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Structural joint modeling and identification: numerical and experimental investigation

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Abstract. In the present work, structural joints have been modeled as a pair of translational and rotational springs and frequency equation of the overall system has been developed using sub-structure synthesis. It is shown that using first few natural frequencies of the system, one can obtain a set of over-determined system of equations involving the unknown stiffness parameters. Method of multi-linear regression is then applied to obtain the best estimate of the unknown stiffness parameters. The estimation procedure has been developed first for a two parameter joint model and then for a three parameter model, in which cross coupling terms are also included. Two cases of structural connections have been considered, first with a cantilever beam with support flexibility and then a pair of beams connected through lap joint. The validity of the proposed method is demonstrated through numerical simulation and by experimentation.

Keywords: vibration; sub-structure synthesis; joint stiffness identification; linear parameters; multi-linear regression; cantilever; single lap joint

1. Introduction

Many mechanical structures can be seen as an assembly of subsystems. These substructures or subsystems usually are connected to each other by joints such as bolts, welds, rivets etc. Modeling of composite structures has become a challenging task due to uncertainty in system parameters, particularly those associated with structural joints. It is widely accepted that the behavior of the whole structure can be significantly affected by the way joints are modeled. To conduct an accurate dynamic analysis, it is first necessary to model the joints accurately and then identify their structural parameters. Much work has been done to extract joint properties from measurement data.

In an effort to model joints in a structure, various methods are suggested by the researchers. Most of the methods are based on FRF (Frequency Response Function) measurements and modal parameters. The most popular method of identifying joint structural parameters is to use modal parameters which have been obtained experimentally. Joints have significant effects on the dynamic response of the assembled structures due to existence of two non-linear mechanisms in

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their interface, namely slipping and slapping. These mechanisms affect the structural response by adding considerable damping into the structure and lowering the natural frequencies due to the stiffness softening (Ahmadian *et al.* 2007). Tsai and Chou (1988) proposed a formulation based on receptance method to calculate the properties of a single bolt joint directly from the measured FRFs of a structure. Yang and Park (1993) demonstrated a method of identifying joint structural parameters using subset FRF measurements. Mottershed *et al.* (1996) applied updating of geometric parameters in a cantilever plate. However parameterization of joints and boundary conditions was not carried out. Hwang (1998) derived the identification method for stiffness parameters of connections between structures from measured FRF data. The estimation is carried out for continuous beam with only translational stiffness as the joint parameter. The linearised joint structural parameters are then identified by minimizing the loss function derived from measured and estimated FRFs. A method for joint stiffness determination has been developed by Patricia *et al*. (1999), based upon rigid-body dynamics and FRF measurements. Stiffness components in six coordinate directions were estimated.

A T-shaped structure was considered by Kim and Park (1997), for joint stiffness identification from selected degrees of freedom. The natural frequencies estimated with identified joint parameters were compared with exact values. The updated natural frequencies using identified joint stiffness were close to the measured natural frequencies. Ratcliff and Lieven (2000) examine a technique that calculates the properties of structural joints by minimizing the difference between substructure FRFs and assembly FRFs. A technique which relies on the comparison of the overall dynamics of the bolted structure to that of a similar but unbolted one was presented by Ma *et al.* (2001).

A non parametric model for the joint dynamics was proposed by Wu and Li (2006), assuming that the difference in the dynamics of the two systems is attributed to the joint. An eigensensitivity-based FE (Finite Element) model updating was developed for identification of the structural parameters, in which connection stiffness of semi-rigid joints was estimated through FE model updating and in FE updating procedure it was assumed that the measured natural frequencies are more reliable than measured mode shapes. Celic *et al.* (2008) presented a method for establishing a theoretical model of a joint from the substructures and assembly FRF data. A non parametric model was used in the joint identification. Sjovall and Thomas (2008) presented a procedure for substructure identification using test data from larger system. The procedure was applied to real test data. An accurate non-parametric model was identified. A Finite element based analytical model has the advantage of being complete and precise. On the other hand, the experimental data are generally considered to be more accurate given the availability of reliable data acquisition and measuring equipment (Modak *et al.* 2002, Wang *et al.*2012). Domínguez and Pérez-Mota (2014) studied how the quantity and distribution of the steel reinforcement array inside the RC beam-column connection affects its structural response when a strong cyclic loading is applied. Asgarian *et al.* (2014) derived the equations which can be used in global analyses of offshore structures to account for the Local Joint Flexibility (LJF) effects on overall behavior of the structure.

In the present work, a simple and efficient method based on sub structure synthesis for identification of boundary condition parameters is proposed, in which only first few measured natural frequencies are used in the identification procedure. The method is demonstrated for a cantilever beam with two parameter (2-P) and three parameter (3-P) joint model and then for a single lap jointed (SLJ) beam, in which the joint is modeled with spring stiffness only. The numerical simulation has been carried out with non-dimensional stiffness parameters so that the results are applicable for any size or dimension of the actual test specimen. An experimental identification for real test structure is also presented using measured natural frequencies; the estimated joint parameters are then incorporated in the FE model. The FE model updated with the joint parameters gave natural frequencies in close agreement with the measured frequencies.

2. Sub-structure synthesis of assembled systems

Structural and machine systems are often assembled from several sub-systems. Although the sub-structure dynamics may be well known, dynamics of the assembled system cannot be predicted accurately unless the interface conditions are properly modeled and identified. The concept of substructure synthesis (Bishop and Johnson 1960) provides a technique for obtaining the dynamic behavior of the assembled system from the knowledge of Frequency Response Functions (FRFs) of the sub-systems interfaced through one or more degrees of freedom. As shown in Fig. 1(a), a composite system Z is assembled from two sub systems P and R connected by two joint co-ordinates X and θ at the interface. The forcing functions corresponding to these two co-ordinates are F and M respectively.

