency response of vertical cantilever composite shells containing fluid. Amabili (1996, 1999) has investigated free vibrations of circular cylindrical shells and tubes completely and partially filled with a dense medium and partially immersed in a different fluid having a free surface. Askari *et al*. (2011) investigated the effects of a rigid internal body on bulging and sloshing frequencies. Zhang *et al*. (2001a, b) studied the vibration characteristics of empty and fluidfilled cylindrical shells. Koo *et al*. (2002) proposed a simpler approach for the interface boundary Condition by placing a virtual rigid-wall between the external and internal fluids in the gap. Zhang (2002) employed the wave propagation approach to investigate the coupled frequency of submerged cylindrical shells in a dense medium. Kwak *et al*. (2011) carried out a theoretical study on a clamped-free thin cylindrical shell partially submerged in a fluid and found that the natural frequencies of the shell decreased significantly with the risein the water level. Arshad *et al*. (2010) analyzed the natural frequencies of the bi-layered cylindrical shells with layers of different materials. The layers of the shell were structured from isotropic as well as functionally graded materials.

In this paper vibration analysis of functionally graded cylindrical shell submerged in an incompressible fluid is precented. Initially the stability equations considering axial and pressure of fluid obtained based on first order shear deformation theory of shell motion using the wave propagation approach and classic Flugge shell equations. To study accuracy of the present analysis, a comparison carried out with a known data and the finite element package ABAQUS. And finally effects of shell parameters and diferent boundary conditions on the natural frequencies are investigated.

2. Formulation

2.1 Material properties

The Functionally graded cylindrical shell as shown in Fig. 1 that is assumed to be thin and of uniform thickness. It is of length L, thickness h, and radius R, Yong's modulus E, Poisson's ratio υ, and density ρ_T, is considered to be submerged in a fluid of density ρ_c where the velocity of sound is c_f.

The cylindrical coordinates system (x, φ, z) is applied in our work to define the position of points in the region. The coordinate axis x is chosen to coincide with the cylindrical shell centerline, while the coordinate axes z and φ respond to the radial and circumferential directions respectively. The displacements of shell are defined by u, v, w in the coordinates system (x, φ, z) respectively.

For a functionally graded material with two constituent materials, the Youngs modulus E, Poisson ratio υ and the mass density ρ are (Loy *et al*. 1999)

$$
\begin{aligned}
E &= (E_1 - E_2)\left(\frac{2z+h}{2h}\right)^N + E_2 \\
\nu &= (\nu_1 - \nu_2)\left(\frac{2z+h}{2h}\right)^N + \nu_2 \\
\rho_T &= (\rho_1 - \rho_2)\left(\frac{2z+h}{2h}\right)^N + \rho_2
\end{aligned}
\tag{1}
$$

Where N is the power-law exponent, $0 \le N \le \infty$, from these equations, when $z=-h/2$, $E=E_2$, $\upsilon=\upsilon_2$ and $\rho=\rho_2$, and when $z=h/2$, $E=E_1$, $\upsilon=\upsilon_1$ and $\rho=\rho_1$. The material properties vary continuously from material 2 at the inner surface of the cylindrical shell to material 1 at the outer surface of the cylindrical shell. In the next section, a formulation, based on First order shear deformation theory and the classic Flugge shell equations (Flügge 1973), for a functionally graded cylindrical shell is carried out.

2.2 Motion equations of the shell and fluid

The problem of the cylindrical shell is three-dimensional and is transformed into a two-dimensional problem by assuming the plane stress condition, i.e., the strain and stress components are neglected in the z-direction. First order theory is used to deal with the influence of shear forces on the frequencies of the thick shell. According to the theory the displacement fields are

$$
\begin{aligned}
& u(x,\varphi,z,t) = u_0(x,\varphi,t) + z\phi_x(x,\varphi,t) \\
& v(x,\varphi,z,t) = v_0(x,\varphi,t) + z\phi_\varphi(x,\varphi,t) \\
& w(x,\varphi,z,t) = w_0(x,\varphi,t) \\
& \phi_x = \frac{\partial u}{\partial z}, \qquad \phi_\varphi = \frac{\partial v}{\partial z}
\end{aligned}
\tag{2}
$$

Where u_0, v_0, w_0, ϕ_x and ϕ_φ are unknowns to be determined and also are the displacements on the surface $z=0$ and the rotations of transverse normal about its φ and x axis respectively.

