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A practical neuro-fuzzy model for estimating modulus of elasticity of concrete

İdris Bedirhanoglu*

Department of Civil Engineering, Dicle University, 21280, Diyarbakir, Turkey

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Abstract. The mechanical characteristics of materials are very essential in structural analysis for the accuracy of structural calculations. The estimation modulus of elasticity of concrete (E_c), one of the most important mechanical characteristics, is a very complex area in terms of analytical models. Many attempts have been made to model the modulus of elasticity through the use of experimental data. In this study, the neuro-fuzzy (NF) technique was investigated in estimating modulus of elasticity of concrete and a new simple NF model by implementing a different NF system approach was proposed. A large experimental database was used during the development stage. Then, NF model results were compared with various experimental data and results from several models available in related research literature. Several statistic measuring parameters were used to evaluate the performance of the NF model comparing to other models. Consequently, it has been observed that NF technique can be successfully used in estimating modulus of elasticity of concrete. It was also discovered that NF model results correlated strongly with experimental data and indicated more reliable outcomes in comparison to the other models.

Keywords: concrete; elastic modulus; fuzzy logic; neuro-fuzzy systems

1. Introduction

The modulus of elasticity of concrete is one of the main parameters in estimating structural elements' deformations. E_c must be determined correctly for the realistic analysis of structures. It can be accurately determined from tests of standard cylinder specimens by using well known sophisticated equipment. However, such a process is difficult and time-consuming. Since engineers need an easy way to determine the modulus of elasticity, researchers have tried to develop easy formulas to calculate it. Concrete is not a homogenous material, so its mechanical behavior is very complex and shows great variation. As a consequence, analytically determining concrete's modulus of elasticity is also a challenging task; researchers tend to develop empirical formulas by using experimental data. Many formulas have been proposed to define a relationship between modulus of elasticity and various other properties of concrete. As reported by Li and Zeng (2007) many parameters such as concrete compression strength, type of aggregate, weight of concrete and cement type affect concrete's modulus of elasticity. As observed however, it is not only concrete compression strength that affects concrete's modulus of elasticity. For simple

*Corresponding author, Assistant Professor, E-mail: idrisbed@gmail.com

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calculations, researchers have tried to determine the modulus of elasticity very generally, only based on concrete compression strength. In terms of concrete strength, related studies can be divided into two categories: studies on normal strength and studies on high strength concrete.

It is common knowledge that concrete types with different concrete strengths have different characteristics. Relationships proposed for normal strength concrete do not normally give acceptable results for high and/or low strength concrete. Some of the models proposed by different researchers and national building codes will be discussed. For normal weight concrete with a density of 2300 kg/cm^3, the American Concrete Institute ACI 318 (2008) provides the following relationship for modulus of elasticity

$$E_c = 4.73\sqrt{f'_c} \tag{1}$$

in which f'_c is the standard cylinder compression strength of concrete. However ACI Committee 363 (1984) states that this equation overestimates the modulus of elasticity of concrete when the concrete has excessive compression strength, and states another equation for such high strength concrete (> 41 MPa) as follows

$$E_c = 3.32\sqrt{f'_c} + 6.9 \tag{2}$$

The Turkish code TS 500 (2000) provides Eq. (3) for normal strength concrete

$$E_c = 3.25\sqrt{f'_c} + 14 \tag{3}$$

Wee *et al*. (1994) proposed the following relationship based on experimental data which has a range of cylinder strengths from 50 to 120 MPa

$$E_c = 10.2 \times (f'_c)^{1/3} \tag{4}$$

and Attard and Stewart (1998) suggested Eq. (5) based on statistical analysis of experimental data, which has a range of cylinder strength from 20 to 120 MPa and was reported by Setunge (1993).

$$E_c = 4.3703 \times (f'_c)^{0.5164} \tag{5}$$

In the above equations the units are in MPa and GPa for f'_c and E_c, respectively. Noguchi *et al*. (2007) also proposed a practical equation to calculate elastic modulus.

As we can see, different researchers and building codes (Li *et al*. 2007, ACI 318 2008, ACI 363 1984, TS 500 2000, Wee *et al*. 1994, Setunge 1993, Attard and Stewart 1998, Noguchi *et al*. 2007) proposed a number of empirical equations to predict the modulus of elasticity of concrete. Regression analysis techniques have been used to establish most of these relationships. Recently, some techniques in artificial neural networks, fuzzy systems, and evolutionary computation have been successfully combined, and new techniques called soft computing or computational intelligence have been developed (Zadeh 1965, Zadeh 1973, Mamdani and Assillan 1975, Takagi and Sugeno 1985, Castellano 2000, Wang 1994, Shafahi 2003). These techniques are attracting more attention in several research fields because they tolerate a wide range of uncertainty. Use of Neural Network and Fuzzy Systems has been developed in a range of civil engineering applications in recent decades (Castellano 2000, Wang 1994, Shafahi 2003). Neuro-fuzzy systems, Adaptive Neuro Fuzzy Inference System (ANFIS) in particular, are an effective practice for data processing in laboratory work (Lee 2003, Vahdani 2003, Shekarchizadeh *et al*. 2004, Demir 2005, Toprak 2009, Choi *et al*. 2009). But this system has been rarely employed for concrete.

Reinforced concrete buildings built with low-strength concrete (f'_c<10 MPa) are common in Turkey (Koru 2002, Bedirhanoglu 2009, Bedirhanoglu *et al*. 2010) as well as in other developing countries. On the other hand available formulas in the literature, especially the one given by TS 500 (2000), for predicting elastic modulus of concrete does not give reasonable results particularly for low strength concrete. This study was carried out to develop a simple and practical model based on NF logic to estimate the elastic modulus of concrete including different concrete compression strength levels (i.e., low, normal and high strength). However, the models in the existing studies do not include all strength levels. Another aim is to show the usability of NF model for civil engineering studies, i.e., evaluating experimental data.