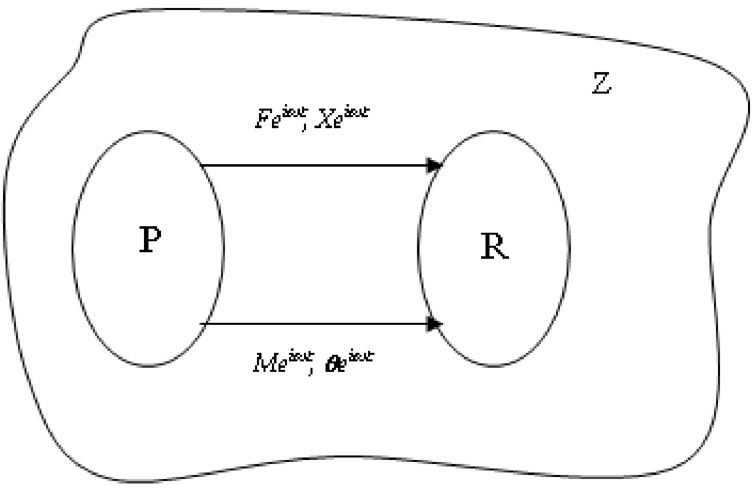


Fig. 1(a) A composite system Z, with sub-systems P and R

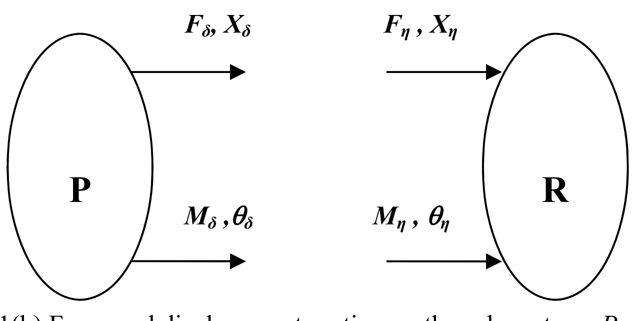


Fig. 1(b) Force and displacements acting on the sub-systems P and R

Fig. 1(b) shows the sub-systems separated with corresponding displacements and forcing functions at the interface.

Let the direct receptance functions δ_{ii} and η_{ii} and cross receptance functions δ_{ii} and η_{ii} for the two sub-systems be defined as

$$\delta_{11}=\frac{X_\delta}{F_\delta} \quad ; \quad \delta_{22}=\frac{\theta_\delta}{M_\delta}; \qquad \delta_{12}=\frac{X_\delta}{M_\delta}; \qquad \delta_{21}=\frac{\theta_\delta}{F_\delta} \tag{1a}$$

$$\eta_{11}=\frac{X_\eta}{F_\eta} \quad ; \quad \eta_{22}=\frac{\theta_\eta}{M_\eta}; \qquad \eta_{12}=\frac{X_\eta}{M_\eta}; \qquad \eta_{21}=\frac{\theta_\eta}{F_\eta} \tag{1b}$$

Then, considering the dynamic equilibrium of the sub-systems *P* and *R* separately, one can obtain

$$X_\delta = \delta_{11}F_\delta + \delta_{12}M_\delta \quad ; \quad X_\eta = \eta_{11}F_\eta + \eta_{12}M_\eta \tag{2a}$$

$$\theta_\delta = \delta_{21}F_\delta + \delta_{22}M_\delta \quad ; \quad \theta_\eta = \eta_{21}F_\eta + \eta_{22}M_\eta \tag{2b}$$

Eqs. (2a)-(2b) can be written in a matrix form as

$$\begin{Bmatrix} X_\delta \\ \theta_\delta \end{Bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{Bmatrix} F_\delta \\ M_\delta \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} X \\ \theta \end{Bmatrix}_\delta = [\delta] \begin{Bmatrix} F \\ M \end{Bmatrix}_\delta \tag{3}$$

$$\begin{Bmatrix} X_\eta \\ \theta_\eta \end{Bmatrix} = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} \begin{Bmatrix} F_\eta \\ M_\eta \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} X \\ \theta \end{Bmatrix}_\eta = [\eta] \begin{Bmatrix} F \\ M \end{Bmatrix}_\eta \tag{4}$$

For the assembled system *Z*, the direct and cross receptance functions can similarly be defined as

$$v_{11} = \frac{X}{F} \quad ; \quad v_{22} = \frac{\theta}{M}; \qquad v_{12} = \frac{X}{M}; \qquad v_{21} = \frac{\theta}{F} \tag{5}$$

which gives

$$\begin{Bmatrix} X \\ \theta \end{Bmatrix} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \begin{Bmatrix} F \\ M \end{Bmatrix} \quad \text{or} \quad \begin{Bmatrix} X \\ \theta \end{Bmatrix} = [v] \begin{Bmatrix} F \\ M \end{Bmatrix} \tag{6}$$

Now, compatibility requirement of forces and displacements at the joint co-ordinates gives

$$X = X_\delta = X_\eta \qquad \text{and} \qquad \theta = \theta_\delta = \theta_\eta \tag{7a}$$

$$F = F_\delta + F_\eta \qquad \text{and} \qquad M = M_\delta + M_\eta \tag{7b}$$

which gives

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{Bmatrix} F \\ M \end{Bmatrix}_\delta + \begin{Bmatrix} F \\ M \end{Bmatrix}_\eta = \left\{ [\delta]^{-1} + [\eta]^{-1} \right\} \begin{Bmatrix} X \\ \theta \end{Bmatrix} \tag{8}$$

$$[v] = \left\{ [\delta]^{-1} + [\eta]^{-1} \right\}^{-1} \tag{9}$$

Simplifying Eq. (9) with Eq. (3) and Eq. (4), FRF matrix $[v]$ is obtained as

$$v_{11} = \frac{\delta_{11}(\eta_{11}.\eta_{22} - \eta_{12}{}^2) + \eta_{11}(\delta_{11}.\delta_{22} - \delta_{12}{}^2)}{(\delta_{11} + \eta_{11})(\delta_{22} + \eta_{22}) - (\delta_{12} + \eta_{12})^2}$$

$$v_{22} = \frac{\delta_{22}(\eta_{11}.\eta_{22} - \eta_{12}{}^2) + \eta_{22}(\delta_{11}.\delta_{22} - \delta_{12}{}^2)}{(\delta_{11} + \eta_{11})(\delta_{22} + \eta_{22}) - (\delta_{12} + \eta_{12})^2}$$

$$v_{12} = \frac{\delta_{12}(\eta_{11}.\eta_{22} - \eta_{12}.\delta_{12}{}^2) + \eta_{12}(\delta_{11}.\delta_{22} - \eta_{12}.\delta_{12}{}^2)}{(\delta_{11} + \eta_{11})(\delta_{22} + \eta_{22}) - (\delta_{12} + \eta_{12})^2} = v_{21}$$

The frequency equation for the assembled system is obtained by setting the denominator of the receptance functions as zero, which gives

$$(\delta_{11} + \eta_{11})(\delta_{22} + \eta_{22}) - (\delta_{12} + \eta_{12})^2 = 0 \tag{10}$$

Thus if the FRFs at the interface co-ordinates are known for the individual sub-systems, then the natural frequencies of the assembled system can be obtained by solving the frequency Eq. (10). If sub-system P is a joint sub-system and sub-system R is a structural component, then Eq. (10) provides a basis to study the effect of joint parameters on the system natural frequencies. However, Eq. (10) is exploited for an inverse analysis, where one can estimate the joint parameters from the over determined set of measured frequency data, using multi-linear regression technique (Draper and Smith 1998).