The constitutive relations for a thin cylindrical shell are stated by the two dimensional Hooke's law as (Loy *et al*. 1999)

$$\begin{Bmatrix} \sigma_x \\ \sigma_\varphi \\ \sigma_{x\varphi} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\varphi \\ \gamma_{x\varphi} \end{Bmatrix} \tag{3}$$

Q_{ij} (i, j=1, 2, 6) are the reduced stiffnesses, and for isotropic materials they are expressed as

$$\begin{aligned} Q_{11}(z) &= Q_{22}(z) = \frac{E(z)}{1-\upsilon^2} \\ Q_{12}(z) &= \frac{\upsilon E(z)}{1-\upsilon^2} \\ Q_{66}(z) &= \frac{E(z)}{2(1+\upsilon)} \end{aligned} \tag{4}$$

From Love's shell theory (Love 1952), the components in the strain vector $\{\varepsilon\}$ are defined as

$$\begin{aligned} \varepsilon_x &= \varepsilon_1 + zk_1, \\ \varepsilon_\varphi &= \varepsilon_2 + zk_2, \\ \varepsilon_{x\varphi} &= \gamma + 2z\tau \end{aligned} \tag{5}$$

Where ε_1, ε_2 and γ are the reference surface strains, and k_1, k_2 and τ are the surface curvatures. These surface strains and curvatures are defined as

$$\begin{aligned} \{\varepsilon_1, \varepsilon_2, \gamma\} &= \left\{ \frac{\partial u}{\partial x}, \frac{1}{R}\left(\frac{\partial v}{\partial \varphi} + w\right), \frac{\partial u}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \varphi} \right\} \\ \{k_1, k_2, \tau\} &= \left\{ -\frac{\partial^2 w}{\partial x^2}, -\frac{1}{R}\left(\frac{\partial^2 w}{\partial \varphi^2} - \frac{\partial v}{\partial \varphi}\right), -\frac{1}{R}\left(\frac{\partial^2 w}{\partial x\,\partial \varphi} - \frac{\partial v}{\partial x}\right) \right\} \end{aligned} \tag{6}$$

The force and moment resultants are defined as

$$\begin{aligned} (N_x, N_\varphi, N_{x\varphi}) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\sigma_x, \sigma_\varphi, \sigma_{x\varphi}\} dz, \\ (M_x, M_\varphi, M_{x\varphi}) &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \{\sigma_x, \sigma_\varphi, \sigma_{x\varphi}\} z dz \end{aligned} \tag{7}$$

Where

$$\begin{Bmatrix} N_x \\ N_\varphi \\ N_{x\varphi} \\ M_x \\ M_\varphi \\ M_{x\varphi} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{11} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & C_{11} & C_{12} & 0 \\ B_{12} & B_{22} & 0 & C_{22} & C_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma \\ k_1 \\ k_2 \\ \tau \end{Bmatrix} \tag{8}$$

Where A, B, C, are

$$(A,B,C)=\int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}(1,z,z^2)dz \tag{9}$$

Governing shell motion equations are given by

$$\begin{aligned} &N_{x,x}+\frac{1}{R}N_{x\varphi,\varphi}=\rho_T\frac{\partial^2 u}{\partial t^2}\\ &N_{x\varphi,x}+\frac{1}{R}N_{\varphi,\varphi}+\frac{2}{R}M_{x\varphi,x}+\frac{1}{R^2}M_{\varphi,\varphi}=\rho_T\frac{\partial^2 v}{\partial t^2}\\ &\frac{\partial^2 M_x}{\partial x^2}+\frac{2}{R}\frac{\partial^2 M_{x\varphi}}{\partial x\partial\varphi}+\frac{1}{R}\frac{\partial^2 M_\varphi}{\partial\varphi^2}-\frac{N_\varphi}{R}=\rho_T\frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{10}$$

By substituting Eqs. (4)-(6), into Eq. (10), the equations of motion of the shell can be written with the displacement component u,v and w in a matrix form as

$$\begin{bmatrix} L_{11} & L_{12} & L_{13}\\ L_{21} & L_{22} & L_{23}\\ L_{31} & L_{32} & L_{33}\end{bmatrix}\begin{Bmatrix} u\\ v\\ w\end{Bmatrix}=\begin{Bmatrix} 0\\ 0\\ 0\end{Bmatrix} \tag{11}$$

Where the differential operator L_{ij} with respect to x and θ.