2. Experimental data

The test data used in the proposed NF model includes a wide range of compression strengths varying from 6.6 to 126 MPa. It should be noted that by using the formulas in the mentioned literature, concrete's modulus of elasticity can usually be calculated for a certain range of compression strength. It was quite a challenge to collect the mentioned data. Some of the data, low strength concrete's in particular (f'_c≤15 MPa), was obtained from studies carried out at the Istanbul Technical University, Structural and Earthquake Eng. Lab. where the author has carried out extensive experimental work. It is important to use familiar and reliable data in the modeling process. The experimental data presented in Table 1 was used to develop the NF model while the experimental data presented in Table 2 was used to test the model. The data group includes experimental results those described in references (Oktar *et al*. 1984, Ozturan 1984, Cusson and Paultre 1997, Iravani 1996, Turan and Iren 1997, Yalcin 1997, Ilki 2000, Dahl 1992, Gesoglu *et al*. 2002, Sengul *et al*. 2002, Watanabe *et al*. 2004, Demir 2004, Altun *et al*. 2004, Ilki *et al*. 2004, Yilmaz 2004, Karamuk 2005, Akgun 2005). Due to the fact that concrete's modulus of elasticity in the proposed model was only calculated in terms of compression strength, the elastic modulus and compressive strength were taken from the data base for each data point. In some of the experimental studies the elastic modulus of each specimen was directly given by the researchers while in some of the experimental studies these values were obtained from the slope of the linear portion of the stress-strain curves given by researchers by using the procedure described in ACI 318 (2008). According to ACI 318 (2008), E_c is defined as the slope of the line drawn from a stress of zero to a compressive stress of $0.45f'_c$. In this study for some of the specimens given in Table 1 the elastic modulus was obtained from the stress-strain relationships per the slope of the line drawn from a compressive stress of 0.05 f'_c to a compressive stress of $0.40f'_c$ where the stress-strain relationship is linear. It is quite a challenge and a lot of work to digitize stress-strain curves and define the modulus of elasticity. The following relationship proposed by Lydon and Balendran (1986) was used to estimate the static elastic modulus (E_c) from the dynamic elastic modulus (E_d) of normal strength concrete given by Ozturan (1984).

$$E_c=0.83E_d \tag{6}$$

More details relating to the experimental data can be found in the related references.

Table 1 Training and checking data*

No	Researcher	f'_c	E_c (Exp.)	No	Researcher	f'_c	E_c (Exp.)
1	Akgun (2005)	6.9	14422	66	Turan *et al*. (1997)	22.1	21800
2	Akgun (2005)	7.5	14034	67	Ilki *et al*. (2004)	22.6	21150
3	Oktar *et al*. (1984)	8.2	13000	68	Ilki *et al*. (2004)	22.7	27800
4	Yilmaz (2004)	8.7	7324	69	Turan *et al*. (1997)	22.9	26500
5	Ilki *et al*. (2004)	9.1	14600	70	Sengul *et al*. (2002)	23.0	27700
6	Yilmaz (2004)	9.6	5826	71	Karamuk (2005)	23.1	27637
7	Ilki *et al*. (2004)	9.6	13400	72	Ozturan (1984)	23.2	23900
8	Oktar *et al*. (1984)	9.6	16500	73	Ilki *et al*. (2004)	23.3	17650
9	Ilki *et al*. (2004)	10.2	16300	74	Sengul *et al*. (2002)	23.3	25300
10	Oktar *et al*. (1984)	10.5	20500	75	Sengul *et al*. (2002)	23.4	31400
11	Ilki *et al*. (2004)	11.3	12500	76	Ilki *et al*. (2004)	23.5	18250
12	Yilmaz (2004)	11.9	7751	77	Ilki *et al*. (2004)	23.6	18500
13	Oktar *et al*. (1984)	12.0	29400	78	Ilki *et al*. (2004)	23.6	20950
14	Ilki *et al*. (2004)	12.3	13100	79	Ozturan (1984)	23.6	32100
15	Oktar *et al*. (1984)	12.4	16500	80	Turan *et al*. (1997)	23.7	27200
16	Yilmaz (2004)	12.6	11525	81	Ilki *et al*. (2004)	23.8	21900
17	Ilki *et al*. (2004)	12.9	15300	82	Sengul *et al*. (2002)	23.9	27500
18	Yilmaz (2004)	13.3	10405	83	Ozturan (1984)	23.9	30500
19	Yilmaz (2004)	13.9	10156	84	Ilki *et al*. (2004)	24.2	20100
20	Ilki *et al*. (2004)	13.9	16000	85	Ilki *et al*. (2004)	24.3	20700
21	Ilki *et al*. (2004)	14.4	17000	86	Karamuk (2005)	24.7	27795
22	Ilki *et al*. (2004)	14.4	18700	87	Turan *et al*. (1997)	25.3	28100
23	Oktar *et al*. (1984)	14.5	23000	88	Ilki *et al*. (2004)	25.3	18400
24	Ilki *et al*. (2004)	15.3	18000	89	Ilki *et al*. (2004)	25.3	20250
25	Karamuk (2005)	16.1	9311	90	Ozturan (1984)	25.8	28600
26	Ozturan (1984)	16.2	23300	91	Ilki *et al*. (2004)	25.9	22050
27	Ilki (2000)	16.4	15851	92	Turan *et al*. (1997)	26.1	24900
28	Ilki *et al*. (2004)	16.4	15850	93	Ilki *et al*. (2004)	26.3	18000
29	Ozturan (1984)	16.9	20500	94	Turan *et al*. (1997)	26.3	24000
30	Ilki (2000)	17.0	18775	95	Turan *et al*. (1997)	26.4	30000
31	Ozturan (1984)	17.1	26300	96	Turan *et al*. (1997)	26.4	26500
32	Oktar *et al*. (1984)	17.7	31200	97	Ilki *et al*. (2004)	26.9	20700
33	Ilki *et al*. (2004)	17.8	18450	98	Ozturan (1984)	27.1	24700
34	Ilki (2000)	17.8	18448	99	Turan *et al*. (1997)	27.1	23900
35	Ozturan (1984)	17.9	18000	100	Turan *et al*. (1997)	27.3	26500
36	Turan *et al* (1997)	18.4	21900	101	Turan *et al*. (1997)	27.4	27090
37	Ilki *et al*. (2004)	18.5	19850	102	Ilki *et al*. (2004)	27.5	27400
38	Ilki *et al*. (2004)	18.5	20650	103	Turan *et al*. (1997)	27.7	25600
39	Ozturan (1984)	18.5	30100	104	Ilki *et al*. (2004)	27.8	26951
40	Oktar *et al*. (1984)	18.6	27100	105	Turan *et al*. (1997)	27.8	29100
41	Ilki (2000)	18.8	18647	106	Turan *et al*. (1997)	27.8	26000
42	Ilki *et al*. (2004)	19.1	19400	107	Turan *et al*. (1997)	27.8	25300
43	Akgun (2005)	19.4	24108	108	Turan *et al*. (1997)	27.9	26200

