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A modified multidisciplinary feasible formulation for MDO using integrated coupled approximate models

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Abstract. This paper is concerned with the modification of multidisciplinary feasible formulation for MDO problems using the integrated coupled approximate models. A drawback of conventional MDFs is the numerical difficulty in decomposing the design variables and deriving the coupled equations of state. To overcome such a drawback of conventional methods, the coupling in analysis and design is resolved by approximating the state variables in each discipline by the response surface method and by modifying the optimization formulation using the corresponding integrated coupled approximate models. The validity, reliability and effectiveness of the proposed method are illustrated and verified through two optimization problems, a mathematical MDF problem and the multidisciplinary optimum design of suspension unit of wheeled armored vehicle.

Keywords: multidisciplinary design optimization (MDO); successive iterative design; Integrated coupled approximate model; response surface method; wheeled armored vehicle

1. Introduction

The advances in the computer performance and the numerical analysis and design technologies made one challenge to the design of large-scale complex systems such as aircrafts, automobiles, and others. In such multidisciplinary systems involving more than one discipline, the constituent disciplines are dependent on each other in the complicated interrelation. Thus, differing from the single-discipline design problem, the design variables, objective functions and constraints in multidisciplinary simulation and optimization are highly coupled. Besides the increase of the number of design variables, the inherent interdisciplinary coupling in multidisciplinary systems dramatically increases the CPU time for the analysis and optimization, requiring costly nonlinear methods even if each discipline is linear problem (Berkes 1990).

In early days, the design solutions of such multidisciplinary system were sought by a simple iterative design approach in which the design process is performed discipline by discipline in sequence until the target performance is satisfied. However, this early uncoupled approach is not only time-consuming but it can not account for the complex coupling among disciplines. In order to resolve such problems of the traditional design approach, a linear decomposition approach was introduced in 1982 by Sobieszczanski-Sobieski (1982), in which the design of a multidisciplinary

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system is subdivided into a number of more manageable subsystems. This design approach motivated the subsequent investigators to introduce current multidisciplinary design optimization (MDO) methods, a concurrent engineering design approach to the integrated simultaneous design of multidisciplinary systems, which decompose the system into its constituent subsystems and link through design, function and performance by evaluating the design feasibility (Mcallister *et al.* 2005). The design of multidisciplinary systems requires iterative cycles composed of design initialization, system analysis, sensitivity analysis, and design optimization. The key issue in MDO lies in formulating the multidisciplinary design problem so that the design feasibility is guaranteed by effectively resolving the interdisciplinary coupling and managing the memory resources and the CPU time (Coelho *et al.* 2008, Cramer *et al.* 1993). The formulation strategies introduced so far are broadly classified into two categories, single-level and multi-level approaches (Mcallister *et al.* 2005, Coelho *et al.* 2008), according to how the design and coupling variables are managed.

In single-level formulations in which a global optimizer manages the design variables together, three fundamental approaches have been introduced, the most familiar multidisciplinary feasible (MDF) approach, the all-at-once (AAO) approach, and the individual discipline feasible (IDF) approach. In the first approach, the complete multidisciplinary feasibility is enforced at each optimization iteration, at least once every time any problem function or constrain or derivative is evaluated (Cramer *et al.* 1993). The optimization problem in this approach treats only the design variables as the optimization variables, but a great deal of time becomes its main drawback (Hulme and Bloebaum 2000). The second approach, also referred to as SAND (simultaneous analysis and design), has been introduced by Haftka (1985), Cramer *et al*. (1993) and Balling and Sobieszczanski-Sobieski (1994) as one of alternative methods to MDF. This approach treats the entire multidisciplinary design cycle as a single large optimization problem by converting the system analysis equations into equality constraints and by treating the system design variables and the subsystem outputs as the optimization variables. It can eliminate iterative design cycles by eliminating the costly iterative analysis evaluations, but it leads to much more complicated optimization problem involving more optimization variables and equality constraints. Meanwhile, in the third approach (Hafka *et al.* 1992, Cramer *et al.* 1992), another alternative solution procedure to MDF, each single discipline is feasible on every design cycle, while driving all the disciplines toward multidisciplinary feasibility as the iterative computation converges. As an intermediate approach between MDF requiring the full disciplinary feasibility at each and every optimization cycle and AAO enforcing the disciplinary feasibility only at the final solution, IDF includes all the coupling variables into optimization variables.

Multi-level formulations have been developed as an overall management of all disciplines is difficult by single-level ones, in which a system-level coordinator rather than an optimizer separates the variables in a global level and a local one. Representatives of these multi-level formulations are collaborative optimization (CO) (Alexandrov and Lewis 1999, Braun and Kroo 1997), concurrent subspace optimization (CSSO) (Sobieszczanski-Sobieski 1988), bi-level integrated system synthesis (BLISS) (Sobieszczanski-Sobieski *et al.* 1998).

