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Control of chaotic dynamics by magnetorheological damping of a pendulum vibration absorber

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Abstract. Investigations of regular and chaotic vibrations of the autoparametric pendulum absorber suspended on a nonlinear coil spring and a magnetorheological damper are presented in the paper. Application of a semi-active damper allows controlling the dangerous motion without stooping of system and additionally gives new possibilities for designers. The investigations are curried out close to the main parametric resonance. Obtained numerical and experimental results show that the semi-active suspension may reduce dangerous motion and it also allows to maintain the pendulum at a given attractor or to jump to another one. Moreover, the results show that, for some parameters, MR damping may transit to chaotic motions.

Keywords: chaos; bifurcation; MR damping; control; attractor

1. Introduction

The problem of undesired vibration reduction has been known since many years ago and becomes more attractive nowadays. The dynamic vibration absorbers (DVA) are special devices, consisting of masses suspended on springs and dampers. In the classical theory of DVA, the primary structure is modelled as a spring mass system. However, other dynamic vibration absorption models also have high interest in research and engineering application. In particular, the pendulum type systems can play an important role in many fields such as machinery, transportation and civil engineering. But, dynamic behavior of a pendulum absorber is significantly more complex than it is supposed by the widely used additional simple dynamical dampers.

Systems with time-varying parameters belong to a very important class in the field of structural dynamics. Many mechanical engineering problems are described by differential equations with periodically changing parameters (Kapitaniak *et al*. 2013). Therefore, vibrations generated in such systems are called parametric vibrations. An autoparametric system represents a special class of nonlinear systems. Such system is composed of at least two subsystems (primary and secondary), i.e. at least two degrees of freedom model has to be considered. The secondary subsystem is coupled to the primary system in a nonlinear way, and moreover it may become a source of

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internal parametric excitation. In such a case we deal with self-parametric vibrations called autoparametric. In autoparametric vibrations, a small excitation can produce a large response basically when the frequency of excitation is close to one of the natural frequencies of the system. In actual engineering problems, the loss of stability depends on frequency tuning of the various components of the system, and on the interaction between them.

The mass-spring systems with an attached pendulum represent an interesting dynamical physical structure which is used in many mechanical and civil engineering applications. In a large number of problems it is used to diminish the vibration amplitudes (the dynamical vibration dampers). For example, the pendulum absorber is applied to helicopters as one of the vibration suppression devices of helicopters blades (Nagasaka *et al*. 2007). Moreover, special dampers working against earthquake are mounted in high buildings, mounted on bridges against river vortex or on high chimneys where they are designed to reduce vibration induced by the wind (Spencer and Sain 1997). Vibrations absorption of the dynamical dampers is possible in the system due to the pendulum swinging. Harmonically excited pendulum systems may undergo complicated dynamics, in particular if the pendulum and the oscillator are coupled by inertial resonance condition (Kecik and Warminski 2012a). It has been found that the system generates various type of motion, from simple periodic oscillation to complex dynamics including chaos and rotation of the pendulum (Kecik and Warminski 2012b). The presence of the coupling terms can lead to a certain type of instability which is referred as the autoparametric resonance. However, for some parameters the situation may worsen and the pendulum vibrations may increase dramatically, and then the protection of the structure is lost.

The control of the pendulum motions without seems difficult because of inertial coupling both systems. Therefore, intelligent and adaptive material systems and structures have become very important in engineering applications. A new class of materials with promising applications in structural and mechanical systems is the magnetorheological (MR) dampers (Tang *et al*. 2004, Yoshioka *et al*. 2002). MR dampers are attractive elements in structural control which have capability to provide large controllable damping forces and may change their properties to accommodate varying loading conditions. Therefore, magnetorheological fluids devices are the most promising for control of vibrations and for the vibration isolation. Application of a smart damper to regular and chaotic dynamics control and also for reduction of the force transmitted on the ground is investigated in this paper.

The purpose of this paper is to study possible dynamical phenomena of a coupled oscillator-pendulum system for realistic data, and to present a method of semiactive reduction of dangerous vibrations, mainly the chaotic motions and rotation. We propose to use the magnetorheological damper, which is installed between the oscillator and the ground to provide controllable damping for the system. Application of a smart damper allows control dynamics without stopping of system (online control). It is shown numerically and experimentally that MR damping can efficiently reduce chaotic oscillations. Additionally by activation of MR damping the change of motion is possible. Moreover this concept can be applied to control the pendulum for energy generation (energy harvester (Horton and Wiercigroch 2008)).

