Structural Engineering and Mechanics, *Vol. 51, No. 4 (2014) 531-546*
DOI: http://dx.doi.org/10.12989/sem.2014.51.4.531

Multimode pushover analysis based on energy-equivalent SDOF systems

Grigorios E. Manoukas*, Asimina M. Athanatopoulou[a] and Ioannis E. Avramidis[b]

Department of Civil Engineering, Aristotle University, University Campus, 54124, Thessaloniki, Greece

(Received July 27, 2012, Revised March 14, 2014, Accepted March 20, 2014)

Abstract. In this paper the extension of a recently established energy-based pushover procedure in order to include the higher mode contributions to the seismic response of structures is presented and preliminary evaluated. The steps of the proposed methodology in its new formulation are quite similar to those of the well-known Modal Pushover Analysis. However, the determination of the properties of the 'modal' equivalent single-degree-of-freedom systems is achieved by a rationally founded energy-based concept. Firstly, the theoretical background and the assumptions of the proposed methodology are presented and briefly discussed. Secondly, the sequence of steps to be followed for its implementation along with the necessary equations is systematically presented. The accuracy of the methodology is evaluated by an extensive parametric study which shows that, in general, it provides better results compared to those produced by other similar procedures. In addition, the main shortcoming of the initial version of the methodology now seems to be mitigated to a large extent.

Keywords: inelastic seismic response; advanced pushover procedures; energy based procedures; equivalent single-degree-of-freedom system; higher mode effects; multimodal pushover analysis; planar frames

1. Introduction

In recent years an increasing interest for pre-earthquake assessment and rehabilitation of existing buildings has been observed. For this purpose the Performance Based Design methodology, which is adopted by almost all modern seismic codes (e.g., ASCE 41-06, ATC-40, EC-8 Part 3), is usually applied. Performance Based Design consists of a set of provisions, rules, design criteria and methods which aim at a predefined performance of the structure for a specific earthquake hazard level. In order to check the achievement of this goal some critical response parameters have to be calculated through the implementation of a linear or nonlinear analysis procedure. It is obvious that linear procedures are suitable only for high target performance levels (Operational or Immediate Occupancy), i.e., for structures which are expected to respond (nearly) elastically for the design earthquake hazard level (Avramidis 2006). There is no doubt that the

∗Corresponding author, Ph.D., E-mail: grman7@otenet.gr
[a]Professor, E-mail: minak@civil.auth.gr
[b]Professor, E-mail: avram@civil.auth.gr

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

most rational analysis procedure for structures expected to sustain extensive inelastic deformations under strong earthquake excitations is the nonlinear dynamic analysis (NDA). However, this procedure involves many shortcomings such as significant computational cost, lack of adequate number of representative accelerograms for each area of interest, dependence on the choice of the accelerograms' scaling procedure, etc.

Static Pushover Analysis (SPA) as it is called in some publications (e.g., Krawinkler and Seneviratna 1998), or Nonlinear Static Procedure (NSP) as it is named in seismic codes, seems to be a useful alternative tool for the approximate estimation of the inelastic performance of buildings under strong seismic excitations. Initially, SPA has been developed in some more or less similar variants called 'conventional' procedures. The main idea of all of these procedures is that the inelastic response of a structure can be related to the response of an equivalent single-degree-of-freedom (E-SDOF) system. As a first step, the structure is subjected to incremental lateral forces with constant distribution along the height and the base shear versus roof displacement diagram is plotted (capacity or pushover curve). The capacity curve is then idealized to a bilinear curve from which the fundamental properties of an E-SDOF system are determined. On the basis of several additional simplifying assumptions, the peak roof displacement of the structure (target displacement) is correlated to the peak response of the E-SDOF system which is estimated with the aid of a selected design or response spectrum. All other response quantities are determined by conducting pushover analysis up to the already calculated target displacement.

Nevertheless, as it has already been stressed by many researchers (e.g., Krawinkler and Seneviratna 1998, Goel and Chopra 2004) this procedure has many shortcomings and can provide reasonable results only for low- and medium-rise planar systems. This is mainly due to the fact that the determination of the structure's response is based on the assumption that the dynamic behaviour depends only on a single elastic vibration mode. In addition, this elastic mode is supposed to remain constant despite the successive formation of plastic hinges during the seismic excitation. Also, the choice of roof displacement instead of any other displacement is arbitrary and it is doubtful whether the capacity curve is the most meaningful index of the nonlinear response of a structure, especially for irregular and spatial systems. In order to overcome these shortcomings many researchers have proposed modified pushover analyses called 'advanced' procedures. Thus, 'multimode' procedures (e.g., Modal Pushover Analysis (Chopra and Goel 2001, Reyes and Chopra 2011), Incremental Response Spectrum Analysis (Aydinoglou 2003, Lin and Tsai 2007, Fujii 2007, Jan *et al.* 2004, Moghadam 2002) take into account the higher mode contributions to the response, while 'adaptive' procedures (e.g., Antoniou and Pinho 2004, Kalkan and Kunnath 2006, Requena and Ayala 2000) require modification of the lateral load pattern at each step of analysis.

A major category of advanced variants of pushover analysis are the so-called 'energy-based' procedures (e.g., Hernadez-Montes *et al.* 2004, Parducci *et al.* 2006, Oliveto *et al.* 2001, Jiang *et al.* 2010, Hashemi and Mofid 2010, Kotanidis and Doudoumis 2008, Leelataviwat *et al.* 2008), whose main idea is that the strain energy of the structure or, equivalently, the work done by the external loads is the most representative index of its nonlinear response. According to those variants, the definition of the E-SDOF system is based on the equalization of the external work of the lateral loads acting on the multi-degree-of-freedom (MDOF) system under consideration to the strain energy of the E-SDOF system. This equalization is used to derive a virtual energy-based displacement of the E-SDOF system which can be used in the capacity curve instead of the roof displacement. More specifically, in each step of the pushover procedure, the work done by lateral loads is computed using an incremental formulation. The corresponding increment in the energy-based displacement is calculated by dividing the increment of work at each step by the base shear at that step. The incremental displacements are accumulated to obtain the energy-based displacement of the E-SDOF system. Thus, a modified capacity curve is plotted for each mode taken into account, which is used in lieu of the traditional pushover curve.