The purpose of this study is to introduce a modified multidisciplinary feasible approach to MDO using integrated coupled approximate models, in order to obtain the optimum solution with the reduced number of analyses. The above-mentioned main drawback of conventional MDF is caused by the difficulties in decomposing the design variables and deriving the coupled equations of state. In this context, in the current study, the coupling in analysis is resolved by approximating the response in one discipline in terms of the coupled state variables in the other disciplines as well as the uncoupled local and coupled global design variables. While, the coupling in design is resolved by modifying the conventional MDO formulation using the integrated coupled approximate models, considering the interaction between design variables, objective functions and constraints which share the design conditions. The validity of the proposed method is justified through the numerical experiment of a mathematical MDO problem having an analytical optimum solution. Also, the proposed method is applied to the multidisciplinary design optimization of the suspension unit of wheeled armored vehicle for enhancing both the firing stability and the vehicle mobility.

2. Standard formulation of MDF approach

For the sake of simplicity, let us consider a MDO problem in which two disciplines are coupled in design variables, objective functions and constraints. Two single-discipline optimization problems are formulated as following

Find x_I, x_c

to minimize $f_I(x_I, x_c, y_I)$

subject to

$$g_I(x_I, x_c, y_I) \le 0 \tag{1}$$

$$y_I = A_I(x_I, x_c) \tag{2}$$

with I being designated as disciplines 1 and 2. Where, x_I and xc_I are the uncoupled and coupled design variables, f_I the objective functions, g_I the constraints, and A_I the analyzers, respectively. The objective functions, constraints and analyzers in two disciplines are coupled through the coupling variable x_c. The subscript and superscript c are used to indicate the coupled variables hearafter.

By considering the interaction between two disciplines, one can convert two single-discipline optimization problems to the following multidisciplinary design optimization problem

Find x_2, x_2, x_c

to minimize $f_1(x_1, x_c, y_1) + f_2(x_2, x_c, y_2)$

$f_c(x_1, x_2, x_c, y_1, y_2)$

subject to

$$g_1(x_1, x_c, y_1) \le 0 \tag{3}$$

$$g_2(x_2, x_c, y_2) \le 0 \tag{4}$$

$$y_1 = A_1(x_1, x_c, y_2^c) \tag{5}$$

$$y_2 = A_2(x_2, x_c, y_1^c) \tag{6}$$

The objective functions and constraints are expressed in terms of two local design variables x_1 and x_2, a global coupling variable x_c, and two coupled state variables (subsystem outputs) y_1 and y_2. Fig. 1 schematically represents the couplings between design variables, objective functions and

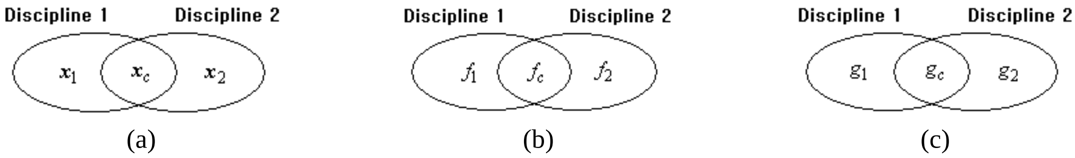

Fig. 1 Coupling in the design: (a) between design variables, (b) between objective functions, and (c) between constraints

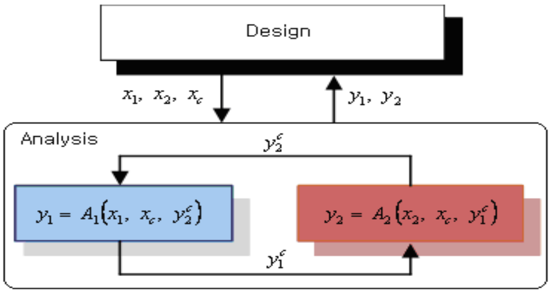

Fig. 2 Coupling in the analysis

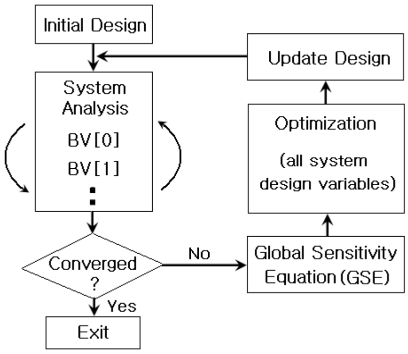

Fig. 3 Numerical procedure of conventional MDF

constraints in the design. Meanwhile, two analyzers are coupled each other in terms of the counterpart state variable y_2 or y_1 as represented in Fig. 2.

Fig. 3 represents the numerical procedure of conventional MDF approach which directly solves the multidisciplinary optimization problem in the non-hierarchic structure without decomposition. The system analysis associated with the overall design cycle is carried out with an initial design, and the entire design cycle repeats until the convergence criterion is satisfied. The sensitivity analysis can be done by the finite difference scheme or by the analytical procedure, namely the global sensitivity equation (GSE) method (Sobieszczanski-Sobieski 1990). The optimization is carried out by defining all the system variables as the design variables, and the current design is updated. The performance of the updated design is evaluated by the system analysis and the convergence is checked.

