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Contact forces generated by fallen debris

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Abstract. Expressions for determining the value of the impact force as reported in the literature and incorporated into code provisions are essentially quasi-static forces for emulating deflection. Quasi-static forces are not to be confused with contact force which is generated in the vicinity of the point of contact between the impactor and target, and contact force is responsible for damage featuring perforation and denting. The distinction between the two types of forces in the context of impact actions is not widely understood and few guidelines have been developed for their estimation. The value of the contact force can be many times higher than that of the quasi-static force and lasts for a matter of a few milli-seconds whereas the deflection of the target can evolve over a much longer time span. The stiffer the impactor the shorter the period of time to deliver the impulsive action onto the target and consequently the higher the peak value of the contact force. This phenomenon is not taken into account by any contemporary codified method of modelling impact actions which are mostly based on the considerations of momentum and energy principles. Computer software such as LS-DYNA has the capability of predicting contact force but the dynamic stiffness parameters of the impactor material which is required for input into the program has not been documented for debris materials. The alternative, direct, approach for an accurate evaluation of the damage potential of an impact scenario is by physical experimentation. However, it can be difficult to extrapolate observations from laboratory testings to behaviour in real scenarios when the underlying principles have not been established. Contact force is also difficult to measure. Thus, the amount of useful information that can be retrieved from isolated impact experiments to guide design and to quantify risk is very limited. In this paper, practical methods for estimating the amount of contact force that can be generated by the impact of a fallen debris object are introduced along with the governing principles. An experimental-calibration procedure forming part of the assessment procedure has also been verified.

Keywords: quasi-static force; contact force; material dynamic stiffness; impact action; fallen debris

1. Introduction

In traditional engineering practices, impact actions are typically represented by equivalent (quasi- static) forces. This simple format of quantifying impact actions is convenient for structural design purposes. Obviously, the actual amount of force generated by an impact cannot be determined by considering the moving object alone given that it is also dependent on the interaction between the impactor and the target. Nonetheless, the equivalent static force approach is generally accepted as a viable way of ensuring that the barrier (considered herein as "target")

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fulfils the requirements for containment and serves the intended engineering purposes. Limitations of quasi-static force provisions have never been well explained. Consequently, many engineers are not confident in making estimates of impact actions unless specific stipulations are provided by a design code of practices.

The alternative equal energy method is easy to comprehend and appears to be versatile. Expression of the following form as presented in Annex C of Eurocode 1 (2008) for horizontal "hard impact" scenarios is based on equating kinetic energy to strain energy

$$\Delta = \frac{mv_o}{\sqrt{mk}} \tag{1a}$$

The maximum deflection (Δ) of a structural/architectural element such as a column, mullion or cladding, when subject to the impact of a flying object can also be estimated using this expression.

Eq. (1a) can be re-arranged into Eq. (1b) for the determination of the amount of quasi-static force (F_s) that can be applied to the element for emulating the impact generated deflection.

$$F_s = v_o\sqrt{mk} \tag{1b}$$

where m and v_o is the mass and incident velocity of the impactor; k is the stiffness of the element assuming linear elastic behaviour.

In a recent study undertaken by the authors Eq. (1a) has been modified into the following expressions to take into account the mitigating effects of energy losses on impact (Ali *et al.*)

$$\Delta = \frac{mv_o}{\sqrt{mk(1+\alpha)}} \quad \text{for no re-bounce following the impact} \tag{2a}$$

$$\Delta = \frac{mv_o}{\sqrt{mk(1+\alpha)}}\sqrt{\frac{4\alpha}{(1+\alpha)}} \quad \text{for perfect re-bounce following the impact} \tag{2b}$$

where αm is the equivalent (generalised) mass of the element being struck assuming no significant contributions by the higher modes which can result in a distinctive dynamic deflection profile. α is the mass ratio which is defined as the generalised mass of the target divided by the mass of the impactor.

Eqs. (2a)-(2b) have been modified further into the following expression to take into account the effects of contributions by gravity in scenarios of impact by a heavy fallen object

$$\Delta = \Delta_s + \sqrt{\Delta_s^2 + \delta^2} \tag{3}$$

where Δ_s is static deflection resulted purely from gravity (i.e., $\Delta_s = mg/k$); and δ takes the value of Δ as calculated from Eqs. (2a)-(2b).

The interesting size effects that are implied by these expressions are presented in Fig. 1 for simply-supported beams. Take the example of a simply-supported beam which has a generalised mass equal to double the mass of the impactor (i.e., α=2). As the span length of the beam is increased by a factor of 2 (i.e., increasing α from 2 to 4) the magnitude of the reaction force is reduced to 27% the value of the original span length. The bending moment is accordingly reduced to 54% the original value despite having doubled the span length. The deflection at mid-span of

