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FE model updating method incorporating damping matrices for structural dynamic modifications

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Abstract. An accurate finite element (FE) model of a structure is essential for predicting reliably its dynamic characteristics. Such a model is used to predict the effects of structural modifications for dynamic design of the structure. These modifications may be imposed by design alterations for operating reasons. Most of the model updating techniques neglect damping and so these updated models can't be used for accurate prediction of vibration amplitudes. This paper deals with the basic formulation of damped finite element model updating method and its use for structural dynamic modifications. In this damped damped finite element model updating method, damping matrices are updated along with mass and stiffness matrices. The damping matrices are updated by updating the damping coefficients. A case involving actual measured data for the case of F-shaped test structure, which resembles the skeleton of a drilling machine is used to evaluate the effectiveness of damped FE model updating method for accurate prediction of the vibration levels and thus its use for structural dynamic modifications. It can be concluded from the study that damped updated FE model updating can be used for structural dynamic modifications with confidence.

Keywords: damping identification; model updating; structural dynamic modifications

1. Introduction

It is well known that a finite element model will be erroneous due to inevitable difficulties in modeling of joints, boundary conditions and damping. The experimental data are generally considered to be more accurate. This has led to the development of model updating which aims at reducing the inaccuracies present in the analytical model in the light of measured dynamic test data. A significant number of methods, (Berman and Nagy 1983, Baruch 1978), which were first to emerge belonged to the direct category. These methods violate structural connectivity and matrices are difficult to interpret. On the other hand, iterative methods provide wide choices of updating parameters, structural connectivity can be easily maintained and corrections suggested in the selected parameters can be physically interpreted. Iterative methods either use eigendata or FRF data. Collins *et al*. (1974) used the eigendata sensitivity for model updating in an iterative framework. Lin and Ewins (1994) proposed response function method (RFM), which uses measured FRF data to update an analytical model. Wang and Yang (2012) used Modified Tikhonov Regularization (MTR) method in model updating. Govers and Link (2010) used

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stochastic method to update mass and stiffness matrices. Titurus and Friswell (2008) proposed sensitivity-based model updating method with prior information of uncertain parameters. Adhikari and Friswell (2010) updated mass and stiffness matrices using Karhunen-Loeve expansion method. Matta and Stefno (2012) used model updating for large scale building structure. Most of the updating methods neglect the damping. These cannot be used for predicting complex FRFs and damping. Some research efforts have also been made to update the damping matrices. Yong and Zhenguo (2004) proposed a two-step model updating procedure for lightly damped structures using neural networks. In the first step, mass and stiffness are updated using natural and antiresonance frequencies. In the second step, damping ratios are updated. Lepoittevin and Kess (2011) estimated the damping of structure considering modal damping. Zang *et al.* (2012) estimated the unsymmetrical damping of the system using genetic algorithm. Pradhan and Modak (2012) proposed a method of identification of damping matrix using normal FRFs instead of complex FRFs. The normal FRFs are estimated from complex FRFs. Arora *et al*. (2009a) proposed a complex parameter based model updating method in which FE model is updated in such a way that the updated model reflects general damping in the experimental model by considering the updating parameters as complex. Arora *et al*. (2010) proposed a method in which damping matrices are updated along with mass and stiffness matrices. The method is able to update viscous damping as well as structural damping matrices.

A model updating method should able to predict the changes in dynamic characteristics of the structure due to potential structural modifications because of dynamic design. Very little appears to have been done from this aspect though there is lot of work reported on FE model updating itself. Modak *et al*. (2005) compared predictions of dynamic characteristics using undamped updated finite element models. Arora *et al*. (2009b) evaluate the effectiveness of model updating with damping identification method and model updating using complex updating parameter method for dynamic design. This paper deals with the basic formulation for the finite element model updating incorporating damping matrices method (Arora *et al*. 2010) to obtain damped updated model and its use for dynamic design. A case involving actual measured data for the case of F-shaped test structure, which resembles the skeleton of a drilling machine is used to evaluate the effectiveness of model updating incorporating damping matrices method for dynamic design. Structural modifications in the form of lumped masses, which are placed at different locations and beam stiffener, are introduced to check the damped updated FE model for predicting structural modifications because of dynamic design.