Table 1 Continued

44	Ozturan (1984)	19.4	30300	109	Turan *et al*. (1997)	28.0	30800
45	Ozturan (1984)	19.6	23100	**110	Ilki (2000)	28.2	16922
46	Akgun (2005)	19.9	24421	223	Wee *et al*. (1994)	94.0	46300
47	Akgun (2005)	20.0	29472	224	Gesoglu *et al*. (2002)	94.0	48300
48	Turan *et al* (1997)	20.6	23900	225	Sengul *et al*. (2002)	94.5	46100
49	Altun *et al* (2004)	20.7	28250	226	Gesoglu *et al*. (2002)	95.2	50800
50	Altun *et al* (2004)	21.0	28400	227	Wee *et al*. (1994)	95.3	45200
51	Ilki *et al*. (2004)	21.1	18500	228	Sengul *et al*. (2002)	95.3	47400
52	Altun *et al*. (2004)	21.1	28400	229	Sengul *et al*. (2002)	95.3	51100
53	Ozturan (1984)	21.2	26500	230	Wee *et al*. (1994)	96.2	46600
54	Ilki (2000)	21.3	19911	231	Wee *et al*. (1994)	96.6	46500
55	Ilki *et al*. (2004)	21.4	19900	232	Gesoglu *et al*. (2002)	97.6	49300
56	Altun *et al* (2004)	21.4	29000	233	Gesoglu et. al. (2002)	97.7	47000
57	Oktar *et al*. (1984)	21.4	37000	234	Gesoglu *et al*. (2002)	99.7	47600
58	Ilki (2000)	21.6	18715	235	Gesoglu *et al*. (2002)	102.0	48800
59	Ilki *et al*. (2004)	21.6	18700	236	Wee *et al*. (1994)	102.1	46100
60	Ilki *et al*. (2004)	21.6	20050	237	Wee *et al*. (1994)	102.8	46700
61	Altun *et al* (2004)	21.6	29250	238	Wee *et al*. (1994)	104.2	46300
62	Ilki *et al*. (2004)	21.7	28400	239	Sengul *et al*. (2002)	104.5	50600
63	Ilki *et al*. (2004)	21.7	17250	240	Wee *et al*. (1994)	106.3	48400
64	Ozturan (1984)	21.8	20900	241	Wee *et al*. (1994)	119.9	49100
65	Ilki *et al*. (2004)	22.0	17750	242	Wee *et al*. (1994)	125.6	50900

*Note: All the units are in MPa.
**Note: Due to size limitation the data from 111 to 223 were removed from the table.

Table 2 Test data results*

No	Researcher	f'_c	E_c (Exp.)	E_c (NF)	E_c (Wee)	E_c (Attard and Setunge)	E_c (ACI 318 - ACI 363)	E_c (TS 500)
1	Akgun (2005)	7.2	13632	13383	19696	12113	12692	22721
2	Yilmaz (2004)	9.4	7452	14945	21519	13893	14494	23959
3	Oktar *et al*. (1984)	9.5	14000	15022	21603	13977	14579	24017
4	Karamuk (2005)	10.4	11745	15642	22254	14635	15244	24474
5	Yilmaz (2004)	13.1	9504	17465	24014	16467	17087	25741
6	Ilki *et al*. (2004)	13.7	9200	17874	24383	16860	17482	26012
7	Ilki *et al*. (2004)	13.8	10700	17947	24448	16930	17552	26060
8	Yilmaz (2004)	14.4	8596	18359	24810	17319	17943	26329
9	Ilki *et al*. (2004)	15.3	18650	18958	25322	17877	18501	26712
10	Ilki *et al*. (2004)	17.0	18800	20058	26227	18876	19502	27400
11	Ozturan (1984)	18.0	28800	20694	26732	19442	20068	27789
12	Ozturan (1984)	20.9	23900	22489	28096	21001	21624	28858
13	Oktar *et al*. (1984)	21.5	37000	22852	28363	21310	21932	29070
14	Altun et.al. (2004)	22.1	29500	23200	28615	21605	22226	29272
15	Ilki *et al*. (2004)	22.1	16150	23212	28624	21615	22236	29278
16	Ilki (2000)	22.6	21169	23530	28853	21884	22504	29462