A new energy-based procedure for the approximate estimation of the seismic response of structures has been developed recently (Manoukas *et al.* 2011). This procedure uses the strain energy which is considered as a more meaningful index of the structural response than the base shear. This is due to the fact that the strain energy depends on the values of all forces acting to the structure as well as on the values of the displacements of all the system's degrees of freedom. According to this procedure the definition of the E-SDOF system is based on the equalization of the external work of the lateral loads acting on the MDOF system under consideration to the strain energy of the E-SDOF system. In contrast to other energy-based procedures, the energy equivalence is used to derive a modified resisting force of the E-SDOF system, instead of an energy-based displacement. Thus, through a very simple approach, a modified capacity curve is plotted which is used for the establishment of the E-SDOF system. The procedure has been formulated in a manner that takes into account only the predominant vibration mode, so it can be rigorously applied only to low- and medium-rise planar systems. The preliminary evaluation of the procedure has shown that, in general, it is more accurate than other similar variants of pushover analysis. However, a failure in providing a reasonable estimation for drifts at the upper storeys of frames has been observed. Obviously, this shortcoming arises because of the higher mode effects.

The objective of this paper is the extension of the latter approach to take into account the higher mode contributions to the seismic response of structures, in order to be applicable to structures with significant higher mode effects. The steps of the proposed methodology in its new version are quite similar to those of the well-known Modal Pushover Analysis (Chopra and Goel 2001). However, the determination of the properties of the 'modal' E-SDOF systems is based on the aforementioned rationally founded concept. In the following paragraphs, firstly, the theoretical background and the assumptions of the proposed methodology are presented and briefly discussed. Taking into account the basic assumptions and applying well-known principles of structural dynamics, some fundamental conclusions are derived and, on that basis, an alternative, energy-equivalent SDOF system for each mode taken into account is established, which can be used for the estimation of the target displacement. Secondly, the sequence of steps to be followed for the implementation of the proposed methodology along with the necessary equations is systematically presented. Finally, the accuracy of the proposed methodology is evaluated by an extensive parametric study. The paper ends with comments on results and conclusions. The whole investigation proved that the here proposed methodology gives, in general, better results as compared to other similar procedures. In addition, the main shortcoming of the initial version of the methodology now seems to be mitigated to a large extent.

2. Theoretical background

It is well known that the linear elastic response of a MDOF system can be decomposed into responses of SDOF systems, one for each elastic vibration mode (modal analysis). Although this concept lacks a theoretical basis in the inelastic range of behaviour, it has been widely used by many researchers (e.g., Chopra and Goel 2001) in order to develop approximate, simplified nonlinear static procedures which are widely accepted by the scientific community. It is obvious that this approach includes some fundamental assumptions. A major assumption is that the nonlinear response of a MDOF system can be expressed as superposition of the responses of appropriate SDOF systems just like in the linear range. Of course, such an assumption violates the very logic of nonlinearity, as the superposition principle is not valid to nonlinear systems. However, it must be thought as a fundamental postulate, which constitutes the basis on which many simplified pushover procedures are built. Thus, each SDOF system corresponds to a vibration 'mode' i with 'modal' vector $\boldsymbol{\varphi}_i$ (the quotation marks indicate that the application of the superposition principle is not strictly valid). The displacements $\boldsymbol{u}_i$ and the inelastic resisting forces $\boldsymbol{F}_{si}$ are supposed to be proportional to $\boldsymbol{\varphi}_i$ and $\boldsymbol{M\varphi}_i$, respectively (where $\boldsymbol{M}$ is the mass matrix of the system). Furthermore, 'modal' vectors $\boldsymbol{\varphi}_i$ are supposed to be constant, despite the successive development of plastic hinges. Finally, it is supposed that Rayleigh damping is present.

The response of a MDOF system with N degrees of freedom to an earthquake ground motion $\ddot{u}_g(t)$ is governed by the following equation

$$\boldsymbol{M\ddot{u}}(t) + \boldsymbol{C\dot{u}}(t) + \boldsymbol{F}_s(t) = -\boldsymbol{M\delta}\ddot{u}_g(t) \tag{1}$$

where $\boldsymbol{u}(t)$ is the displacement vector of the N degrees of freedom (translations or rotations) relative to the ground, $\boldsymbol{M}$ is the NxN diagonal mass matrix, $\boldsymbol{C}$ is the NxN symmetric damping matrix, $\boldsymbol{F}_s$ is the vector of the resisting forces (or moments), i.e., the forces that would have to be applied to the structure in order to obtain displacements $\boldsymbol{u}(t)$ (for the sake of simplicity (t) is left out in all following expressions) and $\boldsymbol{\delta}$ is the influence vector that describes the influence of support displacements on the structural displacements. The terms of $\boldsymbol{\delta}$ corresponding to translational degrees of freedom parallel to the excitation direction are equal to unity, while the rest are equal to zero. Taking into account the aforementioned assumptions and applying modal analysis, just like in the linear range, N independent equations, each one corresponding to a vibration mode i, are derived (Manoukas *et al.* 2011)

$$M_i^* \ddot{D}_i + 2M_i^* \omega_i \zeta_i \dot{D}_i + V_i = -M_i^* \ddot{u}_g \tag{2}$$

where M_i^*, ω_i, ζ_i, and V_i, are the effective modal mass, the natural frequency, the damping ratio or fraction of critical damping and the 'modal' base shear parallel to the direction of excitation of mode i, respectively, while D_i is the displacement of the corresponding E-SDOF system. Eq. (2) shows that, due to the aforementioned assumptions, the nonlinear response of a MDOF system with N degrees of freedom subjected to a horizontal earthquake ground motion $\ddot{u}_g$ can be expressed as superposition of the responses of N SDOF systems, each one corresponding to a vibration 'mode' having mass equal to M_i^*, displacement equal to D_i and inelastic resisting force equal to V_i. Obviously, this definition of the SDOF systems is not unique, e.g., the mass could be taken equal to unity and the resisting force equal to the quantity V_i/M_i^*.

Furthermore, it can be proved that the external work of 'modal' forces $\boldsymbol{F}_{si}$ on the differential displacements $d\boldsymbol{u}_i = \nu_i \boldsymbol{\varphi}_i \, dD_i$ (where ν_i is the modal participation factor of mode i) is given by Eq. (3) (Manoukas *et al.* 2011)

$$dE_i = V_i dD_i \tag{3}$$

Eq. (3) shows that the external work of 'modal' forces is equal to the work of the resisting force (or the strain energy) of the corresponding SDOF system for the displacement dD_i.