In the following sections, two types of structural joint systems are demonstrated with numerical simulation and experimentation. First, a cantilever beam with two models of the boundary condition so called, 2-parameter and 3-parameter model,

Model 1: $[P] = \begin{bmatrix} K_t & 0 \\ 0 & K_r \end{bmatrix}$, 2-parameter model.

Model 2: $[P] = \begin{bmatrix} K_t & K_{tr} \\ K_{rt} & K_r \end{bmatrix}$, 3-parameter model.

and then a single lap jointed structural system has been demonstrated.

2.1 Two parameter joint model of cantilever beam

In a two parameter joint system, a cantilever beam is modeled with elastic constraint at the fixed end represented by a translational spring (stiffness K_t) and a rotational spring (stiffness K_r) as shown in Fig. 2, the beam is considered as a composite system consisting of elastic springs as sub-system P and a free-free beam as sub-system R.

The FRF matrix for P is represented by a diagonal matrix

$$[\delta] = \begin{bmatrix} 1/K_t & 0 \\ 0 & 1/K_r \end{bmatrix} \tag{11}$$

and the FRF matrix for R can be obtained as (Bishop 1960)

$$[\eta] = \begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} \tag{12}$$

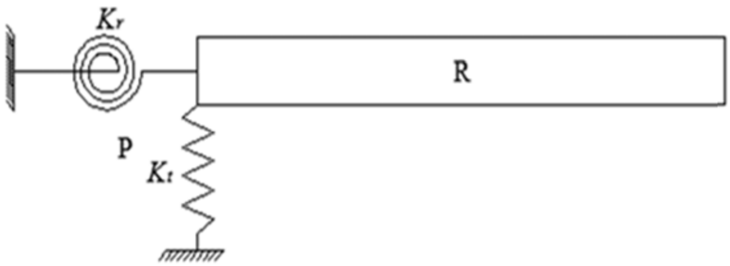


Fig. 2 Cantilever beam modeled with elastic support at fixed end

Where,

$$\eta_{11} = \left(\frac{-L^3}{EI}\right)\left(\frac{1}{\lambda^3}\right)\left(\frac{F_5}{F_3}\right) \quad ; \quad \eta_{22} = \left(\frac{L}{EI}\right)\left(\frac{1}{\lambda}\right)\left(\frac{F_6}{F_3}\right)$$

$$\eta_{12} = \left(\frac{L^2}{EI}\right)\left(\frac{1}{\lambda^2}\right)\left(\frac{F_1}{F_3}\right).$$

Where,

$$F_1 = sin(\lambda).sinh(\lambda) \quad ; \quad F_3 = cos(\lambda).cosh(\lambda) - 1$$

$$F_5 = cos(\lambda).sinh(\lambda) - sin(\lambda).cosh(\lambda) \quad ; \quad F_6 = cos(\lambda).sinh(\lambda) + sin(\lambda).cosh(\lambda)$$

Further defining the joint stiffness through non-dimensional parameters, the frequency Eq. (10) for a composite system shown in Fig. 2 can be obtained as

$$K_x.K_y + K_x\lambda\frac{F_5}{F_4} - K_y.\lambda^3.\frac{F_6}{F_4} - \lambda^4\frac{F_3}{F_4} = 0 \tag{13}$$

Where,

$$F_4 = 1 + cos(\lambda)cosh(\lambda) \quad ; \quad K_x = \frac{K_t}{(EI/L^3)} \quad ; \quad K_y = \frac{K_r}{(EI/L)} \quad \text{and} \quad \lambda = \left(\frac{\omega^2 \rho A L^4}{EI}\right)^{1/4}.$$

Now for each measured natural frequency, $\lambda_i = \omega_i^2$, Eq. (13) presents a nonlinear relationship between the unknowns K_x, K_y and λ_i. If we experimentally measure 'n' natural frequencies ω_i (i=1,2…..n), and λ_j, λ_k be the values corresponding to ω_j and ω_k (j, k =1,2….n; $j \neq k$), then, Eq. (13) gives

$$K_x.K_y + K_x.\lambda_j\left(\frac{F_5}{F_4}\right)_{\lambda=\lambda_j} - K_y.\lambda_j^3\left(\frac{F_6}{F_4}\right)_{\lambda=\lambda_j} - \lambda_j^4\left(\frac{F_3}{F_4}\right)_{\lambda=\lambda_j} = 0 \tag{14}$$

$$K_x.K_y + K_x.\lambda_k\left(\frac{F_5}{F_4}\right)_{\lambda=\lambda_k} - K_y.\lambda_k^3\left(\frac{F_6}{F_4}\right)_{\lambda=\lambda_k} - \lambda_k^4\left(\frac{F_3}{F_4}\right)_{\lambda=\lambda_k} = 0 \tag{15}$$

Subtracting Eq. (15) from Eq. (14), one obtains

$$K_x.\left\{\lambda_j\left(\frac{F_5}{F_4}\right)_{\lambda=\lambda_j}-\lambda_k\left(\frac{F_5}{F_4}\right)_{\lambda=\lambda_k}\right\}+K_y\left\{\lambda_k^{\,3}\left(\frac{F_6}{F_4}\right)_{\lambda=\lambda_k}-\lambda_j^{\,3}\left(\frac{F_6}{F_4}\right)_{\lambda=\lambda_j}\right\}$$
$$=\lambda_j^{\,4}\left(\frac{F_3}{F_4}\right)_{\lambda=\lambda_j}-\lambda_k^{\,4}\left(\frac{F_3}{F_4}\right)_{\lambda=\lambda_k} \tag{16}$$