Table 2 Contined

17	Ilki *et al*. (2004)	22.9	26300	23704	28978	22030	22650	29563
18	Turan *et al*. (1997)	23.4	26300	23986	29179	22267	22886	29725
19	Ozturan (1984)	24.2	33600	24446	29503	22652	23269	29988
20	Turan *et al*. (1997)	25.7	25700	25323	30112	23381	23993	30486
21	Ilki *et al*. (2004)	26.7	20650	25868	30486	23832	24441	30793
22	Oktar *et al*. (1984)	26.7	37400	25868	30486	23832	24441	30793
23	Turan *et al*. (1997)	27.0	27200	26035	30600	23970	24578	30887
24	Ozturan (1984)	27.3	32900	26201	30713	24107	24714	30981
25	Turan *et al*. (1997)	27.5	25200	26290	30773	24180	24786	31031
26	Ilki *et al*. (2004)	28.2	16900	26695	31047	24514	25118	31259
27	Turan *et al*. (1997)	29.4	33000	27339	31477	25043	25643	31619
28	Ozturan (1984)	30.3	35900	27822	31799	25441	26036	31890
29	Turan *et al*. (1997)	31.4	30400	28398	32179	25913	26505	32212
30	Sengul *et al*. (2002)	32.5	34400	28963	32551	26378	26965	32528
31	Ilki *et al*. (2004)	33.3	18300	29367	32816	26712	27295	32754
32	Ozturan (1984)	35.0	35600	30209	33365	27407	27983	33227
33	Sengul *et al*. (2002)	35.6	32400	30501	33554	27649	28222	33391
34	Oktar *et al*. (1984)	35.6	46500	30501	33554	27649	28222	33391
35	Ozturan (1984)	36.6	39300	30979	33866	28047	28616	33662
36	Ilki *et al*. (2004)	37.3	29500	31286	34065	28304	28869	33836
37	Sengul *et al*. (2002)	42.0	38000	33419	35455	30113	28416	35062
38	Yalcin (1997)	43.9	29129	34219	35982	30809	28897	35534
39	Ozturan (1984)	47.7	29600	35727	36992	32159	29830	36446
40	Cusson (1993)	55.7	45050	38496	38948	34832	31673	38250
41	Gesoglu *et al*. (2002)	62.2	45400	40362	40414	36883	33084	39632
42	Wee *et al*. (1994)	63.2	41800	40616	40629	37188	33293	39837
43	Wee *et al*. (1994)	65.8	40800	41237	41179	37970	33831	40363
44	Sengul *et al*. (2002)	66.3	43100	41979	41283	38119	33933	40463
45	Gesoglu *et al*. (2002)	66.5	46800	42400	41324	38178	33974	40503
46	Wee *et al*. (1994)	73.9	41600	43770	42804	40316	35440	41939
47	Gesoglu *et al*. (2002)	77.2	47100	44348	43432	41236	36071	42556
48	Sengul *et al*. (2002)	77.8	39600	44451	43544	41401	36184	42666
49	Gesoglu *et al*. (2002)	84.5	45300	45554	44760	43206	37419	43875
50	Wee *et al*. (1994)	84.8	47200	45602	44812	43285	37473	43928
51	Wee *et al*. (1994)	85.9	44300	45774	45005	43574	37671	44122
52	Gesoglu *et al*. (2002)	86.9	46100	45929	45179	43835	37849	44297
53	Gesoglu *et al*. (2002)	87.2	41100	45975	45231	43913	37902	44349
54	Gesoglu *et al*. (2002)	87.5	48500	46021	45283	43991	37956	44401
55	Wee *et al*. (1994)	87.6	44500	46036	45300	44017	37973	44418
56	Gesoglu *et al*. (2002)	87.9	43000	46082	45352	44095	38027	44470
57	Wee *et al*. (1994)	90.2	44400	46427	45744	44687	38431	44866
58	Wee *et al*. (1994)	91.7	46000	46646	45996	45069	38692	45122
59	Wee *et al*. (1994)	93.6	47100	46918	46312	45549	39020	45443
60	Gesoglu *et al*. (2002)	96.7	53200	47347	46818	46322	39548	45959
61	Wee *et al*. (1994)	100.6	45800	47861	47439	47277	40199	46597

*Note: All the units are in MPa.

3. Fuzzy inference system

Fuzzy set theory, introduced by Zadeh (1965, 1973) in 1965 and 1973, is a mathematical tool for translating abstract concepts found in natural language into computable entities. Such entities are called fuzzy sets. Fuzzy sets represent vague descriptions of objects, i.e., tall, small, cold, and bright. Mathematically, a fuzzy set A is represented by a membership function defined on a domain X, called the universe of discourse, given by

$$\mu_A : x \rightarrow [0,1] \tag{7}$$

in in which A is the fuzzy label or linguistic (value) term describing the variable x. As an extension to Boolean logic, $\mu_A(x)$ represents the grade of membership of x belonging to the fuzzy set A. It is clear that the definition of fuzzy sets is non-unique to the nature of language, but it is very context-dependent and user-specific. On specifying a membership function $\mu_A(x)$ in its present context the vague fuzzy label A is precisely defined. Hence fuzzy sets can be thought to be accurately measuring the inherent vagueness of language. The properties of these fuzzy sets play an important role in the modeling capabilities of the fuzzy system, and for a model to be truly transparent these sets should sensibly represent terms that describe the input and output variables. Fuzzy sets form a key methodology for representing and processing uncertainty. As such, fuzzy sets constitute a powerful approach not only to deal with incomplete, noisy or imprecise data but also to develop data models that provide better, more efficient and more refined results than traditional modeling techniques. A fuzzy system approximates an unknown mapping by inference from a set of humanly understandable statements or rules such as

IF temperature is cold THEN set output of the heater to high

IF temperature is warm THEN set output of the heater to zero (8)

describing a typical relationship between room temperature and the desired output of the heater (Fig. 1). To cover the complete graph of the mapping being approximated, a collection of such rules known as the rule base is utilized. Hence, fuzzy systems represent the imprecision found in real-world problems using IF-THEN rules expressed in a natural language.