The paper is divided into five sections. Following the above introduction, where a brief survey of literature was given, the second section will describe the analytical model. In the third section the experimental laboratory rig is presented. In the four section the results of numerical simulations and experimental are discussed. The influence of MR damping is detailed studied. For the verification of real chaotic motion the advanced delay method is applied. The last section includes conclusions and the results of the conducted numerical simulations.

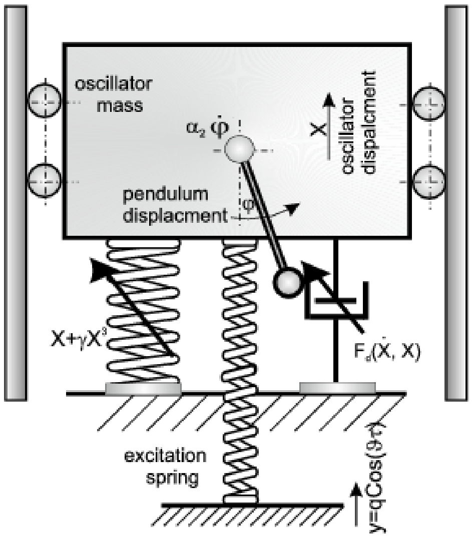


Fig. 1 Model of an autoparmetric pendulum vibration absorber with MR damper

2. An autoparametric system with a magnetorheological damper

2.1 Model of an autoparametric system (AS)

The investigated AS is shown in Fig. 1. The mechanical structure consists of a pendulum and a body of mass suspended on a coil spring with linear or nonlinear characteristic and the MR damper. The dimensionless damping coefficient of pendulum is assumed viscous. The body of mass is subjected to a harmonic vertical excitation by linear spring.

The autoparametric system, presented in Fig. 1, that includes the magnetorheological damper, taken from the work (Kecik and Warminski 2011)

$$\ddot{X}+\alpha_1\dot{X}+F_d\left(\dot{X},X\right)+X+\gamma X^3+\mu\lambda\left(\ddot{\varphi}\sin\varphi+\dot{\varphi}^2\cos\varphi\right)=q\cos\vartheta\tau, \tag{1}$$

$$\ddot{\varphi}+\alpha_2\dot{\varphi}+\lambda\left(\ddot{X}+1\right)\sin\varphi=0. \tag{2}$$

The function F_d describes the nonlinear magnetorheological oscillator damping (explained in next subsection), α_2 denotes damping coefficient of the pendulum, γ is nonlinearity of oscillators spring. Parameters μ and λ describe pendulum parameters (the mass and length as function of natural frequency and static displacement), while q and ϑ identify parameters of excitation (the amplitude and frequency, respectively). If the parameter γ=0, we get a linear oscillators spring. Due to coupling of both coordinates, X and φ, by inertial term, the system is strongly nonlinear. Particular strong interactions between vibration modes occur if the natural frequency of the oscillator is twice higher than the pendulum frequency. Therefore, the resonance 1/2 is studied. A detailed description of the system and derivation of the dimensional equations and their transformation into dimensionless form can be found in (Warminski and Kecik 2009). An analytical solution of Eqs. (2.1)-(2.2) by harmonic balance method (HBM) is presented in (Warminski and Kecik 2012b).

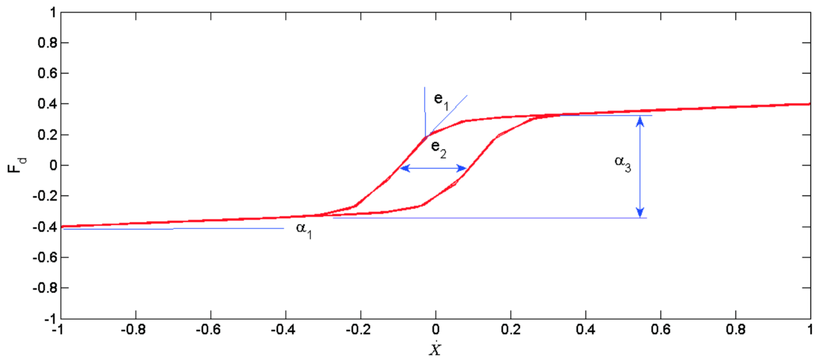


Fig. 2 Attitude of MR damper in force-velocity curve

2.2 Model of MR damper

Damping of the oscillator is studied in two variants, as linear viscous and nonlinear magnetorheological. Our concept on nonlinear damping is realized by application of the magnetorheological damper(MRD). A smooth function of modified Binghams model, to describe of MR damper behavior is proposed. In dimensionless form the dynamic force Fd of MR damper is expressed as