Which can written, using brief notation, as

$$T_i.K_x+U_i.K_y=V_i \tag{17}$$

Above equation is generated for each combination of (j, k), which gives total ${}^nC_2$ equations and thus a set of over determined system of linear equations is generated for estimation which can be used for a least square error based estimation. Taking $r={}^nC_2$, Eq. (17) can be written as

$$\begin{bmatrix} T_1 & U_1 \\ T_2 & U_2 \\ ------- & \\ T_r & U_r \end{bmatrix}\begin{Bmatrix} K_x \\ K_y \end{Bmatrix}=\begin{bmatrix} V_1 \\ V_2 \\ - \\ V_r \end{bmatrix} \quad \text{or} \quad [TU]\{\hat{K}\}=[V] \tag{18}$$

There can be different sources of error in the measured natural frequency values, mainly spectral resolution error in the measuring FFT analyzer. Besides there would be harmonic distortion error in case the stiffness is nonlinear and random noise. For weakly nonlinear stiffness values, harmonic distortion in the measurement of first harmonic frequency will not be significantly high. However random background noise and spectral resolution problem still remain. Spectral resolution can be enhanced by selecting larger data block size and the same has been discussed in later section 3.3. Random error is minimized using least square error estimation method (Norman and Smith 1998). Eq. (18) including the error term can be written as

$$[TU]\{\hat{K}\}+\{\varepsilon\}=\{V\}$$

Minimum error estimation gives

$$\{\hat{K}\}=pinv(TU).V \tag{19}$$

where, *pinv* (Gilot 2003) is the generalized inverse of a matrix.

2.2 Three-parameter joint model of cantilever beam

A cantilever beam with boundary condition (fixed end) modeled as an elastic support consisting of a translational, rotational and cross coupled terms, thus designated as 3-parameter model is shown in Fig. 3. The concept of substructure synthesis, discussed in previous section is used to derive the frequency equation of the composite system consisting of a free-free beam interfaced with a joint at one end. A 3-parameter model i.e., Model 2 of joint is represented by a (2×2) matrix which include cross coupled terms K_{tr} and K_{rt}

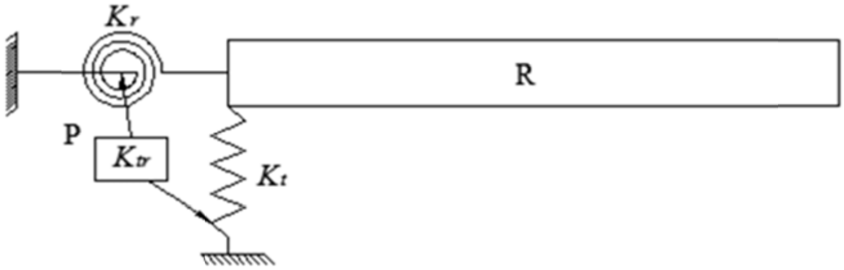


Fig. 3 Cantilever beam modeled with elastic support at fixed end (model 2)

The FRF matrix for joint system P of Model 2 represented by a matrix $[\delta]^* = \begin{bmatrix} 1/K_t & 1/K_{tr} \\ 1/K_{rt} & 1/K_r \end{bmatrix}$.

Now, from FRF matrix $[\delta]^*$ and $[\eta]$, frequency equation, Eq. (20), for 3-parameter joint model of cantilever beam can be obtained from Eq. (10) as

$$\frac{1}{K_x}\left\{\frac{1}{\lambda}\left(\frac{F_6}{F_3}\right)\right\} - \frac{1}{K_y}\left\{\frac{1}{\lambda^3}\left(\frac{F_5}{F_3}\right)\right\} - \frac{1}{K_z}\left\{\frac{2}{\lambda^2}\left(\frac{F_1}{F_3}\right)\right\} = \left\{\frac{1}{\lambda^4}\left(\frac{F_4}{F_3}\right)\right\} - \left(\frac{1}{K_x K_y} - \frac{1}{K_z^2}\right) \tag{20}$$

Where,

$K_x = \dfrac{K_t}{(EI/L^3)}$; $K_y = \dfrac{K_r}{(EI/L)}$ and $K_z = \dfrac{K_{tr}}{(EI/L^2)} = \dfrac{K_{rt}}{(EI/L^2)}$ are non-dimensional stiffness parameters.

Following similar derivation as given for two parameter model, one obtains

$$\frac{1}{K_x}U_i^* + \frac{1}{K_y}T_i^* + \frac{1}{K_z}S_i^* = V_i^* \tag{21}$$

Where the subscript i represents each combination of j and k. Eq. (21) is a linear equation in three unknowns K_x, K_y and K_z; T_i^*, U_i^*, V_i^* and S_i^* are the coefficients which can be computed from the measured natural frequencies. Eq. (21) can be processed through multi linear regression to get a least square error estimate of the unknown joint stiffness parameters.

2.3 Numerical simulation: 2-parameter and 3-parameter model of cantilever beam

Characteristic equation for 2-parameter and 3-parameter model with unknown non-dimensional stiffness parameters is given in Eq. (13) and Eq. (20) respectively and is used in numerical simulation. A computer program for FE model of cantilever beam is developed in MATLAB, for sample values of K_x, K_y in case of Model 1 and K_x, K_y and K_z in case of Model 2. FE modeling has not been presented here for the sake of brevity. Further a ratio $\alpha = K_z/(K_x.\ K_y)$ was defined to indicate the relative strength of the cross coupling terms in the joint matrix. Simulation results are presented in Table 1(a)-(b). It can be observed from simulation results presented in Table 1a that, if the system does not have the presence of cross coupling then both 2-parameter and 3-parameter

Table 1a Simulation results with sample values of non-dimensional stiffness parameters (model 1)

