Structural Engineering and Mechanics, *Vol. 51, No. 5 (2014) 847-866*
DOI: http://dx.doi.org/10.12989/sem.2014.51.5.847

Design analysis of the optimum configuration of self-anchored cable-stayed suspension bridges

Paolo Lonetti* and Arturo Pascuzzo[a]

Department of Civil Engineering, University of Calabria, Via P. Bucci, 87030, Rende, Cosenza, Italy

(Received March 13, 2014, Revised June 20, 2014, Accepted July 2, 2014)

Abstract. This paper describes a formulation to predict optimum post-tensioning forces and cable dimensioning for self-anchored cable-stayed suspension bridges. The analysis is developed with respect to both dead and live load configurations, taking into account design constrains concerning serviceability and ultimate limit states. In particular, under dead loads, the analysis is developed with the purpose to calculate the post-tensioning cable forces to achieve minimum deflections for both girder and pylons. Moreover, under live loads, for each cable elements, the lowest required cross-section area is determined, which verifies prescriptions, under ultimate or serviceability limit states, on maximum allowable stresses and bridge deflections. The final configuration is obtained by means of an iterative procedure, which leads to a progressive definition of the stay, hanger and main cable characteristics, concerning both post-tensioning cable stresses and cross-sections. The design procedure is developed in the framework of a FE modeling, by using a refined formulation of the bridge components, taking into account of geometric nonlinearities involved in the bridge components. The results demonstrate that the proposed method can be easily utilized to predict the cable dimensioning also in the framework of long span bridge structures, in which typically more complexities are expected in view of the large number of variables involved in the design analysis.

Keywords: long span bridges/structures; numerical methods; structural design; finite element method (FEM); post-tension

1. Introduction

Cable-supported bridges are defined by an enhanced combination of the structural components, which lead to notable advantages such as easy erection, efficient utilization of the materials and reduced costs to overcome long spans. In this framework, self-anchored cable-stayed suspension bridges can be considered as an hybrid configuration, which combines the best properties of pure cable-stayed or suspension systems (Zhang *et al*. 2009, Gimsing and Georgakis 2012). In particular, such bridge typologies can be regarded as an extension of the Roebling or the Dischinger schemes, in which the coexistence of the two cable systems, namely suspension and cable-stayed, is able to improve the bridge stiffness, avoiding unstable erection procedures due to the free cantilever arms growing to the half-length of the main span. However, a brief literature

∗Corresponding author, Associate Professor, E-mail: paolo.lonetti@unical.it
[a]Ph.D. Student, E-mail: arturo.pascuzzo@unical.it

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

review denotes that most of the studies developed in the framework of cable supported bridges are mainly concerned to investigate the behavior of pure cable-stayed or suspension systems and only in rare cases hybrid cable-stayed suspension bridges (Bruno *et al*. 2009). As a consequence, the use of self-anchored cable-stayed suspension bridges is quite limited in practice and it still remains in the design proposal phase.

An important task to be achieved in the bridge design concerns the evaluation of the target stress state and the initial geometrical configuration under service loads. In addition, a proper design analysis which quantifies the bridge components, taking into account displacement and stress prescriptions under live loads is much required. As a matter of fact, the design procedures should consistent with the performance based design (PBD), which means the best utilization of the weight utilized in the structure and thus the lowest possible costs in the bridge construction (Liang and Steven 2002). However, such task becomes quite complicated for large scale structures, such as cable-supported bridges, because of the large number of variables to be identified. Moreover, additional complexities arise in the case of self-anchored structures, because of the coupling behavior between the cable system and the girder, which may lead, in some cases, to dangerous instability phenomena especially during the construction phases (Zhang *et al*. 2005; Raftoyiannis 2013). In the literature, most of the existing studies are based on the use of simple design rules obtained by the experience and expertise of the designers, in which relationships typically adopted in the framework of pure cable-stayed or suspension configurations are utilized (Gimsing and Georgakis 2012, Günaydin *et al*. 2012). Refined techniques can be recovered in the framework of the zero displacement methods (ZDMs), where implicit or explicit constraint equations are introduced to achieve the initial configuration. In particular, during the erection procedure, such methods assess the post-tensioning forces under dead loading, enforcing the structure to remain practically undeformed (Greco *et al*. 2013, Liu *et al*. 2012, Ohkubo 1992, Wang *et al*. 2004). The governing equations are expressed as a function of the internal forces of the cable system and introduce a determinate equation system, in which the unknown quantities are obtained, prescribing, at discrete points of the structure, zero displacement conditions. Alternatively to ZDMs, force equilibrium methods (FQMs) consider as control variables to be solved, the internal forces, which are determined reducing bending moments and displacements of the girder, achieving a structural scheme of the girder approximately equivalent to a simply supported continuous beam (Chen 2000, Kim and Lee 2001). Previous models based on ZDMs or FQMs, are mainly developed in the frameworks of cable-stayed or suspension systems, in which typically, the optimum configuration is derived by solving a determinate system of equations. In such context, the main aims of the proposed methodologies are to evaluate mainly the initial post-tensioning forced under dead loading. Alternatively to direct methods, models, developed in the framework of structural optimization (SO), are frequently adopted in the literature. In particular, the optimum configuration is reached by solving a constrained optimization problem, in which the minimization of an objective function can be defined in terms of norm of displacements (Hassan *et al*. 2012), weight of the materials (Lonetti and Pascuzzo 2014) or stress variables (Zhe *et al*. 2010, Pengzhen *et al*. 2014). Moreover, refined techniques concerning robust optimization procedures based neural networks can be recovered in (Barbero and Makkapati 2010). However, such models, especially in the cases of long span bridges, due to the presence of a large number of variables are affected by convergence problems in the solving procedure, which may lead to a local minimum of the objective function and unpractical results in the bridge definition. It is worth noting that, the models described above, developed in the framework of ZDM, FQM or SO, evaluate the initial cable forces directly on the final configuration. However, in the literature, several approaches can be recovered, in which target stress state is predicted by means of step by step procedures based on the actual construction process going from the initial to the final configuration. In particular, computational procedures for the shape finding analysis based on a forward and a backward process are proposed (Behin and Murray 1992, Wang *et al*. 2004), in which different erection stages are investigated in the framework of the free cantilever method. Moreover, in this framework, the unit load model (ULM) developed by Janjic *et al*. (2003) evaluate the desired moment distribution in the final configuration, computing the post-tensioning strategy of the construction method by using additional constraint conditions. An extended and reviewed version of the ULM is proposed by Lee *et al*. (2008) also for asymmetric cable-stayed bridges, in which a two step scheme is proposed for the optimum evaluation of the initial cable forces.

It is worth noting that most of the models presented above are concerned to investigate cable-stayed and, only in rare cases, hybrid cable-stayed suspension bridges; in particular, none of those is involved in the description of self-anchored cable-stayed suspension bridges. Moreover, the optimization analysis is typically developed with the purpose to evaluate only the initial stress state and the corresponding cable forces under dead loads, without verifying if the cable dimensioning is consistent with the design code prescriptions on both stress or deflections, produced by the live load application. Therefore, in the present study, an hybrid model based on two different steps is proposed in which the analysis is developed on the basis of the results obtained by dead and live load configurations. In the former, cable forces are determined in such a way to verify the initial bridge profile. Subsequently the analysis is devoted to analyze the behavior under the action of live loads, enforcing an efficient use of the materials and thus the complete optimization of the material involved in the cable system. The final solution is determined by means of iterative procedure between the above referred steps. In the present formulation, the attention is devoted to analyze the optimum configuration of the elements involved in the cable system, without entering in the optimization of the pylons and girder. Such assumption appears to be quite reasonable especially in the case of long span bridge schemes, where the bridge behavior is mainly dominated by the cable system characteristics. The outline of the paper is as follows. In Section 2, the formulation of the design methodology, bridge modeling, together with the description of the iterative procedure is presented. In Section 3, numerical details of the design method are reported, whereas results on different bridge schemes are presented in Section 4.

2. Formulation of the design model

The structural scheme, reported in Fig. 1, is consistent with a self-anchored cable-stayed suspension bridge in which the cable system is based on the combination of suspension and cable-stayed configurations. In particular, the cable-stayed system is based on discrete stays, which are mainly arranged in the region close to the pylons. Moreover, the suspension system, essentially formed by the hangers and main cable, is connected to the girder by the hangers in the central region of the main span length.