$$F\_d(\dot{X}, X) = \alpha_1 \dot{X} + \alpha_3 \tanh(e_1 \dot{X} + e_2 S) \tag{3}$$

where: α_1 means viscous damping parameter, i.e., the slope of linear part of Eq. (3), α_3 indicates dry friction, i.e., height of hysteretic loop, e_1 describe the slope shape of dry friction and e_2 denotes the width of hysteretic loop.

The influence of parameters of Eq. (3) on the hysteretic loop effect in Fig. 2 is presented. This proposed model consists of a combination of viscous damping (α_1) and a Coulomb friction (α_3). If the parameter α_3 equals zero, then the magnetorheological dampers working as classical viscous.

2.3 Laboratory pendulum-like system

The experiment of the studied two-degree-of-freedom model is performed on a specially prepared test stand presented in Fig. 3(a) and schematically in Fig. 1. The laboratory rig is composed of the pendulum (1), with possibility of full rotation, attached to an oscillator mounted (2) to a base by a linear or nonlinear spring (5) and a damper. The applied can be oil (11) viscous (linear, (12)) or magnetorheological (RD 1097-01, Fig. 3(b) (4)). Motion of the system is generated by a motor (6), and a mechanism which changes rotation of the motor into translational motion. The frequency of the vertical oscillations is controlled by inverter (10). Amplitude of kinematical excitation is fixed by a pitch of a drive shaft (7).

Detailed description and more information about experimental setup, modeling of MR damper and measures apparatus are presented in (Kecik 2012). The spring which connects the oscillator and the base is considered in two variants, linear or nonlinear with different soft or hard stiffness characteristics. Nonlinearity of springs has been reached by designing of a special shape of springs: barrel shape and spiral hourglass helical shape. For data acquisition and for control the

(a) General view (b) MR damper RD1097-01

Fig. 3 The laboratory rig of an autoparametric system

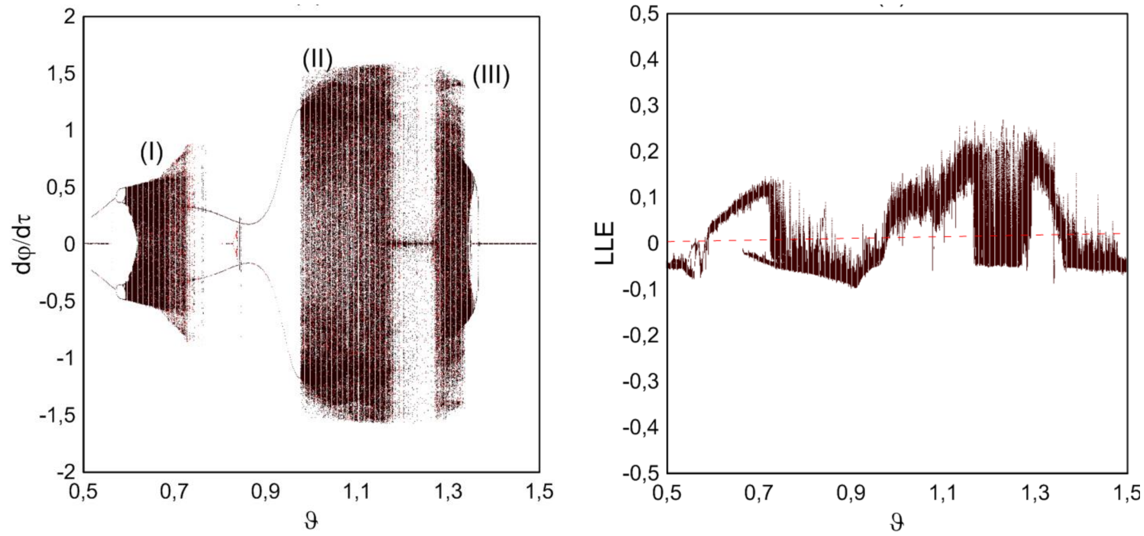
DasyLab system is used. The angle of rotation’ of the pendulum and the displacement X of the oscillator are measured by encoder (3) and sensor (14). Additional sensor (8) mounted on the foundation of an autoparametric system allows measure the force transmitted in the foundation. Additional masses of the pendulum (9) and the oscillator (13) allows change the mass of an autoparametric system. The magnetorheological damper RD 1097-01 is suitable for light structure suspensions and isolation applications. The functional parameters of the damper listed by the manufacturer take values: maximum force 100 N for current 1A and piston velocity 51 mm/s, stroke 25 mm, response time 25 ms. The force in the passive-off-mode 0A is about 9N.