Sample values of K_x and K_y	Non dimensional natural frequency	Estimated K_x and K_y with 2-P	Estimated K_x , K_y and K_z with 3-P
K_x = 100 K_y = 50	f_{r1} =1.8066 f_{r2} = 3.7678 f_{r3} = 5.8097 f_{r4} = 8.6564 f_{r5} = 11.7153	K_x = 99.95 K_y = 50.03	K_x = 100 K_y = 50.07 K_z = -8.11.10^5
K_x = 500 K_y = 100	f_{r1} =1.8500 f_{r2} = 4.4368 f_{r3} = 6.8022 f_{r4} = 9.0929 f_{r5} = 11.915	K_x = 500.154 K_y = 100.270	K_x = 499.99 K_y = 100.24 K_z= -5.67.10^6
K_x = 1000 K_y = 500	f_{r1} = 1.8679 f_{r2} = 4.5755 f_{r3} = 7.2837 f_{r4} = 9.5658 f_{r5} = 12.146	K_x= 1018.64 K_y = 517.310	K_x = 1000.82 K_y = 510.06 K_z = -2.22.10^5

Table 1b Simulation results with sample values of non-dimensional stiffness parameters (model 2)

Sample values of K_x, K_y & K_z	Non dimensional natural freq.	Estimated K_x & K_y with 2-P	Estimated K_x , K_y & K_z with 3-P
K_x = 1000 K_y = 500 K_z = 10000 (α=0.02)	f_{r1} = 1.8684 f_{r2} = 4.5798 f_{r3} = 7.2930 f_{r4} = 9.5680 f_{r5} = 12.141	K_x = 408.75 K_y = 223.47	K_x = 999.85 K_y = 506.28 K_z = 10038.73
K_x = 500 K_y = 100 K_z = 5000 (α=0.1)	f_{r1} = 1.8510 f_{r2} = 4.4443 f_{r3} = 6.8100 f_{r4} = 9.0876 f_{r5} = 11.9082	K_x= 380.27 K_y = 77.73	K_x = 500.034 K_y = 100.363 K_z = 5021.04
K_x = 1000 K_y = 500 K_z = 500000 (α=1)	f_{r1} = 1.8669 f_{r2} = 4.5751 f_{r3} = 7.2832 f_{r4} = 9.5658 f_{r5} = 12.1463	K_x= 987.51 K_y= 500.61	K_x = 1000.28 K_y = 506.96 K_z= 3.423.10^6
K_x = 1000 K_y = 500 K_z=1000000 (α=2)	f_{r1} = 1.8679 f_{r2} = 4.5755 f_{r3} = 7.2838 f_{r4} = 9.5658 f_{r5} = 12.1463	K_x = 975.9 K_y= 500.47	K_x = 1000.4 K_y = 512.32 K_z = 1.22.10^6

gives good estimates of K_x and K_y ; however estimated K_z gives unrealistic values which may confirms non presence of cross coupled parameter in the system

It can be concluded from simulation results presented in Table 1b, that as α increases, cross-coupling stiffness value also increases and the natural frequencies of 3-parameter model shifts towards the values of 2-parameter model, which means that the 3-parameter model with large α is almost same as the 2-parameter model. Stiffness parameters are estimated here from the so called measured frequency data. Table 1b shows the estimates for different cross coupling ratios. It can be seen that estimates for direct stiffness parameters are in general very good but that for the cross coupling term is good only when α is less than 1. One can thus conclude that a 3-parameter model would achieve good estimate only when cross coupling stiffness is small or for α<1, for α close to or higher than 1, it is better to adopt 2-parameter model.

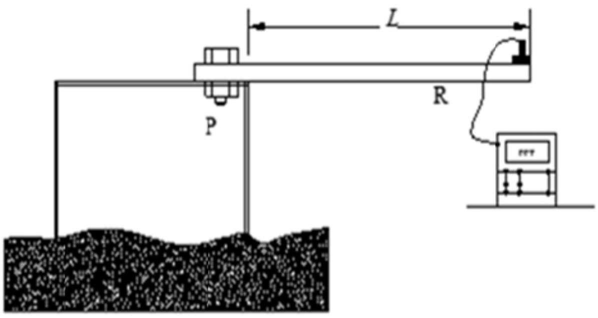
3. Experimental case: cantilever beam

A test set up for measuring the vibration response of a cantilever beam clamped with a bolt to a structure is shown in Fig. 4. The beam was excited through impulse and free vibration response was measured using B&K 4370 (sensitivity 100 pC/g; mass 24 gm) accelerometer and FFT analyzer (DI-22, Diagnostic Instruments, UK). A data block size of 2048 samples with 800 spectral lines was used to measure the natural frequencies of beam. The geometrical and material properties of beam are listed in Table 2.

Table 2 Dimensions and material properties of beam

Dimensions	Material Properties
A: (0.05×0.005) m^2	Elastic modulus, E : 207 GPa
L: 0.423 m	Density, ρ : 7850 Kg/m^3

(a)



(b)

Fig. 4 (a) Experimental setup (b) Schematic diagram of beam with bolted joint

Table 3 Comparison of natural frequency of beam under free-free condition

Natural frequencies	f_{r1}	f_{r2}	f_{r3}	f_{r4}
Measured	105	290.25	571.25	944
FEM	105.57	291.01	571.49	944.1

Table 4 Measured natural frequencies of cantilever beam clamped with bolt

Mode	Measured natural frequency (Hz)	Non-dimensional natural frequency
f_{r1}	17	1.6067
f_{r2}	118	4.2335
f_{r3}	335.625	7.1398
f_{r4}	628.75	9.7724
f_{r5}	994.5	12.2904

The values of the material parameters are obtained by testing a beam under free-free condition and updating the corresponding Finite Element model for measured natural frequencies. Two dimensional beam elements were used in the FE modeling, a program was developed in MATLAB to compute natural frequencies of free-free beam, density of the beam was obtained by measuring its mass and modulus of elasticity was obtained by updating its value in FE model to validate the measured natural frequencies.

Table 3 shows the measured and computed (FEM) natural frequencies of the beam with updated material properties, under unconstrained (free-free) condition. The vibration response of cantilever beam clamped with bolts is measured using FFT analyzer; to improve the accuracy in measurement, different frequency bands were selected in FFT analyzer e.g., for measurement of first two natural frequencies, a frequency band of 0-200 Hz and for higher frequencies a frequency band of 0-1000 Hz was selected.

The measured natural frequencies at the resonance peaks of frequency spectrum are given in Table 4. These measured natural frequencies are less by more than 5% as compared to ideal natural frequencies of cantilever beam; so, one may predict the joint stiffness to fall in sensitive region (i.e., non-stiff region).

