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Probabilistic analysis of micro-film buckling with parametric uncertainty

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Abstract. The intentional buckling design of micro-films has various potential applications in engineering. The buckling amplitude and critical strain of micro-films are the crucial parameters for the buckling design. In the reported studies, the film parameters were regarded as deterministic. However, the geometrical and physical parameters uncertainty of micro-films due to manufacturing becomes prominent and needs to be considered. In the present paper, the probabilistic nonlinear buckling analysis of micro-films with uncertain parameters is proposed for design accuracy and reliability. The nonlinear differential equation and its asymptotic solution for the buckling micro-film with nominal parameters are firstly established. The mean values, standard deviations and variation coefficients of the buckling amplitude and critical strain are calculated by using the probability densities of uncertain parameters such as the film span length, thickness, elastic modulus and compressive force, to reveal the effects of the film parameter uncertainty on the buckling deformation. The results obtained illustrate the probabilistic relation between buckling deformation and uncertain parameters, and are useful for accurate and reliable buckling design in terms of probability.

Keywords: probability and statistics; buckling; thin film; uncertainty parameters; nonlinearity

1. Introduction

The buckling behavior has been applied to micro-structure design and fabrication recently (Bowden *et al*. 1998), for example, the stretchable electronic circuit system (Lacour *et al*. 2003, Khang *et al*. 2006, Xiao *et al*. 2008). The stretchable electric structures have various potential applications such as flexible displays, electronic eye camera, controllable skin sensors and structural health monitoring devices (Ko *et al*. 2008, Lumelsky *et al*. 2001, Nathan *et al*. 2000). The mechanical properties of stretchable electric structures such as micro-films have been presented (Bowden *et al*. 1998, Lacour *et al*. 2003, Shaat *et al*. 2012), and the structural deformation has been studied based on the buckling analysis of beam models (Sridhar *et al*. 2002, Audoly and Boudaoud 2008, Yu and Sun 2012). The buckling designed film has much more elastic

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stretchable capacity than the associated planar film. In general, an electric micro-film such as thin patch is bonded partially, for example, only at two opposite boundaries to a pre-elongated dielectric elastomeric substrate such as rubber polymer, and the other part of the micro-film is not bonded to the substrate. Then release the pre-tension deformation of the substrate so that the micro-film is compressed. A certain large pre-tension strain for the substrate will induce the micro-film to buckle out of the substrate plane (as shown in Fig. 1). The substrate pre-tension strain, buckling amplitude and critical strain of micro-films are important parameters for the buckling design.

Several buckling analysis of micro-films on substrates has been presented (Sridhar *et al*. 2002, Audoly and Boudaoud 2008, Jiang *et al*. 2007, Sato *et al*. 2008), and buckling amplitudes and critical stresses/strains have been calculated. A micro-film was modeled as a clamped-clamped elastic beam and analyzed according to the buckling or instability of linearly elastic beam axially compressed. In many studies, the buckled film shape was described approximately by trigonometric functions and the buckled film amplitude was obtained by minimizing total strain energy or simplified bending equation. In some study, the quadratic nonlinear strain was incorporated in the tension-compression energy (Song *et al*. 2009, Timoshenko and Gere 1985). The critical strain was constant and independent on the buckling deformation, and the buckling amplitude obtained was represented by the pre-tension strain for small buckling deformation. In fact, the critical strain of the buckling film induced by the released pre-tension strain of the elastic substrate and the buckled film shape and amplitude are coupled each other for the nonlinear buckling problem. As the pre-tension strain increases, the dependence of the buckled film shape on the axial compressive force or critical strain enhances. In other words, the coupling between the mechanical deformation and critical strain increases with the pre-tension strain. Recently, the asymptotic analysis based on the rigorous nonlinear differential equation for buckled micro-film deformation has been proposed (Zhao *et al*. 2010, Ying *et al*. 2012). However, for the micro-films, the effects of geometrical and physical parameters uncertainty in manufacturing on the buckling analysis and design become prominent and cannot be neglected (Ohlidal *et al*. 2011, Lal *et al*. 2012, Cavdar 2013). Therefore, the probabilistic nonlinear buckling analysis of micro-films with uncertain parameters is necessary and useful for the buckled micro-film design and manufacture.