3. Modified MDF approach using integrated coupled approximate models

3.1 Integrated coupled approximate models using RSM

Several meta-models (also, called surrogate models or approximate models) such as ANN, the response surface model (RSM) and the kriging model are widely used to approximate the response of engineering problems. The RSM is expressed in terms of low-order polynomials with the coefficients determined by the least-square method, and the choice of the best suitable model is made upon the ANOVA regression analysis. The main advantage of this model is the simplicity and effectiveness (Bucher and Most 2008, Guita and Watson 1998), so that it can be an effective tool unless the response exhibits the high nonlinearity. In usual single-disciplinary optimization, RSM model for a specific response can be approximated in terms of only independent uncoupled design variables.

But, in the multidisciplinary design optimization, the coupling in analysis gives rise to the interrelation between state variables. Thus, the response in one discipline should be approximated by including the coupled state variables in the other disciplines as well as the uncoupled local and coupled global design variables. In order to approximate the state variable y_1 in Eq. (5), a second-order complete regression model expressed in terms of x_1, x_c and $\hat{y}_2$ is adopted

$$\begin{aligned}\hat{y}_1 = \beta_0^1 + \beta_1^1 x_1 + \beta_c^1 x_c + \beta_{11}^1 x_1^2 + \beta_{cc}^1 x_c^2 + \beta_{22}^1 \hat{y}_2^2 \\ + \beta_{1c}^1 x_1 x_c + \beta_{12}^1 x_1 \hat{y}_2 + \beta_{c2}^1 x_c \hat{y}_2\end{aligned} \tag{7}$$

In the same manner, the state variable y_1 in Eq. (6) for discipline 2 is also approximated by

$$\begin{aligned}\hat{y}_2 = \beta_0^2 + \beta_1^2 x_2 + \beta_c^2 x_c + \beta_{11}^2 x_2^2 + \beta_{cc}^2 x_c^2 + \beta_{22}^2 \hat{y}_2^2 \\ + \beta_{2c}^2 x_2 x_c + \beta_{21}^2 x_2 \hat{y}_1 + \beta_{c1}^2 x_c \hat{y}_2\end{aligned} \tag{8}$$

Two meta-models $\hat{y}_1$ and $\hat{y}_2$ are interrelated each other, and the coefficients can be determined by the fixed-point iteration method (Anitescu and Hart 2004) or Newton-Raphson method with the help of design of experiments (DOE).

3.2 Modified MDF using the integrated coupled approximate models

In the multidisciplinary design optimization problem subject to Eqs. (3)-(6), the coupled objective function f_c and the coupled constraint g_c lead to the coupling in design through the coupled global design variable x_c. These coupled objective function f_c and constraint g_c are originally in function of the uncoupled local design variables x_1 and x_2 and a coupled global design variable x_c. But, f_c and g_c can be transformed into a function of $\hat{y}_1$ and $\hat{y}_2$ as well as x_1, x_2 and x_c because $\hat{y}_1$ and $\hat{y}_2$ are also approximated in terms of x_1, x_2 and x_c. Meanwhile, the uncoupled local constraints g_1 and g_2 are kept without modification or could be modified by transforming them into the functions of the uncoupled design variables and the approximate state varaibles in each discipline. Consequently, the objective functions and constraints are transformed using the approximate models $\hat{y}_1$, $\hat{y}_2$ and the coupled global design variable x_c as represented in Table 1, in order to resolve the coupling in design.

The coupling between disciplines is reflected by the coupled approximate models $\hat{y}_1$ and $\hat{y}_2$, and the multidisciplinary optimization problem can be reformulated in terms of the interrelation between the design variables x_1, x_2, x_c and the integrated coupled approximate models $\hat{y}_1$ and $\hat{y}_2$

Table 1 Transformation using the integrated coupled approximate models

Items	Original	Relation between design and state variables	Integrated coupled approximate models
Obj_c1	$f_c(x_1,x_2,x_c)$	$F = f_c(x_1,x_2,x_c)=0$	$\hat{F} = f_c(x_1,x_2,x_c,\hat{y}_1,\hat{y}_2)=0$
Obj_c2	$f_1(x_1,x_c)+f_2(x_2,x_c)$	$F = f_1(x_1,x_c)+f_2(x_1,x_c)=0$	$F = f_1(x_1,x_c,\hat{y}_1,\hat{y}_2) + f_2(x_2,x_c,\hat{y}_1,\hat{y}_2)=0$
Con_1	$g_1(x_1)\le 0$	$g_1 = f(x_1)\le 0$	$\hat{g}_1 = f(x_1,\hat{y}_1)\le 0$
Con_2	$g_2(x_2)\le 0$	$g_2 = f(x_2)\le 0$	$\hat{g}_2 = f(x_2,\hat{y}_2)\le 0$
Con_c	$g_c(x_1,x_2,x_c)\le 0$	$g_c = f(x_1,x_2,x_c)\le 0$	$g_c = f(x_1,x_2,x_c,\hat{y}_1,\hat{y}_2)\le 0$
Dis_1	$y_1 = A_1(x_1,x_c)$	$F_{dis1} = f(x_1,x_c,y_1)=0$	$F_{dis1} = f(x_1,x_c,\hat{y}_1,\hat{y}_2)=0$
Dis_2	$y_2 = A_2(x_2,x_c)$	$F_{dis2} = f(x_2,x_c,y_2)=0$	$F_{dis2} = f(x_2,x_c,\hat{y}_1,\hat{y}_2)=0$