2. Basic theory

The mass, stiffness and structural and viscous damping matrices are updated using damped FE model updating method. The updated mass, stiffness and damping matrices are subsequently used for structural dynamic modifications.

2.1 Damped FE updated method

This method (Arora *et al*. 2010) is a further development of Response function method given by Lin and Ewins (1994), which is an iterative method and uses measured FRF data directly without requiring any modal extraction. In this method, the updating parameters are classified into two classes namely physical parameters and damping parameters. The physical parameters are associated with mass and stiffness matrices and damping parameters are associated with the damping matrices of the system. In this method, both structural and viscous damping matrices are updated. Thus, this method is a hybrid structural-viscous damping updating method. It is assumed that initially there is damping in the analytical model, which results in complex analytical FRFs. Following identities relating dynamic stiffness matrix $[Z]$ and receptance FRF matrix $[\alpha]$ for the analytical model as well as the actual structure respectively can be written as

$$[Z_A]_{(N\times N)}[\alpha_A]_{(N\times N)} = [I] \tag{1}$$

$$[Z_X][\alpha_X] = [I] \tag{2}$$

where subscripts A and X denote an analytical (like an FE model) and experimental model respectively and N is the total number of degrees of freedom in FE model. Details of nomenclature are provided in appendix A. Expressing $[Z_X]$ in Eq. (2) as $[Z_A]+[\Delta Z]$ and then subtracting Eq. (1) from it, following matrix equation is obtained

$$[\Delta Z][\alpha_X] = [Z_A]([\alpha_A]-[\alpha_X]) \tag{3}$$

Pre-multiplying above equation by $[\alpha_A]$ and then using Eq. (1) gives

$$[\alpha_A][\Delta Z][\alpha_X] = [\alpha_A]-[\alpha_X] \tag{4}$$

If only the j^{th} column of measured FRF matrix $[\alpha_X]$, $\{\alpha_X\}_j$, is available then above equation is reduced to

$$[\alpha_A]_{(N\times N)}[\Delta Z]_{(N\times N)}\{\alpha_X\}_{j(N\times 1)} = \{\alpha_A\}_{j(N\times 1)} - \{\alpha_X\}_{j(N\times 1)} \tag{5}$$

Linearizing $[\Delta Z]$ with respect to $\{p\}$, $\{p\}=\{p_1,p_2,\ldots,p_{nu}\}$, where nu is number of updating parameters, being the vector of updating variables associated with individual or group of finite elements, gives

$$[\Delta Z] = \sum_{i=1}^{nu}\left(\frac{\partial[Z]}{\partial p_i}\cdot \Delta p_i\right) \tag{6}$$

Dividing and multiplying above equation by p_i and then writing u_i in place of $\Delta p_i/p_i$, the equation becomes

$$[\Delta Z] = \sum_{i=1}^{nu}\left(\frac{\partial[Z]}{\partial p_i}\cdot p_i\right)\cdot u_i \tag{7}$$

In case of structural and viscous damping, $[\Delta Z]$ in terms of physical and damping parameters can be written as

$$[\Delta Z]_{(N\times N)} = \lfloor \Delta Z_{pp} \rfloor_{(N\times N)} + \lfloor \Delta Z_{sp} \rfloor_{(N\times N)} + \lfloor \Delta Z_{vp} \rfloor_{(N\times N)} \tag{8}$$

where subscripts pp, sp and vp represent physical, structural and viscous damping parameters respectively. The dynamic stiffness matrix of a vibrating system with viscous and structural damping matrices can be expressed as

$$[Z]_{(N\times N)} = [K]_{(N\times N)} + i[D+\omega C]_{(N\times N)} - \omega^2[M]_{(N\times N)} \tag{9}$$

where [D] and [C] are structural and viscous damping matrices and are proportional to mass and stiffness matrices

$$[D] = \alpha_S[M] + \beta_S[K] \tag{10}$$

$$[C] = \alpha_V[M] + \beta_V[K] \tag{11}$$

where α_S, β_S and α_V, β_V are damping coefficients of structural and viscous damping matrices respectively.
[ΔZ_{pp}] for vibrating system with viscous and structural damping matrices can be written as

$$\left[\Delta Z_{pp}\right] = \sum_{j=1}^{np}\left(\frac{\partial\left[[K]-\omega^2[M]\right]}{\partial p_j}\,p_j\right)\cdot u_j \tag{12}$$