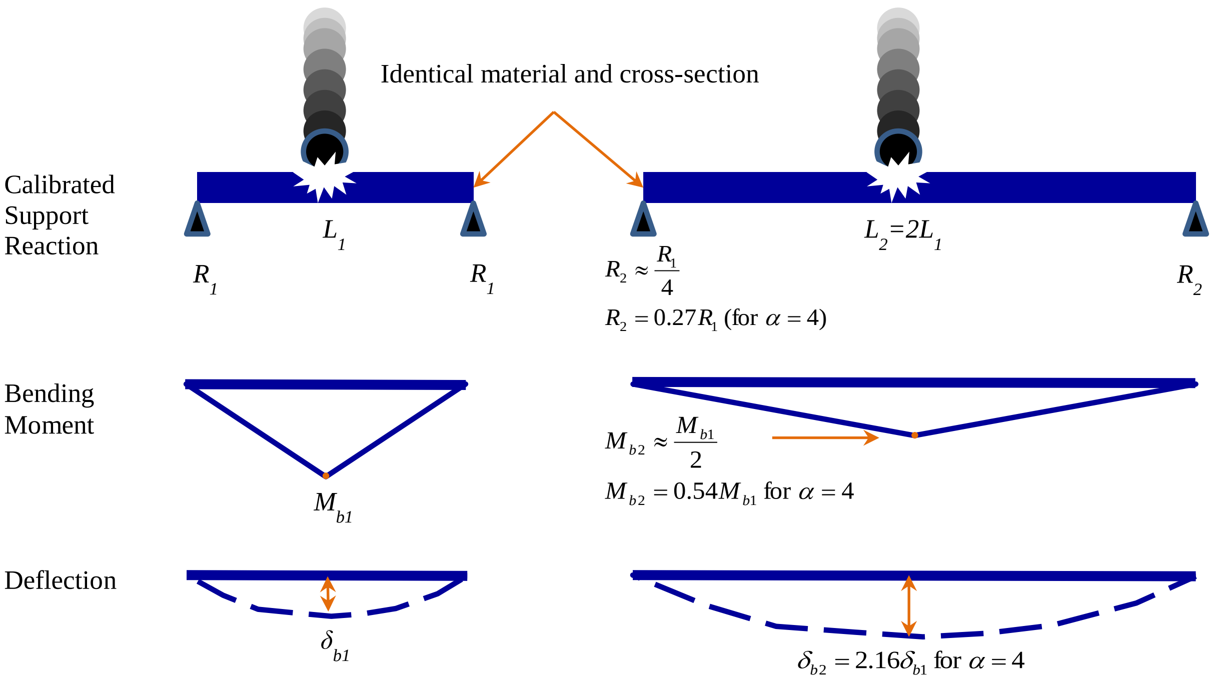


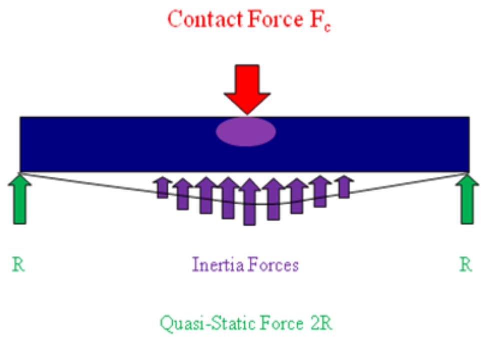
Fig. 1 Scaling relationships for quasi-static impact actions on beams

the beam is increased to 2.16 times the original value. These figures may appear counter-intuitive when interpreted from restrictive force-based principles. The same set of expressions can be used for analysing impact scenarios involving smaller projectiles (i.e., with higher values of α). It is important to review existing risks models to determine if this important trend has been captured to ensure that resources are effectively deployed to address components which are truly vulnerable.

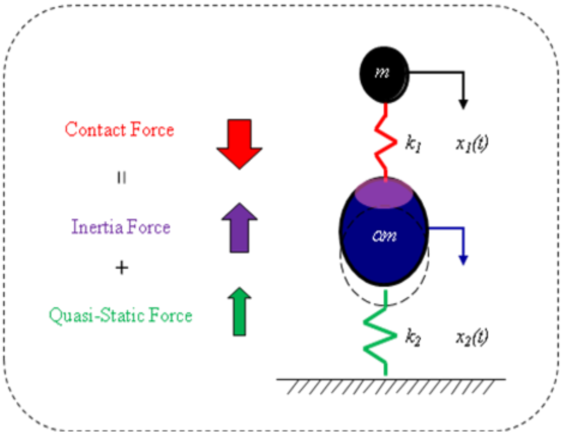
It is noted that all the equations presented in the above are based on the static representation of the impact action wherein the *quasi-static* force is simply used as part of a calculation technique for estimating the maximum deflection of the targeted element. These quasi-static forces are not to be confused with the *impact* force which is generated in the vicinity of the point of contact between the impactor and target, and is responsible for damage featuring perforation and denting. The distinction between the two types of forces in the context of impact actions is not widely understood and few guidelines have been developed for their estimation.

2. Contact forces

The amount of quasi-static force generated by the impact of a dropped object on a simply-supported beam may be taken as the sum of the support reactions (i.e., 2R) and can be exceeded significantly by that of the contact force because of contributions from the inertial resistance of the target (Fig. 2(a)). The two types of forces are best illustrated by a *two-degree-of-freedom* (2DOF) system model comprising two spring-connected lumped masses (Fig. 2(b)). Computer algorithms for simulating the response of the 2DOF model can be found in Lam *et al.* (2010, 2011). The



(a) Impact actions on beam



(b) Spring connected lumped masses

Fig. 2 Contact force versus quasi-static actions

frontal lumped mass represents the impactor whereas the second lumped mass at the rear represents the targeted element. Values of the lumped mass (αm) representing the target and the stiffness of the supporting spring (k_2) at the rear have been derived by the authors in previous publications for beams and plates (Yang *et al.* 2012a, b). The reaction force is linearly correlated with the displacement demand and the shear forces and bending moments, of the beam (or plate) as a whole. In contrasts, the contact force is associated with localised stresses surrounding the point of contact and is responsible for the risks of localised phenomena such as failure by denting, local crushing or perforation.