In the present paper, the effects of parameter uncertainty on the nonlinear micro-film buckling are taken into account. Firstly, for a buckled micro-film with nominal parameters, the nonlinear differential equation for buckling deformation is derived based on the elastic stress-strain relation and forces equilibrium. The asymptotic solution to the nonlinear equation is given to determine the nonlinear buckled amplitude and critical strain, in which the first and second approximate solutions are obtained respectively by solving the corresponding linear equation and applying the least-square method. Then, the film parameters such as span length, thickness, elastic modulus and compressive force are regarded as random variables. The mean values, standard deviations and variation coefficients of the buckling amplitude and critical strain are estimated by using the probability densities of uncertain parameters. Finally, numerical results are given to show the effects of the film parameters uncertainty on the buckling deformation such as buckling amplitude and critical strain.

2. Nonlinear buckling equations for micro-film with nominal parameters

An elastic planar micro-film with nominal parameters has the original length of l_{or}. Under

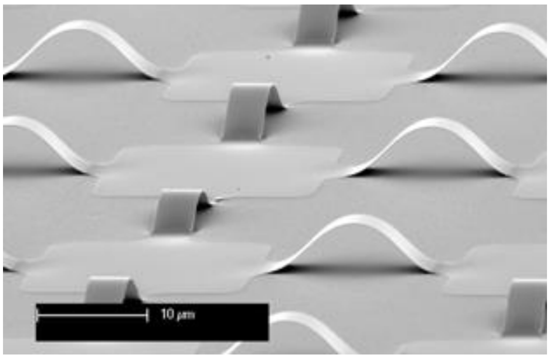


Fig. 1 Buckled micro-films on elastomeric substrate (Xiao *et al*. 2008)

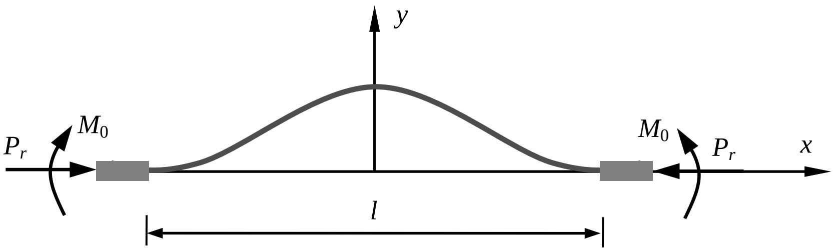


Fig. 2 Buckled film as a flexual clamped-clamped beam

compressive forces along length due to releasing the substrate pre-tension deformation, the film is buckled with span l, as shown in Fig. 2. The so-called pre-tension strain for the buckled film is defined as

$$\varepsilon_{\mathrm{b}} = \frac{l_{\mathrm{or}} - l}{l_{\mathrm{or}}} \tag{1}$$

The buckled film includes compression and bending deformations. If only the bending deformation is returned, the film with only compression has length l_0. Then the compressive strain of the buckled film is

$$\varepsilon_0 = \frac{l_{\mathrm{or}} - l_0}{l_{\mathrm{or}}} = \frac{P_r}{EA} \tag{2}$$

where P_r is the compressive force in buckling state, E is the elastic modulus and EA is the effective compressive rigidity. The change of distance between two ends of the film due to only bending is

$$\frac{\varepsilon_{\mathrm{b}} - \varepsilon_0}{1 - \varepsilon_{\mathrm{b}}} l = \int_0^{l/2} v'^2 \mathrm{d}x \tag{3}$$

where $v(x)$ is the buckling deformation or deflection and superscript "′" denotes the derivative operation with respect to x. Let the x-direction displacement be $u(x)$. The nonlinear strain for the buckled thin film is obtained as (Timoshenko and Gere 1985)

$$\varepsilon(x,y)=\sqrt{\frac{(\sqrt{1+v'^2}+u')^2+v'^2}{1+v'^2}}-\frac{y\theta'(\sqrt{1+v'^2}+u')/(1+v'^2)}{\sqrt{[(\sqrt{1+v'^2}+u')^2+v'^2]/(1+v'^2)}}-1-\varepsilon_0 \tag{4}$$

where $\theta(x)$ is the rotation angle.