* Obj_c1: coupled objective function, Obj_c2: coupled multi-objective function, Con_1 & 2: uncoupled constraints of disciplines 1 & 2, Con_c: coupled constraint, Dis_1 & 2: disciplines 1 & 2.

Find x_2, x_2, x_c
to minimize

$$\hat{F} = \begin{bmatrix} f_c(x_1,x_2,x_c,\hat{y}_1,\hat{y}_2) \\ f_1(x_1,x_c,\hat{y}_1,\hat{y}_2) + f_2(x_2,x_c,\hat{y}_1,\hat{y}_2) \end{bmatrix} \tag{9}$$

subject to

$$\hat{g}_1 = f(x_1,\hat{y}_1)\le 0 \tag{10}$$

$$\hat{g}_2 = f(x_2,\hat{y}_2)\le 0 \tag{11}$$

$$\hat{g}_c = f(x_1,x_2,x_c,\hat{y}_1,\hat{y}_2)\le 0 \tag{12}$$

Discipline 1

$$\hat{y}_1 = A_1(x_1,x_c,\hat{y}_2) \tag{13}$$

Discipline 2

$$\hat{y}_2 = A_2(x_2,x_c,\hat{y}_1) \tag{14}$$

In this context, the above optimization formulation is called the modified multidisciplinary feasible (MDF) formulation for MDO using the integrated coupled approximate models.

4. Numerical experiments

In order for the validation of the proposed optimization method, two numerical examples are considered, a benchmark mathematical problem and an engineering application problem.

4.1 Benchmark problem

The first example is a mathematical multidisciplinary optimization problem composed of two disciplines with three global variables which was previously dealt by Sellar *et al*. (1996). For reference, the optimum solution of global variables is (x_1, x_2, x_3)=(1.9776, 0, 0) and the corresponding system outputs and objective function are (y_1, y_2)=(3.16, 3.7533) and F=3.18339, respectively. The problem is characterized by two coupled variables x_1 and x_3, two system outputs y_1 and y_2, and two constraints g_1 and g_2. The coupling between y_1 and y_2 causes the coupling in analysis and one between x_1 and x_3 gives rise to the coupling in design.

Find x_1, x_2, x_3

to minimize

$$F = x_2^2 + x_3 + y_1 + e^{-y_2} \tag{15}$$

subject to

$$g_1 = 1 - \frac{y_1}{3.16} \leq 0 \tag{16}$$

$$g_2 = \frac{y_2}{24} - 1 \leq 0 \tag{17}$$

$$-10 \leq x_1 \leq 10 \tag{18}$$

$$0 \leq x_2 \leq 10 \tag{19}$$

$$0 \leq x_3 \leq 10 \tag{20}$$

Discipline 1

$$y_1 = x_1^2 + x_2 + x_3 - 0.2 y_2 \tag{21}$$

Discipline 2

$$y_2 = \sqrt{y_1} + x_1 + x_3 \tag{22}$$

The initial values, upper and lower bounds specified for three design variables and two state variables are given in Table 2. In order to construct the approximate models $\hat{y}_1$, $\hat{y}_2$, $\hat{g}_1$, $\hat{g}_2$ and $\hat{F}$ which are in function of three design variables and two state variables, an L_{2^k+1}-type full factorial orthogonal design of experiments is used. Here, k=5 denotes the number of factors (i.e., three design variables and two state variables), “2” the number of levels (i.e., −1 and +1) set for each factors, and “1” an additional experiment case with the factors of level 0. In other words, the total of 33 experiment cases are examined for which the upper bounds and the lower bounds given in Table 2 are chosen for two levels −1 and +1 respectively, for each factor. The center values between the upper and lower bounds are taken as level 0 for one additional experimental case. From the analysis of variance (ANOVA) for examining the reliability of five approximate models, it was found that the linear and interaction terms are significant and the curvature effect should be considered.