3. Chaos control by MR damping

3.1 Influence magnetorheological damping on chaotic motion

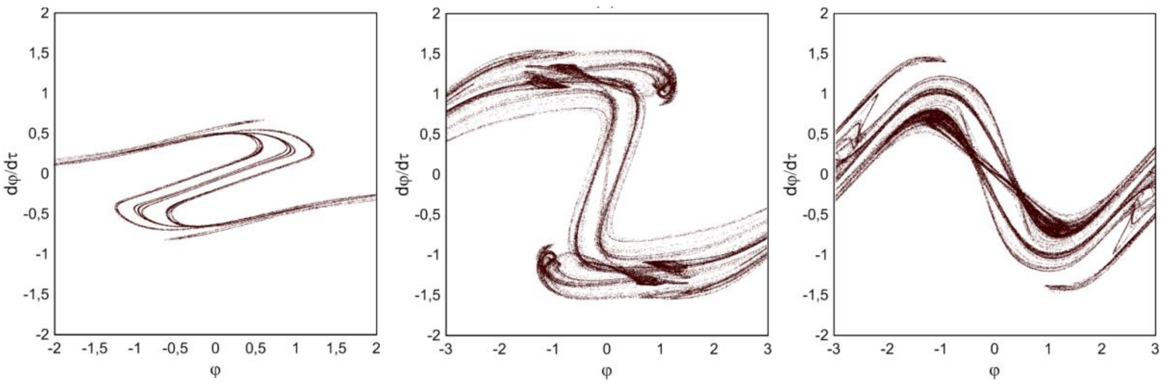
The presence of chaos in physical systems is very common and is a key feature of nonlinear systems. The parameters of an autoparametric system can be tuned in such a way that a small perturbation of initial conditions transits its response to dangerous motion, like a chaotic dynamics. If the pendulum plays a role of a dynamical absorber, this kind of motion is unwanted. This paper proposes to use a MR damper as a tool which fast and easily may prevent dangerous dynamics or in specific situations.

The autoparametric system with an attached pendulum exhibits the resonance behaviour near the frequency ϑ=1. In Fig. 4(a), near the main parametric resonance, the three chaotic regions are discovered (called as: I, II and III). The bifurcation diagram presents solutions obtained for ten various (random) initial conditions of the pendulum. This is done because of such systems coexisting solution are possible.