The axial force and bending moment on each cross section of the buckled film can be calculated by using Eq. (4) and the stress-strain relation. Then the differential equations for displacements u and v can be derived based on the forces equilibrium and moments equilibrium of the film. Eliminating displacement u yields the nonlinear differential equation for deflection v. This equation with cubic nonlinearity in the dimensionless form is

$$[1+2(1+\varepsilon_0)w'^2]w''+[1+2(2+\varepsilon_0)w'^2](\lambda^2\varepsilon_0 w-w_r'')=0 \tag{5}$$

where superscript "′" denotes the derivative operation with respect to z, and

$$w=\frac{v}{l},\quad z=\frac{x}{l},\quad \lambda=\frac{l}{r_I},\quad r_I^2=\frac{EI}{EA},\quad w_r''=w''(\frac{1}{2}) \tag{6}$$

in which EI is the effective bending rigidity. The boundary conditions for the buckled film with two ends fixed on the substrate corresponding to Eq. (5) are

$$w(\pm\frac{1}{2})=0,\quad w'(\pm\frac{1}{2})=0 \tag{7}$$

Governing Eq. (5) and boundary conditions in Eq. (7) constitute a nonlinear buckling problem of the thin film. In small deflection, Eq. (5) can be reduced to the classical buckling equation as given in the classical mechanics of materials.

3. Buckling amplitude and critical strain for micro-film with nominal parameters

The asymptotic technique in dynamics can be applied to the film buckling problem to determine the deflection w (Ying *et al*. 2012). The asymptotic solution to Eq. (5) is expressed as

$$w=w_0+w_1+\cdots \tag{8}$$

where the first approximate solution w_0 and the second approximate solution w_1 are, respectively

$$w_0=\frac{B}{2}(1+\cos 2\pi z) \tag{9}$$

$$w_1=\frac{B_1}{2}(1+\cos 6\pi z)+\frac{B_2}{2}(1+\cos 10\pi z)+\cdots \tag{10}$$

in which B, B_1 and B_2 are constants. The expressions of w_0 and w_1 are determined by the corresponding linear differential equations. According to the least-square method, constants B_1 and B_2 satisfy the algebraic equations

$$\int_0^{1/2}\Delta_{w1}(w_0,w_1)\frac{\partial\Delta_{w1}(w_0,w_1)}{\partial B_1}\mathrm{d}z=0 \tag{11}$$

$$\int_0^{1/2} \Delta_{w1}(w_0, w_1)\frac{\partial \Delta_{w1}(w_0, w_1)}{\partial B_2}\mathrm{d}z = 0 \tag{12}$$

where

$$\begin{aligned}\Delta_{w1} &= [1+2(1+\varepsilon_0)w_0^{'2}]w_1^{''} + 4[(1+\varepsilon_0)w_0^{'}w_0^{''} + (2+\varepsilon_0)(\lambda^2\varepsilon_0 w_0 - w_{0r}^{''})w_0^{'}]w_1^{'} \\ &+ [1+2(2+\varepsilon_0)w_0^{'2}](\lambda^2\varepsilon_0 w_1 - w_{1r}^{''}) + [1+2(1+\varepsilon_0)w_0^{'2}]w_0^{''} \\ &+ [1+2(2+\varepsilon_0)w_0^{'2}](\lambda^2\varepsilon_0 w_0 - w_{0r}^{''})\end{aligned} \tag{13}$$

The substitution of expressions (8)-(10) into Eq. (3) yields the algebraic equation for constants B

$$\frac{\varepsilon_\mathrm{b} - \varepsilon_0}{1-\varepsilon_\mathrm{b}} = \frac{1}{4}\pi^2(B^2 + 9B_1^2 + 25B_2^2 + \cdots) \tag{14}$$

Based on Eqs. (11), (12) and (14), constants B, B_1, B_2 and then the buckled film deflection w depend on the compressive force P_r (by ε_0) and pre-tension strain ε_b. The buckled film amplitude dependent on ε_b and ε_0 or P_r is

$$w(0) = B + B_1 + B_2 + \cdots \tag{15}$$

The buckling critical strain is obtained by the existence of non-zero perturbation to Eq. (5) as

$$\varepsilon_\mathrm{cr} = \varepsilon_0 = \frac{4\pi^2(B + 9B_1 + 25B_2 + \cdots)}{\lambda^2(B + B_1 + B_2 + \cdots)} \tag{16}$$

which is expressed through the buckled film amplitude or constants B, B_1 and B_2. In small deflection, only the first approximate solution w_0 is meaningful so that the critical strain in Eq. (16) is a constant ($4\pi^2/\lambda^2$) which is consistent with the classical results (Song *et al*. 2009, Timoshenko and Gere 1985).