In order to improve the reliability, we further refine the approximate models by adding the

Table 2 Initial values, upper and lower bounds of design and state variables

Items	Design variables			State variables	
	x_1	x_2	x_3	y_1	y_2
Initial values	1.0	5.0	2.0	3.0	3.0
Upper bounds	2.0	10.0	4.0	4.0	4.0
Lower bounds	0.0	0.0	0.0	2.0	2.0

central composite design (CCD). Since CCD is an enriched factorial DOE in which all the axial points are included to enhance the approximation efficiency, it can be constructed by adding the experiment cases corresponding to all the axial points to the previous L_{2^k+1} DOE. The central composite design introduced by Box and Wilson (1951) in 1950's is able to effectively evaluate the quadratic effects of the design variables. Each design variable in the standard CCD has five levels; $-\alpha$,−1,0,+1 and $+\alpha$, where the axial points $\pm\alpha$ are calculated by $\pm\sqrt{k}$. But, we adopt the rotatable central composite design (RCCD) in order to secure the stability of approximate models by providing the rotatability to the CCD. The level α in this enriched design is set by 2.378 from the relation of $\alpha = \sqrt[4]{2^k}$, and the ten additional experiment cases are added to the previous full factorial orthogonal DOE.

According to the analysis of variance for the approximation using 43 experiment cases, it has been found that the linear and quadratic terms are most suitable to approximate the objective function $\hat{F}$, constraint $\hat{g}_1$ and state variable $\hat{y}_1$ while only the linear terms are most suitable for the constraint $\hat{g}_2$ and state variable $\hat{y}_2$. Thus, using the integrated coupled approximate models, the original multidisciplinary optimization problem is rewritten as a modified formulation given by

Find x_1, x_2, x_3

to minimize

$$\begin{aligned}\hat{F} = &1.01x_2^2 + 0.043x_3^2 + 0.00015\hat{y}_1^2 + 0.0012\hat{y}_2^2 \\ &+ 0.318x_2 + 0.693x_3 - 0.389\hat{y}_1 + 0.518\hat{y}_2 - 2.59\end{aligned} \tag{23}$$

subject to

$$\hat{g}_1 = -0.316(x_1^2 + x_2 + x_3) - \hat{y}_1 + 2.14 \le 0 \tag{24}$$

$$\hat{g}_2 = 0.0417(x_1 + x_2) - \hat{y}_2 - 1.05 \le 0 \tag{25}$$

$$-10 \le x_1 \le 10 \tag{26}$$

$$0 \le x_2 \le 10 \tag{27}$$

$$0 \le x_3 \le 10 \tag{28}$$

Discipline 1

$$\hat{y}_1 = 0.999x_1^2 + x_2 + 1.09x_3 - 0.086\hat{y}_2 + 3.71 \tag{29}$$

Discipline 2

$$\hat{y}_2 = 1.01x_1 + x_3 - 0.0037\hat{y}_1 - 1.28 \tag{30}$$

The system analysis is carried out by fsolve of MATLAB and the optimization is performed by the sequential quadratic programming (SQR) (Lim and Arora, 1986), for which the convergence tolerance is set by 1.0×10^{-4} for both the constraints and objective functions. The comparison of the optimization results between the conventional and modified MDFs is given in Table 3. The proposed modified MDF leads to the maximum absolute error of 0.022 in the optimum solution and the relative error of 2.37% in the objective function. And, the proposed method provides the optimum solution with the extremely small number of analyses, when compared with the conventional method. Note that $\hat{F}, \hat{y}_1$ and $\hat{y}_2$ in Eqs. (23)-(30) are approximate functions obtained by the design of experiments (DOE), differing from F,y_1 and y_2 in Eqs. (15)-(22) which are not approximate but exact functions. Eqs. (23)-(30) are solved by the conventional MDF in which three exact functions at each optimization state are sought by Newton-Raphson iteration, thus the total number of analyses increases in proportional to the total number of Newton-Raphson iterations. Meanwhile, the computation of $\hat{F}, \hat{y}_1$ and $\hat{y}_2$ in Eqs. (23)~(30) is straightforward without relying on Newton-Raphson iteration, and the analyses are required only for approximating three functions by the design of experiments.

4.2 Suspension unit of wheeled armored vehicle

As a dynamic system composed of vehicle, bullets and arms, wheeled armored vehicle should

Table 3 Comparison of the optimization results between conventional and modified MDFs

Method	Design variables (x_1, x_2, x_3)			Objective function (F)		Total number of analyses
	Initial	Optimum	Error	Value	Relative error (%)	
Conventional	(1, 5, 2)	(1.978, 0, 0)	-	3.183	-	1,278
Modified		(2, 0.026, 0)	(0.022, 0.023, 0)	3.261	2.37	93 (7.28%)*

* Relative number of analyses with respect to the conventional MDF.