The basic structure of a fuzzy system, as described by Mamdani and Assillan (1975), is shown in Fig. 2. Depending on the particular form of the consequent proposition in fuzzy rules, two categories of fuzzy systems can be identified, Mamdani and Assillan (1975) fuzzy systems and Takagi-Sugeno (TS) (1985) fuzzy systems (Castellano 2000). Takagi and Sugeno's fuzzy inference system has been successfully applied to many practical problems. The advantage of this fuzzy logic system is that it provides a compact system equation. As a result, parameter estimation and order determination methods such as NF algorithms, the most famous ANFIS, or neuro-adaptive

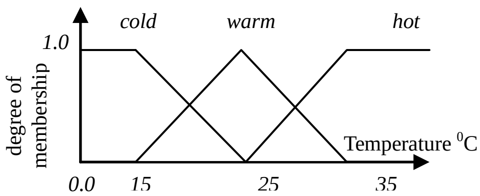

Fig. 1 Typical fuzzy sets defined for a variable

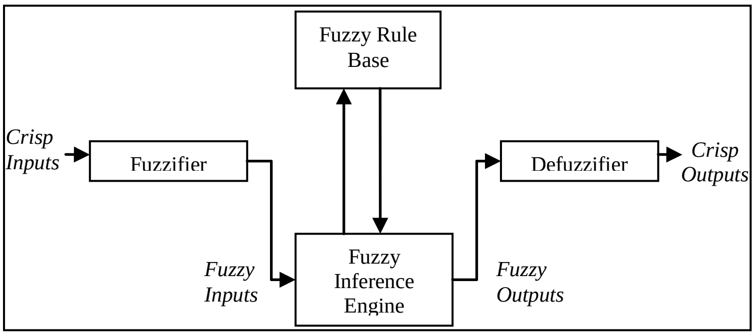

Fig. 2 The basic schema of a fuzzy system

learning techniques can be developed to estimate system parameters. These techniques provide a method for the fuzzy modeling procedure to learn information about a data set, in order to compute membership function parameters that best allow the associated fuzzy inference system to track given input/output data (Matlab, Wang 1994). In this study, the well known adaptive algorithm called ANFIS was used, with the aid of Matlab Fuzzy Logic Toolbox, to train a data set to determine the relationship between the compressive strength and elasticity modulus of unconfined concrete.

3.1 NF modeling

NF modeling refers to the way of applying various learning techniques developed in the neural network literature to fuzzy modeling or to a fuzzy inference system (FIS). NF hybrid systems combine the advantages of fuzzy systems, which deal with explainable and understandable explicit knowledge, and neural networks which deal with acquirable (through the learning procedure) implicit knowledge. Neural network learning provides a good way to adjust the expert's knowledge and automatically generate additional fuzzy rules and membership functions to meet certain specifications and reduce design time and costs. The details on adaptive networks are described by Jang (1993, 1995). In this section, a novel architecture and learning procedure is introduced for the FIS. FIS uses a neural network learning algorithm to generate a set of fuzzy if-then rules from the stipulated input–output pairs with appropriate membership functions (MFs). One of the procedures of developing a FIS through the framework of adaptive neural networks is known as the “Adaptive Neuro Fuzzy Inference system (ANFIS)” (Alturki and Abdennour 1999). As the name suggests, ANFIS combines the fuzzy qualitative approach with neural network adaptive capabilities to achieve a desired performance (Brown and Harris 1994).

3.2 ANFIS architecture

Adaptive Network-Based Fuzzy Inference Systems are fuzzy Sugeno models inserted into the framework of adaptive systems to facilitate learning and adaptation. This framework makes fuzzy models more systematic and less reliant on expert knowledge. In order to present the ANFIS architecture, a two-fuzzy rule will be considered based on a first order Sugeno model

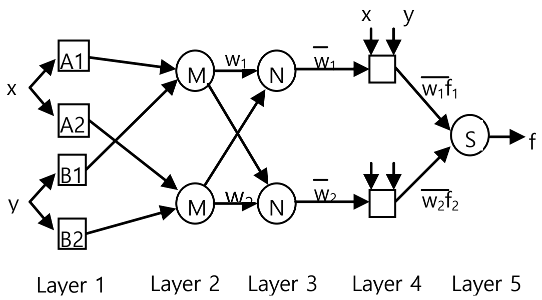

Fig. 3 Anfis architecture

$$\begin{aligned} &Rule1: if\ (x\ is\ A_1)\ and\ (y\ is\ B_1)\ then\ f_1 = p_1x + q_1y + r_1 \\ &Rule2: if\ (x\ is\ A_2)\ and\ (y\ is\ B_2)\ then\ f_2 = p_2x + q_2y + r_2 \end{aligned} \tag{9}$$

A probable ANFIS architecture implementation of these two rules is shown in Fig. 3. Note that circles indicate fixed nodes whereas squares indicate adaptive nodes (the parameters are changed during adaptation or training). In the following presentation, $O_{1,i}$ denotes the output of node i in layer 1.

Layer 1: All the nodes in this layer are adaptive nodes. The output of each node i is the membership degree of the input for the fuzzy membership function represented by the node

$$\begin{aligned} O_{1,i} &= \mu_{A_i}(x), i = 1, 2 \\ O_{1,i} &= \mu_{B_{i-2}}(x), i = 3, 4 \end{aligned} \tag{10}$$

Ai and *Bi* can be any appropriate fuzzy sets in parameter form. For example, if the Gaussian membership function (MF) is used then the membership degree can be derived as

$$\mu_{A_i}(x) = e^{-\left(\frac{x-c_i}{a_i}\right)^2} \tag{11}$$

in which a_i and c_i, are the parameters for the MF.

