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Application the mechanism-based strain gradient plasticity theory to model the hot deformation behavior of functionally graded steels

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Abstract. Functionally graded steels (FGSs) are a family of functionally graded materials (FGMs) consisting of ferrite (α), austenite (γ), bainite (β) and martensite (M) phases placed on each other in different configurations and produced via electroslag remelting (ESR). In this research, the flow stress of dual layer austenitic-martensitic functionally graded steels under hot deformation loading has been modeled considering the constitutive equations which describe the continuous effect of temperature and strain rate on the flow stress. The mechanism-based strain gradient plasticity theory is used here to determine the position of each layer considering the relationship between the hardness of the layer and the composite dislocation density profile. Then, the released energy of each layer under a specified loading condition (temperature and strain rate) is related to the dislocation density utilizing the mechanism-based strain gradient plasticity theory. The flow stress of the considered FGS is obtained by using the appropriate coefficients in the constitutive equations of each layer. Finally, the theoretical model is compared with the experimental results measured in the temperature range 1000-1200°C and strain rate 0.01-1 s-1 and a sound agreement is found.

Keywords: hot deformation; functionally graded steel; dual-layer; mechanism-based strain gradient plasticity theory

1. Introduction

Mechanical behavior of materials under elastic and plastic deformation, creep, fracture, fatigue deserves a great interest in the scientific community and many of researchers work on these topics. The advance of new materials as well as nano-materials and functionally graded materials has increased the number of contributions in these recent research fields.

For example, Shadlou *et al*. (2013) investigated the fracture behavior of epoxy nano-composites reinforced with different carbon nano-reinforcements. They have also studied the fracture mechanisms of nano-composites and the effects of filler shape on these new materials. Ayotollahi and Hashemi (2007) investigated the effects of composite patching on the fracture behavior of an inclined center crack, under different combination of modes I and mode II loading

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by using the generalized maximum tangential stress criterion. The mechanical properties of carbon nanotube reinforced nano-composites subjected to tensile, bending and torsional loading conditions have been investigated in Functionally Graded Materials (FGMs) are characterized by the variation in composition and structure gradually over the volume, resulting in corresponding changes in the properties of the material. These materials can be designed for specific aims and applications (Ayatollahi *et al.* 2011). Chung and Chen (2007) obtained the deflections, strains and stresses of FGM-coated and FGM-undercoated plates with two simply supported opposite edges and two free edges under bending loading. Golmakani and Kadkhodayan (2011) analyzed the nonlinear bending of annular FGM plates utilizing higher-order shear deformation plate theories. The nonlinear von-Karman theory is used in that work. Ganad *et al.* (2012) performed an elastic analysis for axisymmetric clamped-Pressurized thick truncated conical shells made of FGM utilizing the first-order shear deformation theory and the virtual work principle. Dastjerdi and Sotoodeh-Bahreini (2012) performed the static analysis of FGM cylinders subjected to internal and external pressure by a mesh-free method.

The hot deformation behavior of metals and alloys has a great importance for designers of forming processes (hot rolling, forging and extrusion). Recently, the number of Finite Element Method (FEM) simulations to study material forming processes has strongly increased. These simulations could be reliable when a proper material flow stress relationship is selected and used in the FE code to model the forging response of mechanical components under service loading (Ma *et al.* 2009, Shahani *et al.* 2009, Mandal *et al.* 2009, Lin *et al.* 2010, Li *et al.* 2011). The forming temperature and strain rate affect the hardening and the softening mechanisms. In general, increasing the strain rate and decreasing the temperature, the resistance to plastic deformation increases causing also a rise of the flow stress. The effects of the strain rate and the temperature are usually coupled together and should be considered simultaneously to study the behavior of the material. Strong efforts have been recently made to assess the constitutive equations based on the metallurgical factors which are essential to describe the flow stress behavior during hot deformation processes. They are generally influenced by some deformation parameters such as the strain, the strain rate and the temperature (Ju *et al.* 2006, Vo *et al.* 2007, Fang *et al.* 2009, Spigarelli *et al.* 2010, Shao *et al.* 2010, Mirzadeh and Najafizadeh 2010, Momeni and Dehghani 2011). The constitutive or empirical equations require a number of constants that are mainly derived by fitting the experimental data in a single equation.

Lin *et al*. (2008) investigated the compressive deformation behavior of 42CrMo steel at the temperatures from 850℃ to 1150℃ and strain rates from 0.01 to $50s^{-1}$. They also found the optimum hot formation processing parameters for 42CrMo steel. Moreover, a revised model describing the relationships of the flow stress, strain rate and temperature of 42CrMo steel at elevated temperatures was proposed by compensation of strain and strain rate. The stress–strain values of 42CrMo steel predicted by the proposed model showed a well agreement with experimental results.