The main aim of the proposed model is to evaluate optimum set of post-tensioning forces and cable cross-sections, which satisfy structural and design requirements in either dead and live configurations. In particular, under the action of dead loads (DL), the post-tensioning forces of the cables are calculated in such a way that the bridge should behave as a simply supported continuous beam, thus presenting reduced displacements of the girder and pylons (STEP1). Moreover, from

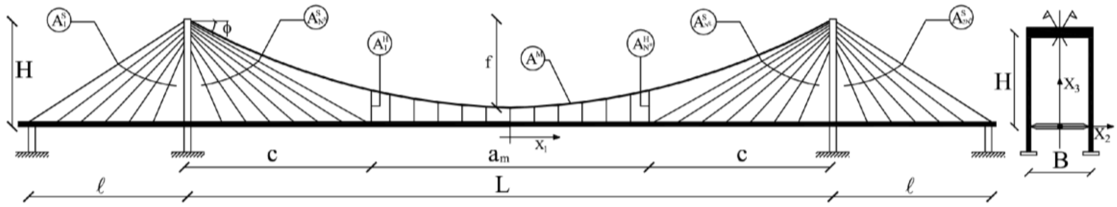

Fig. 1 Structural scheme of the self-anchored cable-stayed suspension bridge

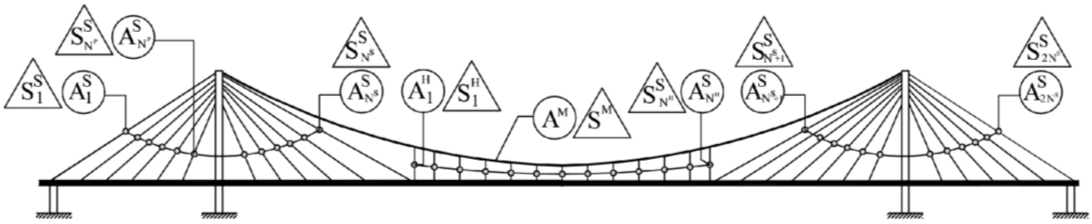

Fig. 2 Identification of the variables involved in the design procedure

the design point of view, the cross-sections of the cable system elements should be designed consistently to the "*maximum performance criterion*", which, basically, consists to verify, under the worst live load LL combinations, the equality condition between the maximum absolute or incremental applied stresses and the corresponding allowable strength values (STEP2). In the proposed formulation, such task is developed by means of an iterative procedure defined by a two-step algorithm, based on the results arising from both DL and LL configurations.

2.1 Analysis under the action of DL (STEP1)

With reference to the bridge scheme reported in Fig. 2, the design variables are represented by the cross-sections (A_i^S, A_i^H A^M) and the post-tensioning forces of the cable system (S_i^S, S_i^H S^M), which are designed by means of the following relationships

$$\underset{\sim}{S}_C = \left\{ S_1^S, ..., S_{2N^S}^S, S_1^H, ..., S_{N^H}^H, S^M \right\}, \quad \underset{\sim}{A} = \left\{ A_1^S, ..., A_{2N^S}^S, A_1^H, ..., A_{N^H}^H, A^M \right\} \tag{1}$$

where N^S is the number of stays of the left or right pylons, N^H is the number of hangers and the superscripts S, H and M refer to the stays, hangers and main cable, respectively. The unknown quantities, reported in Eq. (1), are derived in the DL configuration by solving a set of constraint equations, which enforce the bridge to remain in the undeformed configuration or equal to an initial design profile. In particular, the constraint operators are defined through the following hypotheses, namely H.1 and H.2:

• the displacements of girder and pylons must be zero, under the action of dead loads (H.1);

• the internal stresses of the i-th stay, j-th hanger (with $i=1..N^S$ and $j=1..N^H$) and main cable should be equal to the corresponding initial design values, which are identified as $S_{gi}^{\ S}$, $S_{gi}^{\ H}$ and $S_g^{\ M}$ and are determined on the basis of the results obtained in the subsequent steps, i.e., STEP2, based on the live load analysis (H.2).

The constraint equations, which enforce hypothesis H.1, are expressed in terms of the incremental values of the internal cable tensions, i.e., ΔS_i, measured from the converged ones obtained from the previous step, i.e., $\bar{S}_{i-1}$ or in the case of the first iteration by initial trial values (evaluated in section n.3) In particular, such relationships enforce zero displacement conditions for the following specific kinematic quantities:

• horizontal displacements of the pylons and vertical displacements of the stays in the main cable at the girder connections, i.e., $\underset{\sim}{U}^{S(T)} = \left[U_1^S, U_2^S, .., U_{N_1^S}^S, V_{N_1^S+1}^S, ..., V_{N^S}^S \right]$;

• vertical displacements of the hangers at the girder connections, i.e., $\underset{\sim}{U}^{H(T)} = \left[V_1^H, V_2^H, ..., V_{N^H}^H \right]$;

• vertical displacement of the main cable at the midspan cross section, i.e., U^M.

Therefore, the required equations to verity the kinematic prescriptions are defined as

$$\begin{aligned} &\underset{\sim}{L}_S \left[\left(\bar{S}_1^S + \Delta S_1^S, ..., \bar{S}_{2N_S}^S + \Delta S_{2N_S}^S \right), \underset{\sim}{U}^S \right] = 0, \\ &\underset{\sim}{L}_H \left[\left(\bar{S}_1^H + \Delta S_1^H, ..., \bar{S}_{N^H}^H + \Delta S_{N^H}^H \right), \underset{\sim}{U}^H \right] = 0, \\ &\underset{\sim}{L}_M \left[\left(\bar{S}^M + \Delta S^M \right), U^M \right] = 0, \end{aligned} \tag{2}$$

where $\underset{\sim}{L}_S, \underset{\sim}{L}_H$ and $\underset{\sim}{L}_M$ are multi freedom constraint operators referred to the stays, hangers and main cable variables, respectively. Similarly, additional equations are introduced to verify prescription provided by hypotheses H.2. In particular, the cross-sections of the stays, hangers and main cable elements, i.e., A_i^S, A_i^H and A^C are changed from their previous estimated values, i.e., $\bar{A}_i^S, \bar{A}_i^H$ and $\bar{A}^M$ introducing additive incremental variables, i.e., ΔA_i^S, ΔA_i^H and ΔA^C, as a function of explicit constraint equations, which enforce the stresses in the cables to be equal to the initial design quantity

$$\begin{aligned} &\underset{\sim}{C}_S \left[\left(\bar{A}_i^S + \Delta A_i^S \right), S_i^S - S_{gi}^S \right] = 0, && i = 1, .., 2N^S, \\ &\underset{\sim}{C}_H \left[\left(\bar{A}_i^H + \Delta A_i^H \right), S_i^H - S_{gi}^H \right] = 0, && j = 1, .., N^H, \\ &C_M \left[\left(\bar{A}^M + \Delta A^M \right), S^M - S_g^M \right] = 0, && \end{aligned} \tag{3}$$

where $\underset{\sim}{C}_S, \underset{\sim}{C}_H$ and C_M are the constrain operators, which ensures that the stress variables of the cable elements, i.e., $\left(S_i^S, S_j^H \text{ and } S^M \right)$, are equal to the prescribed values $\left(S_{gi}^S, S_{gi}^H \text{ and } S_g^M \right)$.

2.2 Analysis under the action of LL (STEP2)

In this framework, the analysis is developed with the purpose to verify the bridge behavior under live loads taking into account ultimate, fatigue and serviceability limit states, i.e., ULS, FLS and SLS. In particular, the following conditions, concerning maximum and relative stresses and maximum absolute displacements should be verified

$$\max \left[S_i^{S,H,M} \right]_{ULS} \le S_A, \quad \max \left[\Delta S_i^{S,H,M} \right]_{FLS} \le \Delta S_A \tag{4}$$

$$\max\left[\left|U_3^G\right|\right]_{SLS} \leq \delta_A^{\mathrm{G}}, \ \ \max\left[\left|U_1^P\right|\right]_{SLS} \leq \delta_A^{\mathrm{P}} \tag{5}$$

where S_A and ΔS_A are the maximum or incremental allowable stress values of the cables, $\delta_A{}^{\mathrm{G}}$ and $\delta_A{}^{\mathrm{P}}$ are the maximum allowable displacements of the girder (vertical) and pylons (horizontal), respectively. Similarly, proposed formulation can be generalized in such a way to be consistent with respect to enhanced reliability formulations as the one reported for instance in (Barbero *et al*. 2013). The initial stresses of the cable elements $\left(\underset{\sim}{S}_g^S, \underset{\sim}{S}_g^H, S_g^M\right)$ are evaluated as a function of two sets of factors defined for each cable element, namely Φ_i and Ω_i, which are introduced to verify code prescriptions reported in Eqs. (4)-(5). The basic idea for the definition of such quantities is related to the physical behavior of the cable-supported bridges, which typically can be associated to a continuous beam on multiple elastic supports; the stiffness of the supports is controlled by the mechanical characteristics of the cables, taking also into account the contributions arising from the geometrical nonlinearities of the elements. In light of such considerations, the factors Φ_i and Ω_i are designed in such a way to modify, on the basis of the LL results, the stiffness of the cable system, enforcing the requirements on both strength and deformability of the bridge components. It is worth noting that the factors Φ_i and Ω_i can be regarded as constant quantities, associated to the single element of the suspension system. In particular, the performance factors Φ_i modify the stress distribution in the cable system, by enforcing, consistently with the PBD, that the maximum values under live loads equal to allowable stress arising from the strength of material. On the contrary, the factors Ω_i modify the stiffness of the cable system and thus the cross-section of the elements, by enforcing the pylons and girder to have lower displacements than the maximum permissible values. According to a secant approach, at the generic iteration step, the initial stresses are defined from the previously converged value by the following relationships