As a consequence of the above conclusions, some basic equations correlating the properties of the 'modal' E-SDOF systems to the properties of the MDOF system are derived and summarized

Table 1 Definition of E-SDOF systems

MDOF system		E-SDOF systems
"modal" displacements $\boldsymbol{u}_i^T = \boldsymbol{\varphi}_i^T v_i D_i$ (roof displacement u_{Ni})	$\Rightarrow$	displacement $D_i = u_{Ni}/ v_i \varphi_{Ni}$ (1st)
"modal" base shear V_i	$\Rightarrow$	resisting force $V_{SDOFi} = V_i$ (2nd)
work of "modal" forces on the differential "modal" displacements $d\boldsymbol{u}_i^T = \boldsymbol{\varphi}_i^T v_i dD_i$ $E(d\boldsymbol{u}_i)$	$\Rightarrow$	work of resisting force on the differential displacement dD_i $E(dD_i) = E(d\boldsymbol{u}_i)$ (3rd)

in Table 1. However, these equations are derived on the basis of the aforementioned assumptions and cannot be valid all together at the same time when a pushover analysis is conducted. Thus, Modal Pushover Analysis (Chopra and Goel 2001) leaves out the 3rd equation and uses the two others to establish the 'modal' E-SDOF systems, while the conventional procedures adopted by codes follow a different approach with some additional assumptions. More specifically, they take into account only the predominant vibration mode and permit modifications to the corresponding mode shape vector. On the other hand, the energy-based single or multimodal procedures (e.g., Hernadez-Montes et al. 2004) keep the last two equations and determine the E-SDOF systems' displacements on the basis of the energy equivalence between E-SDOF and MDOF system. In contrast to the above approaches, the proposed method keeps the 1st and the 3rd equation and uses the energy equivalence to determine a modified resisting force of the E-SDOF systems.

3. The proposed methodology

The sequence of steps needed for the implementation of the proposed methodology is as follows:

Step 1: Create the structural model.

Step 2: Apply to the model a set of lateral incremental forces proportional to the vector $\boldsymbol{M\varphi}_1$ of the fundamental elastic vibration mode 1 and determine the (strain energy)-(roof displacement) curve E_1-u_{N1}. E_1 is equal to the work of the external forces.

Step 3: Divide the abscissas of the E_1-u_{N1} curve by the quantity $v_1\varphi_{N1}=u_{N1}/D_1$ and determine the (strain energy)-(displacement) curve E_1-D_1 of the E-SDOF system corresponding to the fundamental vibration mode 1 (Fig. 1).

Step 4: Calculate the work $\Delta E_{1,\lambda}$ (Fig. 1) of the external forces in each of λ discrete intervals between the successive formation of plastic hinges. $dE_{1,\lambda}$, as part of $\Delta E_{1,\lambda}$ (Eq. (4)), is considered to derive from Eq. (5).

$$dE_{1,\lambda} = \Delta E_{1,\lambda} - V_{1,\lambda\text{-}1}(D_{1,\lambda} - D_{1,\lambda-1}) = \Delta E_{1,\lambda} - V_{1,\lambda\text{-}1}\, dD_{1,\lambda} \tag{4}$$

$$dE_{1,\lambda} = \frac{1}{2} k_{1,\lambda}\, dD_{1,\lambda}^2 \Rightarrow k_{1,\lambda} = 2dE_{1,\lambda}/dD_{1,\lambda}^2 \tag{5}$$

where $k_{1,\lambda}$ is the stiffness of the E-SDOF corresponding to mode 1 in the interval λ. The resisting force $V_{1,\lambda}$ is given by Eq. (6)

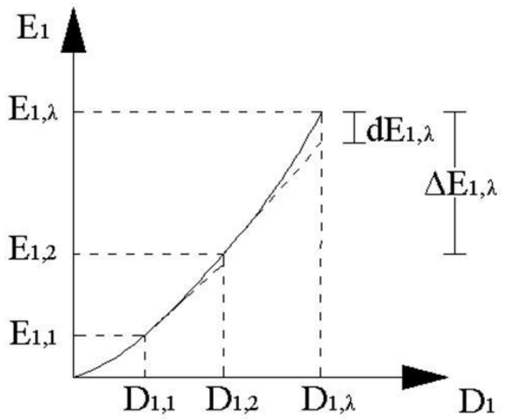

Fig. 1 (Strain energy)-(displacement) curve E_1-D_1

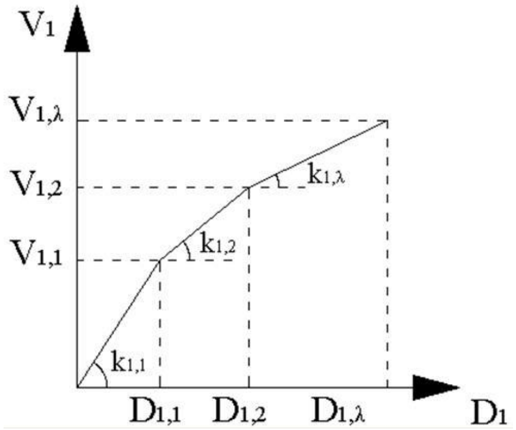

Fig. 2 (Force)-(displacement) curve V_1-D_1

$$V_{1,\lambda} = V_{1,\lambda\text{-}1} + k_{1,\lambda} \, dD_{1,\lambda} \tag{6}$$

For λ=1 (i.e., when the first plastic hinge is created) the force $V_{1,1}$ is equal to the base shear parallel to the direction of excitation. By utilizing Eqs. (4), (5) and (6) for each interval, determine the (resisting force)-(displacement) diagram V_1-D_1 of mode 1 (Fig. 2).

Step 5: Idealize V_1-D_1 to a bilinear curve using one of the well known graphic procedures (e.g., ASCE 41-06, Section 3.3.3.2.5) and calculate the period T_1 and the yield strength reduction factor R_1 of the E-SDOF system corresponding to mode 1 from Eq. (7)

$$T_1 = 2\pi \sqrt{\frac{m_1 D_{y1}}{V_{y1}}} \rightarrow S_{\mathrm{a}}(T_1) \rightarrow R_1 = \frac{m_1 S_a(T_1)}{V_{y1}} \tag{7}$$

where m_1, D_{y1}, V_{y1} are the mass, the yield displacement and the yield strength of the system respectively and $S_a(T_1)$ is the spectral acceleration. It is stated that the mass m_1 is equal to the effective modal mass M_1^* of mode 1.

Step 6: Calculate the target displacement corresponding to mode 1 using one of the well-known procedures of displacement modification (e.g., ASCE 41-06, Section 3.3.3.3.2, FEMA 440, Section 10.4). If the procedure is applied for research purposes using recorded earthquake ground motions, it is recommended to estimate the inelastic displacement of the E-SDOF system by means of nonlinear dynamic analysis, instead of using the relevant coefficients given in various official documents (e.g., C_1 in ASCE 41-06 and FEMA 440). This is due to the fact that the coefficient values given in such documents are based on statistical processing of data with excessive deviations and, therefore, large inaccuracies might result (Manoukas *et al.* 2006).

Step 7: Calculate the 'modal' values of the other response quantities of interest (drifts, plastic rotations, etc.) corresponding to mode 1 by conducting pushover analysis up to the already calculated target displacement.

Step 8: Repeat steps 2 to 7 applying the incremental forces in the opposite direction. It is obvious that this step is necessary only for asymmetric structures.

Step 9: Repeat steps 2 to 8 for an adequate number of modes.