Lin and Chen (2011) presented a critical review on some experimental results and constitutive descriptions for metals and alloys in hot working, which were reported in international publications in recent years. In this review paper, the constitutive models were divided into three categories, including the phenomenological, physical-based and artificial neural network models, to introduce their developments, prediction capabilities, and application scopes, respectively. Moreover, they proposed some limitations and objective suggestions for the further development of constitutive descriptions for metals and alloys in hot working.

Lin *et al*. (2012) predicted the high-temperature deformation behavior of Al-Zn-Mg-Cu alloy.

The hot compression tests were done in the strain rate range of (0.001–0.1) s^{-1} and the forming temperature range of (573–723) °K. Based on the experimental results, Johnson-Cook model was applied to describe the high-temperature deformation behavior of Al-Zn-Mg-Cu alloy. Therefore, a new constitutive model was proposed, considering the coupled effects of strain, strain rate and forming temperature on the material flow behavior of Al-Zn-Mg-Cu alloy.

Functionally graded steels (FGSs) are a family of functionally graded materials (FGMs) consisting of ferrite (α), austenite (γ), bainite (β) and martensite (M) phases co-existing in different configurations and produced via electroslag remelting (ESR). In previous studies, FGSs with various configurations have been produced and their tensile behavior has been investigated experimentally and also modeled considering Vickers micro-hardness profile of the composite (Aghazadeh Mohandesi *et al.* 2006). Recently, Vickers micro-hardness profile of the $\alpha\beta\gamma$ FGSs has been described by means of the mechanism-based strain gradient plasticity (MSG) theory (Nazari *et al.* 2011). In this regard, the density of the statistically stored dislocations has been related to the Vickers micro-hardness profile of each layer. Afterwards, the tensile strength of the $\alpha\beta\gamma$ FGSs has been modeled analytically by MSG theory (Nazari *et al.* 2011). The advantage of this model is that the micro-hardness of each layer (i.e., micro-hardness profile) is not required for determining the mechanical properties. Samareh Salavati Pour *et al.* (2013) presented a new analytical expression to obtain the Charpy impact energy of functionally graded steels considering the plastic volume size.

In (Abolghasemzadeh *et al.* 2012) a theoretical model has been proposed to assess the flow stress of FGSs under hot compression loading based on the rule of mixtures. In that paper, the parameters to be inserted in the constitutive equation of each layer have been determined by using some empirical relationships giving the Vickers hardness profile of the considered structure.

The main aim of the present work is to determine the parameters of the constitutive equation utilizing the mechanism-based strain gradient plasticity theory (MSG) in order to assess the flow stress of dual layer austenitic-martensitic functionally graded steels more accurately than the previous work made by the same authors (Abolghasemzadeh *et al.* 2012). With this aim, some new equations have been proposed here to predict the activation energy of each layer under any combination of the applied load (temperature and strain rate) by using only some appropriate boundary conditions and providing an analytical solution to the problem. The main advantage of the present method if compared with previous models is that only the hardness values corresponding to the boundary layers are required as input of the model and it is not necessary to know a priori all the hardness profile to use the proposed equations. The new model is applied here to a dual layer austenitic-martensitic functionally graded steel. The comparison between the experimental results and the predictive model shows that the MSG theory is appropriate and useful to assess the mechanical properties of functionally graded steels.

2. Experimental procedure

2.1 Production method of functionally graded steel

To fabricate FGSs, a miniature ESR apparatus was employed. In typical ESR process, as illustrated in Fig. 1, the slag is used to heat the solid material and to produce a controlled molten metal bath from the initial electrode, which melts by means of electrical induction occurring between the electrode and the bottom part of the mould.