Table 4 Parameters taken for the half-car dynamic model

Items	Values	Remarks
Vehicle speed, V_s (m/s)	20	-
Mass of suspended mass, m_s (kg)	567	-
Moment of inertia of suspended mass, I_s (kg·m^2)	600	-
Mass of suspension units, m (kg)	72.5	-
Spring coefficient of suspension units, k (kN/m)	28.03	design variable
Damping coefficient of suspension units, c (kN·s/m)	3	design variable
Spring coefficient of tires, k_T (kN/m)	400	design variable
Damping coefficient of tires, c_T (kN·s/m)	0.162	design variable
Distance between front axle and center of gravity, a (m)	0.9	-
Distance between front axle and center of gravity, b (m)	1	-
Radius of bump, R_b (m)	0.1	-

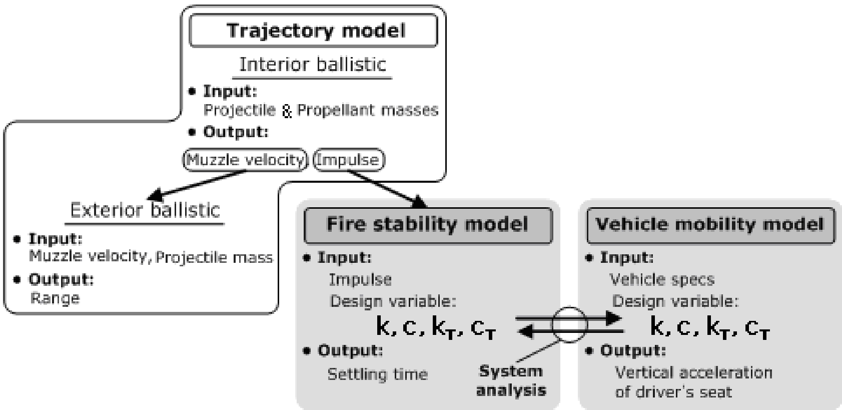

Fig. 4 The integrated coupled analysis system

exhibit good performances in firing and mobility. However, it has been found from our previous study that a strong correlation exists between the firing and mobility performances (Choi *et al.* 2010). In order to maximize these two coupled performances by the modified multidisciplinary optimization, the analysis models for three disciplines, that is, the trajectory, fire stability and vehicle mobility models should be considered as represented in Fig. 4. The firing stability is evaluated by the time required for the vehicle to be reached at the stable state after firing by the firing impulse force providing the maximum range, while the mobility performance is by the maximum vertical acceleration at the driver seat when the vehicle passes over the bump formed on ground.

In order to analyze the stabilizing time and the vertical acceleration, a half-car dynamic model with 4 DOFs shown in Fig. 5 is adopted. The half-car model is widely used to evaluate the basic kinetic behaviors of the concept designs of various kinds of vehicles, which is also employed for the VEHDYN II (Creighton 1985) of the US army, a program to simulate the vehicle mobility. The motion of the dynamic model is restricted to 2-D planar, and both the unsuspended and suspended are assumed to be rigid. The damped elastic properties of both suspensions and tires (Aharp and Crolla 1987, Sleeper and Dreher 1980) are expressed in terms of the Voigt models, and the corresponding spring constants and damping ratios are set by variables to be tailored. Note that spring constants and damping ratios of front and rear parts are set equally such that the total design variables become four, $x_1=k$, $x_2=c$, $x_3=k_T$ and $x_4=c_T$. The detailed numerical values taken for the half-car dynamic model are given in Table 4.

By denoting $\boldsymbol{Z}=\{z_s,\ \theta_s,\ z_1,\ z_2\}^T$ be the rigid body motion of suspended masses, the dynamic response of the half-vehicle model is governed by

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{Z}}(t)\}+[\boldsymbol{C}]\{\dot{\boldsymbol{Z}}(t)\}+[\boldsymbol{K}]\{\boldsymbol{Z}(t)\}=\{\boldsymbol{F}(t)\} \tag{31}$$

with the mass, damping and stiffness matrices defined in Appendix A. While the external load vector $\boldsymbol{F}(t)$ is composed of F_s and F_θ stemming from the impulse at firing, and the road excitations $F_1=k_{T1}z_{g1}$ and $F_2=k_{T2}z_{g2}$. Note that the impulsive force which is measured by experiment at the firing showing the longest firing distance is used. A test Fortran program was coded to numerically solve the above dynamic equations, for which the fourth-order Runge-Kutta method is employed. The reader may refer to the book by James *et al*. (1994) for the detailed numerical solution procedure.

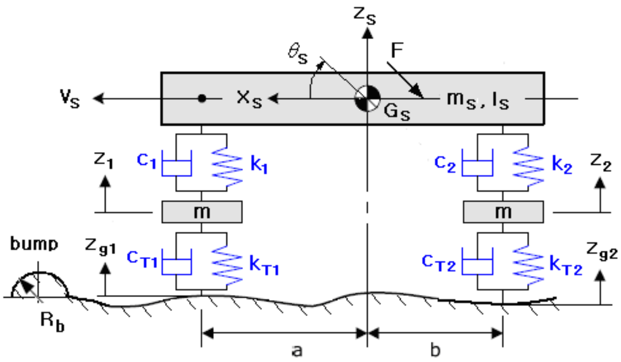

Fig. 5 A half-car dynamic model of the four-wheeled armored vehicle

The optimum design of the damping ratios and spring coefficients of the suspension unit including tires for improving the firing stability y_1 and the mobility y_2 is formulated as