Layer 2: The nodes in this layer are fixed (not adaptive). They are labeled M to indicate that they play the role of a simple multiplier. The outputs of these nodes are given by

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(x) \quad i\text{=1, 2} \tag{12}$$

The output of each node in this layer represents the firing strength of the rule,

Layer 3: Nodes in this layer are also fixed nodes. They are labeled N to indicate that they perform a normalization of the firing strength from the previous layer. The output of each node is given by

$$O_{3,i} = \overline{w}_i = \frac{w_1}{w_1 + w_2} \qquad i\text{=1, 2} \tag{13}$$

Layer 4: All the nodes in this layer are adaptive nodes. The output of each node in this layer is simply the product of the normalized firing strength, in a first order polynomial form (for a first order Sugeno model)

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i) \qquad i=1, 2 \qquad (14)$$

in which, p_i, q_i and r_i are design parameters (referred to as consequent parameters since they deal with the then-part of the fuzzy rule).

Layer 5: This layer has only one node labeled S to indicate that it performs the function of a simple summer. The output of this single node is given by

$$O_{5,i} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \qquad i=1, 2 \qquad (15)$$

The ANFIS architecture is not unique. Some layers can be combined and still produce the same output. In this ANFIS architecture, there are two adaptive layers (Layers 1 and 4). Layer 1 has two modifiable parameters (a_i and c_i) pertaining to the input MFs. These parameters are called premise parameters. Layer 4 also has three modifiable parameters (p_i, q_i and r_i) pertaining to the first order polynomial. These parameters are called consequent parameters as mentioned earlier. The task of the training or learning algorithm for this architecture is to tune all the modifiable parameters to make the ANFIS output match the training data. If these parameters are fixed, the output of the network becomes

$$f = \frac{w_1}{w_1 + w_2} f_1 + \frac{w_1}{w_1 + w_2} f_2 = \bar{w}_1 f_1 + \bar{w}_2 f_2 = \bar{w}_1 (p_1 x + q_1 y + r) + \bar{w}_1 (p_2 x + q_2 y + r)$$

$$= (\bar{w}_1 x) p_1 + (\bar{w}_1 y) q_1 + (\bar{w}_1) r_1 + (\bar{w}_2 x) p_2 + (\bar{w}_2 y) q_2 + (\bar{w}_2) r_2 \qquad (16)$$

and this is a linear combination of the modifiable parameters. Therefore, a combination of gradient descent and the least-square method can easily identify the optimal values for the parameters p_i, q_i and r_i. However, if the MFs are not fixed and are allowed to vary, then the search space becomes larger and, consequently, the convergence of the training algorithm becomes slower (Vahdani 2003).

4. Construction of the NF model

In practice, there are always different sources of uncertainty such as vagueness and ambiguities and/or errors in calculating concrete’s modulus of elasticity. Fuzzy models can describe knowledge in a descriptive human-like manner in the form of simple rules using linguistic variables. This is the main advantage of fuzzy models. Once the fuzzy subsets of compressive strength and the linear equations for the modulus of elasticity are determined by ANFIS in Sugeno Type Fuzzy Systems, it is possible to estimate concrete’s modulus of elasticity from a given concrete strength. Triangular-type MFs were used for both the input and output to develop the model in the simplest and most practical form. It should be noted that special care has been made to ensure that the model is the simplest possible. At the beginning, between three to ten fuzzy sets were chosen for

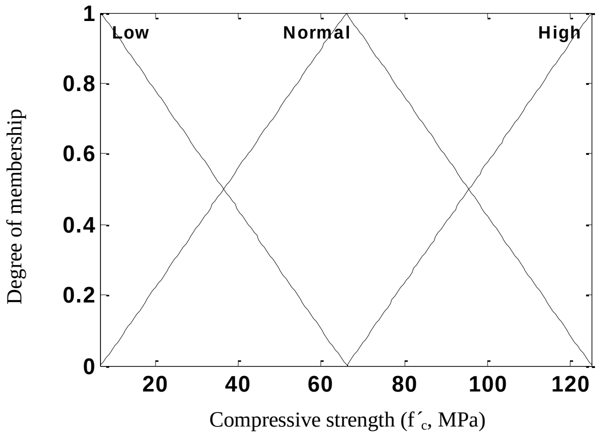

Fig. 4 Membership functions for compressive strength

Table 3 Fuzzy rules

Rules	Antecedent		Consequent
1	*IF f_c is LOW THEN*		y_1=4164*f_c−15567
2	*IF f_c is NORMAL*	THEN	y_2=3918*f_c−217817
3	*IF f_c is HIGH*		y_3=3855*f_c−433553