Step 10: Calculate the extreme values of response parameters by utilizing one of the well established formulas of modal superposition (SRSS or CQC).

Although the proposed methodology is more complicated than the single mode pushover procedures adopted by seismic codes, the computational cost for its implementation does not exceed that of other well-justified multimode pushover methods such as Modal Pushover Analysis (Chopra and Goel 2001). In general, if n is the number of modes taken into account, 2n pushover analyses have to be conducted (step 2) for the two possible directions of the applied lateral loads (according to step 8). Also, 2n target displacements (steps 3 to 6) and 2n 'modal' values of response parameters have to be calculated (step 7). Finally, similarly to other multimode pushover procedures (e.g., Modal Pushover Analysis) 2^n extreme values of response parameters (step 10) are produced.

4. Evaluation of the proposed methodology

In order to evaluate the accuracy of the proposed methodology an extensive parametric study is carried out. In particular, the methodology is applied to a series of 6-, 9- and 12-storey R/C planar frames resembling reinforced concrete buildings used to be constructed in Greece some decades ago (Fig. 3, Table 2). Each frame is characterized by a string symbol comprising one or two letter(s) and a number which indicates the number of its storeys. The meaning of the letter(s) is as follows:

• R - Regular frames

• M - frames with irregular distribution of Mass along the height. (Odd and even storeys have different masses).

• S - frames with irregular distribution of Stiffness along the height. (Odd storeys have greater height).

• SS - frames with Soft Storey. (1st storey has greater height).

The frames are analyzed using SAP2000, considering concentrated plasticity at the ends of beams and columns (plastic hinges) modelled by bilinear moments-rotations diagrams. For each frame three sets of pushover analyses are performed: i) one based on the proposed methodology (PM), ii) a second based on a procedure similar to the existing multimodal energy-based methods, i.e., according to this procedure the energy equivalence between MDOF and E-SDOF systems is

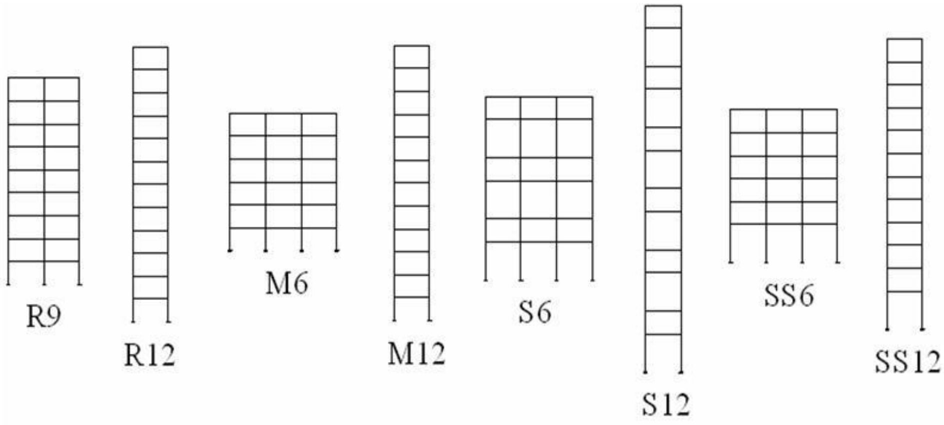

Fig. 3 Geometrical schemes of the analyzed frames

Table 2 Data of the analyzed frames

Data	Frames							
Frame symbol	R9	R12	M6	M12	S6	S12	SS6	SS12
Storey height (m)	3	3	3	3	3/5	3/5	3/5	3/5
Bay width (m)	5							
Restraints	Columns fixed at base							
Constraints	Diaphragm at each level							
Storey mass (t)	30	15	20/40	9/16	25	10	30	13
Damping ratio (%)	5							
Gravity loads	Not considered							
Concrete	C16/20 (f_{ck}=16 MPa)							
Steel	S400 (f_{yk}=400 MPa)							
Column cross-sections (cm)	60/60	60/60	50/50	60/60	50/50	60/60	50/50	60/60
Column reinforcement	8Φ20	8Φ20	8Φ20	8Φ25	8Φ20	8Φ25	8Φ20	8Φ25
Beam cross-sections (cm)	25/50	25/50	25/40	25/50	25/40	25/50	25/40	25/50
Beam reinforcement (over)	2Φ14	2Φ14	2Φ12	2Φ14	2Φ12	2Φ14	2Φ12	2Φ14
Beam reinforcement (under)	2Φ14	2Φ14	2Φ12	2Φ14	2Φ12	2Φ14	2Φ12	2Φ14
Natural period T_1 (sec)	0.947	1.308	0.646	1.211	0.818	1.512	0.738	1.320
Natural period T_2 (sec)	0.292	0.402	0.195	0.373	0.250	0.467	0.220	0.408
Modal participating mass ratio (%)	78.2	76.6	81.0	77.4	81.8	77.5	87.9	81.4
Modal participating mass ratio(%)	10.7	11.5	11.4	11.7	11.7	12.1	8.8	11.5

achieved by modifying the displacements (EB), and iii) a third multimodal procedure based on the conventional displacement modification method (MM). The only difference between the three implemented pushover procedures is the determination of the V_i-D_i diagram (steps 3 and 4), while the remaining steps and assumptions are identical. Namely, for the EB procedure V_i is equal to the modal base shear and D_i is obtained by the energy equivalence between the MDOF and E-SDOF systems, as explained in the introduction. Also, for the CP procedure V_i is equal to the modal base shear and D_i is equal to the roof displacement. V_i-D_i diagram affects the properties of the “modal” E-SDOF systems and, as a consequence, the estimation of the target displacements. The variation

Table 3 List of seismic excitations

Excitation	Date	Magnitude (Ms)	Peak Ground Acceleration (m/sec^2)	Peak Spectral Acceleration (m/sec^2)
Aeghio (longitudinal)	06/15/1995	6.4	4.918	12.099
Aeghio (transverse)			5.326	14.157
Thessaloniki (longitudinal)	06/20/1978	6.5	1.389	4.477
Thessaloniki (transverse)			1.430	4.809
Alkyonides (longitudinal)	02/24/1981	6.7	2.336	6.023
Alkyonides (transverse)			2.989	8.155
Kalamata (longitudinal)	09/13/1986	6.0	2.170	6.648
Kalamata (transverse)			2.913	10.125
Patras (longitudinal)	07/14/1993	5.5	1.402	4.455
Patras (transverse)			3.936	12.151
Pirgos (longitudinal)	03/26/1993	5.5	1.466	5.887
Pirgos (transverse)			4.455	7.705