The vibrational equations of cylindrical shell in which the hydrostatic pressure is modeled as the static prestress terms in the shell equations based on the classic Flugge shell equations (Flügge 1973), can be rewritten as

$$\begin{bmatrix} L_{11} & L_{12} & L_{13}\\ L_{21} & L_{22} & L_{23}\\ L_{31} & L_{32} & L_{33}\end{bmatrix}\begin{Bmatrix} u\\ v\\ w\end{Bmatrix}=\frac{(1-\mu^2)R^2}{Eh}\begin{Bmatrix} 0\\ 0\\ P_m\end{Bmatrix} \tag{12}$$

Where P_m is the fluid acoustic pressure.
Where

$$\begin{aligned} L_{11}&=(1+T_1)R^2\frac{\partial^2}{\partial x^2}+\left[T_2+\frac{1-\mu}{2}(K+1)\right]\frac{\partial^2}{\partial\varphi^2}-\frac{\rho_T R^2(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2}\\ L_{12}&=L_{21}=R\frac{1+\mu}{2}\frac{\partial^2}{\partial x\partial\varphi}\\ L_{13}&=L_{31}=R(\mu-T_2)\frac{\partial}{\partial x}-KR^3\frac{\partial^3}{\partial x^3}+KR\frac{1-\mu}{2}\frac{\partial^3}{\partial x\partial\varphi^2}\\ L_{22}&=R^2\left[T_1+\frac{1-\mu}{2}(3K+1)\right]\frac{\partial^2}{\partial x^2}+(1+T_2)\frac{\partial^2}{\partial\varphi^2}-\frac{\rho_T R^2(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2}\\ L_{23}&=L_{32}=(1+T_2)\frac{\partial}{\partial\varphi}-KR^2\frac{3-\mu}{2}\frac{\partial^3}{\partial x^2\partial\varphi}\\ L_{33}&=1+K+(2K-T_2)\frac{\partial^2}{\partial\varphi^2}+K\nabla^4-R^2T_1\frac{\partial^2}{\partial x^2}+\frac{\rho_T R^2(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2} \end{aligned}$$

$$\nabla^4 = \left(R^4 \frac{\partial^4}{\partial x^4} + 2R^2 \frac{\partial^4}{\partial x^2 \partial \varphi^2} + \frac{\partial^4}{\partial \varphi^4} \right), \qquad K = \frac{h^2}{12R^2} \tag{13}$$

Where

$$T_1 = \frac{R(1-\mu^2)}{2Eh} P_0, \qquad T_2 = \frac{R(1-\mu^2)}{Eh} P_0 \tag{14}$$

T_1 and T_2 including the impacts of the hydrostatic pressure which comprise of an axial strengthen component and a radial strengthen component and P_0 is the external hydrostatic pressure.

The equation of the motion of the fluid around the cylindrical shell is assumed non-viscous and isotropic which satisfy the acoustic wave equation can be written by Morse and Ingard (1968) in the cylindrical coordinate system (x, φ, r).

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial P}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial^2 P}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} \tag{15}$$

Where P is the acoustic pressure and c is the sound speed of the fluid. The x and φ coordinates are the same as those of the shell, where According to Fig. 1 the r coordinate is taken from the z-axis of the shell.

3. Wave propagation method

In the wave propagation approach the displacements of the shell are expressed in the format of wave propagation, associated with the longitudinal wave number k_x and the circumferential modal parameter n and defined

$$\begin{aligned} u_0 &= U_m \cos(n\varphi) e^{(i\omega t - ik_m x)} \\ v_0 &= V_m \sin(n\varphi) e^{(i\omega t - ik_m x)} \\ w_0 &= W_m \cos(n\varphi) e^{(i\omega t - ik_m x)} \end{aligned} \tag{16}$$

Where U_m, V_m and W_m are respectively the displacement amplitudes in x, φ, z directions, the rotation amplitudes of transverse normal about φ and x-axis, ω is circular frequency.