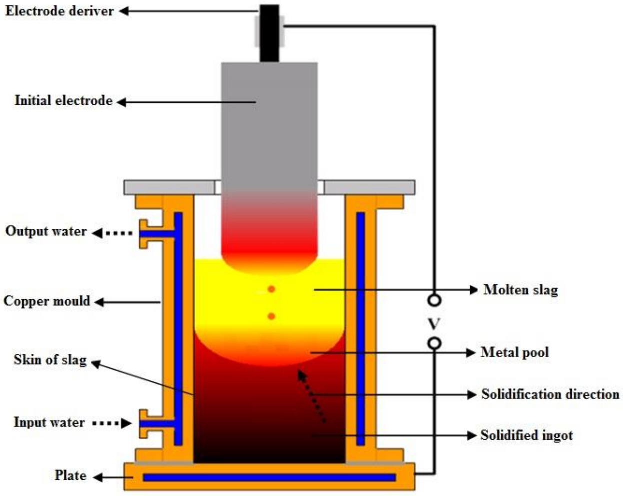

Fig. 1 ESR device used for specimens manufacturing

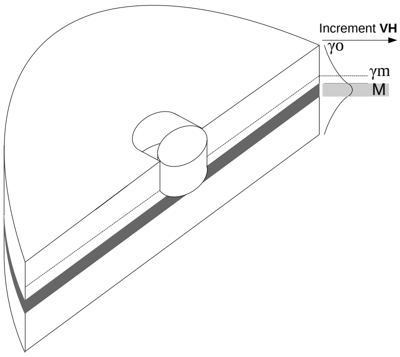

Fig. 2 Dual-layer compression test specimens


Fig. 3 Hot compression test specimens before testing

The slag used was a mixture of 50 pct CaO, 50 pct Al_2O_3. The consumable electrodes were made with arrangement of slices of AISI 1020 and AISI 316 steels (called in the present work as initial ferritic and initial austenitic, respectively).

The height of initial ferritic and austenitic slices was 26 and 92 mm. The two parts were joined by means of spot welding and were so prepared for the remelting stage. The remelting process was carried out under a constant power supply of 16KVA. After remelting, the composite ingots were forged at 980℃ and the specimens were air-cooled.

By following the procedure described in (Abolghasemzadeh *et al.* 2012), a martensitic phase was produced during the remelting stage approximately in the middle of the forged specimen. Therefore, two series of dual-layer specimens were produced from the two austenitic graded regions (see Fig. 2).

For metallographic examinations, the plates were sliced, ground, polished, and etched in a “Kalling” solution and 1 pct “Nital”.

In order to determine the Vickers hardness corresponding to the boundary edges in the obtained compression specimens, some tests were carried out. The average Vickers hardness of the austenitic and the martensitic edges was found to be 195 and 380 Vickers, respectively.

2.2 Test technique

The flow stress of FGSs was evaluated by means of compression testing. Standard sized specimens were produced following the recommendations provided by ASTM E209. The cylindrical specimens were wire cut to reach a height to diameter ratio equal to 1.5. In fact, this is the ratio required by the standard in force for compression tests (ASTM E209). The diameter was equal to 6.5 mm and the height 10 mm (Fig. 3). The compression load was applied during the tests normally to the interface layers. Moreover, the upper and lower surfaces of the specimens were smoothly polished to reduce the friction during the tests adding also some graphite powder with the aim to improve lubrication (Abolghasemzadeh *et al.* 2012).

The compression tests were carried out in a range of temperature between 1000℃ to 1200℃ and three different strain rates 0.01, 0.1 and $1s^{-1}$. Fig. 4 shows the induction furnace for obtaining the desired temperatures. The compression tests were carried out at a constant value of the strain rate and of the temperature. The servo-hydraulic press used for the tests is shown in Fig. 5. During the tests the load-displacement diagrams have been recorded by a computer connected to the test

Fig. 4 Induction furnace

Fig. 5 Hot compresion test devices

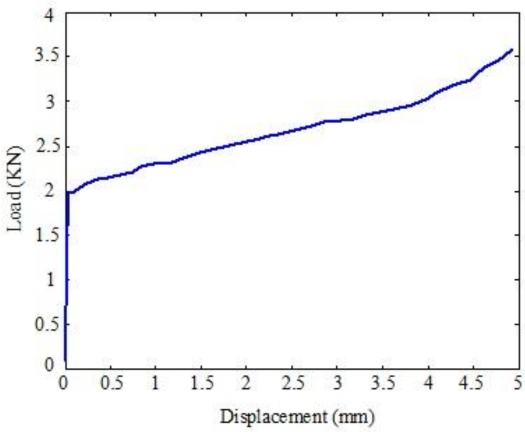

Fig. 6 Stress-strain curve of a test specimen

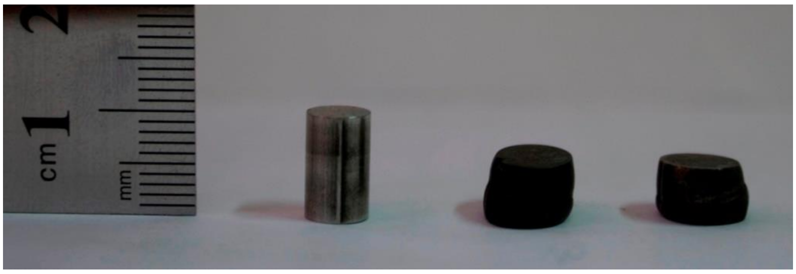

Fig. 7 Hot compression test specimens after testing

machine. The data have been then elaborated to obtain the stress-strain curves. An example of a typical load-displacement curve corresponding to the temperature 1000℃ and strain rate 0. 1 s^{-1} is shown in Fig. 6.