in the values of the response quantities produced by the three procedures reflects clearly this influence. Each set of analyses comprises 12 strong earthquake motions recorded in Greece (Table 3). The analyzed frames - which have been designed according to older seismic codes - sustain extensive inelastic deformations even for the less strong seismic excitation. The maximum response of the E-SDOF system is calculated by means of nonlinear dynamic analysis (NDA) for each excitation. Then, the target roof displacement is either estimated by multiplication of the resulting response by the quantity $v_i\varphi_{Ni}$ (PM, MM) or obtained by the 'roof displacement' - 'energy-based displacement' correspondence (EB) (Hernadez-Montes *et al.* 2004). All the three procedures are performed taking into account the first two elastic vibration modes, which possess a total modal participating mass ratio around 90% (Table 2) As it has been demonstrated in the past (e.g., Reyes and Chopra 2011), in general, using two (for uniaxial excitation) or two pairs (for biaxial excitation) of translational modes is adequate for the determination of storey displacements and drifts even of very tall buildings, while using the higher modes does not significantly affect the results. The modal superposition of response parameters carried out using the SRSS rule. Due to the fact that the analyzed frames are symmetric, step 8 is not applied (see also the previous section) and, as a consequence, only one extreme (absolute) value for each response parameter is produced.

The storey displacements and drifts of the frames under consideration are compared with those produced by the NDA, which is considered as reference solution. For each response parameter $R_{j,s}$ estimated by the three applied NSPs for an excitation j, the error with regard to the NDA results E_j is calculated from the following relation

$$E_j(\%) = 100\frac{R_{j,s} - R_{j,d}}{R_{j,d}} \tag{8}$$

where $R_{j,d}$ is the value of the response parameter obtained by NDA. Furthermore, the mean error ME_j for the 12 excitations used in this study is given by Eq. (9)

$$ME(\%)=\frac{1}{12}\sum_{1}^{12}E_j=100\frac{1}{12}\sum_{1}^{12}\left(\frac{R_{j,s}-R_{j,d}}{R_{j,d}}\right) \tag{9}$$

In Figs. 4-5 the mean errors of storey displacements and drifts for the 12 seismic excitations are shown. Notice that the positive sign (+) means that the response parameters obtained by NSPs are greater than those obtained by NDA. Conversely, the negative sign (-) means that the response