The associated form of the acoustic pressure filed in the interior fluid, which satisfies the acoustic wave Eq. (15) can be expressed in the cylindrical coordinate system, associated with an axial wave number k_x, radial wave number k_r and circumferential modal parameter n, given as

$$P = P_m \cos(n\varphi) J_n(k_r r) e^{(i\omega t - ik_m x)} \tag{17}$$

Where J_n is the Bessel function of order n. This function is replace with second kind Hankel function of order n when considering exterior acoustic medium, then the acoustic pressure satisfying wave Eq. (17) given as

$$P = P_m \cos(n\varphi) H_n^{(2)}(k_r r) e^{(i\omega t - ik_m x)} \tag{18}$$

Where P_m is the fluid acoustic pressure amplitude; k_r is the radial wave numbers respectively, which have the relation $k_r^2 + k_m^2 = k_0^2$ where $k_0 = \omega / c_f$ that is the free wave number and $H_n^{(2)}$ is the second kind of Hankel function.

To ensure that fluid remains in contact with the shell wall, the fluid radial displacement and the shell radial displacement must be equal both at the interface of shell with outer wall and the fluid (Zhang 2001a, b). This coupling condition is then

$$-\left\{\frac{1}{i\omega\rho_c}\right\}\left(\frac{\partial P}{\partial r}\right)\bigg|_{r=R} = \left(\frac{\partial \omega_0}{\partial t}\right)\bigg|_{r=R} \tag{19}$$

Consequently, the above condition reduces to (Zhang *et al*. 2002)

$$P_m = \left[\frac{\omega^2 \rho_c}{k_r H_n^{'(2)}(k_r R)}\right] W_m \tag{20}$$

Where ρ_c is the density of fluid and the prime on the $H_n^{'(2)}$ denotes differentiation with respect to the argument $k_r R$.

Substituting Eqs. (16), (17) or (18) into Eq. (12), with consideration of coupling Eq. (20) results in the equation of motion of coupled system in matrix form as

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} U_m \\ V_m \\ W_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

The elements of the matrix [L] is

$$\begin{aligned} L_{11} &= \Omega^2 - (1+T_1)\lambda^2 - \left[T_2 + (1+K)(1-\mu)/2\right]n^2 \\ L_{12} &= L_{21} = -i\lambda n(1+\mu)/2 \\ L_{13} &= L_{31} = -i\left[(\mu - T_2)\lambda + K\lambda^3 - K(1-\mu)\lambda n^2/2\right] \\ L_{22} &= \left[T_1 + (1+3K)(1-\mu)/2\right]\lambda^2 + (1+T_2)\mathrm{n}^2 - \Omega^2 \\ L_{23} &= L_{32} = (1+T_2)n + Kn\lambda^2(3-\mu)/2 \\ L_{33} &= 1 + K + K\lambda^4 + 2Kn\lambda^2 + Kn^4 - (2K - T_2)\mathrm{n}^2 + T_1\lambda^2 - \Omega^2 + FL \end{aligned} \tag{22}$$

Ω is the non-dimensionless frequency, $\Omega = \sqrt{\dfrac{\rho_T R^2(1-\mu^2)}{E}}$, $\lambda = k_m R$ and FL is the fluid-loading term that for a submerged cylindrical shell is (Zhang *et al*. 2002)

$$FL = -\frac{\rho_c \omega^2}{k_r}\frac{H_n^2(k_r R)}{H_n^{'2}(k_r R)} \tag{23}$$

When the FL term is removed, the frequency equation is reduced to the uncoupled cylindrical shell case. The eigenvalues in Eq. (21) are associated with the natural frequencies.

Table 1 Material properties of the cylindrical shell and surrounding fluid

	E (N/m^2)	υ	ρ (Kg/m^3)	c (m/s)
Nickel	2.1×10^{11}	0.3	8900	-
Steel	2.1×10^{11}	0.3	7850	-
Water	-	-	1000	1500

Table 2 Comparison of natural frequencies of a clamped/clamped viscoelastic cylindrical shell between ABAQUS, Love's theory and present method (L/R=20, h/R=0.002, m=1)

Order	Modal shape	Frequency(Hz)			
		ABAQUS	Love (Zhang *et al*. 2002)	SYSNOISE (Zhang *et al*. 2002)	Present
1	(1,2)	5.00	4.95	4.92	5.21
2	(1,3)	9.62	8.95	8.95	9.98
3	(2,3)	11.22	10.66	10.66	11.36
4	(2,2)	11.39	11.54	11.54	11.64
5	(3,3)	15.18	14.73	14.73	14.98
6	(1,4)	20.58	18.26	18.26	19.01
7	(2,4)	20.96	18.71	18.71	19.47
8	(3,4)	22.10	20.00	20.00	21.1

4. Numerical result and discussion

4.1 Material properties

The material properties of the cylindrical shell and fluid as shown in Table 1. In the present study, the material is assumed to be functionally graded in the radial direction that nickel is taken at the outer surface and steel is at inner surface in the shell.