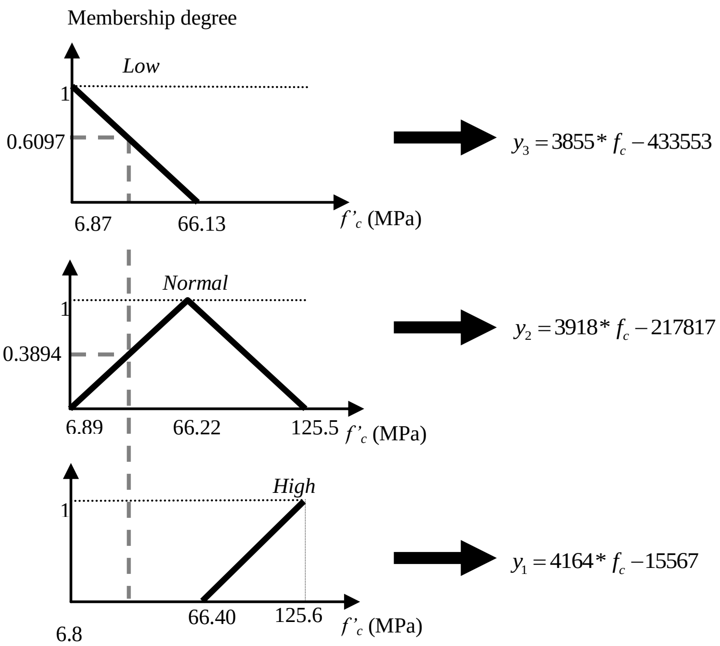

Fig. 5 Fired rules and corresponding membership degrees for f'_c=30 MPa

the input MFs. After training each model (to get the simplest one), the best model was selected corresponding to three fuzzy sets for the input MFs and three rules governing the system. The modeler’s experience was used, in addition to the training process, to determine the MFs, i.e., similar MFs were removed from the model in accordance with instructions from the modeler.

The resulting fuzzy subsets for the input (f'_c, MPa) are shown in Fig. 4. The available data was obtained from tests of 303 standard cylinder specimens randomly divided into two subgroups. 242 of them were used for model training, while the remainders (61 of 303) were used for model testing (validation). Rules relating to the proposed model concrete’s modulus of elasticity are shown in Table 3. These rules and the related fuzzy sets of MFs are shown in Fig. 5.

5. Flow chart

Elastic modulus can be calculated from concrete strength in three simple steps which are calculation of rules and membership degrees and finally elastic modulus. For better clarifying these steps (flow chart) were given as follow:

1—Rule outputs;

y_1=4164.7* $f_c^{'}$-15567

y_2=3918.4901* $f_c^{'}$-217817.3293

y_3=3855.8152 $f_c^{'}$-433553.9011

2—Membership degrees (Fig. 5) and result;

Mlow = (66.13- $f_c^{'}$)/(66.13-6.87) : Low membership function

Mnormali=($f_c^{'}$-6.899)/(66.22-6.899) : Increasing branch of normal membership function

Mnormald=(125.5- $f_c^{'}$)/(125.5-66.22) : Decreasing branch of normal membership function

Mhigh=($f_c^{'}$-66.4)/(125.6-66.24) : High membership function

3-Result, Ec

```
If    6.87< f'c &&   f'c <66.22
              Ec = (Mlow*y1+Mnormali*y2)/(Mlow+Mnormali)
else if 66.13<=fc && fc <=66.22
                Ec=(y1*Mnormali)/(Mnormali)
else if    66.22<=fc && fc <=66.4
               Ec=(y2*Mnormald)/(Mnormald)
else    if    fc>=66.4 && fc<=125.6
               Ec=(y3*MHigh+y2*Mnormald)/(MHigh+Mnormald)
end
              end
              end
```

Please note that this flow chart can be adapted in any computer program easily. It should also be note that this type step-by-step calculation has not been given in other studies on modeling of elastic modulus with fuzzy or NF approaches. Generally only information on the modeling process was given such as in references Demir (2005), Aydin *et al*. (2006). In reference Aydin *et al*. (2006)

two different models has been formed for normal and high strength concrete separately.

6. Design example

Concrete's modulus of elasticity can be calculated easily from concrete's compressive strength. For example, here is a set of step by step calculations for a given concrete strength:

1. Locate the concrete strength f'_c, say, for instance 30 MPa, on the horizontal axis of Fig. 5. It is possible to find the fuzzy subsets containing this value and the corresponding membership degrees by taking the intersection points. Fired rules and corresponding membership degrees were given in Fig. 5 in detail. For the current value, two rules fired (low and normal) and corresponding membership degrees are $\mu_{low}(30)=0.6097$, $\mu_{nomal}(30)=0.3894$, and $\mu_{high}(30)=0$. Zero membership degree shows that this rule does not fired. Compute the firing strength of each rule, w_i, corresponding to the degrees of membership (Fig. 5),

$w_1=\mu_{low}(30)=0.6097$
$w_2= \mu_{nomal}(30)=0.3894$
$w_3= \mu_{high}(30)=0$

2. Calculate the rules which were given in Table 3 (values of the consequent part), y_i

$y_1=4164* f_c^{'}-15567=109374$,

$y_2=3918* f_c^{'}-217817=-100262.62$,

$y_3=3855* f_c^{'}-433553=-317879$.

3. Finally, the weighted average (simply sum of membership degrees times the corresponding rules' values divided by sum of membership degrees) from Eq. (15) is obtained to produce the final output for concrete's modulus of elasticity with compressive strength of 30 MPa,

$$E_{c,30}=\frac{\sum_i w_i y_i}{\sum_i w_i}=\frac{0.6097*109374-0.3894*100262.62}{0.6097+0.3894}=27668 \text{ MPa.}$$

7. Performance evaluation of the NF model

The results of the NF model were compared with the experimental data and results of the other models as presented in Table 2. Fig. 6 shows the performance of the models in predicting the modulus of elasticity against the data from 303 standard cylinder specimens. As can be seen in this figure, the NF model matched the data well in all ranges of compressive strength. The scatter diagrams of the predicted and measured E_c for all the models are shown in Fig. 7 and the NF model clearly matches experimental data better than other models. In order to evaluate the performance of the generated NF model some statistical indexes were also used, namely the error measures root mean square error (RMSE), the relative error, and the average of the ratio of predicted to experimental results. Here, for the best model, the RMSE value should be the lowest, the relative error should have the smallest percentage and the average of ratio of predicted to experimental results should be the nearest to one. As can be seen in Table 4, the prediction accuracy of the NF model in terms of all indices is better for the validation data group than for the training data group, which indicates good generalization capabilities for the NF model.