Importantly, the magnitude of the contact force can vary greatly even when values of the impact parameters encapsulated in Eqs. (1)-(3) are all held constant. The value of the contact force can be many times higher than that of the quasi-static force and lasts for a matter of only a few milli-seconds whereas the deflection of the target can evolve over a time span of tens, or hundreds, of milli-seconds depending on the natural period of vibration of the targeted element. The stiffer the impactor the shorter the period of time to deliver the impulsive action onto the target and consequently the higher the peak value of the contact force. None of the equations presented so far in this paper is able to model this phenomenon.

Because of anomalies associated with the generation of the contact force some impact scenarios on slender targets featuring denting, or perforation, is not well represented by any quasi-static force model (which only emulates flexural stresses and deflection) nor by any codified model which is amenable to hand calculations. Computer software such as LS-DYNA has the capability of accurately predicting contact force provided that the material properties of both the impactor and the surface of the target have been well represented in the computer model (e.g., Nguyen *et al.* 2005, Heimbs *et al.* 2009). Thus, the dynamic stiffness parameters of the impactor material will need to be input into the program. In addition, the variability of relevant material properties would also need to be known to cover for uncertainties. Clearly, this type of information has not been documented for debris materials which are the most common form of impactor in accident and natural disaster scenarios.

The alternative, direct, approach for an accurate evaluation of the damage potential of an impact scenario is by physical experimentation. In a *quasi-static* experiment the test load is typically applied in increments up to the limit of ultimate failure in order that the strength capacity of the component can be determined readily along with the load-deflection behaviour. Thus, plenty of information can be retrieved from a *quasi-static* experiment. However, this is not possible with an impact experiment wherein the *forcing function* to be applied is not defined *in priori* given that much depends on the interaction of the impactor with the target when contact is made at the termination of a free fall.

Whilst maximum deflection of the target is always measured in an impact experiment the amount of maximum contact force developed at the point of contact during the course of the impact is often not measured (as is difficult to do so without interfering with the impact). What can be found from a typical impact experiment is whether the specimen fails, or remains intact, in a designated impact scenario. The only other observation that can be made are the amount of damage that has been inflicted onto the specimen. Whist experimentations involving the use of the gas gun is common and test data is abundant the amount of potentially useful information that can be retrieved from those experiments to guide design and to quantify risk is very limited. Experimental data on contact force in particular is very scarce because of challenges with taking representative measurements. Forces are normally measured by load cells in physical experiments (e.g., Sjoblom *et al*. 1988, Zineddin *et al*. 2007). However, placing the load cell in front of the target for the direct measurement of the contact force can result in damage to the instrument following repetitive testings because of the abrasive nature of impact actions. Any attempt to protect the load cell such as the use of a shield, or cushion, would only compromise the accuracies of the measurements if the actual impact to be modelled is without any protection. Placing the load cell behind the target would only measure the reaction force, and not the contact force.

Many of the unknowns stem from the lack of knowledge on contact force and the correlation of its values with parameters characterising the impactor and the target. Consequently, it is very difficult to predict the risk of damage to a component that is exposed to certain impact hazards and to quantify uncertainties.

The objective of the paper is to introduce inexpensive methodologies for estimating the dynamic compressive stiffness properties of an impactor material in order that the value of the contact force can be estimated with a reasonable degree of accuracies. The rationale of the proposed experimental-calibration procedure is first explained (Section 3). Illustrations and verifications of the proposed procedure for estimating contact force using a cricket ball as an example impactor is next presented (Section 4). A cricket ball was chosen in view of its size in order that confirmatory impact tests could be performed by the authors on a custom made (drop tube) apparatus to measure contact forces. The choice of a commonly available spherical object such as a cricket ball enables results presented in this paper to be checked independently. The authors are primarily interested in the amount of contact force generated by the impact of a solid object and NOT on the motion behaviour of the cricket ball or any other types of sport balls in a game. The application of the method is illustrated herein by the example of a piece of concrete debris hitting the surface of a concrete slab (Section 5). Finally, recommendations are made for day-to-day engineering applications (Section 6).

3. Determination of stiffness parameters and contact force

The *contact force – displacement* (F_c - δ) relationship is expressed in the following form

$$F_c = k\delta^P \tag{4}$$

where displacement (δ) is defined as the amount of movement of the centre of the impactor object resulted from both the “squashing” of the object and “indentation” into the surface of the target; and k and P are coefficients to be determined by calibration.

This F_c-δ behaviour is represented by the *frontal* spring in the 2DOF spring connected lumped mass system and is not to be confused with the behaviour of the *rear* spring which is in support of the target lumped mass.

If the values of k and P are known then the maximum value of the displacement (δ) can be estimated using the following equation which is expressed in terms of the basic impact parameters (m and v_o)

$$\frac{1}{2}mv_o^2 = \int_0^{\delta_o} F_c.d\delta = \int_0^{\delta_o} k\delta^P.d\delta = \frac{k}{P+1}\delta_o^{P+1} \tag{5a}$$

$$\delta = \left(\frac{P+1}{2k}mv_o^2\right)^{\frac{1}{P+1}} \tag{5b}$$

Eqs. (5a)-(5b) are based on the assumption that no energy is dissipated in the loading phase of the impact (when the impactor is compressed). Mitigating effects which are derived from interactions between the impactor and the target have also been ignored. In spite of these assumptions reasonable results can be obtained from these equations in a typical scenario where the duration of contact is an order of magnitude shorter than the time taken by the target to displace.