(a) Bifurcation diagram (b) Largest Lyapunov exponents

Fig. 4 The bifurcation diagram and Lyapunov exponent

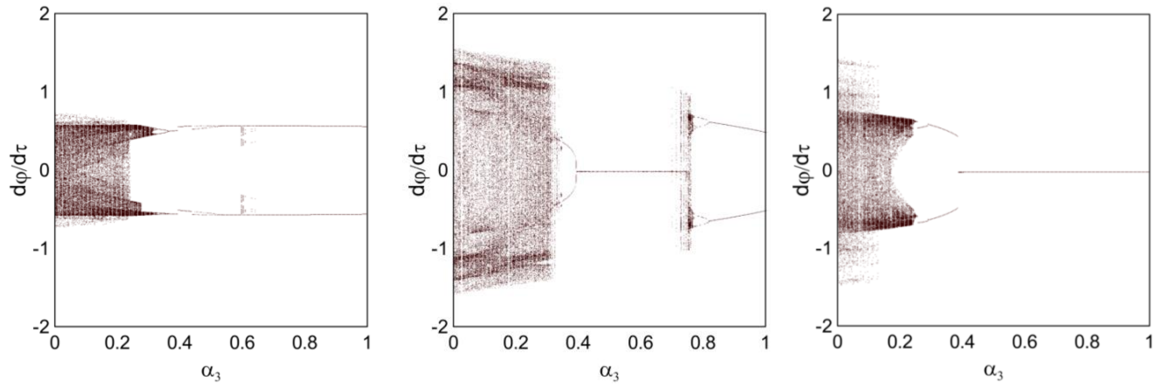


(a) ϑ=0.7 (b) ϑ=1.1 (c) ϑ=1.32

Fig. 5 Strange attractors in the set of chaotic regions

For the same set of data a few solution may exist depending on initial conditions. Therefore, all bifurcation diagrams for ten random initial condition of the pendulum are done. The initial conditions of the oscillator (displacement and velocity) are fixed and equal zero. The chaotic regions are identified by positive value of largest Lyapunov exponent Fig. 4(b). In these analyses the following parameters are used: α_1=0.3054, α_2=0.1, μ=14.686, λ=0.134, q=2.324, γ=0, e_1=10 and e_2=0.

Additionally, in these regions, the strange attractors have been done Fig. 5. Shape of strange attractors we can observe in Fig. 5(a)-I chaotic region, Fig. 5(b)-II chaotic region, and Fig. 5(c)-III chaotic region. Comparing the attractors set we can see that the pendulum motion reaches the highest velocity in the widest second chaotic region, however the smallest velocity is obtained in the first chaotic zone. In each in these attractors occurs both swinging and rotation of the pendulum.



(a) ϑ=0.7 (b) ϑ=1.1 (c) ϑ=1.32

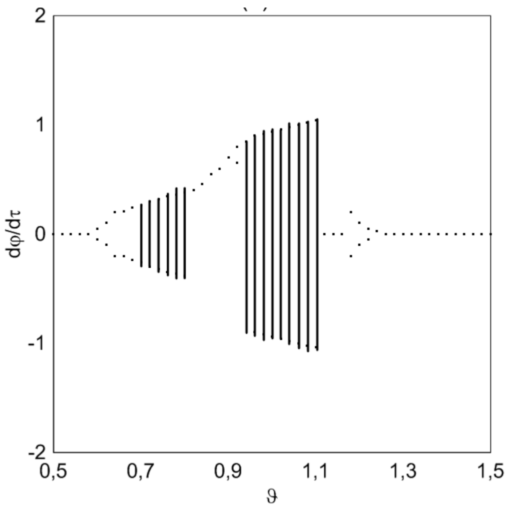
Fig. 6 Influence MR damping on three chaotic regions

Introducing MR damping during first chaotic region I (ϑ=0.7), we can observe that this irregular motion can be eliminate for α_3=0.25, Fig. 6(a), while for the second chaotic region (II) this value is higher and equals α_3=0.3, Fig. 6(b) and for third region α_3=0.2 Fig. 6(c). However, the new chaotic region near the MR damping α_3 =0.75, can be appear (Fig. 6(b)). This denotes that introducing MR damping can lead to a new chaotic behaviour. It follows that the rotation motions of the pendulum changed into chaos.

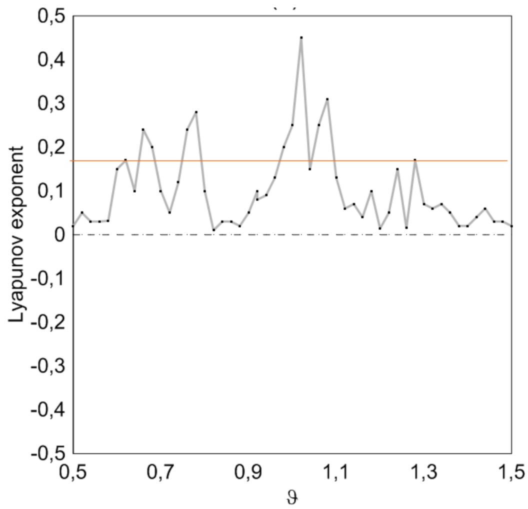
Analysis, of all figures presented in Fig. 6, conclusion can be drawn, that the second chaotic region in the most dangerous. This result from the fact that in second chaotic region has a higher angular velocity of pendulum and it is the widest. Therefore, seems that dynamics of this region is difficult to reduce and eliminate. To improve and control the dynamics in chaotic regions, the magnetorheological damping seems promising. But, applied MR absorption to reduce of dangerous motion should be earlier studied and checked.