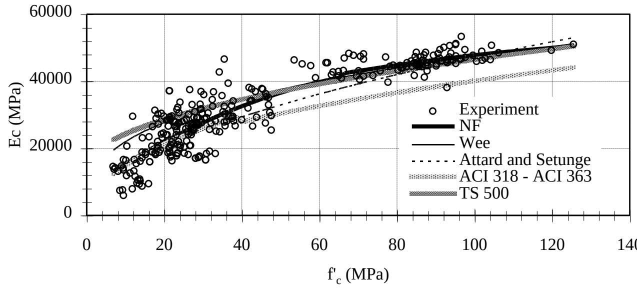

Fig. 6 Relationship between compressive strength and modulus of elasticity for all models

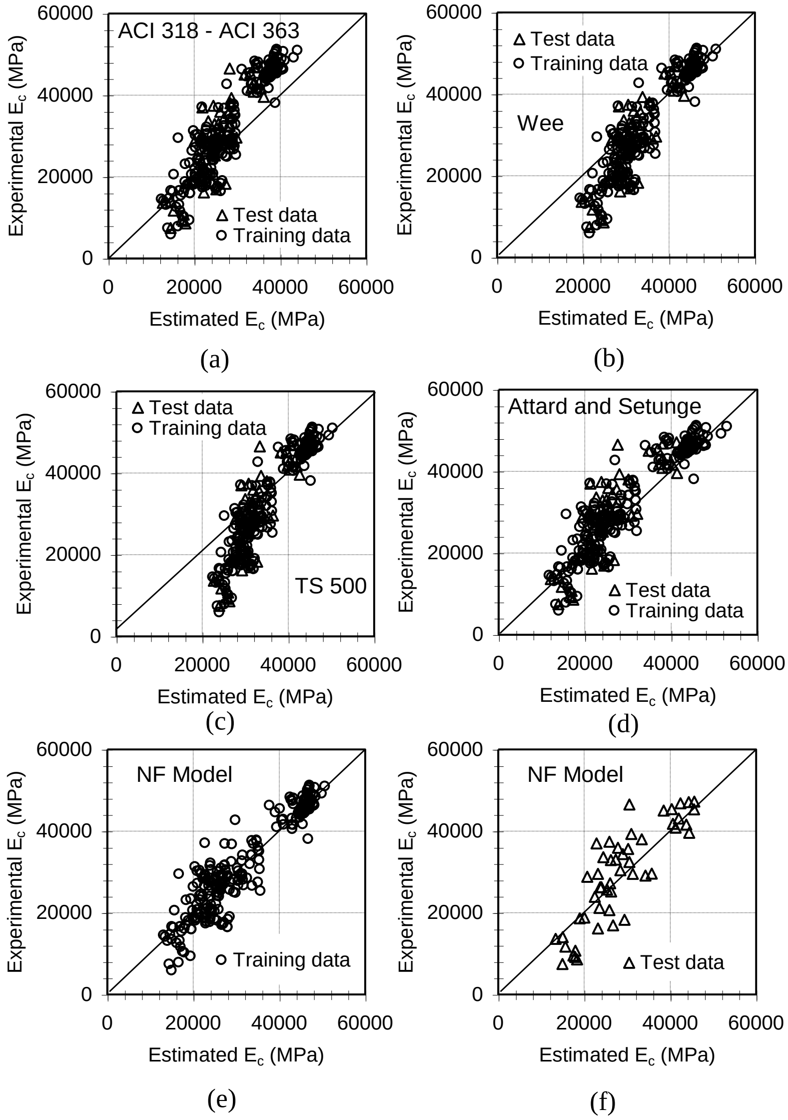

Fig. 7 Comparison of the predicted and experimental values of E_c for (a) ACI 318 and ACI 363 (b) Wee *at al*. (c) TS 500 (d) Attard and Setunge (e) NF model in training data (f) NF model in test data

Table 4 Evaluation the performance of the models

Models	Performance of the model					
	Training and check data			Test data		
	Relative error	Average	RMSE	Relative error	Average	RMSE
NF	0.143	1.051	4.360	0.201	1.064	5.783
Wee	0.240	1.208	6.173	0.302	1.228	6.935
Attard and Setunge	0.146	0.980	4.784	0.207	0.991	6.573
ACI 318 - ACI 363	0.185	0.955	6.409	0.245	0.963	7.895
TS 500	0.266	1.228	6.693	0.333	1.253	7.539

8. Conclusions

In this study, a simple NF Model was derived for predicting concrete's modulus of elasticity and verified with experimental results. The well known learning algorithm, ANFIS, was used to obtain the model parameters. In order to evaluate the performance of the proposed model, various statistical evolutionary criteria were used. Comparison with the experimental data and other models available in the literature shows that concrete's modulus of elasticity can be more accurately estimated from compressive strength of concrete with the developed NF model. In addition, unlike other fuzzy models the developed NF model can be easily adapted in any structural analysis process (i.e. finite element analysis and sectional analysis to obtain moment-curvature relationship) to estimate concrete's modulus of elasticity. It should be noted that one of the greatest obstacles of such models developed by using fuzzy logic, neural network or neuro-fuzzy approaches is the difficulty to put the model into practical use for engineering purposes.

Many parameters that may affect concrete's modulus of elasticity, i.e. aggregate type, unit weight, cement type, water-cement ratio etc., can easily be taken into account by adding related rules to the corresponding NF model. It should be noted that the model can easily be calibrated and modified by adding more data points. In addition, by implementing experimental data the model can be modified for similar uses in different areas, i.e. estimating the elastic modulus of steel fiber reinforced concrete or FRP (fiber reinforced polymer) confined concrete.

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