[ΔZ_{sp}] can be written as

$$\left[\Delta Z_{sp}\right] = \sum_{k=1}^{ns}\left(i\,\frac{\partial\left(\alpha_S[K]+\beta_S[M]\right)}{\partial p_k}\,p_k\right)\cdot u_k \tag{13}$$

$$\left[\Delta Z_{vp}\right] = \sum_{k=1}^{nv}\left(i\,\frac{\partial\left(\alpha_V[K]+\beta_V[M]\right)}{\partial p_k}\,p_k\right)\cdot u_k \tag{14}$$

where np, ns and nv represent number of physical, structural and viscous damping parameters respectively. Total number of updating parameters (*nu*) is the sum of physical parameters (*np*), structural damping parameters (*ns*) and viscous damping parameters.

Eq. (5), after making the substitution for [ΔZ], can be written at various frequency points chosen from the frequency range considered. The resulting equations can be framed in the following matrix form

$$\left[S\right]_{(N\times nf\times nu)}\left\{u\right\}_{(nu\times 1)} = \left\{\Delta\alpha\right\}_{((N\times nf)\times 1)} \tag{15}$$

where [S] is sensitivity matrix and nf is number of selected frequency points for updating. The criterion for selection of frequency points for updating is described in the experimental case study. Because of presence of damping in the analytical model, the analytical dynamic stiffness matrix [Z] and analytical FRFs are complex which results in complex sensitivity matrix, which is then partitioned into real and imaginary parts as.

$$\begin{bmatrix}\Re[S]\\ \Im[S]\end{bmatrix}_{(2\times N\times nf\times nu)}\left\{u\right\}_{(nu\times 1)} = \begin{Bmatrix}\Re\{\Delta\alpha\}\\ \Im\{\Delta\alpha\}\end{Bmatrix}_{((N\times nf)\times 1)} \tag{16}$$

The updating parameter vector $\{u\}$, which consists of correction factor of physical parameters and damping parameters (both structural and viscous) are used to update mass, stiffness and damping matrices. Updating of damping matrix depends upon initial assumption of type of damping in the analytical model. By this procedure the updated damping matrices represent general non-proportional damping in the system. This process is repeated in an iterative way. The performance is judged on the basis of the accuracy with which the FRFs predicted by updated FE model match the experimental FRFs.

2.2 Structural modification using an updated model

Damped updated FE model for a structure (Arora *et al.* 2009b) is available in terms of stiffness, mass and damping matrices. In dynamic design practice, the size of modifications is very small as compared to the structure. It is assumed that there is no effect of structural modifications on the damping of the structure. If $[\Delta K]$ and $[\Delta M]$ represent the modification matrices due to a modification then the modified structure's stiffness and mass matrix denoted by $[K_m]$ and $[M_m]$ respectively can be written as

$$[K_m] = [K] + [\Delta K] \tag{17}$$

$$[M_m] = [M] + [\Delta M] \tag{18}$$

Consider the case of mass modification by assuming that a mass m_o kg is added at i^{th} node. The $[\Delta M]$ is obtained by making the diagonal entries corresponding to the translational degrees of freedom for the i^{th} node equal to '$+m_o$'. The mass modification matrix is given as:

$$[\Delta M] = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & & & & \vdots \\ \vdots & & +m_o & & & \vdots \\ \vdots & & & +m_o & & \vdots \\ \vdots & & & & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{19}$$

For the case of beam modification the $[K_m]$ and $[M_m]$ are essentially obtained by assembling the FE-model for the added beam member with that of the FE-model of the unmodified structure. Predictions on the basis of the updated model can be made by assembling the FE-model for the added beam member with that of the updated FE-model of the unmodified structure. Thus, in general, the number of finite elements, the number of nodes and consequently the size of the modified model will be higher than that for the unmodified model.