Find x_1, x_2, x_3, x_4

to minimize

$$F = wy_1 + (1-w)y_2 \tag{32}$$

subject to

$$25.227 \le x_1 \le 30.833 \tag{33}$$

$$2.7 \le x_2 \le 3.3 \tag{34}$$

$$360 \le x_3 \le 440 \tag{35}$$

$$0.1458 \le x_4 \le 0.1782 \tag{36}$$

with the weighting factor $w(0 \le w \le 1)$. According to our previous work (Choi *et al.* 2010) on this problem by the response surface method, the response surfaces y_1^{rs} and y_2^{rs} of two performances which were approximated making use of an L_{2^k+1}-type full factorial orthogonal DOE plus a rotatable central composite design (RCCD) and the analysis of variance (ANOVA) are given by

$$\begin{aligned} y_1^{rs} = &-0.0792009\,x_1 - 0.91125\,x_2 - 0.00040312\,x_3 + 0.76389\,x_4 + 0.0276489\,x_1x_2 \\ &- 0.024775\,x_1x_4 + 0.00003125\,x_2x_3 - 0.231481\,x_2x_4 + 0.00173611\,x_3x_4 + 3.2308 \end{aligned} \tag{37}$$

$$\begin{aligned} y_2^{rs} = &\,0.000021213\,x_1^2 - 0.0148148\,x_2^2 - 0.0000005208\,33x_3^2 + 4.92176\,x_4^2 + 0.00160542\,x_1 \\ &+ 0.0772222\,x_2 + 0.00153125\,x_3 - 2.75206\,x_4 - 0.00104055\,x_1x_2 + 0.0000089190\,2x_1x_3 \\ &- 0.00275278\,x_1x_4 + 0.00135417\,x_2x_3 + 0.540123\,x_2x_4 - 0.000771605\,x_3x_4 + 0.085 \end{aligned} \tag{38}$$

In case of the suspension unit optimization, there is a coupling in analysis owing to the interaction between two performances, which naturally gives rise to the coupling in design. Considering the coupling in analysis, the response surfaces of two state variables y_1^{rs} and y_2^{rs}

Table 5 Initial values, upper and lower bounds of design variables

Items	Design variables			
	x_1 (k)	x_2 (c)	x_3 (k_T)	x_4 (c_T)
Initial values	28.030	3.0	400	0.162
Upper bounds	30.833	3.3	400	0.1782
Lower bounds	25.227	2.7	360	0.1458

are transformed into the integrated coupled approximate models in terms of x_1, x_2, x_3, x_4 and the counterpart state variable. Note that the transformation of the objective function F is straightforward because it is a simple linear combination of y_1 and y_2. On the other hand, there is no need to transform four constraints because those simply specify the upper and lower bounds of design variables. In order to construct the integrated coupled approximate models $\hat{y}_1$ and $y_{\hat{2}}$, according to the transformation procedure in the current study, 24 experiment cases by a rotatable central composite design (RCCD) are carried out by setting the initial, upper and lower bounds of design variables as given in Table 5.

As in the previous analytic problem, the reliability of six integrated approximate models using linear terms, linear and interaction terms, linear and quadratic terms, simple quadratic terms, second interaction terms or full quadratic terms is evaluated in terms of the coefficients of determination R^2 and $adj.R^2$. It was found from the analysis of variance (ANOVA) that the full quadratic and simple linear models are most suitable for the firing stability y_1 and the vehicle mobility y_2 respectively, with R^2 of 0.998 and $adj.R^2$ of 0.997. Then, using the integrated coupled approximate models, the original multidisciplinary optimization problem is rewritten as a modified one given by

Find x_1, x_2, x_3, x_4

to minimize

$$\hat{F} = w\hat{y}_1 + (1-w)\hat{y}_2 \tag{39}$$

subject to

$$25.227 \le x_1 \le 30.833 \tag{40}$$

$$2.7 \le x_2 \le 3.3 \tag{41}$$

$$360 \le x_3 \le 440 \tag{42}$$

$$0.1458 \le x_4 \le 0.1782 \tag{43}$$

Discipline 1

$$\begin{aligned}\hat{y}_1 = &-0.00187x_1^2 - 19.2x_2^2 - 0.000987x_3^2 + 44.4x_4^2 - 50.2\hat{y}_2^{rs\,2} + 0.226x_1 \\ &+ 91.8x_2 + 0.665x_3 + 56x_4 - 158y_2^{rs} - 0.0689x_1x_2 - 0.000435x_1x_3 \\ &- 0.167x_1x_4 + 0.138x_1y_2^{rs} - 0.267x_2x_3 - 25.3x_2x_4 + 62x_2y_2^{rs} - 0.27x_3x_4 \\ &+ 0.451x_3y_2^{rs} + 51.7x_4y_2^{rs} - 105\end{aligned} \tag{44}$$

Table 6 Comparison of the optimum solutions between conventional and present MDFs