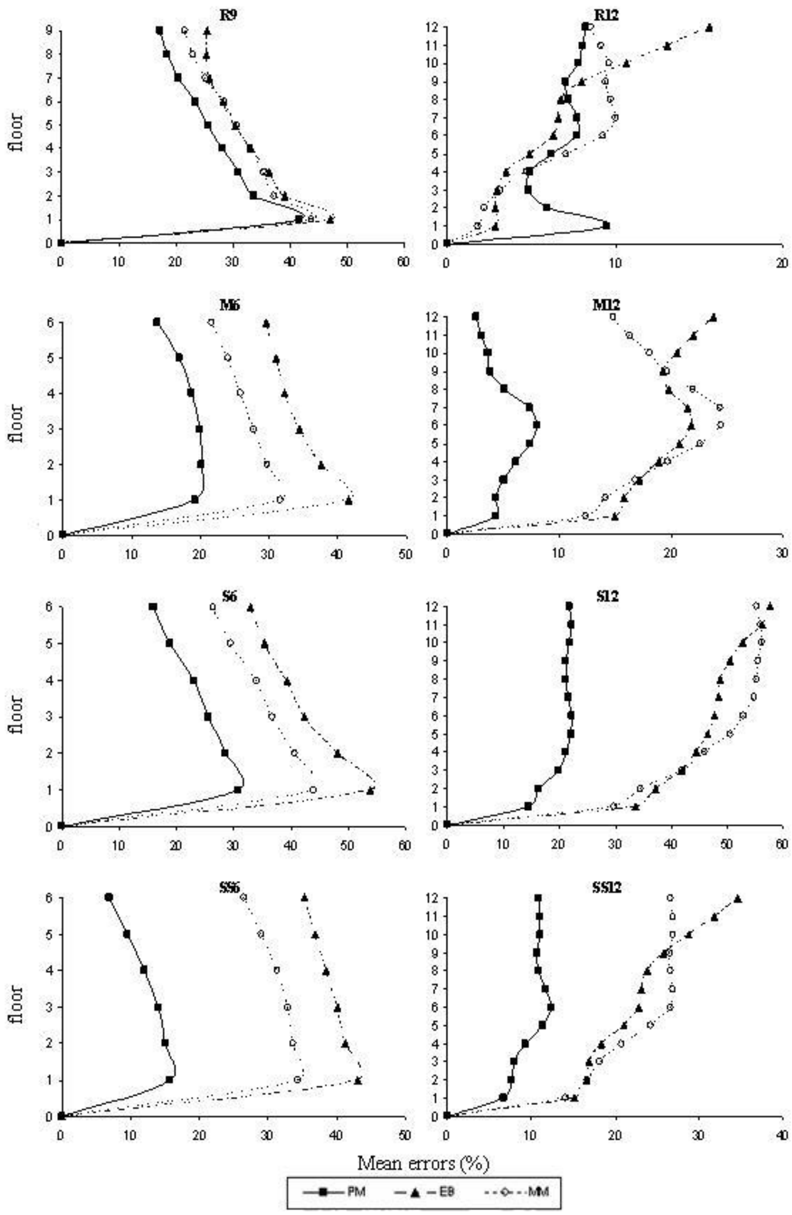

Fig. 4 Mean errors (%) of floor displacements

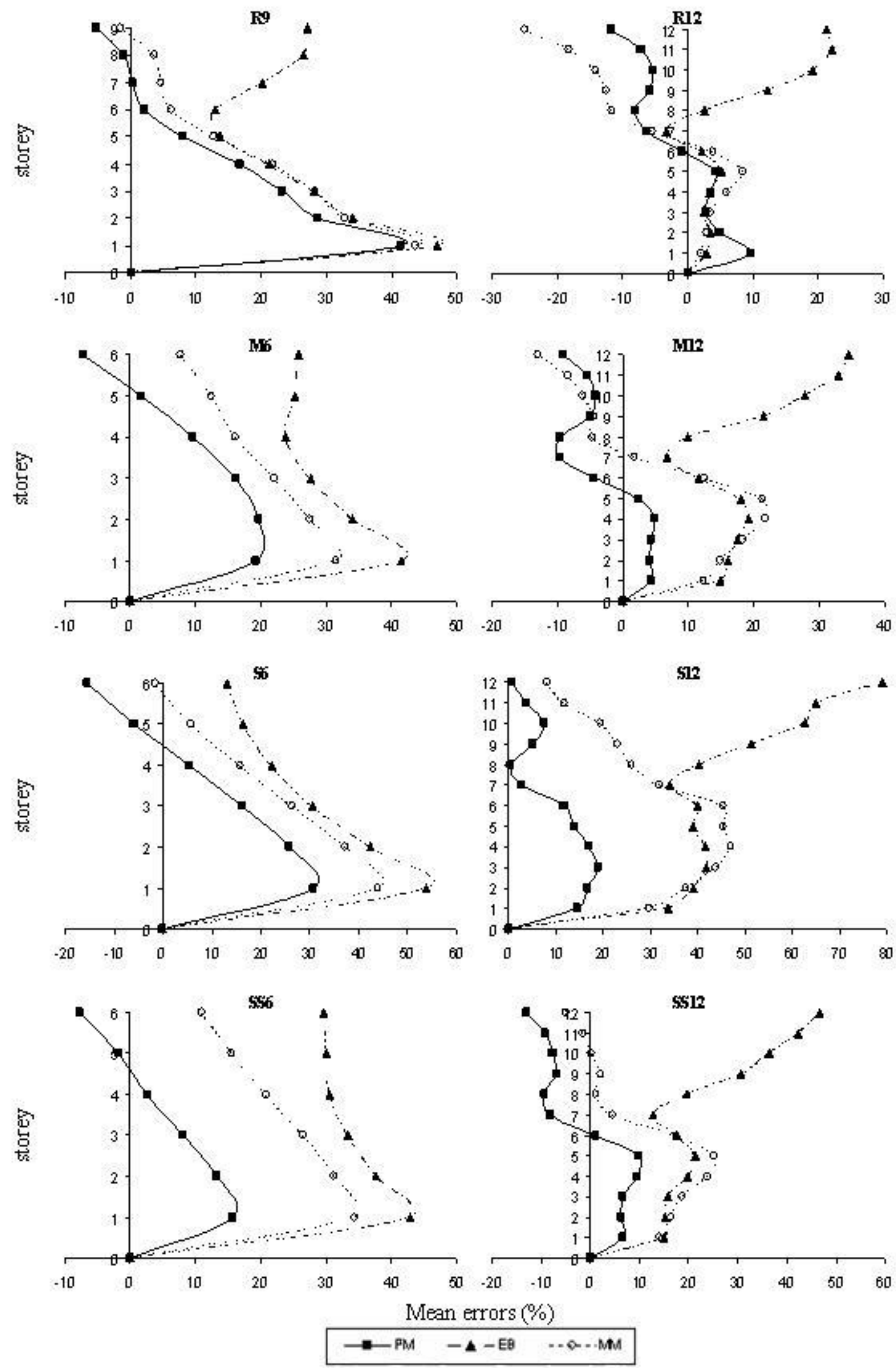

Fig. 5 Mean errors (%) of story drifts

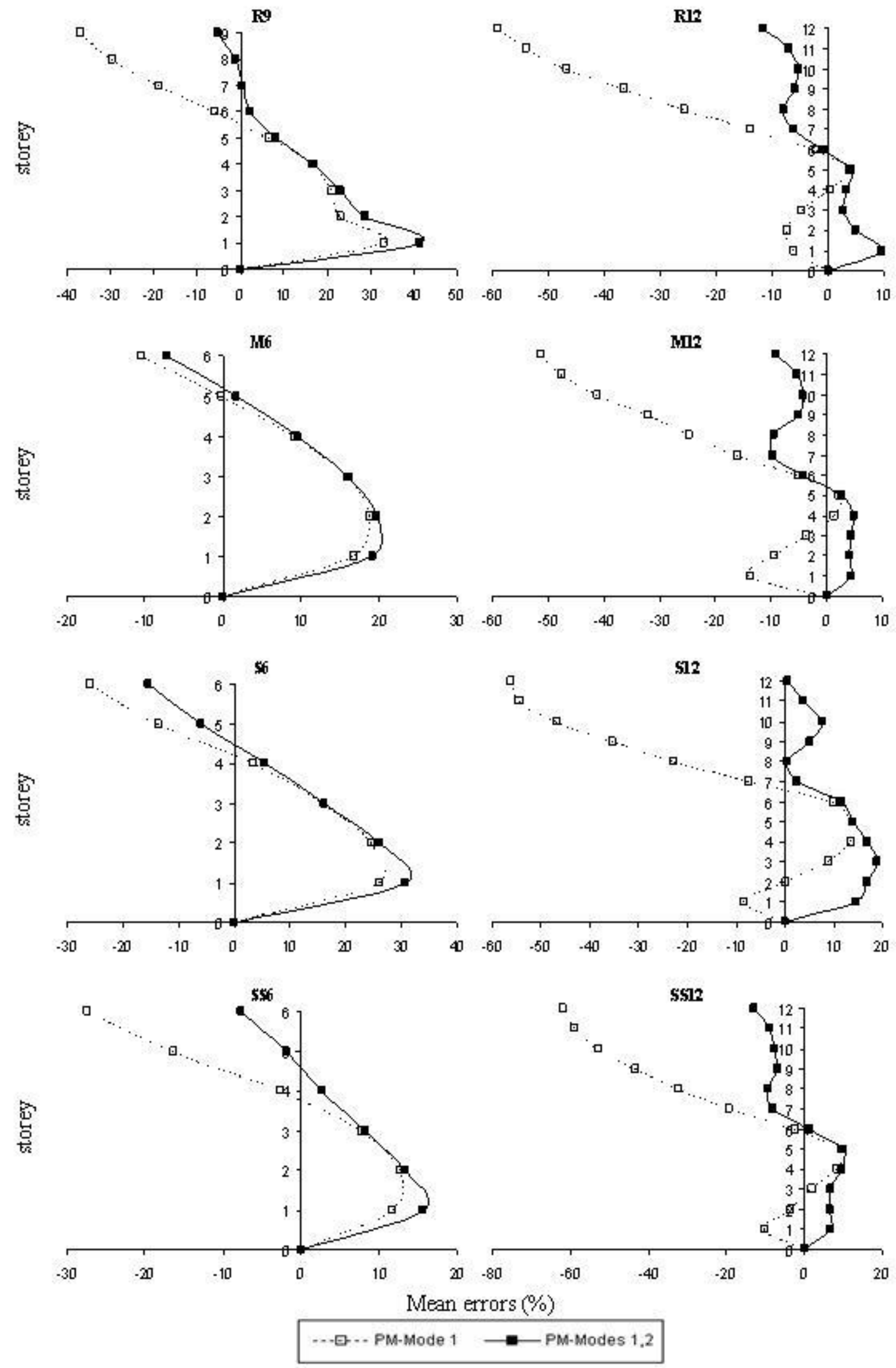

Fig. 6 Mean errors (%) of story drifts resulting from PM

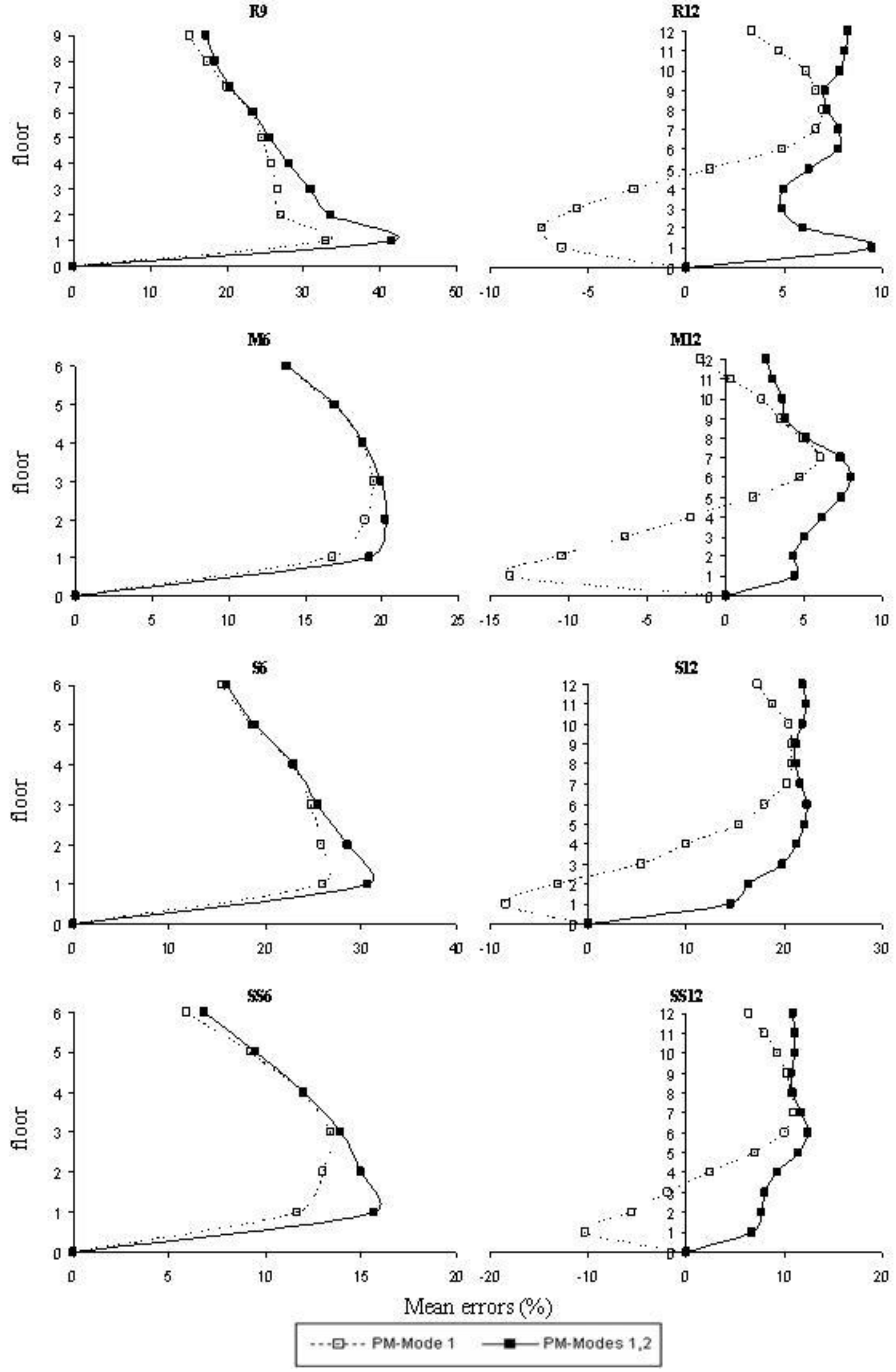

Fig. 7 Mean errors (%) of floor displacements resulting from PM

parameters are underestimated by NSPs. From Fig. 4 it becomes clear that the proposed procedure for the determination of the E-SDOF systems, compared to the other applied procedures, leads to a more accurate estimation of all frames' target roof displacements, with mean errors ranging from 3% to 22% for PM, from 16% to 58% for EB and from 9% to 55% for MM. The resulting values of the other response quantities depend also on the so called MDOF effects, which obviously cause diversifications of the accuracy. However, PM provides a more accurate estimation of the remaining floors' displacements too (except the lower floors of frame R12), with mean errors ranging between 3% and 41% for PM, 3% and 58% for EB and 2% and 56% for MM. It is worth noticing that all mean error's values are positive, i.e., the three applied NSPs provide a conservative estimation of floor displacements. On the contrary, PM as well as MM underestimate the drifts at upper storeys, while EB almost always (with one exception) leads to conservative results. Mean errors range from -15% to 41% for PM, from -3% to 79% for EB and from -25% to 47% for MM. Nevertheless, the absolute values of PM's mean errors are sufficiently smaller in most cases (67 of 75 and 60 of 75 in relevance to EB and MM, respectively). Conclusively, PM leads to a more accurate estimation of the vast majority of computed response parameters (89% and 87% of response parameters in relevance to EB and MM, respectively).