3. Damped FE model updating of F-Shaped structure

Damped FE model updated method is evaluated for the case of an F-shaped structure (Arora *et al*. 2009b), as shown in Fig. 1, using experimental data. The F-shape structure has been constructed by bolting the two beam members horizontally to a vertical beam member, which in turn, has been welded to the base plate at the bottom. All the beam members have a square cross-

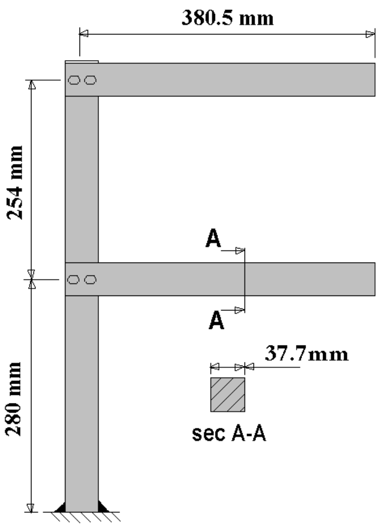


Fig. 1 F-shaped structure

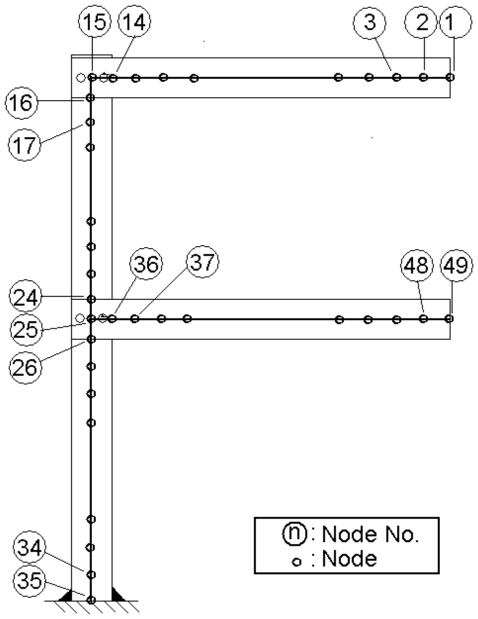


Fig. 2 Initial FE model

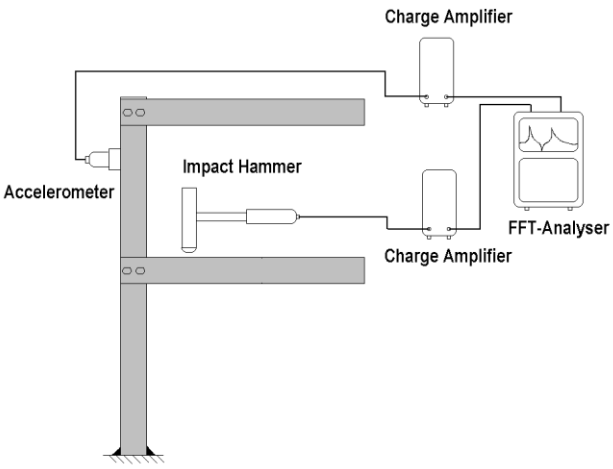


Fig. 3 Instrumentation set-up for modal test using impact excitation

section with 37.7 mm side.

A Finite element model of the F-structure is built, as shown in Fig. 2, using 48 two dimensional frame elements (Two translational degrees of freedom in x and y direction and one rotational degree of freedom, per node) to model in-plane dynamics. In the F- shaped structure, there are three joints, which are modeled by taking coincident nodes at each of them. Thus the, two nodes which are geometrically coincident are taken as one joint instead of one node. A horizontal, a vertical and a rotational spring couple the two nodes at each of such a coincident pair. The stiffnesses of these springs are $K_{x,}$ K_y and K_r respectively.

The modal test is performed by exciting the structure with an impact hammer at 16 locations and response is measured at one location using accelerometer as shown in Fig. 3. The frequency response functions so acquired are analyzed using a global curve fitting technique available in to obtain experimental sets of modes in the range of 0-1000Hz. A comparison of the corresponding experimental and analytical natural frequencies, the percentage difference between them and the corresponding MAC-value for first five modes are given in Table 1. An overlay of the measured FRFs and the corresponding FE model FRFs are shown in Fig. 4. The FRF 14*x*17*x* represents excitation at node 14 and response at node 17 both in x-direction. It is observed that the shape of the FE model FRF-curve is similar to the measured curve. It therefore infers that though the FE model is in error it is, in principle, of updatable quality.