The value of F_c is accordingly defined by the following expression based on substituting Eq. (5b) into Eq. (4)

$$F_c = k\left(\frac{P+1}{2k}mv_o^2\right)^{\frac{P}{P+1}} \tag{5c}$$

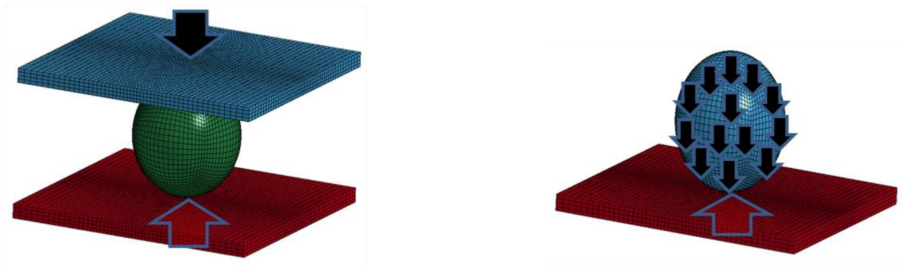
If the value of the material constants namely the Young’s Modulus (E) and Poisson’s ratio (ν) are known then the following expressions which are well established in the field of contact mechanics may be used for estimating the value of k

$$k = \frac{4}{3}E\sqrt{R} \tag{6a}$$

$$\frac{1}{E} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \tag{6b}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{6c}$$

where E_1 and ν_1 and E_2 and ν_2 are the *Young’s* Modulus and *Poisson’s* ratio of the impactor and



(a) Model 01 (b) Model 02

Fig. 3 Finite element models for calibrations

target material respectively; and R_1 and R_2 are their respective radius of curvature (i.e., $1/R_2=0$ if surface of the target is flat).

In situations where these parameter values are unknown the F_c-δ relationship would need to be determined experimentally. The objective of this paper is to introduce such an experimental calibration procedure which involves only quasi-static testings. It is noted that a conventional loading rig commonly employed for quasi-static testing would not reproduce the boundary conditions of a fallen object when impacting on the surface of a floor slab at the end of a free fall. Thus, results from a quasi-static test would need to be modified in order to be representative of an impact scenario.

First, a finite element (FE) model of the impactor and the loading platens would need to be developed in LS-DYNA software package. Values of the E and ν parameters in the FE model are then calibrated to ensure that the simulated and recorded F_c-δ relationship match (Fig. 3(a)). Second, the calibrated FE model is modified to have the upper platen removed in order that inertia forces (that are assumed to be uniformly distributed within the impactor object) are in equilibrium with the reaction force applied from the lower platen (Fig. 3(b)). The boundary condition of the impact is then emulated by this modified calibrated model. The F_c-δ relationship so simulated by this model is then curve-fitted using the functional form of Eq. (4) for determination of the values of parameters P and k. Given these unknowns the maximum value of the contact force (F_c) can be found using Eq. (5c). Details of this calibration procedure are illustrated in the next section using cricket ball as an example impactor material.

4. Illustration and verification of calibration procedure

4.1 Modelling the cricket ball

The composition of the cricket ball is quite complex as it is made up of a central rubber core, layers of wool packing and a stitched leather cover. A 3D finite element model which was used in the study to represent the compressive properties of the cricket ball was made up of *constant stress eight-node hexahedral solid elements* which were configured into a spherical object of diameter equal to 72.5 mm. Different classes of material models have been described in the literature for modelling the compressive behaviour of spherical objects. The simplest model to adopt is a homogeneous linear elastic material model as used in a previous study by Crisco *et al.* (1997) for modelling the behaviour of a base ball which is similar in structure to that of a cricket ball. Such a