4. Probabilistic analysis of buckled micro-film deformation

In practice, the geometrical and physical parameters of the micro-film are uncertain due to manufacturing. Let the film span length (l), thickness (h), elastic modulus (E) and compressive force ($P_b=P_r/b$, b is the film width) be random variables with probability densities $p(l)$, $p(h)$, $p(E)$ and $p(P_b)$, respectively. These random variables are independent of each other generally (Lal *et al*. 2012). Then the mean values and mean square values of the buckled amplitude and critical strain are expressed as

$$E[w(0)] = \int_{-\infty}^{+\infty} w(0)p(l)p(h)p(E)p(P_b)\mathrm{d}l\mathrm{d}h\mathrm{d}E\mathrm{d}P_b \tag{17}$$

$$E[w^2(0)] = \int_{-\infty}^{+\infty} w^2(0)p(l)p(h)p(E)p(P_b)\mathrm{d}l\mathrm{d}h\mathrm{d}E\mathrm{d}P_b \tag{18}$$

$$E[\varepsilon_\mathrm{cr}] = \int_{-\infty}^{+\infty} \varepsilon_\mathrm{cr}\, p(l)p(h)p(E)p(P_b)\mathrm{d}l\mathrm{d}h\mathrm{d}E\mathrm{d}P_b \tag{19}$$

$$E[\varepsilon_{\rm cr}^2] = \int_{-\infty}^{+\infty} \varepsilon_{\rm cr}^2 p(l)\,p(h)\,p(E)\,p(P_b) \mathrm{d}l \mathrm{d}h \mathrm{d}E \mathrm{d}P_b \tag{20}$$

where E[.] denotes the expectation operation. The standard deviations of the buckling amplitude and critical strain can be estimated further by using Eqs. (17)-(20). The influences of the parameter uncertainty on the buckling amplitude and critical strain increase with the standard deviations. In the deterministic case, mean square value (18) or (20) is equal to the square of mean value (17) or (19), and its standard deviation is equal to zero. For the random parameters with the Gaussian distribution, the probability densities are

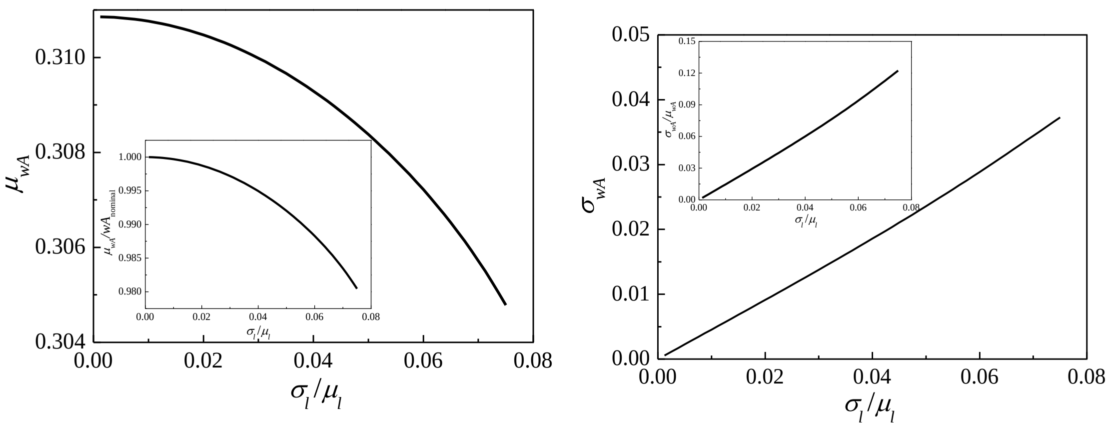
$$p(l) = \frac{1}{\sqrt{2\pi}\sigma_l} \exp[-\frac{(l-\mu_l)^2}{2\sigma_l^2}] \tag{21}$$

$$p(h) = \frac{1}{\sqrt{2\pi}\sigma_h} \exp[-\frac{(h-\mu_h)^2}{2\sigma_h^2}] \tag{22}$$

$$p(E) = \frac{1}{\sqrt{2\pi}\sigma_E} \exp[-\frac{(E-\mu_E)^2}{2\sigma_E^2}] \tag{23}$$