$$\left(S_{gi}^S\right)^k = \left[\Phi^S\Omega^S\right]_i^k \left(S_g^S\right)_i^{k-1}, \ \left(S_{gi}^H\right)^k = \left[\Phi^H\Omega^H\right]_i^k \left(S_g^H\right)_i^{k-1}, \ \ \left(S_g^M\right)^k = \left[\Phi^M\Omega^M\right]^k \left(S_g^M\right)^{k-1} \tag{6}$$

where the superscript k or k-1 refer to the current or the previous iteration step and the subscript i refers to the generic i-th cable element. In particular, the factors Φ_i concerning the stresses for the i-th stay, hanger or the main cable are be derived by the ratios between the allowable stress and the maximum value observed in the LL combinations

$$\left(\Phi^S\right)_i^k = \frac{S_A}{\max\limits_{LL}\left(S_{LL}^S\right)_i^k}, \quad \left(\Phi^H\right)_i^k = \frac{S_A}{\max\limits_{LL}\left(S_{LL}^H\right)_i^k}, \quad \left(\Phi^M\right)^k = \frac{S_A}{\max\limits_{LL}\left(S_{LL}^M\right)^k}, \tag{7}$$

where S_{LL} is the value of the stress observed during the LL combinations. Moreover, the definition of the factors Ω_i is strictly connected to the following limit functions, which indicate the distance from the limit allowable values of the current displacement and stress quantities

$$\mathrm{g}_{\mathrm{U}_{1i}^k}^P = \frac{\delta_{1A}^P}{\max\left(\left|U_{1LL}^P\right|\right)_i^k} - 1, \ \ \mathrm{g}_{\mathrm{U}_{3i}^k}^G = \frac{\delta_{3A}^G}{\max\left(\left|U_{3LL}^G\right|\right)_i^k} - 1, \ \ g_{S_{Ai}^k} = \frac{\left(S_A^j\right)_i^{k-1}}{S_A} - 1, \quad j = S, H, M \tag{8}$$

As far as the kinematic functions, namely $\mathrm{g}_{\mathrm{U}_{1i}^k}^P$ for the pylon and $\mathrm{g}_{\mathrm{U}_{3i}^k}^G$ for the girder, are negative, an increment of stiffness is required, since the displacements in the structure are expected to be larger than the allowable quantity. Similarly, when the allowable function g_{S_A} is

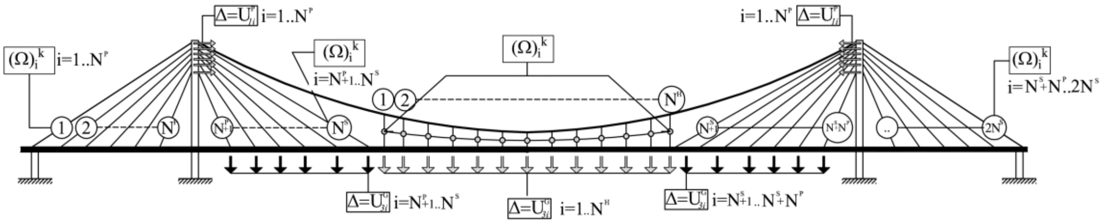

Fig. 3 Synoptic representation of the factors Ω_i and the variables Δ_i involved in the design procedure

negative it means that the cable stiffness can be potentially reduced, since the stress is lower than allowable strength and thus a lower stiffness in the cable is permitted than the current value. As a results, the definition of the Ω_i for a generic element is based on the following piecewise functions

$$(\Omega)_i^k = \begin{cases} \dfrac{|\Delta_{\max}|}{\max\limits_{LL}\left[\left(|\Delta|\right)_i^k\right]} \dfrac{\left(S_A\right)_i^{k-1}}{S_A} & \text{if} \quad g_\Delta \le 0 \\ \dfrac{|\Delta_{\max}|}{\max\limits_{LL}\left[\left(|\Delta|\right)_i^k\right]} \dfrac{\left(S_A\right)_i^{k-1}}{S_A} & \text{if} \quad g_{S_{Ai}^k} < 0 \text{ and } g_\Delta > 0 \\ 1 & \text{if} \quad g_{S_{Ai}^k} \ge 0 \text{ and } g_\Delta > 0 \end{cases} \tag{9}$$

where Δ_{max} and Δ are the allowable and maximum LL displacements, respectively, and g_Δ the displacement limit function. It is worth noting that Eq. (9).1 is essentially utilized when the displacements are larger than the allowable quantity and thus an increment of stiffness is required; when $g_\Delta>0$ and $g_{S_{Ai}^k} < 0$, namely in the case of Eq. (9).2, prescriptions on displacements are satisfied and thus the current allowable stress level of cable can be increased to enforce the equality with strength value (S_A). Finally, the case defined by Eq. (9).3, is related to the condition in which values of the stress and the displacement limit functions are both positive and thus there is no need to modify the stiffness or the allowable stress level of the element, thus the factor is assumed to be equal to the unity. However, the quantities Δ and Δ_{max} are different in each part of the bridge, since the elements of the cable system are differently related to the bridge deformability. As a matter of fact, the stay elements in the lateral spans are typically designed to reduce the horizontal displacements of the pylons, whereas the stays and the hangers in the main span or the main cable are dimensioned to reduce vertical deflections of the girder. Therefore, starting from such considerations, the following relationships identify the variables Δ, Δ_{max} and g_Δ introduced in Eq. (9)

$$\Delta = U_1^P, \quad \Delta_{\max} = \delta_A^P, \quad g_\Delta = g_{U_{1i}^k}^P \qquad (\textit{Stays – lateral span}) \tag{10}$$

$$\Delta = U_3^G, \quad \Delta_{\max} = \delta_A^G, \quad g_\Delta = g_{U_{3i}^k}^G \qquad (\textit{Stays – central span}) \tag{11}$$

$$\Delta = U_3^G, \quad \Delta_{\max} = \delta_A^G, \quad g_\Delta = g_{U_{3i}^k}^G \qquad (\textit{Hangers}) \tag{12}$$

$$\Delta = \max\left(U_3^G\right), \quad \Delta_{\max} = \delta_{3A}^G, \quad g_\Delta = \max\left(g_{U_{3i}^k}^G\right) \qquad (\textit{Main cable}) \tag{13}$$

A synoptic representation of the variables defined in Eqs. (10)-(13) is reported in Fig. 3.

2.3 Convergence condition and iterative scheme

The design analysis defined by the previous steps is based on an iterative procedure, in which the initial stresses of the cable elements, namely $\left(\underset{\sim}{S}_g^S, \underset{\sim}{S}_g^H, S_g^C\right)$, are modified during the iterations. Therefore, once the new values of the stress levels in the DL configuration are evaluated by means of Eq. (6), the procedure goes back to find the prediction of the cross-sections and the post-tensioning forces on the basis Eqs. (2)-(3). This procedure is repeated until achieving the convergence conditions of the algorithm defined on the basis of the following expression

$$\max\left\{\sum_{i=1}^{N_S}\left[\frac{\left(S_{gi}^S\right)^k - \left(S_{gi}^S\right)^{k-1}}{\left(S_{gi}^S\right)^{k-1}}\right], \sum_{i=1}^{N_H}\left[\frac{\left(S_{gi}^H\right)^k - \left(S_{gi}^H\right)^{k-1}}{\left(S_{gi}^H\right)^{k-1}}\right], \frac{\left(S_g^M\right)^k - \left(S_g^M\right)^{k-1}}{\left(S_g^M\right)^{k-1}}\right\} \le toll. \tag{14}$$

3. Numerical implementation

The numerical algorithm was implemented by using an external subroutine, which interacts with the three-dimensional FE modeling created in Comsol Multiphysics (Comsol 2012) to calculate the optimum configuration. The iteration procedure is developed by using an external subroutine, which combines LivelinkTM for Excel package and Comsol Multiphysics (Comsol 2012). The former is introduced to evaluate the design factor Φ_i, whereas the latter is required to calculate the current solution on the basis of the FE method. Therefore, both steps can be easily developed by using several computational frameworks, since they are based on quite standard and simple mathematical operations.