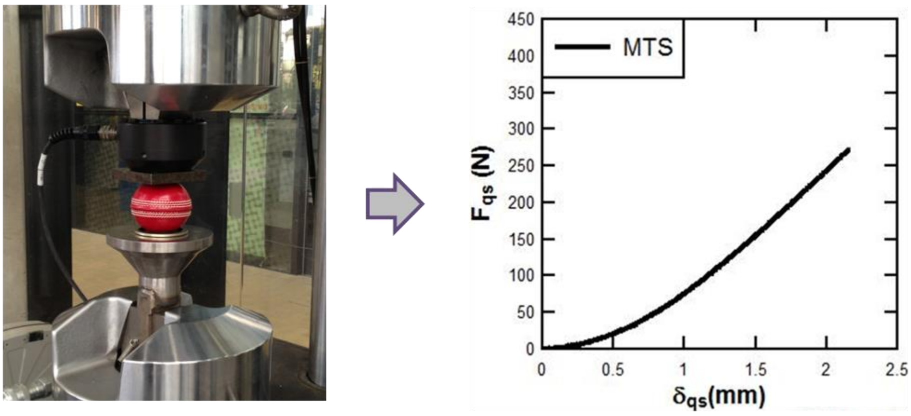


Fig. 4 Quasi-static test on cricket ball for determination of F_c-δ relationship

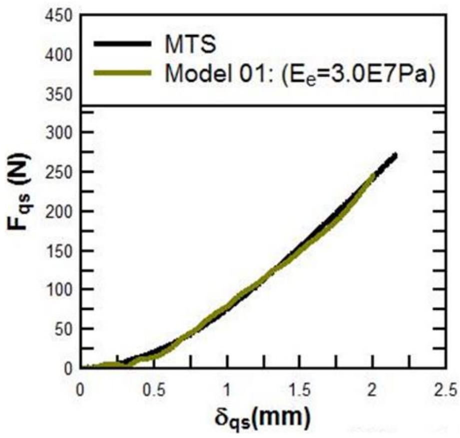


Fig. 5 Calibration of FE model to achieve a good match with experimental results

simplified model can be characterised fully by the value of the Young's Modulus (E), Poisson's ratio (ν=0.3) and density (ρ=781.8 kg/m^3). The value of E (as if the ball was made of a homogenous material) was determined by calibration. The simple assumption of homogeneity, and the adoption of linear elastic material properties, means that the model is only valid if the ball is not compressed by more than 10% of its diameter. More complex material models namely linear visco-elastic models (e.g., Smith 2001, Nicholls *et al.* 2006) and multi-layered finite element models involving the use of different materials have also been employed (e.g., Cheng *et al.* 2011).

4.2 Modelling the loading platens

The upper and lower platens, each measuring 100 mm×100 mm×10 mm thick can both be modelled as linear elastic material with Young's Modulus (E=200 GPa), Poisson's ratio (ν=0.3) and density (ρ=7850 kg/m^3). An automatic surface contact algorithm was specified to define the interaction between the impactor (the cricket ball) and the loading platens. An hourglass control type (IHQ=3) and QM hourglass coefficient of 0.01 was also specified based on recommendations by Day (2006).

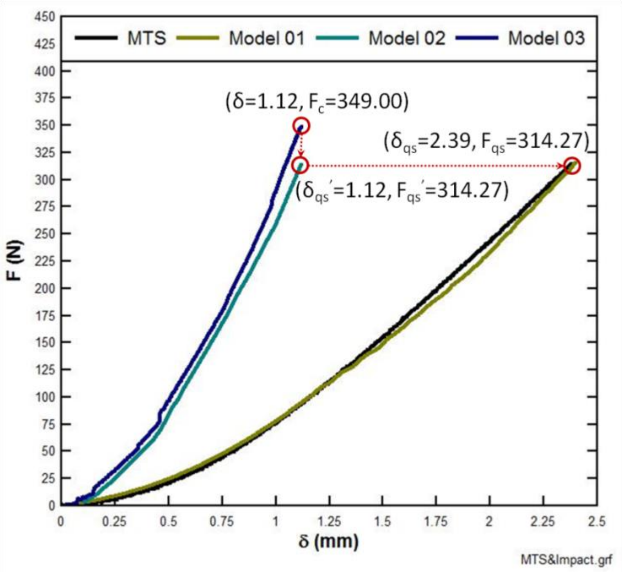


Fig. 6 F_c-δ relationships from calibrated *Model 01*, *Model 02* and *Model 03*

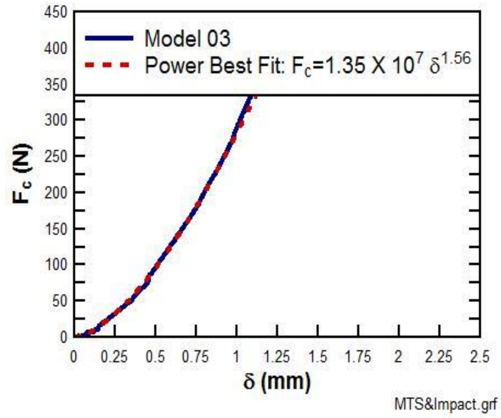


Fig. 7 Curve-fitting of simulated F_c-δ relationship for cricket ball

4.3 Calibration of stiffness parameters

The cricket ball was first tested on a MTS machine which applied a load of up to 300 N with a 5 mm/min loading velocity to result in a 2 mm compression as shown in Fig. 4. The F_c-δ relationship was then simulated by a finite element model for comparison with the recorded relationship. To achieve a good match of the simulated with the recorded F_c-δ relationships the value of E characterising the compressive behaviour of the cricket ball was calibrated to a value of 30 GPa (Fig. 5). Such a set-up is denoted as *Model 01*.

Next, the calibrated FE model is modified to have the reaction force from the upper platen replaced by inertia forces that are applied uniformly on the cricket ball (Fig. 3(b)). This modified calibrated model is denoted as *Model 02*. Both *Model 01* and *Model 02* are based on quasi-static conditions. With the third model (*Model 03*) an incident velocity was specified as the initial condition at the start of the analysis to simulate an impact action. The boundary condition of *Model 03* is identical to that of *Model 02* except that *Model 03* is a dynamic model simulating the impact action. In summary, there are a multitude of reasons causing the differences between quasi-static test results and real behaviour in dynamic conditions, and this includes boundary conditions *and* distribution of inertial force resistance within the impactor. Thus, a configuration of three models (1, 2 and 3) is introduced herein to isolate these individual factors.