The same eight frames have been analyzed for the same seismic excitations in a previous study (Manoukas *et al.* 2011) applying the proposed methodology in its original formulation, i.e. taking into account only the fundamental elastic vibration mode. The results of the proposed procedure have been compared with those obtained by the conventional displacement modification method as well as by a single-mode energy-based procedure similar to the here applied EB. In general, the proposed methodology led to more accurate results in most cases (80% and 73% of cases in relevance to the conventional and energy-based procedure, respectively). However, all the three applied procedures failed to provide a reasonable estimation for drifts at the upper storeys of frames. Taking into account the second vibration mode leads to a great improvement of drifts' estimation, especially at the upper storeys, so the main drawback of the previous formulation of the proposed methodology, as well as the other single-mode pushover procedures, seems to be mitigated to a large extent. In Fig. 6 the mean errors of storey drifts resulting from the proposed methodology in its original (PM-Mode 1) and in its current (PM-Modes 1, 2) formulation are shown. It is obvious that the mean errors of PM-Mode 1 at upper storeys reaching -60% are now significantly reduced. At the same time, the response parameters predicted with sufficient accuracy by the single-mode procedure are not much affected. This also becomes clear from Fig. 6 as well as Fig. 7 where the mean errors of floor displacements are shown.

5. Conclusions

An extended version of a recently developed energy-based Nonlinear Static Procedure (NSP) is presented and evaluated in this paper. The initial formulation of the proposed methodology takes into account only the fundamental vibration mode of the structure. According to this:

• The properties of the E-SDOF system are determined by equating the external work of the lateral loads acting on the MDOF system under consideration to the strain energy of the E-SDOF system.

• In contrast to other energy-based procedures, this energy equivalence is used to derive a modified resisting force of the E-SDOF system, instead of an energy-based displacement.