In order to reduce the computational efforts in the numerical calculations, the FE model is based on beam elements for girder and pylons and truss elements for the cable system. Specifically, the bridge deck is replaced by a longitudinal spline with equivalent cross-section and material properties, whereas the pylons are composed by two columns linked at their top by horizontal beam elements. The bridge deck is connected to the suspension system by means of explicit constraint equations, which are imposed between the off-set nodes of the girder and those associated to the cable elements. The cable system, which is connected to the pylons and girder, is essentially defined by the combination of stays, hangers and main cable. In particular, the cable system is modeled according to the Multi Element Cable System (MECS) approach, in which each cable is discretized using multiple truss elements (Greco *et al*. 2013; Ren and Gu, 2010). Large deformations are reproduced by using Green Lagrange formulation and the axial strain is calculated by expressing the global strains in tangential derivatives and projecting the global strains on the cable edge. Additional details on the approach here adopted to model nonlinear behavior of the cable elements can be found in (Bruno *et al*. 2013; Lonetti and Pascuzzo, 2014). The subroutine utilized to calculate the optimum solution, is based according to Sec.3 on the following different steps, which are executed iteratively:

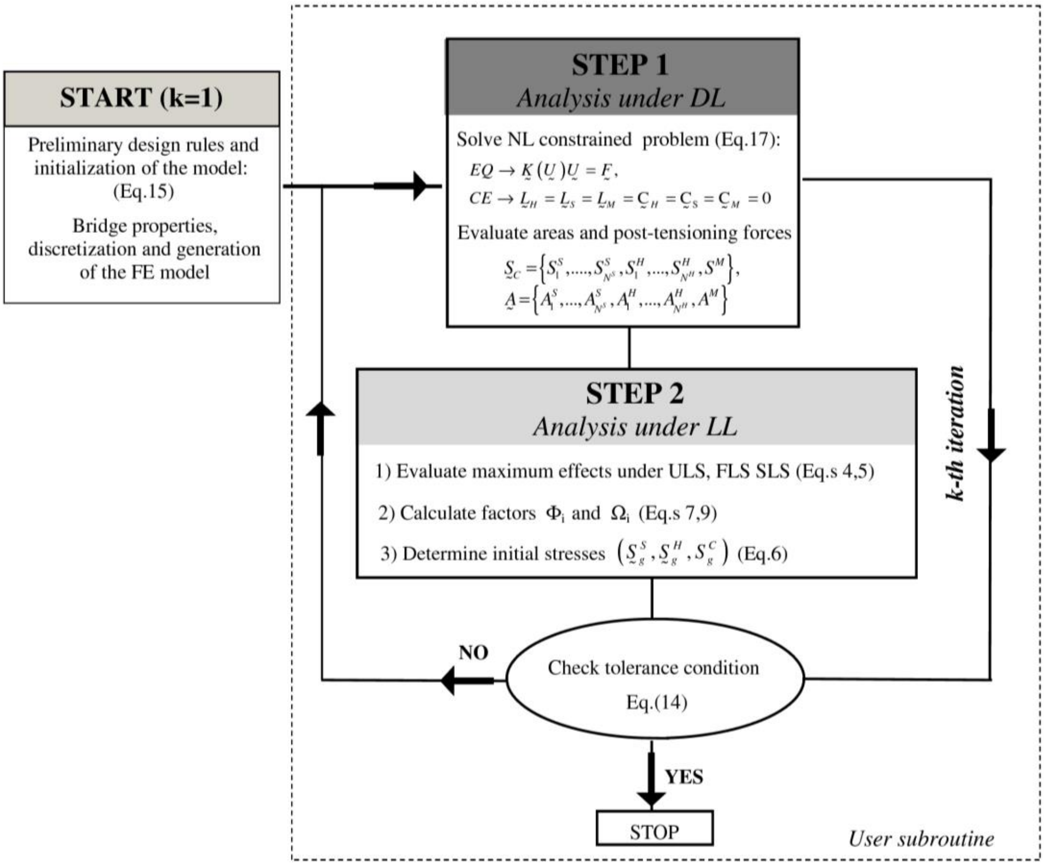

Fig. 4 Flow chart of design procedure

• STEP 1: generation of the finite element model, evaluation of the initial post-tensioning forces and the cable cross-sections in the cable system by means of the optimization procedure;

• STEP 2: calculation of the maximum stresses and displacements under live loads and prediction of the new set of the initial stress quantities on the basis of the performance factors.

For the sake of brevity, the proposed formulation is presented with reference to the final configuration of the bridge. However, the algorithm can be easily applied during the erection procedure of the bridge for the definition of the current configuration at the *k*-th construction step just solving relationships concerning STEP n.1. In this contest, the unknown quantities are represented by the post-tensioning stresses of the cables, i.e., Eq. (1).1, whereas only constrain equations concerning the undeformed geometry, namely Eq. (2), must be considered.

At first, only for the first iteration, the generation of the FE and its initial configuration is designed on the basis of practical design rules, typically, accepted in the framework of cable-supported bridges (Gimsing and Georgakis 2012, Zhang *et al*. 2009). In particular, the initial cable cross-sections for stays $\left(\overline{A}_i^S\right)$, hangers $\left(\overline{A}_i^H\right)$, main cable $\overline{A^C}$ and the initial stresses $\left(S_g^S, S_g^H\right)$ are estimated from the equilibrium relationships based on the truss assumption of the cable stress distribution on the girder (Bruno *et al*. 2008, Gimsing and Georgakis 2012)

$$\overline{A_i^S} = \frac{g\Delta_i}{S_g^S \sin\alpha_i}, \quad \overline{A_i^H} = \frac{g\Delta_i}{S_g^H}, \quad \overline{A^C} = \frac{H_{t0}}{S_A \cos(\phi)}, S_g^{S,H} = \frac{g}{g+p} S_A, \tag{15}$$

where p is the per unit length live loads, α_i is the angle between stay and girder, ϕ is the orientation angle formed by the main cable tangent at pylon intersection and the horizontal direction, H_{t0} horizontal axial force, with $H_{t0} = 1/f \cdot \left[0.25a_m \cdot (q+p)\left(L - 0.5a_m\right)\right]$ with a_m the projection length on the girder of the hanger position (see Fig. 1). It is worth noting that Eq. (15) are based on practical relationships obtained from an engineering point of view and provide reasonable values of the design variables, which are modified during the iterative procedure by the proposed numerical algorithm. Such quantities represent initial trial estimates, which helps the iterative procedure to have a better convergence behavior. More details on the definition and the assumptions of the relationships reported in Eq. (15) can be recovered in (Hui-Li *et al*. 2010).

The evaluation of the post-tensioning forces and the optimum cable cross-sections is developed by solving the governing equations concerning equilibrium (EQ) and constrain equations, (CE) defined according to Eqs. (2)-(3), whose compact form can be expressed as follows

$$\begin{aligned} &EQ \rightarrow \underset{\sim}{K}\left(\underset{\sim}{U}\right)\underset{\sim}{U} = \underset{\sim}{F}, \\ &CE \rightarrow \underset{\sim}{L}_H = \underset{\sim}{L}_S = \underset{\sim}{L}_M = \underset{\sim}{C}_H = \underset{\sim}{C}_S = \underset{\sim}{C}_M = 0 \end{aligned} \tag{16}$$

where $\underset{\sim}{K}$ is the global stiffness of the structure, $\underset{\sim}{U}$ is the global displacement vector, $\underset{\sim}{F}$ is the external force vector associated to the dead loading configuration. It is worth noting that Eq. (16) introduce as a nonlinear constrained optimization problem, in which the unknown quantities are represented by displacements, cable dimensioning and post-tensioning forces. Once the configuration under dead loads is evaluated, the analysis is developed to determine maximum effects on the bridge components produced by the live loads in terms of stresses and displacements. However, since the bridge behavior is essentially nonlinear, the analysis under live loads should be considered as a continuation from the previously converged configuration under the dead load. In particular, all the variables involved in the solving procedure or in the definition of the structural elements of the bridge are taken from the last converged solution, i.e. under dead loads. At this point, the solving procedure is defined by a restarting analysis, in which the initial values are scaled as a function of the current solution (Comsol 2012). The solution is performed for a fixed number of loading conditions, i.e., N^{LL}, which collect, for all bridge components, maximum stress and displacement effects

$$\underset{\sim}{K}\left(\underset{\sim}{U}_i\right)\Delta\underset{\sim}{U} = G_i - P_0 \qquad \text{i=1,..,N}^{LL} \tag{17}$$

where P_0 is the vector of nodal point forces corresponding to the increment in element displacements and stresses from the dead load to the live load configurations, $\Delta\underset{\sim}{U}$ are the incremental displacement vector and G_i is the live load vector force of the i-th loading configuration. From the loading combinations defined by Eq. (17), the new estimates of initial stresses are derived on the basis of Eq. (6) by using Eq. (7) and Eq. (9). Subsequently, convergence conditions, defined by Eq. (14), are checked and if they are not satisfied, the analysis goes back to evaluate the configuration of the bridge under dead loading with the new values of the initial stresses. Such procedure can be handled by using an external subroutine whose flowchart is reported synoptically in Fig. 4.