4.2 Validation

Variation of natural frequencies of a clamped/clamped viscoelastic cylindrical shell between ABAQUS, Love,s theory and present method are shown in Table 2 and Fig. 2 . According to Table 1 and Fig. 2 the results with present theory are higher than those of Love's theory and results of ABAQUS package and good agreement are seen.

4.3 Results

In Fig. 3, two frequency curves display the variations of natural frequencies of submerged and not submerged FGM cylindrical shells with the circumferential wave number n for simply supported boundary condition is shown. The axial wave number has been assumed to be equal to unity. It is clear from this figure that the frequencies of the cylindrical shell were considerably lowered by submerging it in fluid.

Table 3 shows the variations of the natural frequencies with the circumferential wave numbers n for submerged FGM cylindrical shells. The influence of the value of N, which affects the

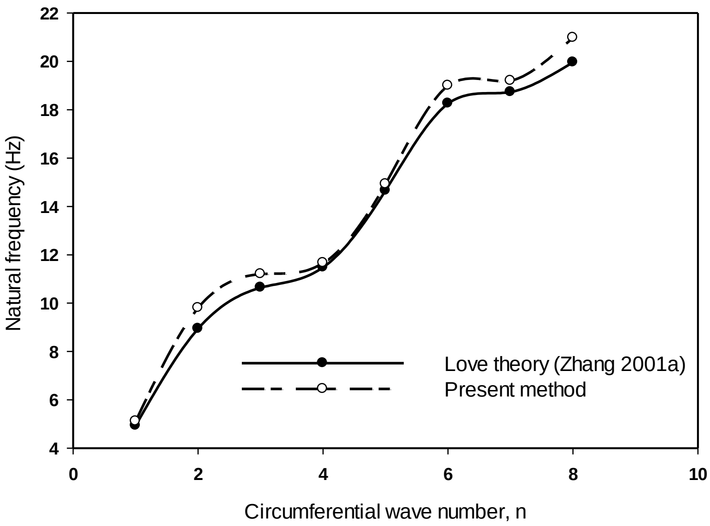

Fig. 2 Comparison of coupled natural frequencies of fluid-filled isotropic cylindrical shell between Love's theory and present method (L/R=20, h/R=0.002, m=1)

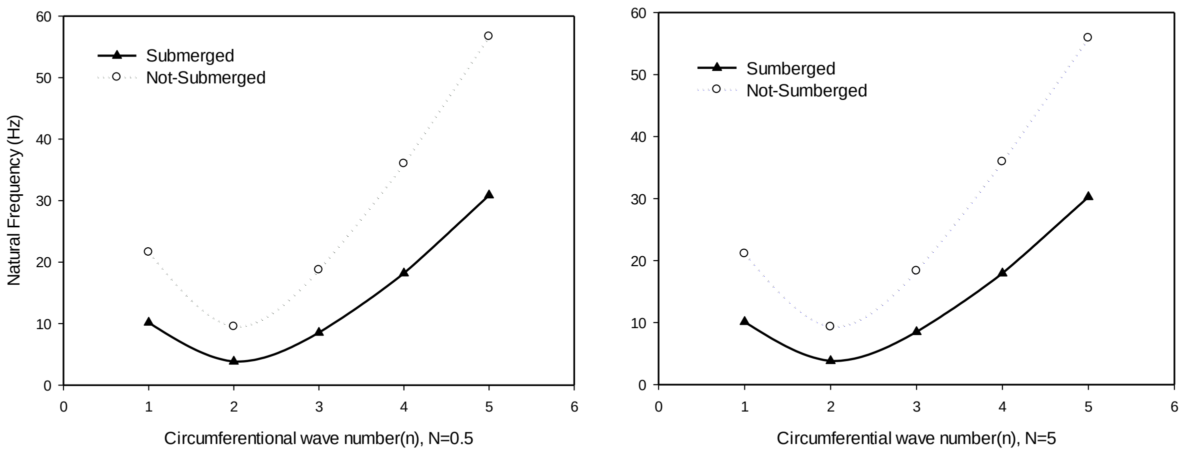

Fig. 3 Comparison of natural frequencies of submerged and not submerged thin FGM cylindrical shells for m=1, L/R=20, h/R=0.01

constituent volume fraction, can be seen in Table. 3 and can be seen as N increased, the natural frequencies decreased.