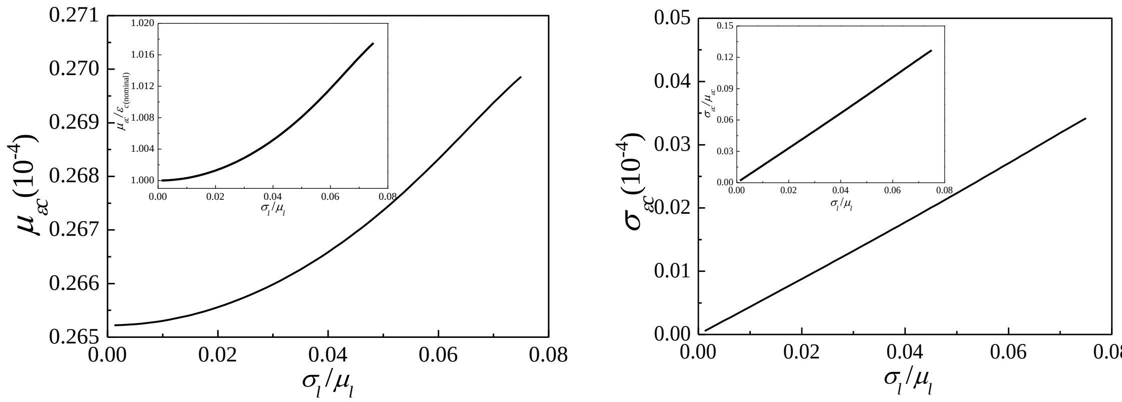
$$p(P_b) = \frac{1}{\sqrt{2\pi}\sigma_{Pb}} \exp[-\frac{(P_b-\mu_{Pb})^2}{2\sigma_{Pb}^2}] \tag{24}$$

where exp[.] denotes the exponential function, μ_l, μ_h, μ_E, μ_{Pb} are the mean values and σ_l, σ_h, σ_E, σ_{Pb} are the standard deviations of the film span length, thickness, elastic modulus and compressive force, respectively. The mean values and standard deviations of the buckling amplitude and critical strain (17)-(20) can be calculated by using the probability densities of uncertain parameters (21)-



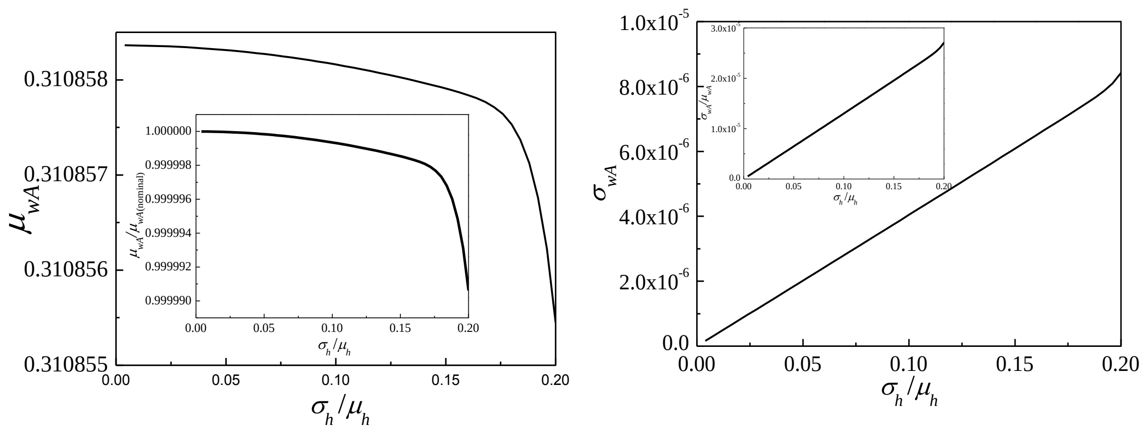
(a) Mean value (μ_{wA}) (b) Standard deviation (σ_{wA})

Fig. 3 Mean value and standard deviation of the buckled amplitude varying with the ratio (σ_l/μ_l) of standard deviation to mean value of the film span length (l)



(a) Mean value ($\mu_{\varepsilon c}$) (b) Standard deviation ($\sigma_{\varepsilon c}$).