The F_c-δ relationships associated with *Model 01*, *Model 02* and *Model 03* are presented in Fig. 6. It is shown that the change in boundary conditions from *Model 01* to *Model 02* has resulted in a significant increase in stiffness of the impactor by a factor of 2.1 (i.e., 2.39/1.12) which can be described as the *boundary factor* (F_B). The change from *Model 02* (quasi-static) to *Model 03* (dynamic) conditions has only resulted in a modest increase in stiffness by a factor of 1.1 (i.e., 349/314) which can be described as the *dynamic factor* (F_D). The total increase in stiffness from *Model 01* to *Model 03* is 2.3 (i.e., 2.1×1.1).

Table 1 Contact force and compression of the cricket ball

Impacting Velocity (m/s)	Indentation (mm)	Contact Force (N)
1.4	1.1	350
3.1	2.1	920
4.3	2.7	1370
6.0	3.6	2050

Both the F_B and F_D factors have values exceeding unity meaning that the value of the compressive stiffness of the impactor in dynamic conditions is higher than that measured in a *quasi-static* experiment. The difference in the boundary conditions is a major contributory factor which is represented by F_B. The much higher rate of loading in dynamic condition is another factor which is represented by F_D. For incident velocity of up to 50 m/s the boundary condition is usually the dominating factor.

The F_c - δ relationship so obtained from *Model 03* was curve-fitted using the functional form of Eq. (4) for determination of the values of parameters P and k (Fig. 7). It was found that for the cricket ball k=13.5×10^6 N/m^P and P=1.56.

4.4 Simulation of contact force

Substitution of the value of k=13.5×10^6 N/m^P and P=1.56 into Eqs. (5b)-(5c) results in Eqs. (7a)-(7b) respectively.

$$\delta = 8.74\times10^{-4} v_o^{\,0.78} \quad \text{(units in m and seconds)} \tag{7a}$$

$$F_c = 230 v_o^{\,1.22} \quad \text{(units in N, m and seconds)} \tag{7b}$$

The amount of compression and contact force generated by the impact of the cricket ball can be found using Eqs. (7a)-(7b) respectively for any given value of the incident velocity. Values predicted for an incident velocity of 1.4 m/s, 3.1 m/s, 4.3 m/s and 6.0 m/s can be found using the listed relationships (refer Table 1). The term “indentation” used herein refers to the sum of displacements resulted from the compression of the cricket ball and the actual indentation into the surface of the target.

4.5 Experimental verification of simulated results

As explained earlier, the reason for choosing a cricket ball as an example impactor is that the simulated forces as listed in Table 1 can be compared with results obtained from miniature experimentations which were conducted by the authors using a custom made device that is designed to measure the contact force. Agreement of the estimated with the experimentally measured values gives credibility of the proposed method of estimation.

The custom made tubular device is essentially made up of a spring connected dummy lumped mass as shown in Fig. 8. The dummy mass (1.57kg) is made of mild steel which has Young’s Modulus of 210 GPa and density of 7850 kg/m^3 approximately. It has a diameter of 77 mm and thickness of 43 mm. A clear PVC pipe with diameter of 80 mm is used to guide the impactor (the

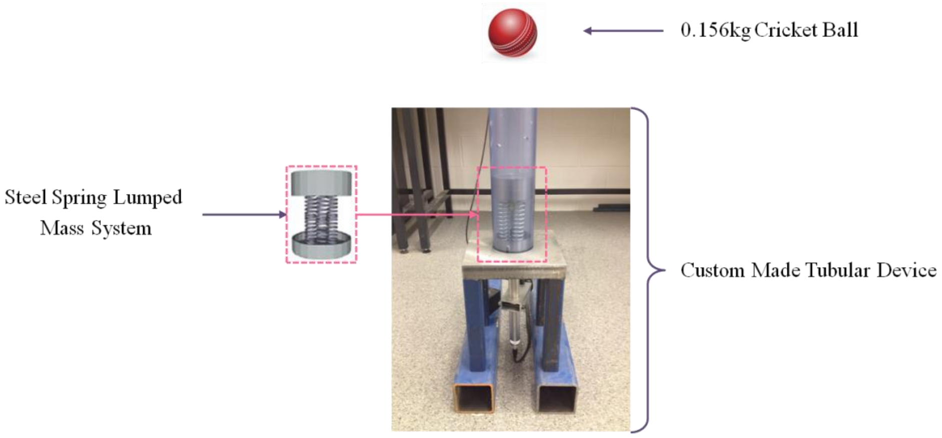


Fig. 8 Custom made tubular device for measuring contact forces

cricket ball) towards the target. Holes have been drilled on the pipe wall in order to partially relieve aerodynamic drag on the falling object. The supporting spring has a total stiffness value of 23.4 kN/m. A strain gauge linear displacement transducer (LDT) with range of 100 mm was mounted at the base of the target to measure the amount of retraction of the rear spring. Data from the displacement transducer was read by a high speed data logger and then displayed on the computer screen. Essentially, the tubular device is to simulate the 2DOF lumped mass system as depicted in Fig. 2. Full details of the device and its use in measuring contact forces generated by the dropping of spherical objects can be found in Yang (2013). A brief description of the operational principles of the device is given below.