Due to results in Fig. 3 and Table 3, the difference between the frequency of not submerged and submerged FGM cylindrical shells is shown that the natural frequencies of submerged cylindrical shells are less than that of the natural frequencies of the not submerged cylindrical shells. The results shows that natural frequencies first decrease up to circumferential wave number n=2 and then increase concurrently with increase the values of n and the natural frequencies decreased when N increased.

Table 3 Comparison of natural frequencies (Hz) of submerged thin FGM cylindrical shells with circumferential wave number n for m=1, L/R=20, h/R=0.01

n	N=0.5	N=0.7	N=1	N=2	N=5	N=30
1	10.2825	10.2316	10.1768	10.1118	10.0922	9.7263
2	4.0678	3.9887	3.9511	3.8552	3.7663	2.9857
3	8.5875	8.0795	8.0369	8.0132	8.0026	7.5432
4	18.3616	18.2963	18.1233	18.0921	18.0472	17.4468
5	30.9040	30.7813	30.5945	30.4987	30.2645	29.8654

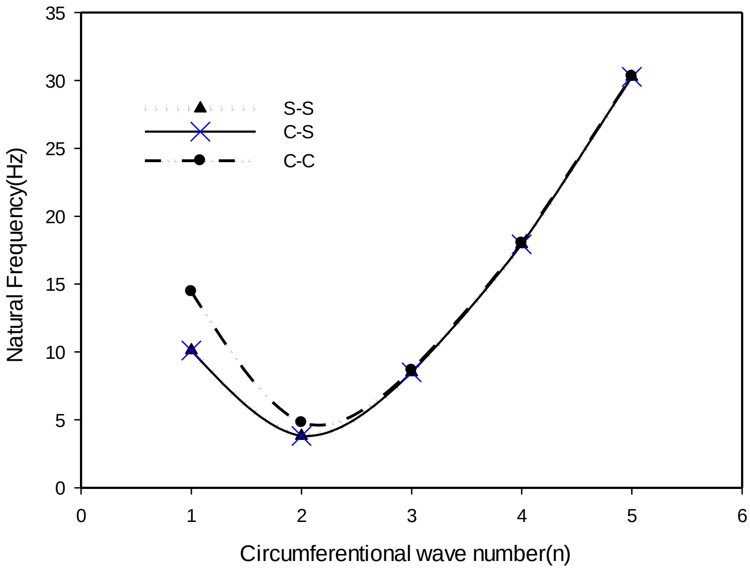

Fig. 4 Comparison of natural frequencies of submerged thin FGM cylindrical shells with diffrence boundary condition. (m=1, L/R=0, h/R=0.01, N=0.5)

In Fig. 4, variation of natural frequencies of submerged cylindrical shells have been showed with circumferential wave number n that the boundary conditions considered are simply-simply supported (S-S), clamped-clamped (C-C) and clamped-simply supported (C-S).

The results in Fig. 4 shows that the influence of boundary conditions on the shell frequencies gets more pronounced by adding the extra constraints. The effects of boundary conditions can be seen to be more significant at small circumferential mode n than at large circumferential modes that the natural frequencies of the (C-C) cylindrical shell is clearer and has the highest natural frequencies at low circumferential wave number amongst the (C-S) and (S-S) boundary conditions. So that, at high circumferential wave number, boundary conditions have little effect on the natural frequencies, So that after n=3 the results are very close together. This results are near to the results of other research (See (Zhang 2002, p. 1269)).

In Fig. 5 and Fig. 6, the natural frequencies for a submerged FGM cylindrical shell against the circumferential wave numbers n are determined for nondimensional L/R and h/R, respectively with simply-simply supported (S-S) boundary condition.