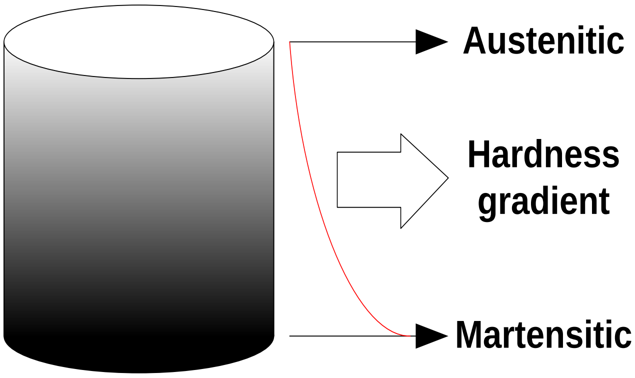

Fig. 8 Variation of Vikers hardness along the investigated cylindrical FGS specimens

Fifty percent of height reduction was applied to each specimen during each single step of pressure loading (see Fig. 7).

The experimental results showed that the flow stress increased by increasing the applied strain rate and decreased by increasing the investigated temperature.

3. Model

The main aim of the present work is to provide a theoretical model able to assess the flow stress of a dual layer FGS. In the tested cylindrical FGS specimens, Vickers hardness raises slowly from the austenitic edge to the martensitic edge along the specimen height (see Fig. 8).

While dealing with hot compression behavior modeling of conventional material, constitutive equations have been utilized to describe the flow stress as a function of the temperature and the strain rate. These equations are usually based on parameters determined by means of empirical results requiring also an accurate and time wasting set up. As the empirical parameters are difficult to be obtained and the experimental errors could affect these parameters, different results could be gained for the same specific alloy by using different equations (Abolghasemzadeh *et al.* 2012). For this reason, it is very important to propose a theoretical model able to estimate the flow stress of FGSs under hot deformation limiting at minimum the use of empirical parameters set on the basis of experimental results.

3.1 Establishing constitutive equation

The relationship between the flow stress, the strain rate and the temperature, especially at high temperature, can be expressed by means of the well-known Arrhenius equation. The effects of the temperature and strain rate on the deformation behavior can be described by the hyperbolic sine function of Zener-Hollomon (Z-H) parameter (Majzoobi *et al.* 2010, Li *et al.* 2011, Golmakani and Kadkhodayan 2011, Abolghasemzadeh *et al.* 2012)

$$Z = \dot{\varepsilon}\exp\left(Q/RT\right) = A[\sinh(\alpha\sigma)]^n \tag{1}$$

In Eq. (2) $\dot{\varepsilon}$, Q, T and R are strain rate (s^{-1}), the deformation process activation energy ($J.mol^{-1}$), the absolute temperature (°K) and the universal gas constant (8.314 $J.mol^{-1}.K^{-1}$), respectively and n, A and α are material constants. As visible from the equation, Z is a parameter related to the temperature and the strain rate and remains constant if the applied temperature and strain rate do not vary. The term *exp(Q/RT)* is linked to the process of heat activation. Meanwhile, n, A and α are the material constants. While some exponential-based and exponent-based relationships have been used for representing Z, the hyperbolic sine function is usually adopted to describe the flow stress under hot deformation loading conditions with high strain rates. This representation is also used for creep at low strain rates. Eq. (2) is used in the present investigation and adopted in the new developed analytical frame.

3.2 Previous model

As explained in (Nazari *et al.* 2011) for modeling the mechanical behavior of FGS, the composite is divided into many layers placed in the direction normal to the applied load. Each layer can be considered as homogenous and characterized by the same properties (Abolghasemzadeh *et al.* 2012). The subdivision of the specimen in the layers is carried out considering 5 units rise or fall of the Vickers hardness value along the height of the specimen. A theoretical model has been formulated to predict the flow stress of graded steel under hot compression conditions based on the Reuss model for the overall strains. The model proposed in (Abolghasemzadeh *et al.* 2012) allows us to take into account the variation of the volume fraction of individual layers of the graded composite. The following expression has been proposed for the composite strain rate (Nazari *et al.* 2011)

$$\dot{\varepsilon} = \frac{\sum_{i=1}^{I} \langle f_{vi}\,\dot{\varepsilon}_i \times [1+\xi\varepsilon_i\,] \rangle}{1+\chi\sum_{i=1}^{I} \langle f_{vi}\,\varepsilon_i \rangle} \tag{2}$$