Fig. 4 Mean value and standard deviation of the buckling critical strain varying with the ratio (σ_l/μ_l) of standard deviation to mean value of the film span length (l)

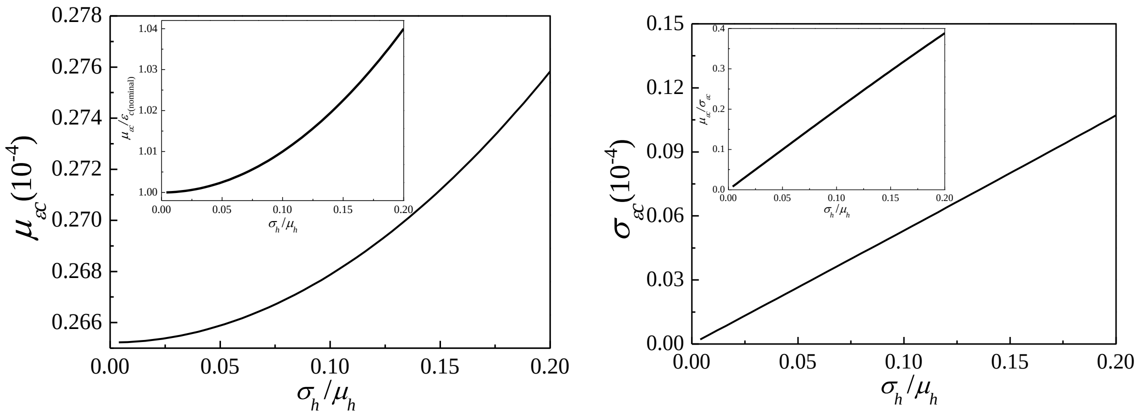


(a) Mean value (μ_{wA}) (b) Standard deviation (σ_{wA})

Fig. 5 Mean value and standard deviation of the buckled amplitude varying with the ratio (σ_h/μ_h) of standard deviation to mean value of the film thickness (h)

(24) to determine the influence of parameter uncertainty on the film buckling.

Consider a micro-film with nominal length l_{or}=20μm, width b=4μm, thickness h=50nm and elastic modulus E=200GPa. Let the buckled film span length l=16μm. The buckled deflection, amplitude and critical strain of the film with nominal parameters have been given in the reference (Ying *et al*. 2012). For the micro-film with uncertain parameters, the film span length (l), thickness (h), elastic modulus (E) and compressive force (P_b) are regarded as random variables with Gaussian probability densities (21)-(24), respectively. Numerical results on the statistics of the buckled film amplitude and critical strain are shown in Figs. 3-10. Figs. (a) and (b) depict the mean values and standard deviations, respectively. The insets in Figs. (a) plot the mean values normalized by the nominal values, while those in Figs. (b) plot the standard deviations normalized by the mean values.

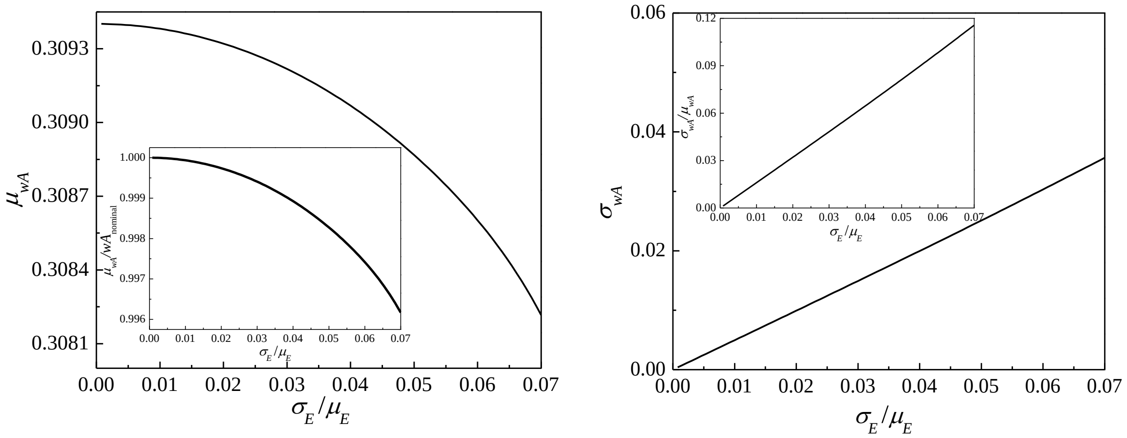


(a) Mean value ($\mu_{\varepsilon c}$) (b) Standard deviation ($\sigma_{\varepsilon c}$)