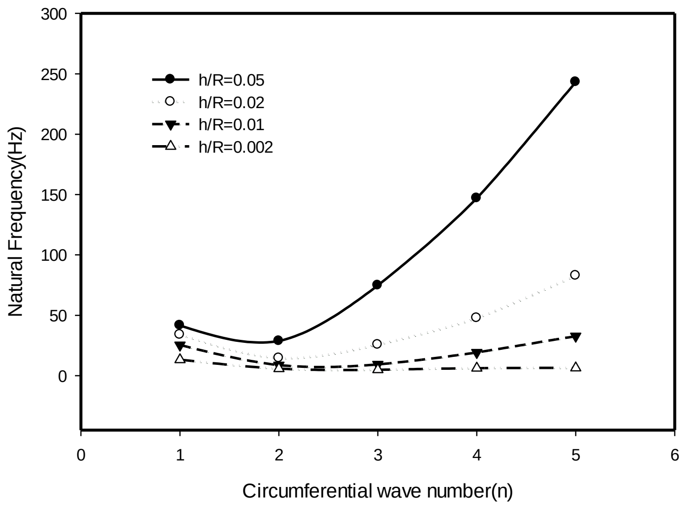

Fig. 5 Comparison of natural frequencies of submerged thin FGM cylindrical shells with for different values of *h/R* ratios with shell parameters *m*=1, *L/R*=20

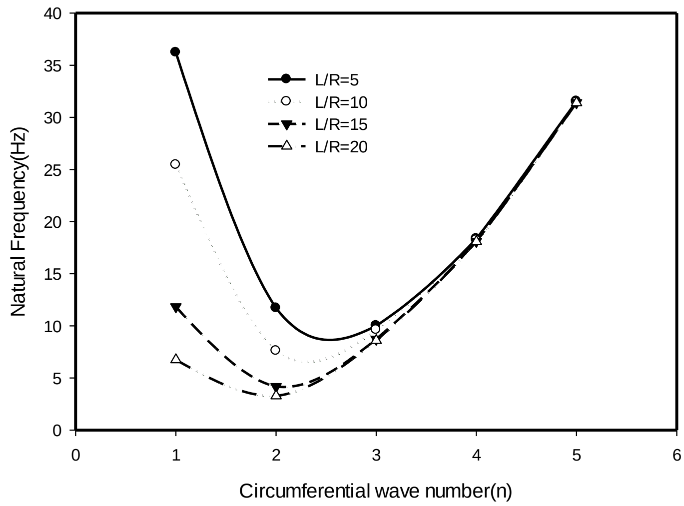

Fig. 6 Comparison of natural frequencies of submerged thin FGM cylindrical shells for different values of *L/R* ratios with shell parameters *m*=1, *h/R*=0.002

The Fig. 5 show that as h/R is decreased, the frequencies also decrease. So that in smaller ratio of thickness to radius, for example in *h/R = 0.002* the natural frequencies in subtle increase but in larger ratio of thickness to radius (especially as h/R = 0.05) from n=2 the frequencies have a further increase. This is the same behavior beholded for isotropic cylindrical shells as in the case of a shell with not sumberged in the fluid on the exterior surface of the shell. This means that the natural frequencies of thin shells are lower than that of the thick shells, which is similar to the classical results.

The Fig. 6 show that as *L/R* is increased, the frequencies decrease. In variations as ratio of length the radius, whatever *L/R* ratios increases, the natural frequencies decreases. In all of the ratio of length the radius, since *n=2* observed that the frequencies are increased , so that after *n=3* the natural frequencies have a little differences. Where *n* is increased, the natural frequencies will be closer together. This means that the natural frequencies of long shells are lower than those of short shells.

5. Conclusions

A study on the vibration of submerged functionally graded circular cylindrical shells has been presented. The analysis was carried out with first order shear deformation theory of shell motion using the wave propagation approach and classic Flügge shell equations. The natural frequencies of the system under different hydrostatic pressures are obtained by solving the coupled dispersion equation.This pressure is studied by the acoustic wave motion equation in cylindrical coordinates. A validation of the analysis has been carried out by comparing results with those in the literature and ABAQUS packege and has found to be accurate. The difference between the frequency of not submerged and submerged FGM cylindrical shells is very prominent. Natural frequencies of submerged cylindrical shells are lower than that of the natural frequencies of the not submerged cylindrical shells. The natural frequencies of long shells are lower than those of short shells, whereas natural frequencies of thin shells are lower than that of thick shells, which is similar to the classical results. The influence of the value of *N*, which affects the constituent volume fraction, can be seen from the tables. As *N* increased, the natural frequencies decreased. Furthermore, the shell frequency is affected by the variation of power law exponent so that the shell frequency decreases initially and increases with the ascending power law exponent values submerged FGM cylindrical shells.

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