In Eq. (3) $\dot{\varepsilon}$ is the composite stain rate, $\dot{\varepsilon}_i$ is strain rate of the i^{th} layer; f_{vi} is the volume fraction of the i^{th} layer. Parameter $\xi=(1+\varepsilon_{flow})^{-1}$ is the constraint factor which depends only on the given composite strain under the applied compression loading. Finally, the strain, ε_i, corresponding to the i^{th} layer is obtained following the method presented in (Aghazadeh Mohandesi *et al.* 2006) and assuming the Zener-Hollomon's relationship in the plastic zone and the Hooke's law in the elastic zone for the corresponding stress-strain curves

$$\varepsilon_i = \varepsilon_{yi} \times \left(\frac{\sigma_i}{\sigma_{yi}}\right)^{\frac{1}{n_i'}} = \frac{\sigma_{yi}}{E} \times \left(\frac{\sigma_i}{\sigma_{yi}}\right)^{\frac{1}{n_i'}} \tag{3}$$

In Eq. (4) σ_i, σ_{yi}, ε_{yi} and n'_i are the stress, the yield strength, the yield strain and the strain-hardening exponent of the i^{th} layer, respectively and E is the Young's modulus that is approximately constant along the graded region of the FGS composite.

In (Abolghasemzadeh *et al.* 2012), the mechanical properties, such as the strain-hardening exponent, n', and the constitutive equation parameters, have been assumed as exponential functions versus the layer position in the graded region. In the present paper, the inverse of exponent n and the logarithm of *A* are assumed to be exponential functions with respect to the layer position in the graded region. Meanwhile, the values of these parameters are ln *A*=32 and *n*=72 for austenitic boundary and ln *A*=37.1 and *n*=5 for martensitic boundary.

3.3 Implementation of MSG theory to predict flow stress distribution

The flow stress (yield stress or ultimate stress) at each point (or elementary volume) of the functionally graded austenitic steel was related to the density of the statistically stored dislocations of that element by using MSG as follows (Nazari *et al.* 2011)

$$\left(\sigma_{flow}\right)_i = \sqrt{3}\alpha G b \sqrt{\left(\rho_S^{flow}\right)_i} \tag{4}$$

In Eq. (5), $(\sigma_{flow})_i$ indicates the flow stress (yield stress or ultimate stress) of a generic point *i* inside the material, *G* is the elastic shear modulus (equal to 80 GPa for steel specimens); α is an empirical coefficient ranging between 0.2 and 0.5 and considered here equal to 0.3. Moreover, *b* is the Burgers vector which is equal to $a_0\sqrt{2}/2$ for FCC crystals (such as austenitic steels) and $a_0\sqrt{3}/2$ for BCC crystals (such as ferritic steel). The parameter a_0 is the lattice parameter and is equal to $4R/\sqrt{2}$ for FCC crystals and $4R/\sqrt{3}$ for BCC crystals whereas *R* is the atomic radius which is equal to 1.27 Å for iron. The dislocation density around the point *i* is indicated as $(\rho_s^{flow})_i$.

The hardening mechanisms which contribute to the gradual variation of the hardness profile in a certain layer, such as the solid solution hardening and the hardening effect due to an increase of the density of the statistically stored dislocations, may be considered as a pseudo dislocation density, ρ^*_S (Nazari *et al.* 2011). Therefore, instead of analyzing a functionally graded composite in which the strengthening effect of each element is obtained by means of the solid solution mechanism, we analyze a functionally graded composite with gradual density of dislocation having the same strengthening effect. Therefore, the flow stress of each layer is determined by the following expression

$$\left(\sigma_{flow}\right)_i = \sqrt{3}\alpha G b \sqrt{\left(\rho_S^{*flow}\right)_i} \tag{5}$$

ρ_s^{*flow} in the graded region has been considered to vary exponentially. In agreement with that reported in (Nazari *et al*. 2011) as follow

$$\rho_s^{*flow}(x_i) = \left(\rho_s^{*flow}\right)_1 e^{\frac{x_i - x_1}{x_2 - x_1}.\ln\frac{\left(\rho_s^{*flow}\right)_2}{\left(\rho_s^{*flow}\right)_1}} \tag{6}$$

In Eq. (6) x_i is the position of each element in the graded region and x_1, x_2 are the position of the boundary layers. The pseudo dislocation density for the boundary layers, $(\rho_s^{*flow})_1$ and $(\rho_s^{*flow})_2$, could be expressed as

$$\left(\rho_S^{*flow}\right)_1 = \left[\frac{\left(\sigma_{flow}\right)_1}{\sqrt{3}\alpha G b}\right]^2 \tag{7a}$$

$$\left(\rho_S^{*flow}\right)_2 = \left[\frac{\left(\sigma_{flow}\right)_2}{\sqrt{3}\alpha Gb}\right]^2 \tag{7b}$$

where $(\sigma_{flow})_1$ and $(\sigma_{flow})_2$ are the flow stresses of the boundary layers in the graded region.