3.1 Joint stiffness identification: 2-P model

The linear parameters of the joint, i.e., translational stiffness K_t and rotational stiffness K_r are identified using first four measured frequencies only. The support parameters are most important in lower modes and are less significant in higher modes (Ahmadian *et al.* 2001), for these reason first four modes were chosen in the identification procedure. Linear equation in two unknowns K_x and K_y is obtained from Eq. (13); Eq. (19) was then used for best estimate of non dimensional stiffness parameters. The estimated non dimensional stiffness parameters and corresponding linear joint parameters are given in Table 5. Next, the identified joint parameters are incorporated in FE model and natural frequencies are computed; these frequencies are compared with experimentally measured natural frequencies, Table 6 shows the comparison.

Considerable error in measured and computed natural frequencies is observed, the reasons for error in natural frequencies, may be due to insufficient finite elements in FE modeling & over simplification of joint model (Model 1, 2-P model) where cross coupling terms are neglected. The

Table 5 The identified joint parameters (2-p)

Non-dimensional joint stiffness parameters	Joint stiffness parameters
$K_x = 969.8064$	$K_t = 1.381\times10^6$ (N.m^{-1})
$K_y = 1.2723$	$K_r = 324.28$ (N.m.rad^{-1})

Table 6 Comparison of natural frequencies measured experimentally and computed with updated FE model

Mode	Experimentally Measured (Hz)	Computed from FE Model (Hz)	Percentage Error
f_{r1}	17	11.276	33.67
f_{r2}	118	106.03	10.14
f_{r3}	335.625	315.44	6.01
f_{r4}	628.75	599.07	4.72
f_{r5}	994.5	941.52	5.32

Table 7 The identified joint parameters (3-p)

Non-dimensional joint stiffness parameters	Joint stiffness parameters
$K_x = 1928.972$	$K_t = 2.75\times10^6$ (N.m^{-1})
$K_y = 4.6964$	$K_r = 1196.99$ (N.m.rad^{-1})
$K_z = 876.731$	$K_{rt} = 5.283\times10^5$ (N.rad^{-1})

Table 8 Comparison of natural frequencies measured experimentally and computed with updated FE model

Mode	Experimentally Measured (Hz)	Computed from FE Model (Hz)	Percentage Error
f_{r1}	17	16.96	0.23
f_{r2}	118	117.56	0.37
f_{r3}	335.625	337.7	-0.618
f_{r4}	628.75	635.77	-1.11
f_{r5}	994.5	1011.3	-1.68

first reason could be excluded easily by increasing number of finite elements; the second reason was studied by using joint Model 2 i.e., 3-P model.

3.2 Joint stiffness identification: 3-P model

The measured natural frequencies given in Table 4, are now used in Eqs. (20)-(21) to estimate three parameters of the joint. The identified joint parameters are presented in Table 7.

These identified joint parameters are again incorporated in FE model and computed natural frequencies are compared with experimentally measured natural frequencies (see Table 8).The FE model updated with identified joint parameters using 3-parameter model gives better estimate of natural frequencies than 2-parameter model, the ratio α defined in previous section has the value <1 for the joint demonstrated in this paper. Since α<1, 3-P gives better estimates of joint parameters than 2-P, it is evident from simulation and experimentation results. Although only first four measured natural frequencies are considered in regression analysis, the fifth measured natural frequency is also close to natural frequency computed with FE model.

Table 9a Estimation error in the identified joint parameters with ±0.05% frequency perturbation

Non-dimensional Joint Parameters	K_x	K_y	K_z
Identified Values	1928.972	4.6964	876.7310
Estimated with +0.05% frequency perturbation	1875.211	4.7024	840.7858
Estimated with -0.05% frequency perturbation	1940.823	4.6772	909.1771
Average percentage (%) error	1.7	0.268	3.9

Table 9b Estimation error in the identified joint parameters with ±0.1% frequency perturbation

Non-dimensional Joint Parameters	K_x	K_y	K_z
Identified Values	1928.972	4.6964	876.7310
Estimated with +0.1% frequency perturbation	1844.136	4.7152	822.3736
Estimated with -0.1% frequency perturbation	1983.690	4.6651	923.1977
Average percentage (%) error	3.6	0.53	5.75

3.3 Error sensitivity analysis

The estimates of K_x K_y and K_z presented in Table 7 are obtained with data without any measurement error. However, for practical measurement of natural frequencies, effect of measurement noise also needs to be considered. Natural frequencies are generally measured using FFT analyzers. The major source of error with these instruments is spectral resolution error, which depends on data block size for FFT processing. For a typical 2048 data block size, 800 spectral lines are displayed in the given frequency range f_r, which means two adjacent frequency data will be separated by f_r /800. The error in the measurement then becomes f_r /(2*800)=0.0625%. With a data block size of 4096, there will be 1600 spectral lines in the frequency range and measurement error will be f_r /(2*1600)=0.03125%. Over and above this resolution error, there may be random noise also. Hence the frequency values measured experimentally are perturbed by ±0.05% and ±0.1% to test the effect of measurement error on the joint stiffness estimation. The estimation with perturbed natural frequencies is presented for experimental case in Tables 9(a)-(b).

The estimated error in the identified joint stiffness parameter is within 6%. As the cross coupling stiffness K_z is more sensitive to the natural frequencies of the joint system the error is relatively more compared to that of translational and rotational stiffness.

4. A Single Lap Jointed (SLJ) beam

In a SLJ beam demonstrated in this section, a joint is modeled as an elastic support consisting of translational and a rotational spring. The concept of substructure synthesis, discussed in the previous section, is used to derive the frequency equation of the composite system consisting of two free-free beams; one embedded with springs K_1 and K_2. An over determined system of linear equations involving the unknown joint parameters are formulated from the frequency equation using a set of measured natural frequency data. The equations are solved for best estimation of the support parameters using multi-linear regression procedure.