4. Results

Results are presented to validate the proposed model in terms of convergence of the solution during the iterations steps and efficiency of the formulation to predict the cable system dimensioning and post-tensioning stresses of the cable elements. The analysis is developed with respect to two different bridge schemes, characterized by a different scale length. In particular, the first example refers to a case of a bridge with a small central span and a reduced number of elements, in which severe working conditions in terms of design requirements are considered. Moreover, the second case is presented with respect to a typical long span bridge scheme, in which more complexities are expected in relationship to the large number of variables involved in the analysis. However, for both cases, the geometry and the characteristics of the structure as well as the distribution on the girder of both stays and hangers are defined by means of dimensionless parameters, commonly accepted, in the literature, for self-anchored schemes, (Zhang *et al*. 2009), which are summarized as follows:

• the ratio between rise span of the cable and main span length, i.e., f/L, is between 0.1 and 0.2;

• the ratio between height of the pylon (H) and length of the cable segment length (cL), i.e., H/cL, is larger than 0 and lower than 0.5.

Without loss of generality, in both analyses only live loads concerning traffic loads are considered, which are combined with dead loading by using factored or unfactored loading combinations equal to $1.2DL+1.7LL$ or to $DL+LL$ in cases of ULS or SLS, respectively. However, the generalization of the proposed model to consider also the effects of seismic or wind forces, can be easily developed just introducing additional loading combinations to the ones concerning live loads.

At first, results are developed with reference to a bridge structure with central span (L) and total length (L_T) equal to 100 *m* and 180 m (Fig. 5), whose aspect ratios f/L and H/cL are equal to 0.12 and 0.45, respectively. The main purpose is to present a benchmark analysis, involving a low number of the stays and hangers having a large spacing step. In such configuration, it is quite difficult to verify prescriptions on girder and pylon deformability or in terms of maximum or incremental cable strength. The cable system consists of a double layer formed by only 10 stays and 3 hangers and the main cable, whose elements present an allowable stress (S_a) equal to 6.4×10^8 Pa. The displacement limits of both girder and pylons are assumed to be equal to 1/600 of the corresponding lengths. The girder and pylons present steel rectangular single box sections, whose data are reported in Table 1. Moreover, the dead loads on the girder (Q_{DL}) include contributions arising from the weight of structural and non-structural elements, which are equal to 44.15 kN/m and 60 kN/m, respectively. Moreover, the ratio between live and dead loads, i.e., Q_{DL}/Q_{LL}, is equal to 0.57. Such value is consistent with specification reported in (AASHTO 2013, Yoo and Choi 2009) in which approximately a number of six uniform traffic lanes are considered. Finally live loads are applied on the structure to produce maximum effects in terms stresses or displacements by using the loading combinations reported synoptically in Fig. 5.

Results concerning the cross-section area distribution of all cable elements are reported in Fig. 6, whereas the convergence behavior of the predicted solution is reported in Fig. 7. The analysis denotes that with respect to the initial values, essentially based on preliminary design rules summarized in Sec.3, the cross-section areas are strongly modified. In particular, in the cable-system, the anchor stays and the main cable are dimensioned with larger values than the remaining elements. The convergence of the solution toward the optimum solution is reached with a relatively low number of iterations especially for the internal cable elements, i.e., from cable 2 to 5,

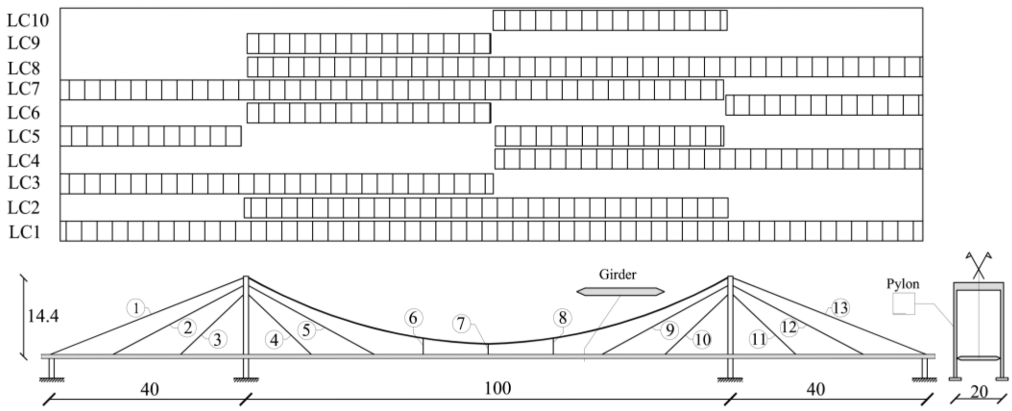

Fig. 5 Synoptic representation of the structural scheme

Table 1 Bridge parameters of the girder and pylons

	Cross sectional area [m^2]	Second moment of cross section [m^4]
Girder	0.57	0.024
Pylon (vertical strut)	0.30	0.24

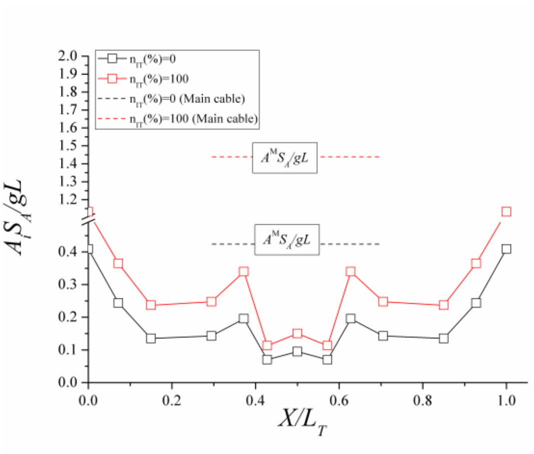

Fig. 6 Cross-section distribution: initial and final evaluations

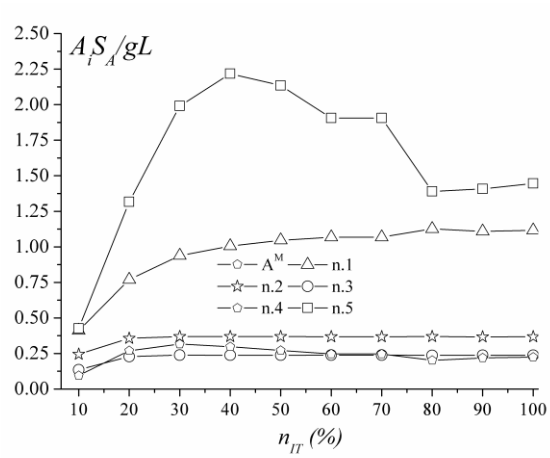

Fig. 7 Convergence behavior of the cross-sections as a function of the iteration number

whereas for the anchors stays and the main cable a larger number of iterations is required to verify prescriptions on maximum displacements.

In Figs. 8, 9 results in terms of bridge deformability under DL and LL are presented for both girder and pylons. In particular, in Fig. 8, the analysis shows that under DL the deflections of the girder are practically negligible in comparisons to those observed in the case of LL. The initial configuration, predicted by the proposed model in terms of the cable forces, is able to constraint to zero the displacements of the girder at the connections with the cables. Moreover, in Fig. 9, the bridge deformability is presented in terms of convergence behavior to evaluate the optimum

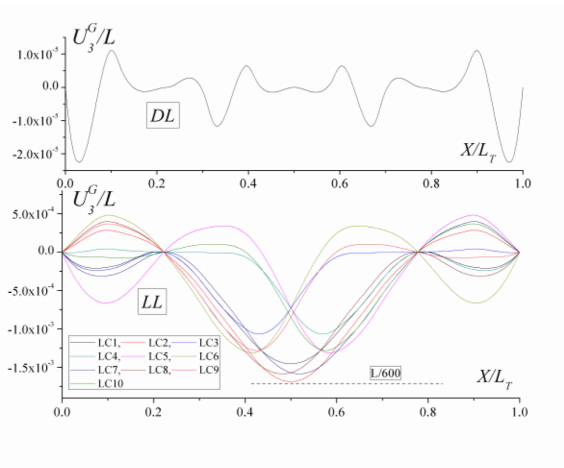

Fig. 8 Envelope of the girder deflections under DL and LL

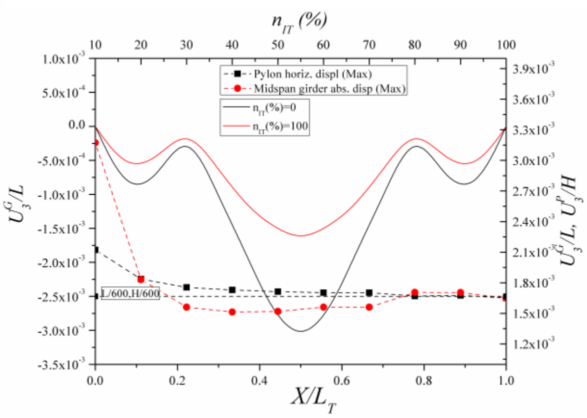

Fig. 9 Comparisons between initial and final evaluations of the girder deflections, convergence behavior of the top pylon horizontal displacement and midspan vertical deflections