3.4 Implementation of MSG theory to predict activation energy distribution

In (Abolghasemzadeh *et al.* 2012), the properties of the layers to model the flow stress of FGS under hot deformation have been determined using the hypothesis that the variation of the activation energy is proportional to the yield strength. In the present study, with the aim to improve the previous model, the activation energy of the layers in the graded region has been determined utilizing the MSG theory. For doing so, at any specified loading conditions (temperature and strain rate), by combining Eq. (2) and Eq. (6), the following expression has been obtained to assess the pseudo dislocation density of a monotonic material under hot compression

$$\rho_S^{*flow} = \left(\frac{1}{\alpha\sqrt{3}Gb}.\sinh^{-1}\left[\frac{\dot{\varepsilon}_{\mathrm{Appl}}}{A}.\exp\left(\frac{Q}{KT_{\mathrm{Appl}}}\right)\right]\right)^2 \tag{8}$$

In Eq. (8) , $\dot{\varepsilon}_{\mathrm{Appl}}$ and T_{Appl} state for the applied strain rate and temperature, respectively.

Moreover, the activation energy of each layer considering an exponential variation of the pseudo dislocation density along the graded region can be obtained as follows

$$Q_i = KT_{Appl}\ln\left\{\frac{A}{\dot{\varepsilon}_{Appl}}.\sinh^n\left[\alpha\sqrt{3}Gb\sqrt{\left(\rho_s^{*flow}\right)_1 e^{\frac{x_i - x_1}{x_2 - x_1}.\ln\frac{\left(\rho_s^{*flow}\right)_2}{\left(\rho_s^{*flow}\right)_1}}}\right]\right\} \tag{9}$$

By determining the necessary parameters in the *Z-H* equation for each layer, the strain rate corresponding to each layer can be expressed as a function of the temperature and the flow stress by updating Eq. (2)

$$\dot{\varepsilon}_i = A_i[\sinh(\alpha_i\sigma)]^{n_i}.\exp(-Q_i/RT) \tag{10}$$

By substituting Eqs. (4) and (10) into Eq. (3), one obtains

$$\dot{\varepsilon}_{\mathrm{FGS}} = \frac{\sum_{i=1}^{I} f_{vi}A_i\left[\sinh(\alpha_{\gamma_o}\sigma)\right]^{n_i}\exp(-Q_i/RT)\left(1+\chi\left\langle\frac{\sigma_{yi}}{E}\left(\frac{\sigma_{flow}}{\sigma_{yi}}\right)^{n_i'}\right\rangle\right)}{1+\frac{\chi}{E}\sum_{i=1}^{I} f_{vi}\left(1+\chi\left\langle\frac{\sigma_{yi}}{E}\left(\frac{\sigma_{flow}}{\sigma_{yi}}\right)^{n_i'}\right\rangle\right)} \tag{11}$$

Index *i* corresponds to the mechanical properties of the i^{th} layer in the graded region while I is the number of total layers in the considered region. It can be easily observed that Eq. (11) is an

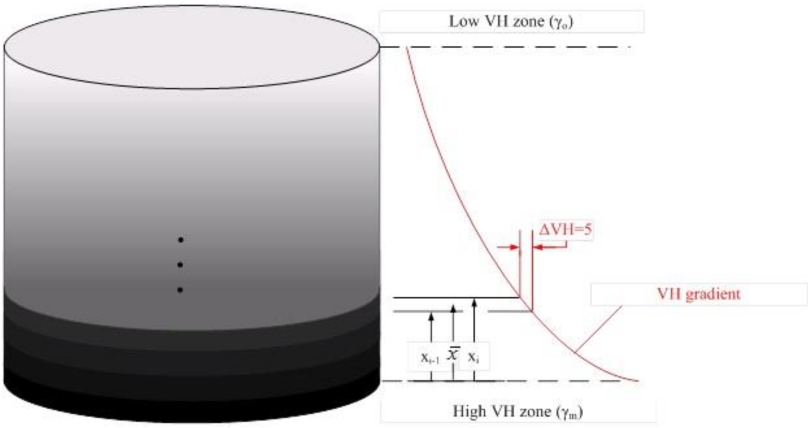

Fig. 9 Subdivision of the FGS specimen in the layers

equation with a single unknown parameter to be determined, the flow stress (σ_{flow}). Then for each applied temperature (T), strain rate ($\dot{\varepsilon}$), the flow stress can be determined from Eq. (11)

4. Implementation of proposed model to austenite-martensite functionally graded steel

In order to investigate the accuracy of the new proposed model, Eq. (11) is applied here to assess the flow stress of the composite made of two phases, described in the previous sessions and tested under compression loading.