4.1 Two parameter model of SLJ

In a two parameter joint system, the SLJ beam is modeled with elastic constraint, represented by a translational spring (stiffness K_1) and a rotational spring (stiffness K_2) as shown in Figs. 5(a)-(b). The beam is considered as a composite system consisting of a free-free beam embedded with translational & rotational spring as sub-system P and other free-free beam as sub-system R as shown in Fig. 5(b). The elements of FRF matrix [δ] for sub system P are,

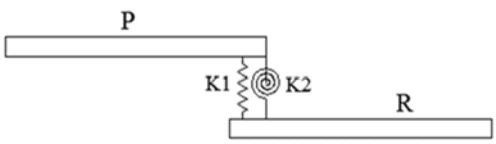
$$\delta_{11} = \frac{1}{K_1} - \left(\frac{L^3}{EI}\right)\left(\frac{1}{\lambda^3}\right)\left(\frac{F_5}{F_3}\right)$$

$$\delta_{22} = \frac{1}{K_2} + \left(\frac{L}{EI}\right)\left(\frac{1}{\lambda}\right)\left(\frac{F_6}{F_3}\right)$$

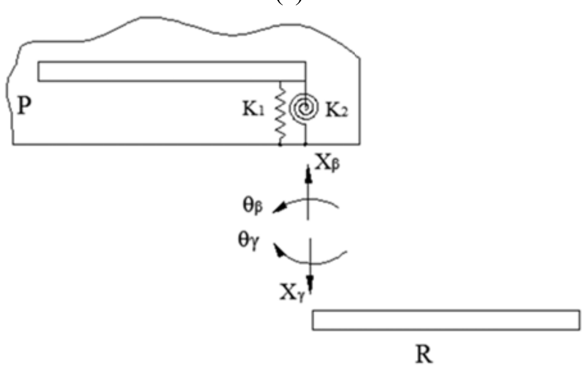
$$\delta_{12} = \left(\frac{L^2}{EI}\right)\left(\frac{1}{\lambda^2}\right)\left(\frac{F_1}{F_3}\right) = \delta_{21}$$

The elements of FRF matrix [η] for sub system R are same as given in previous section. Now, simplifying frequency Eq. (10) by substituting direct and cross receptances of substructure P and R, gives

$$\begin{aligned} &-4\left(\frac{L^2}{EI}\right)^2\left(\frac{1}{\lambda^4}\right)K_1.K_2 - 2\left(\frac{L^3}{EI}\right)\left(\frac{1}{\lambda^3}\right)\left(\frac{F_3}{F_6}\right)K_1 \\ &+2\left(\frac{L}{EI}\right)\left(\frac{1}{\lambda}\right)\left(\frac{F_3}{F_5}\right)K_2 + \frac{F_3{}^2}{F_5F_6} = 0 \end{aligned} \tag{22}$$



(a)



(b)

Fig. 5 (a) Modeling of single lap joint (b) Sub-systems P and R of single lap jointed beam

The joint stiffness parameters can be represented as non-dimensional parameters as,

$$K_x = \frac{K_1}{(EI/L^3)} \qquad \text{and} \qquad K_y = \frac{K_2}{(EI/L\ )}$$

Eq. (22) then becomes

$$4K_x.K_y + 2K_x.\lambda.\frac{F_3}{F_6} - 2K_y.\lambda^3.\frac{F_3}{F_5} - \lambda^4\frac{F_3^{\,2}}{F_5F_6} = 0 \tag{23}$$

The joint parameters K_x and K_y can be estimated from measured natural frequencies using the similar procedure discussed in previous section.

4.2 Finite dlement modeling of SLJ beam

A single lap jointed free-free beam comprises of two beams jointed in between, a joint matrix with two unknowns K_1 & K_2 representing a joint element is given in Eq. (24).

$$[K]_{\text{joint}} = \begin{bmatrix} K_{11} & 0 & K_{13} & 0 \\ 0 & K_{22} & 0 & K_{24} \\ K_{31} & 0 & K_{33} & 0 \\ 0 & K_{42} & 0 & K_{44} \end{bmatrix} \tag{24}$$

where, K_{11}=K_{33}=-K_{13}=-K_{31}=K_1 and K_{22}=K_{44}=-K_{24}=-K_{42}=$K_{2.}$ Cross coupling terms are neglected in joint modeling. A program is developed in MATLAB to model single lap joint using FEM. The dimensions of the beams are: L=1 m, A=2×10^{-4} m^2, ρ=7800 Kg/m^3, E=2.07×10^{11} N/m^2 and I=3×10^{-9} m^4. The natural frequencies were computed (see Table 10a) for three sets of assumed values of non dimensional stiffness K_x and K_y; the frequencies thus obtained are used in Eq. (23) to estimate K_x and K_y. The estimated K_x and K_y values are given in Table 10(b). These values obtained using sub structure synthesis & multi linear regres sion are very close to assumed values.

4.3 Numerical simulation and discussion

For numerical simulation, Eq. (23) is considered here again. When K_x and K_y values are zero,

Table 10a Non-dimensional frequencies λ_i of a SLJ beam for different joint parameters using FEM

Joint Parameters	λ_1	λ_2	λ_3	λ_4	λ_5
K_x=1000 K_y=100	4.7007	7.5928	10.9430	12.5820	16.9299
K_x=500 K_y=50	4.6722	7.3354	10.8937	11.5757	16.2911
K_x=50 K_y=20	4.5912	5.1595	9.7041	10.7630	15.7590

Table 10b Estimates of joint stiffness parameters using sub-structure synthesis

Exact joint parameters	Estimate of K_x	Estimate of K_y
K_x=1000 K_y=100	1002.63	100.4
K_x=500 K_y=50	500.270	50.11
K_x=50 K_y=20	50.0500	20.00

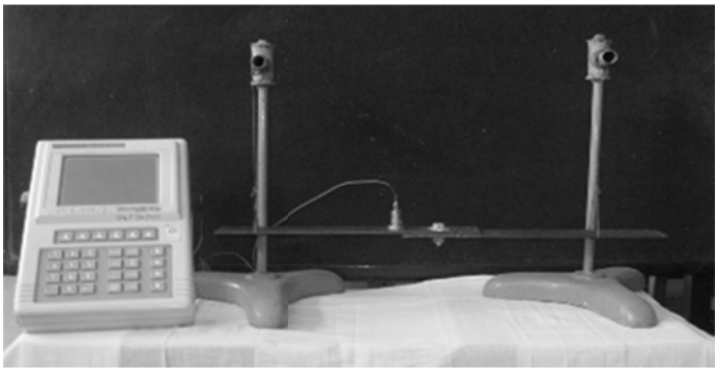


Fig. 6 Test set up - single lap bolted joint

the equation reduces to F_3=cos(λ)cos$h$(λ)−1=0, which is the frequency equation of free-free beam, for which λ values are 4.7300, 7.8532, 10.9956,14.1371 and so on; similarly for very high values of K_x and K_y , these λ values tends to ideal values of free-free beam. However, in practical cases, K_x and K_y will have some finite values depending on type of the joint and λ values will be less than those of an ideal SLJ beam. For a given set of support stiffness, non-dimensional natural frequencies λ can be solved from Eq. (23).