Choice of updating parameters on the basis of engineering judgment about the possible locations of modeling errors in a structure is one of the strategies to ensure that only physical meaningful corrections are made. In case of F-structure, modeling of stiffness of the joints is expected to be a dominant source of inaccuracy in the FE model assuming that the values of material and the geometric parameters are correctly known. Analytical sensitivity analysis of the joint springs shows that the rotational stiffness is the most important variable affecting the FRFs. Rotational springs of stiffness K_{r1}, K_{r2} and K_{r3} coupling the rotational degrees of freedom of the coincident nodes at the three joints are taken as updating parameters. The other two degrees of freedom of the coincident nodes are taken as rigidly coupled. The joints are the major source of energy dissipation (Bert 1973). The major source of damping in the system is assumed to be at the joints. The structural damping coefficient proportional to the rotational stiffness of each joint are

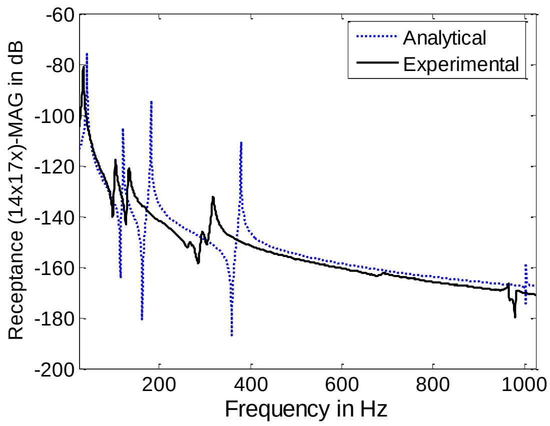


Fig. 4 Overlay of the measured FRFs and the corresponding FE model FRFs before updating

Table 1 Correlation of measured and FE-model based modal data of F-shaped structure before updating

Mode No.	Measured Frequency in Hz.	FE-Model Predictions Frequency in Hz.	FE-Model Predictions % Error	MAC-Value
1	34.95	43.05	23.17	0.9650
2	104.02	123.67	18.89	0.9364
3	133.96	185.21	38.26	0.9311
4	317.52	385.17	21.30	0.9141
5	980.16	1020.06	4.07	0.6908

Table 2 Values of rotational springs stiffness and damping coefficients of each joint after updating of the F-shaped structure

Updating Variable	Initial Value (N m rad^{-1})	Updated values (N m rad^{-1})	Structural damping coefficients
K_{r1}	3.28E+06	2.61E+05	7.01E+03
K_{r2}	3.28E+06	2.69E+05	3.51E+03
K_{r3}	3.28E+06	3.15E+05	1.15E+04

Table 3 Correlation between the measured and updated model

Mode No.	Measured		Updated Model Predictions			
	ω_n (Hz)	ζ	ω_n (Hz)	ζ	% ω_n Error	MAC-Value
1	34.95	0.022	34.25	0.021	-2.0	0.9923
2	104.02	0.016	100.27	0.018	-3.60	0.9693
3	133.96	0.014	134.42	0.014	0.34	0.9675
4	317.52	0.007	313.73	0.0065	-1.19	0.9423
5	980.16	0.005	973.44	0.0051	-0.68	0.4370

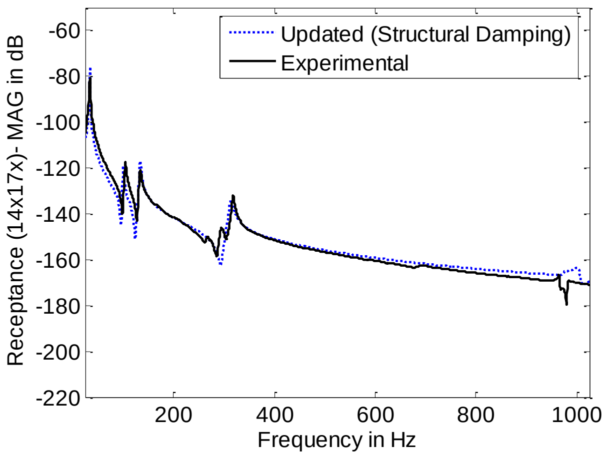


Fig. 5 Overlay of the measured FRF and the corresponding updated model FRF of F-shaped structure after model updating incorporating structural damping matrix

updated assuming initially a very small value. The initial and final values of the rotational spring stiffness of each joint are given in the Table 2.