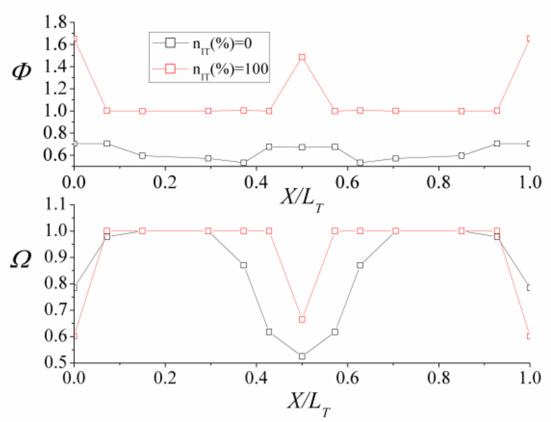

Fig. 10 Stress and displacement optimization factors: initial and final prediction

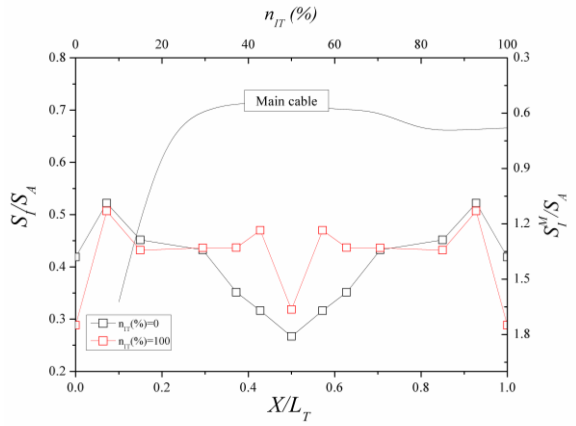

Fig. 11 Distribution of the initial stresses (S_I) under DL and convergence behavior of the values observed in the main cable as a function of the percentage value of the iteration steps (n_{IT}%)

solution. Such results show how the maximum value of the normalized girder displacements, which is equal to -0.003 in the initial configuration, is reduced to a value almost close to its half and thus in agreement with design prescriptions on girder deflections. Moreover, the analysis denotes that the iterative procedure is based on a quite convergent and stable behavior, without jumps or singularities of the solution toward the final configuration.

The distribution of the optimization factors Ω_i and Φ_i for all elements of the cable system from the initial to the final configurations is reported in Fig. 10, whereas the initial stresses under DL for each cable element are analyzed in Fig. 11. It is worth noting that factors Φ_i have the role to constrain the equality of the stresses with the allowable quantity, whereas the factors Ω_i are

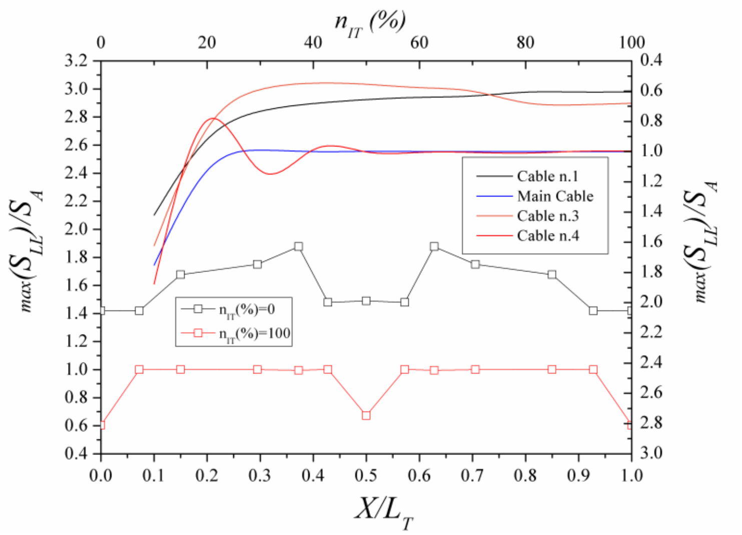

Fig. 12 Distribution of the maximum stresses under LL and convergence behavior of the maximum stresses as a function of the iteration percentage number *(n_{IT}%)*

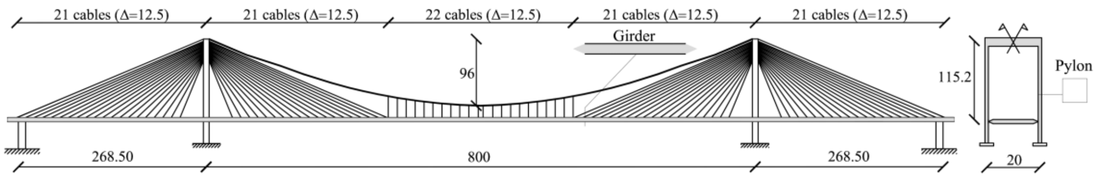

Fig. 13 Structural model of the long span cable-stayed suspension bridge

responsible of the prescriptions regarding bridge deformability. Both of them during the iterations, modify the stiffness of the cables, reducing or increasing the quantity of material involved in the cable system. During the iterative procedure, the internal cables, namely from 2 to 6, present increasing values of the factors Φ_i and Ω_i, except for the anchor stays and the main cable, since both of them are directly responsible of the midspan girder deflections and the horizontal top pylon displacements. Therefore, in order to verify prescriptions on bridge deformability, sin such elements lower values than the unity of Ω_i are predicted by the iterative procedure.

Similarly, the initial stresses presented in Fig. 11, are modified from the initial distribution, in such a way that the anchor stays and main cable during the iterations are forced to have lower post-tensioning forces, larger stiffness and cross-section than the remaining elements. In particular, in such cables, the maximum allowable stresses are modified from the standard value arising from the material characteristics, i.e., S_A, introducing lower thresholds under live loads. Such behavior can be also discussed on the basis of the results presented in Fig. 12, in which the distribution of the maximum stresses under LL for all cable elements is considered. In particular, during the iterations the most of the elements of the cable system are designed consistently with PBA, since the ratio between actual and allowable stresses, i.e., max(S_{LL})/S_A, is close to the unity; contrarily for that elements with values lower than one, in order to verify design constrains on bridge deformability, larger values of cross-sections than the remaining elements are predicted.

Table 2 Bridge parameters of the girder and pylons

	Cross sectional area [m^2]	Second moment of cross section [m^4]
Girder	1.60	4.96
Pylon (vertical strut)	2.72	49.64

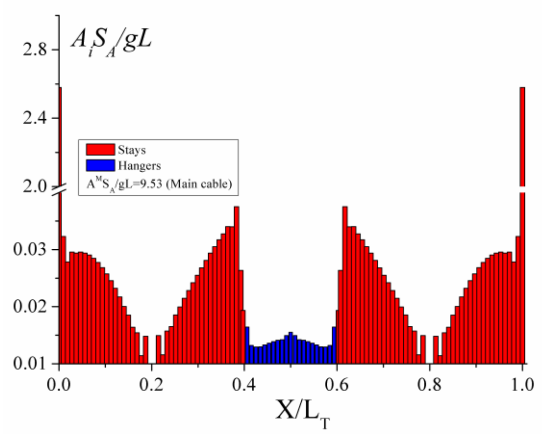

Fig. 14

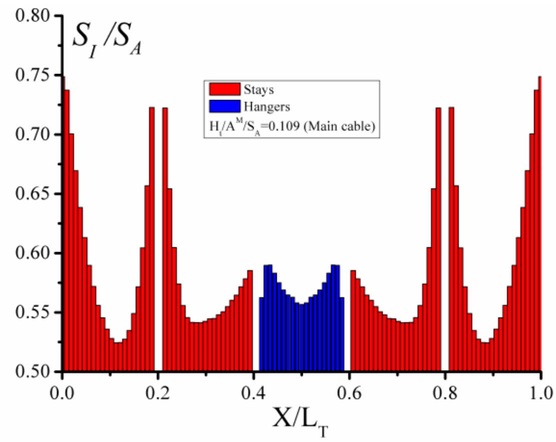

Fig. 15

Figs. 14-15 Normalized cross-sections distribution ($A_i S_A/gL$) and initial stresses (S/S_A) in the cable-system as a function of the normalized position on the girder projection (X/L_T)