The estimates of K_x and K_y presented in Table 10b are obtained with data without any measurement error. However, for practical measurement of natural frequencies, effect of measurement noise and spectral resolution error of measuring instrument needs to be considered, as discussed in section 3.3. The estimation with perturbed natural frequencies is presented for experimental case in next section.

5. Experimental case: single lap jointed beam

Fig. 6 shows the test set up for measuring the vibration response of a SLJ free-free beam. The beam was excited through impulse and free vibration response is measured using B&K 4370 accelerometer and FFT analyzer. A data block size of 2048 samples with 800 spectral lines is used to measure the natural frequencies of beam. The material properties and cross sectional area of beam is listed in Table 2, length of beam is taken as 0.6 m. The measured natural frequencies at the resonance peaks of frequency spectrum is given in Table 11; to minimize the resolution error of FFT, different frequency bands were selected in FFT analyzer. It is observed that the measured natural frequencies are less by more than 6% compared to natural frequencies of ideal SLJ beam; so one can predict that the joint stiffness will be in sensitive region (non stiff region).

5.1 Identification of joint parameters

The joint parameters K_1 and K_2 are identified using first four natural frequencies given in Table 11.

Eq. (23) is used for estimation of non dimensional stiffness parameters using non dimensional frequencies. The estimated non dimensional stiffness parameters and corresponding linear joint parameters is given in Table 12.

Table 11 Measured natural frequencies of SLJ Beam

Mode	Measured natural frequency (Hz)	Non-dimensional natural frequency
f_{r1}	72	4.3929
f_{r2}	197.25	7.271
f_{r3}	397.25	10.3185
f_{r4}	528.75	11.9045
f_{r5}	945.7	15.9207

Table 12 The identified joint parameters

Non-dimensional joint stiffness parameters	Linear joint stiffness parameters
$K_x = 824.3349$	$K_1 = 5.348\text{x}10^5$ (N.m^{-1})
$K_y = 15.4084$	$K_2 = 3599.46$ (N.m.rad^{-1})

Table 13 Comparison of natural frequencies measured experimentally and computed with FEM

Mode	Experimentally Measured (Hz)	Computed from FE Model (Hz)
f_{r1}	72	72.61
f_{r2}	197.25	199.42
f_{r3}	397.25	398.8
f_{r4}	528.75	526.7
f_{r5}	945.7	951.49

Table 14 The identified joint parameters using perturbed natural frequencies

Non-dimensional joint parameters		Linear joint parameters		Percentage (%) error in -	
K_x	K_y	K_1	K_2	K_1	K_2
823.79	15.39	$5.34\text{x}10^5$	3594.89	0.05	0.12

Now, the estimated joint parameters are incorporated in FE model to validate the sub-structure synthesis model by comparing experimentally measured natural frequencies with those computed from FE model. Table 13 shows the comparison. The result of FE model agrees well with the proposed model of SLJ beam. Although only first four measured natural frequencies are considered in regression analysis, the fifth measured natural frequency is also close to that computed with FE model. This shows that the higher natural frequencies, those not considered in regression analysis, also validate the proposed algorithm.

Considering FFT resolution error (as discussed in section 3.3) corresponding to measured natural frequencies and frequency band selected in measurement; all four natural frequencies are perturbed, first on higher side and then on lower side to study the robustness of the model. The identified joint parameters with perturbed natural frequencies are given in Table 14.

With perturbed natural frequencies, the estimated error in joint stiffness parameter is very less; thus the model is robust for error in frequency measurement.

6. Conclusions

A new procedure for joint stiffness identification has been proposed in this work. The procedure is based on natural frequency measurement and hence is very much convenient in practical applications. Using method of sub-structure synthesis, a frequency equation in terms of the joint stiffness parameters is developed. With the measured natural frequencies, one can obtain an over determined set of equations, which is then processed through multi-linear regression to obtain the best estimates of the joint parameters. It is shown that the procedure gives accurate estimates for a wide range of stiffness values. Unknown joint stiffness parameters are identified for physical structure using both 2-parameter and 3-parameter model in case of cantilever beam. It was observed for the demonstrated structure that the 3-parameter model gives better estimate than 2-parameter model.

Similar identification technique has also been developed for single lap joint. It is seen that the experimentation results agree well with the updated FE model and the method is robust against measurement error. The procedure can be used for stiffness identification of various other joints in structures.

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Nomenclature

A	cross sectional area of each beam
E	Young's modulus
F	force, corresponds to coordinate X.
FRF	Frequency Response Function
F_1 –F_6	functions defined in reference (Bishop 1960)
I	moment of inertia
K_1	translational stiffness of single lap joint
K_2	rotational stiffness of single lap joint
K_r	rotational stiffness of cantilever joint
K_t	translational stiffness of cantilever joint
K_x	non dimensional translational stiffness of joint
K_y	non dimensional rotational stiffness of joint
K_z	non dimensional cross coupled stiffness of joint
L	length of beam
M	moment, corresponds to coordinate θ
P	sub structure (sub system) I
R	sub structure (sub system) II
X	joint coordinate, translational.
Z	composite system (sub system I+II)
pinv	generalised inverse of matrix.
θ	joint coordinate, rotational.
δ	receptance function for sub system P of cantilever.
δ^*	receptance function for sub system P of SLJ.
η	receptance function for sub system R.
v	receptance function for composite system Z.
$[\delta]$	FRF matrix for P.
$[\eta]$	FRF matrix for R.
$[v]$	FRF matrix for Z.
$\left\{\hat{K}\right\}$	non dimensional joint parameter vector.
λ	non dimensional natural frequency of Z
ρ	mass density

ω radian frequency.
f_r natural frequency, Hz