It is observed that the values of stiffness of the rotational springs corresponding to three joints are reduced and also values of three springs are not very different from each other while the damping coefficients value of each rotational spring stiffness represents damping in the system. A comparison of the correlation between the measured and the updated model natural frequencies is given in the Table 3. It is observed from the Table 3 that for the model updating method incorporating damping matrices, there is a significant reduction in the error in natural frequencies. ω_n and ζ represent natural frequencies and damping ratios respectively.

Fig. 5 shows the overlay of measured and updated FRF. It is noticed shape of the updated FRFs is same as that measured FRFs. The quantitative index of matching AEFRF of 14×17× reduces

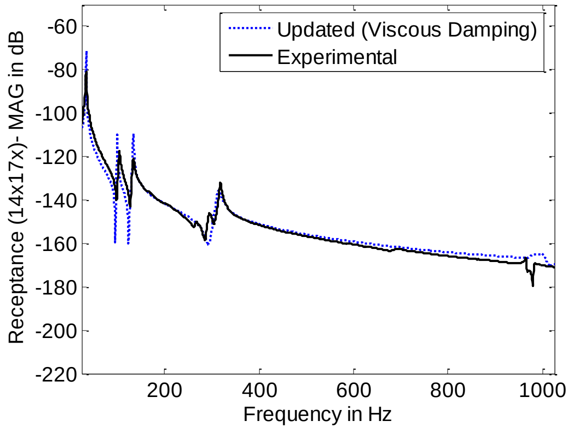
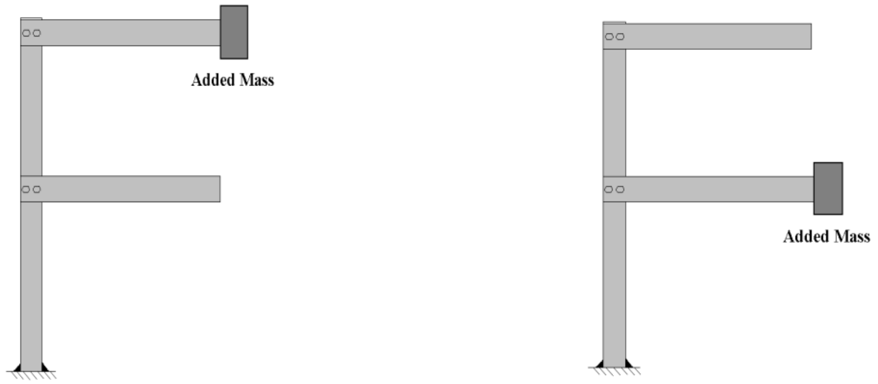


Fig. 6 Overlay of the measured FRF and the corresponding updated model FRF of F-shaped structure after model updating incorporating viscous damping matrix



Mass Modification at the tip of the upper horizontal beam member

Mass modification at the tip of the lower horizontal beam member

Fig. 7 F-Shape-Structure with mass modifications

from 46.24% to 3.31%. Viscous damping coefficients of rotational stiffness of each joint are also considered for updating. Fig. 6 shows the overlay of measured and updated FRF. It can be noticed from Figs. 5 and 6 that structural damped updated model gives better FRF prediction compared to viscous damped updated model as shown by the amplitude predicted by both updated models.

4. Structural dynamic modifications using damped updated FE model

The damped updated model obtained above is used for predicting the effects of structural modifications made to the structure due to dynamic design process. This section gives a comparison of the measured changes in dynamic characteristics due to structural dynamic modifications with those predicted using the updated model incorporating damping matrices. The comparison is performed first for a mass modifications, which are placed at two different locations, and then for a beam modification.

4.1 Mass modifications

Two different types of mass modifications are introduced by attaching a mass of 1.8 kg as shown in Fig. 7.

1. At the tip of the upper horizontal beam member
2. At the tip of the lower horizontal beam member

The FRFs for the each mass-modified structure are then acquired. The mass modifications are also introduced analytically in the damped updated models. The mass matrix for each modified structure, and subsequently its modal data and FRFs, corresponding to the updated model incorporating damping matrices are obtained.