Additional results are presented for a long span bridge structure, similar to the one proposed in (Wang *et al*. 2013), whose main span length (L) and total length (L_T) are equal to 800 m and 1337 m, respectively (Fig. 13). Moreover, the aspect ratios f/L and H/cL are equal to 0.12 and 0.46. The stays and the hangers present a distance equal to 12.5 m and an allowable stress (S_a) equal to 6.4×10^8 Pa, whereas the geometrical properties of girder and pylons are reported in Table 2. Dead loads of the girder including also the permanent contributions are equal to $Q_{DL}=\gamma_s A_s+60=2.06E5$ kN/m, whereas live loads, according to bridge design specifications reported in (AASHTO 2013, Yoo and Choi 2009), consist of eight uniform traffic lanes with a value equal to Q_{LL}=76.2 kN/m. In the analysis, the displacement limits of both girder and pylons are assumed equal to 1/800 and 1/600 of the corresponding lengths. From the mechanical and geometric characteristics defined above, dimensionless parameter concerning the ratio between stiffness of the girder (G) and the cable system, i.e., $\bar{\alpha}^2 = EI^G / H_t L^2$ and $\bar{\lambda}^2 = (8f/L)^2 \times (L/L_e) \times (E^C A^C / H_t)$, are equal to $\bar{\alpha}^2 = 0.092$ and $\bar{\lambda}^2 = 223$, where EI^G is the bending stiffness of the girder and H_t is the dead load horizontal component of the cable tension. Such values are consistent with data available from the literature, whose range, obtained on existing cable-supported bridge structures are equal to $\bar{\alpha}^2 = [0.5-10]\times10^{-3}$ and $\bar{\lambda}^2 = [170-800]$ (Luco and Turmo 2010).

Results in terms of cross-sections of the cable and post-tensioning stresses as a function of the normalized positions on the girder projection are reported in Fig. 14 and Fig. 15, respectively. The analysis denotes that the design procedure defines a cross section distribution, whose maximum values are observed in the main cable and anchor stays. Moreover, the remaining stays present their largest values at the extremities of the cable-stayed system. Finally, the hangers are affected essentially by a constant distribution of the cross-sections. Such dimensioning predicted by the

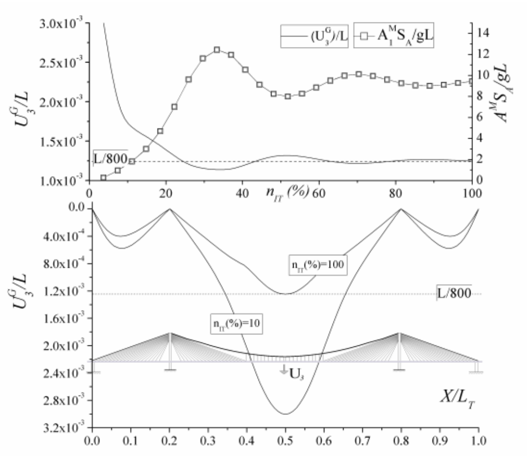

Fig. 16

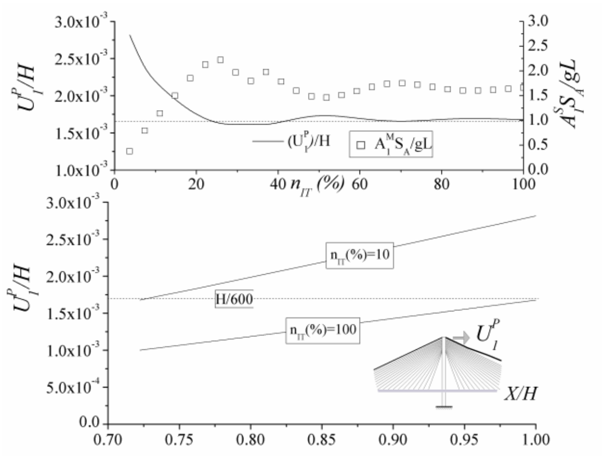

Fig. 17

Figs. 16-17 Normalized girder $\left(U_3^G/L\right)$ and pylon $\left(U_1^G/L\right)$ displacements produced by live loads; convergence behavior of the cross-sections and the displacements as a function of the percentage value of the iteration steps *(n_{IT}%)*

proposed modeling can be assumed quite reasonable from the structural and physical viewpoints. In particular, the high values of the cross-section in the anchor stays are due to the development of the internal force arising from the main cable, which carries out most of the loading contributions arising from the central span. For this reason, in self-anchored schemes, the cross-sections of both anchor stays and main cable should be comparable. Moreover, the reduced values of the cross-sections of the stays located close to the pylons can be explained by the fact that in such regions the elements present their maximum efficiency in terms of loss stiffness due to sag effects. In addition, the prescriptions on girder or pylon displacements are not restrictive as the ones at the midspan girder cross-section, because such cables are placed in proximity to the girder support. As a consequence, a low cable dimensioning is required.

In Figs. 16-17, results concerning bridge deformability are presented. In particular, girder deflections or horizontal displacements of the pylons are reported as function of the percentage number of iterations to obtain the final configuration. Moreover, in the same figure the evolution of the cross-sections of the main cable and the anchor stays are analyzed. The results denote that the anchor stays are mostly responsible of the horizontal displacements of the pylons, which is reduced as far as the cross-sections of such cable elements increase. Moreover, the girder deformability of the central span at the midspan cross-section is mostly affected by the stiffness of the main cable. Actually, the hangers have the role to transfers the loads from the girder to the main cable, without providing any important contributions in terms of stiffness improvement; the stays, which are the most stiffness elements in combined cable-stayed suspension bridge schemes, are far from the midspan and thus their effect on the girder deformability is negligible. In both cases, the results show that during the iterations the solutions is strongly modified from the initial trial solution. In particular, the cross-sections of the cable elements are increased to verify prescriptions on bridge deformability. Such behavior can be also analyzed by the envelope of maximum displacements reported in the same figures, which show how all the displacements at the final step are below the corresponding allowable limits typically adopted for design purposes.

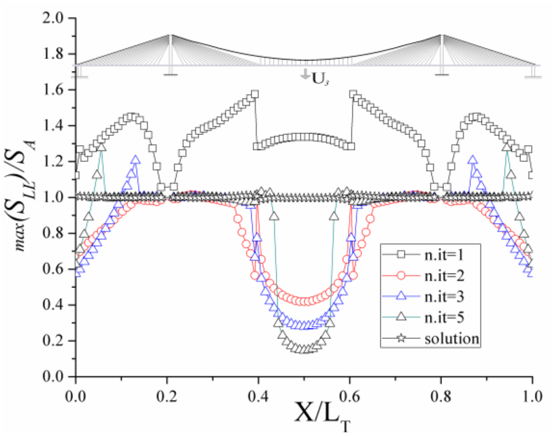

Fig. 18

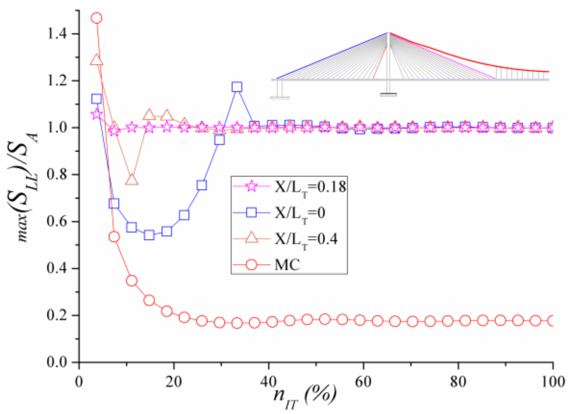

Fig. 19

Figs. 18-19 Normalized maximum tensile stresses $\left(\max\left(S_{LL}\right)/S_A\right)$ under live loads and evolution as a function of the percentage value of the iteration steps (n_{IT}%)

Additional results are presented in Fig. 18, in which tensile stresses in the cables resulting from the ULS combinations are reported as function of the iteration steps. In particular, consistently with the PBA, during the iterations, maximum cable stresses are forced to reach the equality with the allowable strength value. Such condition cannot be reached in the first iterations since initially the conditions regarding the bridge deformability are more restrictive than the ones related to the cable strength. However, once a number of iterations is developed, the cable dimensioning as well as the post-tensioning forces are calibrated to verify such proscriptions. Finally, in Fig. 19, the convergence behavior of the solution is investigated, by means of the relationships between the maximum stresses of some cable elements and percentage number of iterations developed by the iterative procedure. Except for the main cable, whose dimensioning is quite constrained by the displacements prescriptions on bridge deformability, all the cables after a low number of iterations present a stress rate equal to the allowable strength value, reaching, consistently with the PBA, the optimum design.