3.2 Experimental verification of the bifurcation diagram

Chaotic behaviour is defined just as sensitive dependence on initial conditions and characterizes randomness of solutions and unpredictability. Existence of chaos can be found in many numerical simulations and experimental tests. Usually, in most cases, the analysis of experimentally observed chaotic behaviour is confined to numerical simulations of appropriate mathematical models. However, showing that a mathematical model exhibits chaotic behaviour is no proof that chaos is also present in the corresponding experimental system. To show convincingly that an experimental system behaves chaotically, chaos has to be identified directly from the experimental data. For this purpose, the different methods are used to identify experimentally observed chaos are applied. The most popular is the maximal Lyapunov exponent which is an effective chaos indexes (Abarbanel 1996). Besides, the classical Lyapunov exponents, Poincare maps and phase space, novel methods based on the nonlinear signals analysis, like recurrence plots (RP) or recurrence quantification analysis (RQA) are used recently (Kecik and Warminski 2010). A reconstructed bifurcation diagram, as shown in Fig. 7(a) made by apply step by step of fixed excitation frequency at ϑ=0.02. The experimental research based on the angular velocity of the pendulum signals. This choice of signal makes the analysis easier because in the

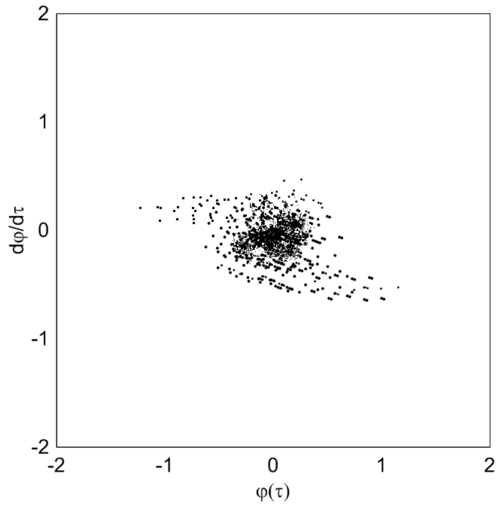


(a) Bifurcation diagram

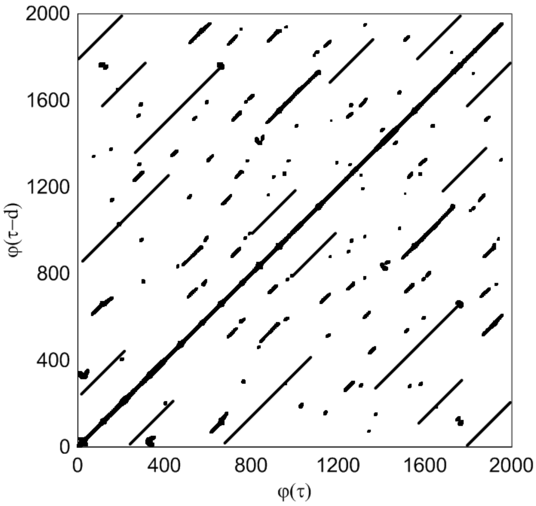


(b) Lyapunov exponent

Fig. 7 Experimental bifurcation diagram and Lyapunov exponent obtained from experimental time series