A comparison of the modified FRFs as predicted by damped updated model is shown in Figs. 8 and 9 while a comparison of natural frequencies is given in Tables 4 and 5. It is observed from Figs. 8, 9 and Tables 4 and 5 that the predicted dynamic characteristics of damped updated model are closer to the measured characteristics of the modified structure even at resonance and anti-resonance frequencies. The average percentage error in the predictions for the first five natural frequencies for mass modification on upper horizontal beam members based on the FE-model and

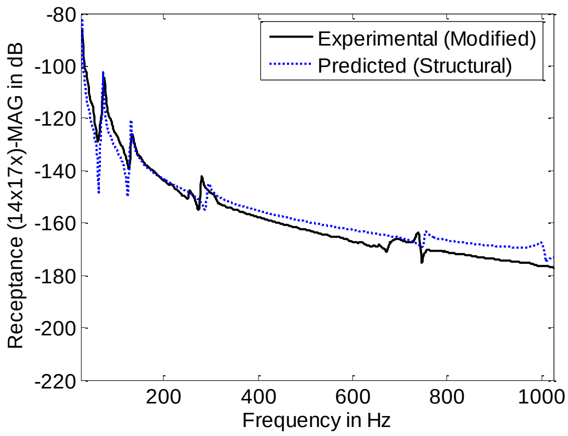


Fig. 8 Overlay of the measured FRFs and the corresponding predicted FRFs after mass modification at the tip of the upper horizontal beam member

Table 4 Comparison of the predictions of the modified dynamic characteristics based on the updated models with the measured changes for the case of mass modification at the tip of the upper horizontal beam member

Mode No.	Measured		FE Model-based predictions			Updated model based predictions			
	ω_n (Hz)	ζ	ω_n (Hz)	% ω_n error	MAC-Value	ω_n (Hz)	ζ	% ω_n error	MAC-Value
1	27.32	0.025	34.93	27.8	0.9764	28.45	0.023	4.13	0.9856
2	74.53	0.023	91.49	22.7	0.9665	72.38	0.019	-2.87	0.9950
3	133.38	0.021	178.97	34.1	0.9792	131.58	0.019	-1.34	0.9926
4	280.11	0.014	357.51	27.6	0.7803	293.65	0.012	4.83	0.7622
5	745.12	0.01	805.02	8.0	0.7170	753.01	0.008	1.05	0.6482

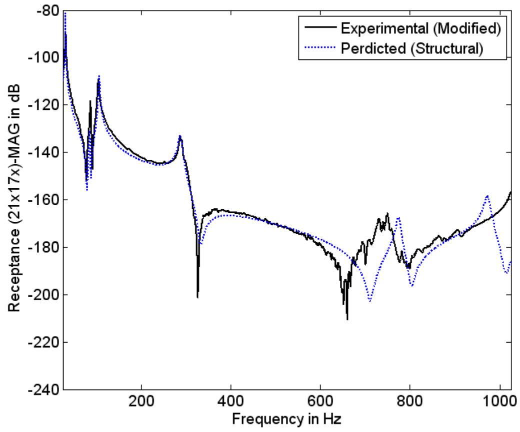


Fig. 9 Overlay of the measured FRFs and the corresponding predicted FRFs after mass modification at the tip of the lower horizontal beam member

Table 5 Comparison of the predictions of the modified dynamic characteristics based on the updated models with the measured changes for the case of mass modification at the tip of the lower horizontal beam member

Mode No.	Measured		FE Model-based predictions			Updated model based predictions			
	ω_n (Hz)	ζ	ω_n (Hz)	% ω_n Error	MAC-Value	ω_n (Hz)	ζ	% ω_n Error	MAC-Value
1	31.14	0.028	39.41	26.5	0.9902	31.35	0.029	0.66	0.9939
2	85.87	0.026	104.44	21.6	0.9301	85.00	0.025	-1.01	0.9843
3	102.96	0.025	144.24	40.0	0.9104	106.65	0.026	3.58	0.9809
4	284.56	0.015	351.24	23.4	0.8985	287.95	0.014	1.19	0.8353
5	735.83	0.012	850.16	15.5	0.7554	774.68	0.011	5.28	0.6524

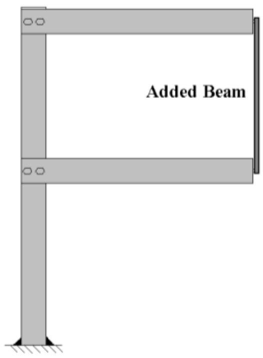


Fig. 10 F-Shape-Structure with Beam Modification

damped updated model are 24.04% and 2.84% respectively. Average percentage error in the predictions for the first five natural frequencies for mass modification on lower horizontal beam members based on the FE-model and damped updated model are 25.4% and 2.34 %. It can be concluded from mass modifications studies that updated model incorporating damping matrices is able to predict mass modifications at different locations accurately.