Method	Weight w	Optimum solutions						Number of analyses
		Design variables				Performances		
		x_1	x_2	x_3	x_4	y_1 (sec)	y_2 (g)	
Conv. MDF	0.3	30.445	2.70	360	0.168	0.519 (0.54%)	1.927 (-0.05%)	2,898
	0.5	25.241	2.88	360	0.146	0.519 (-4.93%)	2.010 (0.06%)	378
	0.7	25.259	3.30	360	0.146	0.418 (1.36%)	2.232 (0.0%)	558
Mod. MDF	0.3	25.240	2.88	360	0.173	0.519 (-4.42%)	2.017 (-0.06%)	50
	0.5	25.227	2.89	360	0.176	0.519 (-5.47%)	2.018 (-0.04%)	50
	0.7	25.227	3.30	360	0.153	0.419 (1.29%)	2.233 (-0.04%)	50

(*) indicate the relative errors measured with respect to the solutions obtained by the test Fortran program with the optimum design variables.

Discipline 2

$$\hat{y}_2 = 0.00284x_1 + 0.585x_2 + 0.0053x_3 + 0.077x_4 - 0.0261y_1^{rs} - 1.66 \tag{45}$$

The PLBA(Pshenichy-Lim-Belegundu-Arora) algorithm (1986), which is based on recursive quadratic programming, is used to seek the optimum solutions. The optimum solutions sought by conventional and present modified MDFs for different weighting factors w are compared in Table 6, where the values in parenthesis indicate the approximation accuracy of the integrated coupled approximate models $\hat{y}_1$ and $\hat{y}_2$. To evaluate the approximation accuracy, two performances y_1 and y_2 corresponding to the optimum design variables are directly solved by our test Fortran program which was coded to solve the rigid body dynamic Eq. (31). It is clearly justified that the integrated coupled approximate models are reliable with the maximum relative error of 5.47%. From the optimum design variables, it is found that the tire spring coefficient k_T(=x_3) is kept at the lower bound for both MDFs for all the weighting factors, but the other three optimum design variables show the dependence on the type of MDFs. Nevertheless, the differences in two performances y_1 and y_2 between conventional and modified MDFs are found to be negligible for all the weighting factors, implying that the present multidisciplinary optimization problem is characterized by multi-peaks (Cho *et al.* 2002). Meanwhile, it has been also observed that the proposed method provides us the optimum solution with the extremely small number of analyses, when compared with the conventional method.

As illustrated in the previous example on the suspension unit design of wheeled armored vehicle, the proposed method could be applied to the MDO problems in various engineering applications by utilizing the finite element analyses (FEA). The main process for implementing the proposed method is to approximate the MDO formulation, and the objective function, constraints and state variables which are coupled in more than one discipline could be approximated by the finite element analyses according to the design of experiments.

5. Conclusions

A modified multidisciplinary feasible formulation for MDO using the integrated coupled approximate models has been introduced in this paper. The interrelation between the design and coupling variables in disciplines was approximated by response surface method to resolve the coupling in analysis. Meanwhile, the coupling in design between disciplines was resolved by transforming the MDO formulation through the interaction between the design and coupling variables, the objective functions and the constraints. The validity and effectiveness of the proposed method have been verified through two benchmark MDO problems, a mathematical MDO problem and the weighted multi-objective optimization of suspension unit of wheeled armored vehicle. Through the numerical results, it has been observed that the modified method seeks an optimum solution with the extremely small number of analyses. In the mathematical MDO problem, the present method provides the optimum design variables and the corresponding objective function which are almost the same with those obtained by the conventional method, with the maximum relative error of 2.37%. Meanwhile, in the optimum design of suspension unit, the present method leads to the optimum design variables which are slightly different from those sought by the conventional method, nevertheless the difference in the objective functions is found to be negligible.

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Appendix

Matrices in Eq. (31)

The mass, damping and stiffness matrices included in the rigid body dynamic motion (39) of the half-car model are defined by

$$[\boldsymbol{M}] = \begin{bmatrix} m_S & 0 & 0 & 0 \\ 0 & I_S & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{bmatrix} \tag{A1}$$

$$[\boldsymbol{C}] = \begin{bmatrix} c_1 + c_2 & ac_1 - bc_2 & -c_1 & -c_2 \\ ac_1 - bc_2 & a^2c_1 + b^2c_2 & -ac_1 & bc_2 \\ -c_1 & -ac_1 & c_1 + c_{T1} & 0 \\ -c_2 & bc_2 & 0 & c_2 + c_{T2} \end{bmatrix} \tag{A2}$$

$$[\boldsymbol{K}] = \begin{bmatrix} k_1 + k_2 & ak_1 - bk_2 & -k_1 & -k_2 \\ ak_1 - bk_2 & a^2k_1 + b^2k_2 & -ak_1 & bk_2 \\ -k_1 & -ak_1 & k_1 + k_{T1} & 0 \\ -k_2 & bk_2 & 0 & k_2 + k_{T2} \end{bmatrix} \tag{A3}$$