(a) Reconstructed attractor

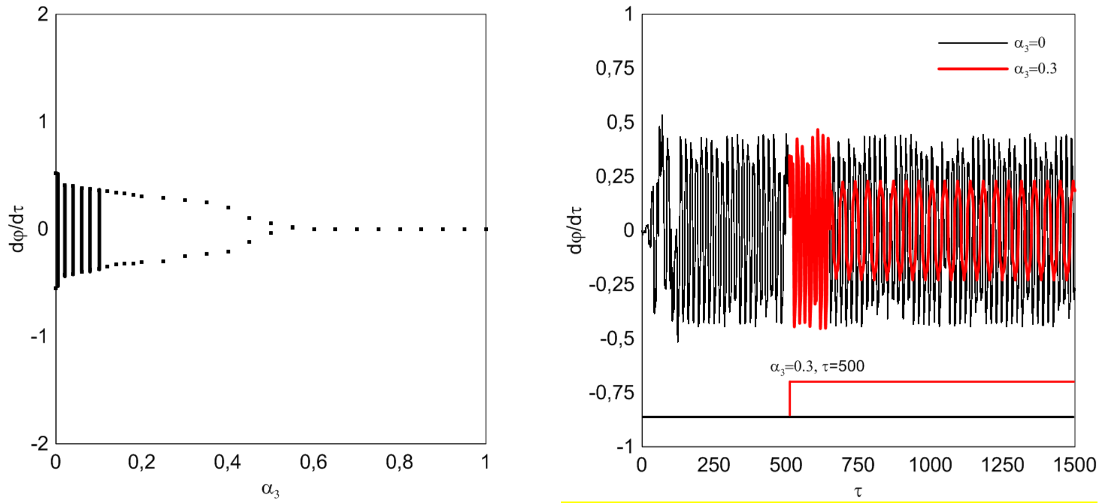


(b) Recurrence plot

Fig. 8 Results of delay method

velocity domain the rotation of the pendulum is eliminated. This research discovered only two chaotic regions, identified by estimation of largest Lyapunov exponent Fig. 7(b) and additionally confirmed by reconstructed attractor Fig. 8(a) and recurrence plot Fig. 8(b). It should be noted, that Lyapunov exponent gives good results with numerical simulation, if zero value will be shifted up (the orange line in Fig. 8(b)). For the analyzed results the zero is shifted to the value equals 0.18. This probably comes from the noise included in the analyzed experimental signals and disturbances.

Therefore, to confirm chaotic motion, the reconstructed attractor is done. The attractor reconstruction is carried out using a time series analysis application (Kantz and Schreiber 1999).



(a) Experimental influence of MR damping, for ϑ=0.7 (b) Time histories of the pendulum, for ϑ=0.7

Fig. 9 Control of chaos by MR damper in on-off method

This software for time series analysis based on the theory of nonlinear deterministic dynamical systems. The reconstructed attractor consists of swinging and rotation of the pendulum, too (similar to numerical attractor).

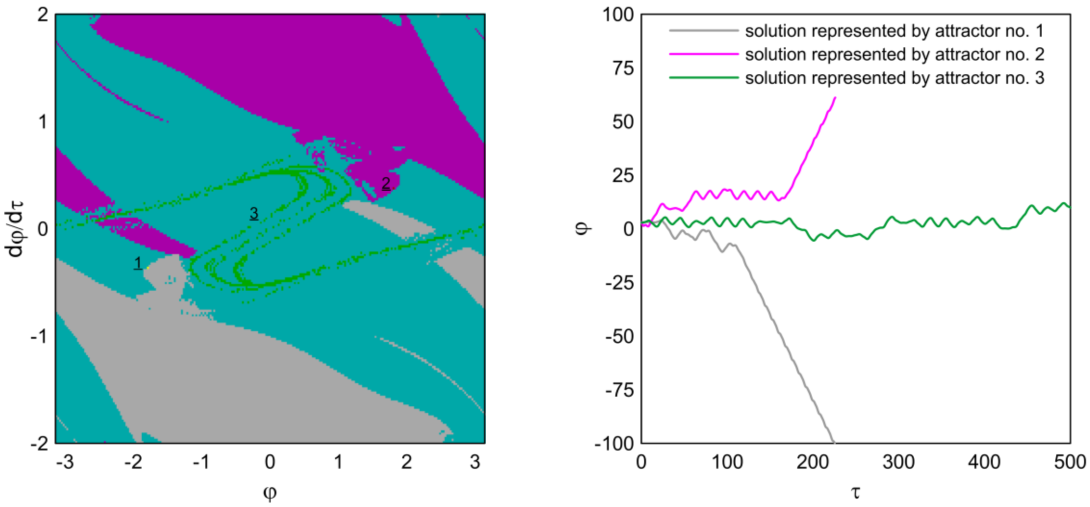
In order to undertake attractor reconstruction, the time delay (d) is calculated using the mutual information method (AMI), providing important information about reasonable delay times, while the false neighbors (FNN) statistics applied for estimating the embedding dimension. An exact mathematical description of these functions is given in Refs. (Hegger and Kantz 1999, Kantz and Schreiber 1999). The reconstructed attractor in Fig.8a is presented. The numerical (Fig. 5a) and experimental attractors have similar shape, arrangement, and dimension. The reconstructed recurrence diagram gives similar results as Lyapunov exponent.