In the present work, the composite is divided into 38 parts from the martensitic layer (γ_m) to the austenitic layer (γ_1). The subdivision of the specimen in different layers is carried out considering 5 units rise or fall of the Vickers hardness value along the height of the specimen (see Fig. 9). Therefore, the average Vickers micro-hardness value of each element is obtained as follows

$$VH(\bar{x}_i) = \frac{1}{2}\left(VH(x_i) + VH(x_{i-1})\right) \tag{12}$$

Where $VH(\bar{x}_i)$ is the average Vickers micro-hardness of the i^{th} layer, and x_{i-1} and x_i represent the coordinate of the top part and the bottom part of the i^{th} layer.

The value of the pseudo dislocation density ρ_s^{*flow} can be obtained as follows (Nazari *et al.* 2011)

$$\rho_s^{*flow} = \frac{VH^2}{27\alpha^2 G^2 b^2} - \frac{3\sqrt{2}\cot\phi}{\pi b\sqrt{\frac{1.72F}{VH}}} \tag{13}$$

where ϕ is the Vickers indenter semi angle that is here equal to 68° and F is the force applied to the indenter.

By using Eq. (12) and Eq. (13) in combination, the pseudo dislocations density of each element can be obtained. Then, by substituting the obtained pseudo dislocation density of each element into Eq. (6), the position of each layer (x_i) in the graded austenitic regions can be evaluated. Finally, by