The two spring connected lumped masses in the 2DOF system model are represented physically by (i) the object dropping through the tube (the *impactor*) and (ii) a rigid dummy lumped mass (the *target*) which is placed at the end of the tube and supported by a coil spring at the base (the *rear spring*). The magnitude of the reaction force ($F_{reaction}$) was simply taken as the product of the stiffness value of the *rear spring* (k_2) and the amount of spring shortening, $x_2(t)$, whereas the magnitude of the inertia force was taken as product of mass, αm, and acceleration, $\ddot{x}_2(t)$, of the target. By principles of vertical dynamic equilibrium of forces the following equation was used for determining the magnitude of the contact force ($F_{contact}$).

$$F_{contact} = F_{reaction} + F_{\text{Inertia}} = k_2 x_2(t) + (\alpha m)\ddot{x}_2(t) \tag{8}$$

If the time-history of $x_2(t)$ has been digitised accurately at time intervals that were sufficiently closely spaced, its double-differential with respect to time $\ddot{x}_2(t)$ could be calculated with good accuracies. The time-history of the contact force is hence obtained using Eq. (8). The amount of indentation can also be found using Eq. (4).

A cricket ball was dropped at different heights to achieve incident velocities of 3.1 m/s, 4.3 m/s and 6.0 m/s. The experimentally measured contact forces and indentations are listed in Tables 2a & 2b alongside values that were estimated using Eqs. (7a)-(7b). Discrepancies between the estimated and measured values are shown to be well within 20%.

In summary, the simple predictive methodology presented in this paper is supported by the presented experimental measurements.

Table 2a Estimated and experimentally measured indentation

Impacting Velocity (m/s)	**Estimated** Indentation (mm)	**Measured** Indentation (mm)
3.1	2.1	2.0
4.3	2.7	2.8
6.0	3.6	3.9

Table 2b Estimated and Experimentally Measured Contact Forces

Impacting Velocity (m/s)	**Estimated** Contact Forces (N)	**Measured** Contact Forces (N)
3.1	920	840
4.3	1370	1200
6.0	2050	1710

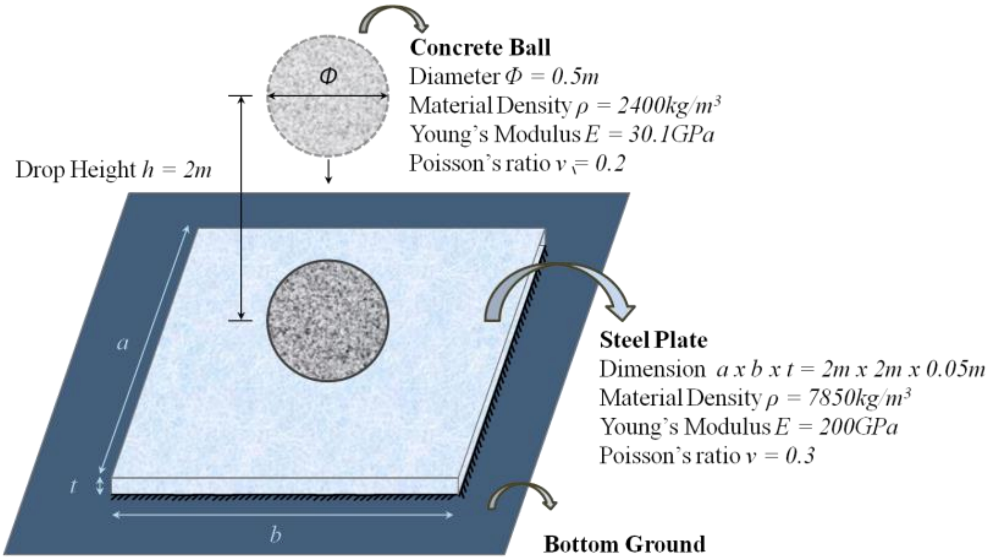


Fig. 9 Custom made tubular device for measuring contact forces

5. Estimation of contact force generated by fallen concrete debris

The proposed estimation methodology is illustrated herein for predicting the amount of contact force that is generated by a piece of concrete debris, weighing 160 kg, and dropping onto a 50 mm thick steel plate from height of 2 m (Fig. 9). The debris material is assumed to be Grade 32 concrete with density (ρ)=2400 kg/m^3, Young's Modulus (E)=30.1 GPa and Poisson's ratio (ν)=0.2.

Given that the Young's Modulus of the concrete is known in this simulated example, FE models can be constructed and F_c - δ relationships determined accordingly (Fig. 10). Considering an impact scenario with an incident velocity of 3.1 m/s the total increase in compressive stiffness from *Model 01* to *Model 03* was found to be very close to the value of 2.3 (which was the value identified earlier for the cricket ball). It was found from a range of simulation studies undertaken by the author that the (*Model 01* to *Model 03*) stiffness modification value was typically within the

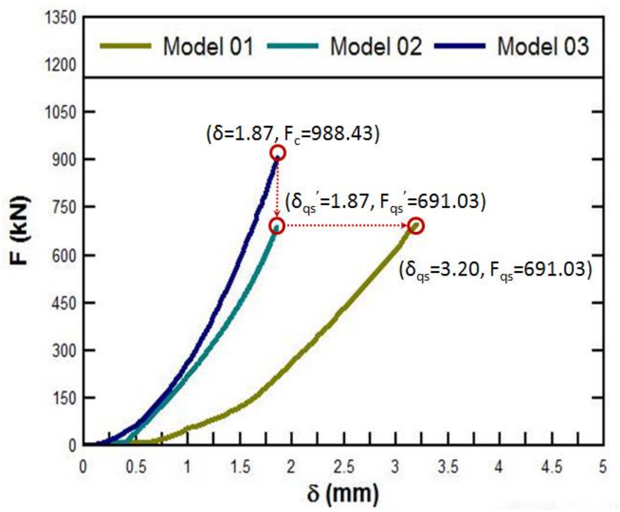


Fig. 10 F_c-δ relationships from *Model 01*, *Model 02* and *Model 03* of the concrete debris