Additionally, from the real chaotic signal recurrence diagram was done Fig. 8(b). The diagram shows different line, much shorter and dashed. The distance between diagonal lines is various because this motion includes components of rotation and oscillation. These results characterize typical chaotic behavior of dynamic systems.

The experimental influences of MR damping on the chaotic behavior is shown in Fig. 9(a). It is observed, that the MR damping, near value of $\alpha_3 \approx 0.15$ causes eliminate of chaotic motion. This result is similar to the numerical results, presented in Fig. 6(a). The experimental time histories - angular velocity of the pendulum, without MR damping (chaos) is marked as the black line and in Fig. 9(b) is shown. The response of the vibration absorber with activation MR damping (α_3=0.3, for time τ>500) is marked as the red line. Note, that reduction of the amplitude vibration and change chaotic into periodic motion occurs later ($\tau \approx 600$). This is due to the inertial coupling both subsytems (the pendulum and oscillator) and delay equipment effects.

3.3 Change of dynamics by MR damper

The autoparametric systems are very sensitive for initial and working conditions. Therefore, even a very small and temporary change in working conditions or slight disturbance may influence



(a) Basins of attractions (b) experimental time histories of the pendulum

Fig. 10 Basins of attractions for α_3=0 and time histories with impulse activation of MR damper (b)

on obtained response. Additionally, in this type of nonlinear systems, the existences of two or more solutions are possible. Dynamics control of an autoparametric structure is very important to keep the pendulum at a given, wanted attractor, or if necessary change it. For this purpose, the MR damper is used.

Fig. 10(a) shows basins of attractions for two sets of initial conditions of the pendulum, that is, its angular displacement (φ) and angular velocity ($d\varphi/d\tau$). The diagram indicates more than one coexisting attractor for the same set of parameters. For each attractor, the set of initial conditions leading to long-time behaviour is plotted in corresponding colours. Attractor no. 1 (dark grey colour) and no. 2 (pink colour) represent negative (clockwise direction) or positive rotation of pendulum, respectively. The attractor no. 3 (blue colour set of initial conditions) represents a chaotic motion consist of a swings and rotation of pendulum. This kind of motions is represented by chaotic attractor, in Fig. 4(a), and by blacked colour in Fig. 3(a), confirmed by positive value of Lyapunov exponent (Fig. 3(b)). This example emphasises a very important aspect of the existence of possible multiple solutions in nonlinear structures. This observation has practical meaning in engineering and physical problems.

Fig. 10(b) shows experimental time histories of pendulum with impulse MR damping activated (α_3=0.3). We observe that impulse turn on of MR damper (value α_3=0.5, activation lasts τ≈10) causes change kind of motion (jump one attractor into another). The response of systems depends on moment (actual initial conditions of pendulum) in which MR damper is turn on. The obtained results, shows that nonlinear suspension with MR damper can be used as special protective systems in dynamical dampers or harvesting energy systems.

Additionally, after proper tuning of the system the response can be modified from chaotic to periodic motion and vice versa. It has been confirmed experimentally that the simple open loop technique, allows for an easy control of the system response. The more complex method of control in closed loop alghotim based on the angular velocity and angular displacement dedicated to change solution between chaos-rotation-swings is presented in (Kecik *et al*. 2014).

4. Conclusions

The paper presented the numerical and experimental study of the autoparametric system with applied MR damper. The chaotic regions are discovered in numerical simulations and confirmed on the special laboratory system. Identification of experimental time series based on delay method (recurrence diagram, reconstructed attractor, estimation Lyapunov exponent). Additionally, the experimental diagram was done. To control of dynamics, the MR damper mounted in suspension is proposed. This solution doesn't reduce the effectiveness of vibration absorption effect. Activation of the MR damper allows for easy an open loop control of the system. Obtained results show, that the application of nonlinear damper may be an effective method of elimination of the chaotic motion, or if necessary to change one attractor into another. Moreover, by applying simple open-loop control, it is possible to fit on-line the structure response to the frequency and amplitude of external excitation. This suggests that MR damper can be used as special device in engineering applications as a system of dangerous motion preventive or as special control dynamics device of harvesting energy applications. However, for some parameters the magnetorheological damping may transit the pendulum from rotation to chaotic motions (Fig. 6). The future work is planned, to use MR damper together with shape memory spring (SMA spring) and apply a closed loop control to prepare a smart dynamical absorber.

Acknowledgments

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