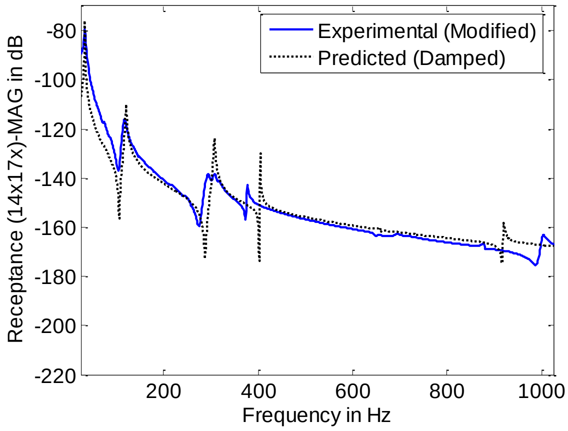


Fig. 11 Overlay of the measured FRF and the corresponding predicted FRF after beam modification at the tip of the upper horizontal beam member

Table 6 Comparison of the dynamic characteristics for the case of beam modification

Mode No.	Measured		FE model-based predictions			Updated model based predictions			
	ω_n (Hz)	ζ	ω_n (Hz)	% ω_n Error	MAC-Value	ω_n (Hz)	ζ	% ω_n Error	MAC-Value
1	33.95	0.0401	42.91	-26.39	0.9743	33.66	0.029	0.85	0.9892
2	117.30	0.0331	165.14	-40.78	0.9797	120.75	0.0273	-2.94	0.9929
3	309.98	0.0291	371.97	-19.99	0.8249	307.78	0.0241	0.71	0.8641
4	376.89	0.024	405.56	-7.60	0.6839	405.23	0.016	-7.52	0.7283
5	648.34	0.021	711.88	9.8	0.9859	659.35	0.012	1.69	0.9845

4.2 Beam modification

A beam modification is introduced in the form of a stiffener of width 38.2 mm and thickness 5 mm. The stiffener is attached between the tips of the lower and the upper horizontal beam members as shown in Fig. 10. The beam is connected to the F- structure by bolted joints.

The FRFs for the beam-modified structure are then acquired. The beam modification will increase the size of mass and stiffness matrices. The mass and stiffness matrices for the modified structure are obtained assuming there is little effect of the beam modification on the damping of the system, and subsequently its modal data and FRFs, corresponding to the updated model are obtained. The overlay of the modified FRF as predicted by damped updated model and measured modified FRF is shown in Fig. 11 while a comparison of dynamic characteristics predicted by damped updated model and FE model is given in Table 6. It is observed from the Fig. 11 and Table 6 that the dynamic characteristics predicted by updated model incorporating damping matrices are closer to the measured characteristics of the modified structure. The average percentage error in the predictions for the first four natural frequencies based on the FE-model is -23.69% while that based on the damped updated model is much less at 3.0 %. The predicted FRFs for the both cases of mass modifications give good results whereas for the case of beam modification the results are poor. It can also be noticed from the Figs. 8, 9 and 11 that predicted FRFs for mass modifications matches better than beam modification. For beam modification no estimation is carried out for the damping and stiffness of the joints. The focus of this study is to evaluate the prediction capabilities of the updated method incorporating damping matrices and it can be concluded with confidence that updated method incorporating damping matrices can be used for structural dynamic modifications.

5. Conclusions

In this paper a damped FE model incorporating damping matrices method has been employed for predicting potential structural modifications because of dynamic design. The dynamic design at the computer level has been demonstrated via mass and beam stiffener using damped updated FE model. It is seen that damped updated FE model predicts accurately not only the natural frequencies but also amplitude of vibration at resonance frequencies. The modified dynamic characteristics due to modifications obtained via damped updated FE-model indicate, on experimental verification, that they are of acceptable accuracy. Thus, it can accordingly be concluded that model updating incorporating damping matrices method can be used for structural dynamic modifications with confidence.

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