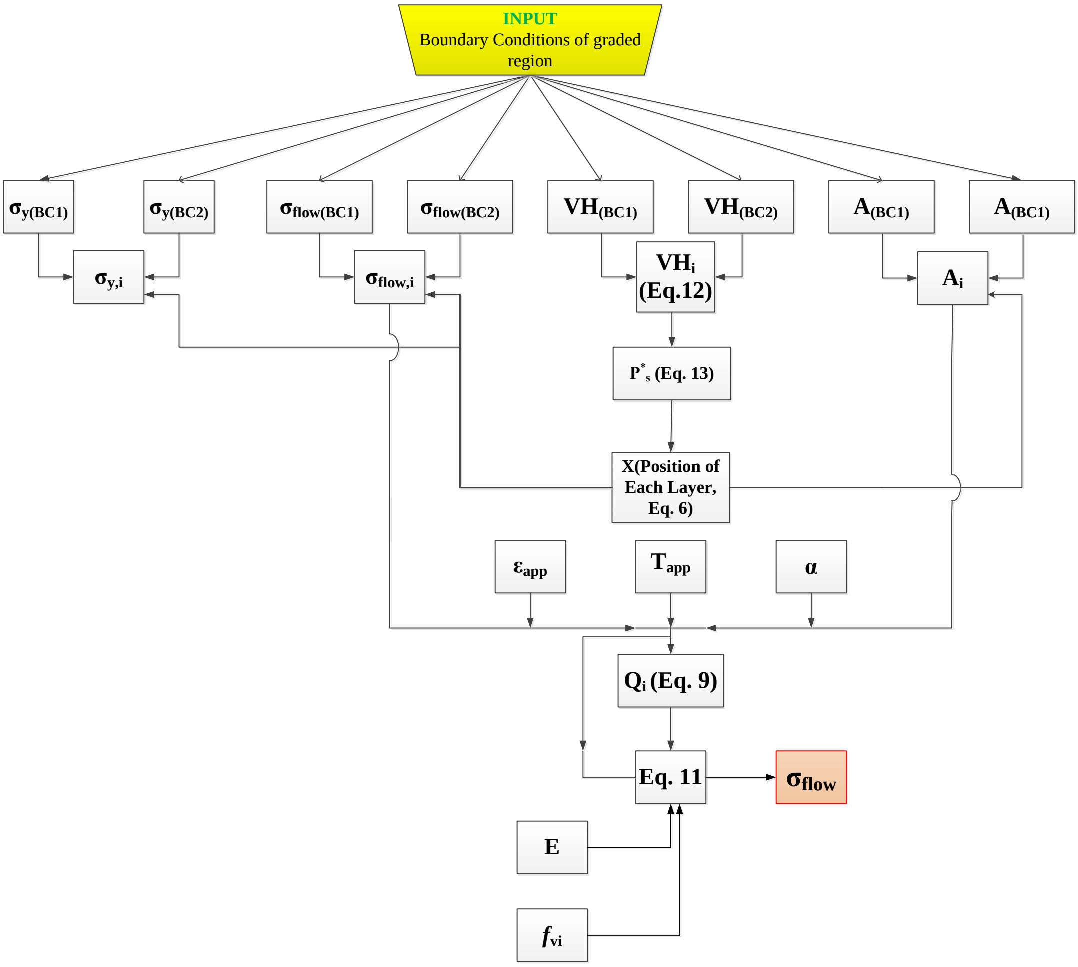

Fig. 10 Flow chart for the application of the proposed method

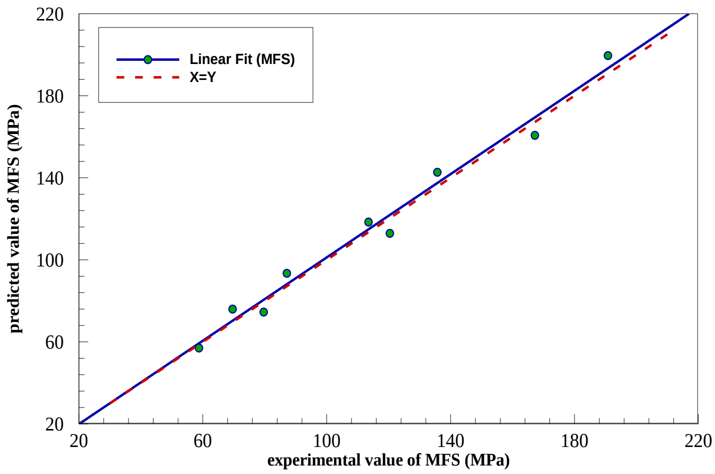

Fig. 11 Comparison between the measured and predicted values of flow stress

Table 1 The comparison of flow stress values predicted by theoretical model and ones measured under hot compression test

Temprature (℃)	Strain rate (s^{-1})	Flow stress of FGS	
		Experimental (MPa)	Model (MPa)
1000	0.01	135.7046	142.7050
1000	0.1	167.2000	160.7068
1000	1	190.7828	199.5887
1100	0.01	79.6464	74.5304
1100	0.1	113.4896	118.4203
1100	1	120.3785	112.9213
1200	0.01	58.7563	57.0128
1200	0.1	69.6090	75.9799
1200	1	87.1166	93.4361
	Mean error		5.5549
	Standard variant		1.9034

solving Eq. (9) in the only unknown parameter, the activation energy and the flow stress can be obtained. All the steps necessary for the application of the present model are reported in the flowchart shown in Fig. 10. The results in terms of the values of the flow stress obtained by using the theoretical model, and in particular Eq. (11) applied to different temperatures and strain rates, have been compared with the experimental values obtained by the compression tests carried out in the present research project (see Table 1). The graphical comparison between the proposed model and the experimental values is shown in Fig. 11. As it is evident from the figure, there is a very small relative deviation between the assessed values of the flow stress and the experimental values. The average error between the predicted hot compression flow stresses and the experimental ones is very limited and generally lower than 5%. This means that the proposed model allows a good estimation of the flow stress without considering empirical results or parameters derived by the hot deformation tests. Moreover, it seems that the hypotheses adopted here, and in particular the exponential variation of the dislocation density inside the FGS, are well suited for describing the material behavior.

Mean error has been determined by the following expression.

$$Mean\ Error = \frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_{flow(Model)i} - \sigma_{flow(\exp)i}}{\sigma_{flow(\exp)i}} \tag{14}$$

The variance is computed as the average squared deviation of each number from its mean. The formula (in summation notation) for the variance in a population is

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} \tag{15}$$

where μ is the mean and N is the number of scores.

5. Conclusions

The hot deformation behavior of a dual layer FGS in a range of temperature between 1000℃ to 1200℃ and three different strain rates 0.01, 0.1 and 1 s^{-1} have been investigated experimentally. A new model has been also proposed to estimate the flow stress of the composite taking advantage of the strain gradient plasticity theory.

The main findings can be summarized as follows:

• A dual layer FGS obtained by two initial electrodes consisting of ferritic and austenitic steels was produced by means of ESR process and the flow stress was measured under hot compression standard test. The results showed that the measured flow stress varied from 58.7 MPa to 190.8 MPa for the tested composite.

• The results showed that the flow stress increases by increasing the applied strain rate and decreases increasing the applied temperature.

• An improved theoretical model based on a previous work by the same authors combined with the strain gradient plasticity theory has been proposed to describe the MFS of FGSs under hot compression conditions.

• The average error between the theoretical assessed values and the experimental data was 5.55 % for the composite and a good agreement is found.

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