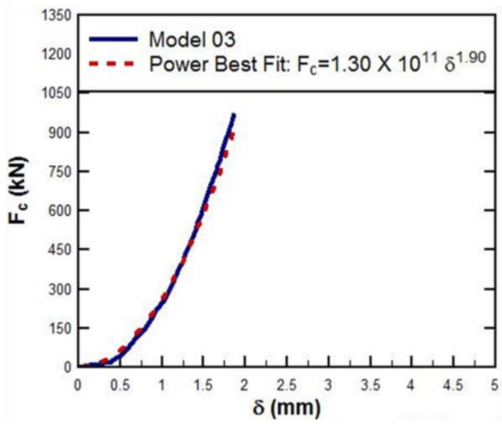


Fig. 11 Curve-fitting of simulated F_c-δ relationship for concrete debris

range 2.0 - 2.5 for spherical objects. The F_c-δ relationship so obtained from *Model 03* was curve-fitted using the functional form of Eq. (4) for determination of the values of parameters P and k (Fig. 11). It was found that for the concrete spherical object being considered: k=2.46×1011 N/m^P and P=1.99.

Substitution of the value of k=2.46×10^{11} N/m^P and P=1.90 into Eqs. (5b)-(5c) results in Eqs. (9a)-(9b) respectively.

$$\delta = 9.62\times10^{-4} v_o^{\,0.67} \quad \text{(units in m and seconds)} \tag{9a}$$

$$F_c = 244\times10^{3} v_o^{\,1.33} \quad \text{(units in N, m and seconds)} \tag{9b}$$

The amount of contact force generated by the impact can be found using Eq. (9b) for any given incident velocity (or drop height). For a drop height of 2 m, the incident velocity is predicted to be 6.3 m/s and the contact force is predicted to be approximately 2640 kN from Eq. (9b). This value can be used for checking against the capacity of the steel plate to resist denting, or that of a concrete slab to resist punching failure.

6. Recommendations for applications

Considering impact actions in many practical engineering constructions, contact force is always required to be estimated in order that the risk of structural damage such as punching failure can be evaluated. Eqs. (5b)-(5c) can be used for obtaining conservative estimates of the indentation and contact force for any given incident velocity (v_0). The assessment of contact force using the aforementioned equations would only require values of the stiffness parameters (k and P) which can be determined by three different approaches to take into account the material properties and shape of the impactor object.

These equations are simple to use and are expected to be adopted widely in engineering design offices. For common engineering materials, such as steel, concrete, or timber which are widely used in design practices the value of k can be found using Eq. (6a) whereas the value of P can simply be taken as 1.5 as per Hertz Law (Machado *et al.* 2012).

In situations where the material properties of the impactor object have not been documented (e.g., cricket ball) values of k and P can be obtained experimentally using the MTS machine. The value of the quasi-static stiffness so obtained for quasi-static conditions (with the MTS) will need to be modified by applying the *boundary factor* (F_B) and the *dynamic factor* (F_D). The product of the two factors has been found to be in the range 2.0-2.5 (and hence the value of 2.3 may be assumed). The value of P as obtained from the MTS test results need not be modified.

These recommendations are for spherically shaped impactors. For irregularly shaped impactors made of materials of unknown properties such as hailstones and windborne debris the value of F_B and F_D will need to be determined using the calibration procedure described in Section 4.3 in which the concept of Model 01, Model 02 and Model 03 was introduced.

7. Conclusions

Contact force is generated in the vicinity of the point of contact between the impactor and target, and is responsible for damage featuring perforation and denting. Computer software such as LS-DYNA has the capability of accurately predicting contact force but the dynamic stiffness parameters of the impactor material which is required for input into the program has not been documented for debris materials. The alternative, direct, approach for an accurate evaluation of the damage potential of an impact scenario is by physical experimentation. However, experimental data on contact force in particular is very scarce because of challenges with taking accurate measurements. Many of the unknowns stem from the lack of knowledge on contact force and the correlation of its values with parameters characterising the impactor and the target.

An experimental-calibration procedure involving quasi-static testings is introduced to determine the compressive stiffness parameter of an impactor object in order that the amount of contact force generated by an impact can be predicted. The force-displacement F_c-δ relationship of the impactor is first obtained by quasi-static testing using the MTS machine. A FE model is then constructed and calibrated, and with the boundary conditions modified. The F_c-δ relationship so simulated by the modified calibrated model is then curve-fitted to determine values of the stiffness parameters k and P. Substituting these parameter values into expressions derived in this paper provides an estimate for the contact force and indentation. The proposed methodology has been verified experimentally using a custom made drop tube device which was used to measure the contact force generated by a cricket ball impacting on a dummy lumped mass. The verified procedure for estimating contact force is then illustrated by the example of a concrete debris, weighing 160 kg, dropping from a height of 2 m onto a steel plate. A contact force of 2640 kN is predicted for this impact scenario. Recommendations for employing this methodology in day-to-day engineering practices are summarised at the end of the paper.

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