This methodology is extended here in order to take into account the higher mode contributions to the seismic response. The procedure in its new formulation comprises multiple implementation of the original version (one application for each mode taken into account) and modal superposition of the resulting response parameters.

This new version of the methodology is evaluated by carrying out an extensive parametric study. Based on the numerical results, the following conclusions can be drawn:

• The proposed methodology always leads to a more accurate estimation of the target roof displacement of the here analyzed frames compared to other similar procedures.

• The proposed methodology also provides a more accurate estimation of the vast majority (about 90%) of the remaining response parameters (storey displacements and drifts).

• Taking into account the 2nd mode contribution to the seismic response of the examined frames mitigates to a large extent the main disadvantage of the initial version of the proposed methodology, i.e., the non-conservative estimation of storey drifts, especially at upper storeys of tall frames.

Conclusively, the whole investigation shows that, in general, the proposed methodology gives better results compared to those produced by the other applied procedures. However, despite the fact that no restrictions are set to the development of the proposed procedure, generalization of the above conclusions to all types of structures requires further investigations, comprising application to a large variety of planar frames as well as 3D-buildings and using an adequately high number of earthquake ground motions. This will be the objective of a forthcoming paper.

References

American Society of Civil Engineers (ASCE) (2008), *Seismic Rehabilitation of Existing Buildings*, ASCE/SEI 41-06 Standard, USA.

Antoniou, S. and Pinho, R. (2004), “Development and verification of a displacement-based adaptive pushover procedure”, *J. Earthq. Eng.*, **7**(5), 643-661.

Applied Technology Council (ATC) (1996), *Seismic Evaluation and Retrofit of Concrete Buildings*, Vol. 1, Report No. ATC-40, Redwood City, CA, USA.

Avramidis, I.E. (2006), “Structural models in research and practice”, *Proceedings of the 15th Hellenic Conference on R/C structures*, Alexandroupoli, Greece, October. (in Greek)

Aydinoglou, N.M. (2003), “An incremental response spectrum analysis procedure based on inelastic spectral displacements for multi-mode seismic performance evaluation”, *Bull. Earthq. Eng.*, **1**(1), 3-36.

Chopra, A.K. and Goel, R.K. (2001), *A Modal Pushover Analysis Procedure to Estimating Seismic Demands of Buildings: Theory and Preliminary Evaluation*, PEER Report 2001/03, Pacific Earthquake Engineering Research Center, University of California, Berkeley, USA.

European Committee for Standardization (2004), *Eurocode 8: Design of Structures for Earthquake Resistance, Part 3: Assessment and retrofitting of buildings*, European standard prEN 1998-3, Brussels, EU.

Federal Emergency Management Agency (FEMA) Applied Technology Council (ATC) (2004), *Improvement of Nonlinear Static Seismic Analysis Procedures*, Report No. ATC-55 (FEMA 440), USA.

Fujii, K. (2007), “Prediction of seismic response of multi-story unsymmetric frame buildings”, *Proceedings of the 8th Pacific Conference on Earthquake Engineering*, Singapore, December.

Goel, R.K. and Chopra, A.K. (2004), “Evaluation of modal and FEMA pushover analyses: SAC buildings”, *Earthq. Spectra*, **20**(6), 225-254.

Hashemi, M.J. and Mofid, M. (2010), “Evaluation of energy-based modal pushover analysis in reinforced concrete frames with elevation irregularity”, *Scientia Iranica, Tran. A: Civil Eng.*, **17**(2), 96-106.

Hernadez-Montes, E., Kwon, O.S. and Aschheim, M.A. (2004), “An energy-based formulation for first- and multiple-mode nonlinear static (pushover) analysis", *J. Earthq. Eng.*, **8**(1), 69-88.

Jan, T.S., Liu, M.W. and Kao, Y.C. (2004), "An Upper-bound pushover analysis procedure for estimating the seismic demands of high-rise buildings", *Eng. Struct.*, **26**(1), 117-128.

Jiang, Y., Li, G. and Yang, D. (2010), "A modified approach of energy balanced concept based multimode pushover analysis to estimate seismic demands for buildings", *Eng. Struct.*, **32**, 1272-1283.

Kalkan, E. and Kunnath S.K. (2006), "Adaptive modal combination procedure for nonlinear static aanalysis of building structures", *J. Struct. Eng.*, **132**(11), 1721-1731.

Kotanidis, C. and Doudoumis I.N. (2008), "Energy-based approach of Static Pushover Analysis", *Proceedings of the 14th World Conference on Earthquake Engineering*, Beijing, China, October.

Krawinkler, H. and Seneviratna, G.D.P.K. (1998), "Prons and cons of a pushover analysis of seismic performance evaluation", *Eng. Struct.*, **20**, 452-464.

Leelataviwat, S., Saewon, W. and Goel S.C. (2008), "An energy based method for seismic evaluation of structures", *Proceedings of the 14th World Conference on Earthquake Engineering*, Beijing, China, October.

Lin, J.L. and Tsai, K.C. (2007), "Simplified seismic analysis of asymmetric building systems", *Earthq. Eng. Struct. Dyn.*, **36**, 459-479.

Manoukas, G., Athanatopoulou, A. and Avramidis, I. (2006), "Comparative evaluation of static pushover analysis' variations according to modern codes", *Proceedings of the 15th Hellenic Conference on R/C Structures*, Alexandroupoli, Greece, October. (in Greek)

Manoukas, G., Athanatopoulou, A. and Avramidis, I. (2011), "Static pushover analysis based on an energy-equivalent SDOF system", *Earthq. Spectra*, **27**(1), 89-105.

Moghadam, A.S. (2002), "A pushover procedure for tall buildings", *Proceedings of the 12th European Conference on Earthquake Engineering*, London, UK.

Oliveto, G., Calio, I. and Marleta, M. (2001), *Seismic Resistance and Vulnerability of Reinforced Concrete Buildings not Designed for Earthquake Action*, *Innovative Approaches to Earthquake Engineering*, WIT Press, UK.

Parducci, A., Comodini, F., Lucarelli, M., Mezzi, M. and Tomassoli, E. (2006), "Energy-based non linear static analysis", *Proceedings of the 1st European Conference on Earthquake Engineering and Seismology*, Geneva, Switzerland.

Requena, M. and Ayala G.A. (2000), "Evaluation of a simplified method for the determination of the non-linear seismic response of RC frames", *Proceedings of the 12th World Conference on Earthquake Engineering*, Auckland, New Zealand.

Reyes, J.C. and Chopra, A.K. (2011), "Three-dimensional modal pushover analysis of buildings subjected to two components of ground motion, including its evaluation for tall buildings", *Earthq. Eng. Struct. Dyn.*, **40**, 789-806.