5. Conclusions

A new design procedure to evaluate post-tensioning forces and cable dimensioning for self-anchored cable-stayed suspension bridges is proposed. In particular, a two-step numerical algorithm is able to evaluate the optimum solution, taking into account design prescriptions of the bridge based on strength and deformability limits. The model is validated by means of parametric studies on different bridge schemes. At first, analyses on short span bridges under extreme loading and deformability conditions are carried out to verify the consistency of the proposed model. Subsequently, a long span bridge scheme involving more complexities in relationship to the large number of variables is investigated. From the analyses, it is possible to drawn the following conclusions:

• The results have shown that the proposed model can be considered as an useful tool in the design procedure, since it is able to provide the optimum design solution, giving a quantification on cross-sectional areas and post-tensioning forces, which satisfy stress and displacement constraints.

• The proposed algorithms is based on a simple procedure, which can be easily handled on the basis of data available by using standard commercial FE software packages.

• The solution presents a stable and convergent behavior, without singularities or jumps in its evolution. Moreover, a low number of iterations is required to obtain the optimum configuration.

• The results show that the cables which require the largest values of steel quantity than the rest of the elements are those associated to the anchor stays, the longest stays in the main span and the main cable. Such cables have an important role to reduce longitudinal deformability of the pylon and vertical displacements of the girder.

• The proposed model is able to find the optimum configuration, consistently with performance based approach, in which most of the cable elements are designed in such a way to reach under the worst LL combinations exactly the allowable design strength.

References

AASHTO (2013), *LRFD bridge design specifications*, Washington, DC.

Barbero, E.J. and Makkapati, S. (2000), "Robust design optimization of composite structures", *The 45th Internaional SAMPE Symposium and Exhibition*, Long Beach, CA, **45**, 1341-1352.

Barbero, E.J., Sosa, E.M., Martinez, X. and Gutierrez, J.A. (2013), "Reliability design methodology for confined high pressure inflatable structures", *Eng. Struct.*, **51,** 1-9.

Behin, Z. and Murray, D. (1992), "A substructure-frontal technique for cantilever erection analysis of cable-stayed bridges", *Comput. Struct.*, **42**, 145-57.

Bruno, D., Greco, F. and Lonetti, P. (2008), "Dynamic Impact Analysis of Long Span Cable-stayed Bridges under Moving Loads", *Eng. Struct.*, **4**, 1160-1177.

Bruno, D., Greco, F. and Lonetti, P. (2009), "A Parametric Study on the Dynamic Behavior of Combined Cable-Stayed and Suspension Bridges under Moving Loads", *Int. Journal for Comp. Methods in Eng. Science and Mechanics*, **10**(4), 243-258.

Chen, D.W., Au, F.T.K., Tham, L.G. and Lee, P.K.K. (2000), "Determination of initial cable forces in prestressed concrete cable-stayed bridges for given design deck profiles using the force equilibrium method", *Comput. Struct*., **74**, 1-9.

Comsol (2012), Reference Manual, Stockholm, Sweden.

Gimsing, N.J. and Georgakis, C.T. (2012), *Cable Supported Bridges Concept and Design*, John Wiley & Sons, New York.

Greco, F., Lonetti, P. and Pascuzzo, A. (2013), "Dynamic behavior of cable-stayed bridges affected by accidental failure mechanisms under moving loads", *Math. Prob. Eng.*, **302706**, 1-20.

Günaydin, M., Adanur, S., Altuniik, A.C. and Sevim, B. (2012), "Construction stage analysis of fatih sultan mehmet suspension bridge", *Struct. Eng. Mech.*, **42**(4), 489-505.

Hassan, M.M., Nassef, A.O. and El Damatty, A.A. (2012), "Determination of optimum post-tensioning cable forces of cable-stayed bridges", *Eng. Struct.*, **44**, 248-259.

Hui-Li, W., Si-Feng, Q., Zhe, Z., Cai-Liang, H. and Wen-Jun, X, (2010), "The basic differential equations of self-anchored cable-stayed suspension bridge", Article ID 805195, 12 pages.

Kim, K.S. and Lee, H.S. (2001), "Analysis of target configuration under dead load for cable supported bridges", *Comput. Struct*., **79**, 2681-2692.

Janjic, D., Pircher, M. and Pircher, H. (2003), "Optimization of cable tensioning in cable-stayed bridges", *J Bridge Eng. ASCE*, **8**, 131-137

Lee, T.Y., Kim, Y.H. and Kang, S. (2008), "Optimization of tensioning strategy for asymmetric cable-stayed bridge and its effect on construction process", *Struct. Multidis. Optim.*, **35**, 623-629.

Liang, Q.Q. and Steven, G.P. (2002), "A performance-based optimization method for topology design of continuum structures with mean compliance constraints", *Comput. Meth. Appl. Mech. Eng*., **191**(13-14), 1471-1489.

Liu, M.Y, Lin, L.C. and Wang, P.H. (2012), "Investigation on deck-stay interaction of cable-stayed bridges with appropriate initial shapes", *Struct. Eng. Mech*., **43**(5), 691-709.

Lonetti, P. and Pascuzzo, A. (2014), "Optimum design analysis of hybrid cable-stayed suspension bridges", *Adv. Eng. Softw.*, **73**, *53-56*.

Luco, E. and Turmo, J. (2010), "Linear vertical vibrations of suspension bridges: a review of continuum models and some new results", *Soil Dyn. Earthq. Eng.*, **30**, 769-781

Ohkubo, S., Taniwaki, K. and Yamano, N. (1992), "Optimum design system for steel cable-stayed bridge dealing with shape, sizing variables and cable prestresses", *Comput. Aid. Civil Infrastr. Eng.*, **7**(3), 201-221.

Peng-Zhen, L., Jianting, C., Jingru Z. and Penglong, L. (2014), "Optimization analysis model of Self-anchored Suspension Bridge", *Math. Prob. Eng*., 403962, 1-34.

Ren, S.Y. and Gu, M. (2010), "Static configurations of cables in cable stayed bridges", *Struct. Eng. Mech*., **34**(4), 545-548.

Raftoyiannis, I.G., Konstantakopoulos, T.G. and Michaltsos, G.T. (2013), "Dynamic response of cable-stayed bridges subjected to sudden failure of stays-the 2D problem", **6**(3), 317-337.

Yoo, H. and Choi, D.H. (2009), "Improved system buckling analysis of effective lengths of girder and tower members in steel cable-stayed bridges", *Comput. Struct*., **87**, 847-860.

Wang, H.L., Tan, Y.B., Qin, S.F. and Zhang, Z., (2013), "Geometric nonlinear analysis of self-anchored cable-stayed suspension bridges", *Sci. World J*., 734387, 1-5.

Wang, P.H., Tang, T. and Zheng, H. (2004), "Analysis of cable-stayed bridges during construction by cantilever method", *Comput. Struct*., **82**, 329-46.

Zhe, Z., Fei-Ran, L., Chang-Huan, K. and Jeng-Lin, T. (2010), "Static Analysis of a Self-anchored Cable-stayed-suspension Bridge with Optimal Cable Tensions", *J. C.C.I.T.*, **39**(2), 1-9.

Zhang, X.J., Sun, B.N. and Xiang, H.F. (2005), "Aerodynamic stability of cable-stayed bridges under erection", *J. Zhejiang Univ. Sci. A*, **6**(3), 175-180.

Zhang, Z., Wang, H., Qin, S. and Ge, X.O (2009), "Limit span of self-anchored cable-stayed suspension cooperation system bridge based on strength", *Front. Archit. Civil Eng. China*, **3**(3), 286-291

Nomenclature

$(\bar{\bullet})$	Values arising from previous iteration step or trial values
$(\cdot)^G$	Stiffening girder variable
$(\cdot)^P$	Pylon variable
$(\cdot)^S$	Stay variable
$(\cdot)^H$	Hanger variable
$(\cdot)^M$	Main cable variable
A	Cross-section area
a_m	Suspended portion of the main span
ΔA	Incremental cross-section area of the cable
B	Girder cross-section width
c	Cable-stayed portion of the main span

DL	Dead Load configuration
E	Material elasticity modulus
f	Main cable sag
g	Self-weight per unit length
H	Pylon height
I_i	Moment of inertia with respect to the i-axis
l	Lateral span length
L	Central span length
L_T	Girder total length
LL	Live Load configuration
ΔS_i	Incremental initial stress in the i-th cable
S_i	Cable stress under self-weight loading
S_{gi}	Initial design stress of the i-th cable
S_a	Allowable cable stress
ΔS_a	Incremental allowable cable stress
U_i	Displacement component with respect to the i-axis
α	Longitudinal stay geometric slope
δ_A	Maximum allowable displacement
Δ	Stay spacing step
ϕ	Maximum longitudinal main cable geometric slope
Φ_i	Stress performance factor of the i-th cable
γ	Material specific weight
Ω_i	Strength performance factor of the i-th cable