Fig. 6 Mean value and standard deviation of the buckling critical strain varying with the ratio (σ_h/μ_h) of standard deviation to mean value of the film thickness (h)

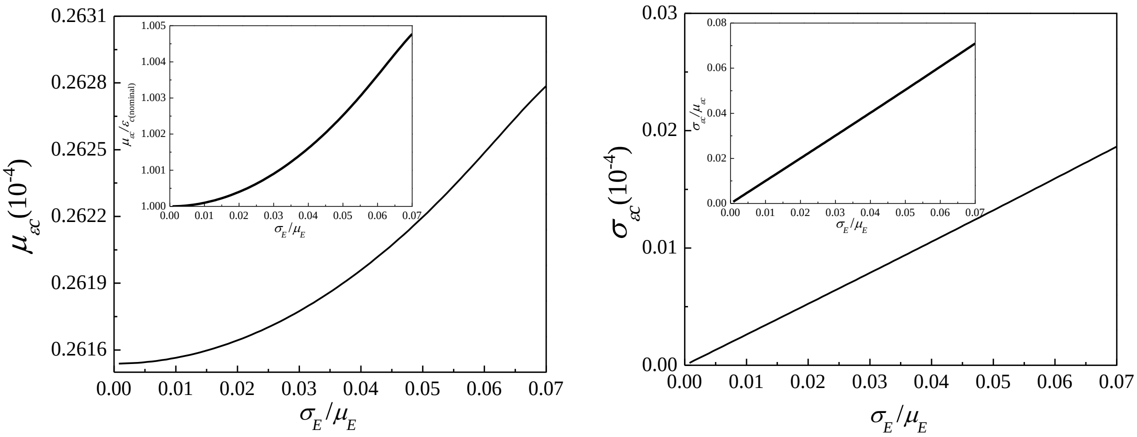
As the ratio of standard deviation to mean value, i.e., the variation coefficient, of the buckled film span length (l) increases, Fig. 3(a) illustrates the nonlinear reduction of the mean buckling amplitude while Fig. 4(a) illustrates the nonlinear heightening of the mean critical strain. Figs. 3(b) and 4(b) show respectively the standard deviations of the buckling amplitude and critical strain linearly increasing with the variation coefficient of the span length. From the insets of Figs. 3(b) and 4(b), it can be concluded that the dispersion of the buckling amplitude and critical strain is more prominent than that of the span length. Fig. 5 illustrates the relatively small change of the mean value and standard deviation of the buckling amplitude compared to the variation coefficient of the film thickness (h). Fig. 6 shows the mean value and standard deviation of the critical strain varying with the variation coefficient of the film thickness similar to Fig. 4. The large dispersion of the critical strain induced by the thickness uncertainty is observed from the inset. As the variation coefficient of the elastic modulus (E) increases, Fig. 7(a) illustrates the small decrease of the mean buckling amplitude while Fig. 8(a) illustrates the small increase of the mean critical strain. Figs. 7(b) and 8(b) show respectively the standard deviations of the buckling amplitude and critical strain linearly increasing with the variation coefficient of the modulus. Fig. 9 illustrates the influence of the compressive force (P_b) uncertainty similar to the modulus uncertainty on the buckling amplitude. Fig. 10 shows the influences of the compressive force uncertainty on the critical strain. It is observed from Fig. 10(a) that the mean critical strain almost keeps constant for the uncertain compressive force with the ratio σ_{Pb}/μ_{Pb} <0.04, or it has the relative stability. Fig. 10(b) illustrates the significant influence of the compressive force uncertainty on the dispersion of the critical strain. To summarize, the uncertainty of the span length, elastic modulus and the compressive force has remarkable influences on the mean value and standard deviation of the buckling amplitude while the influence of the film thickness can be omitted. The uncertainty of the compressive force almost has not the influence on the mean critical strain, while the uncertainty of the four parameters induces the prominent dispersion of the critical strain.

The asymptotic technique establishes the accurate procedure to determine the buckling shape and the critical strain, while the probabilistic parametric sensitivity analysis provides the evaluation of the influence of the crucial parameters on the dispersion of buckling deformation. On



(a) Mean value (μ_{wA}) (b) Standard deviation (σ_{wA})

Fig. 7 Mean value and standard deviation of the buckled amplitude varying with the ratio (σ_E/μ_E) of standard deviation to mean value of the elastic modulus (E)



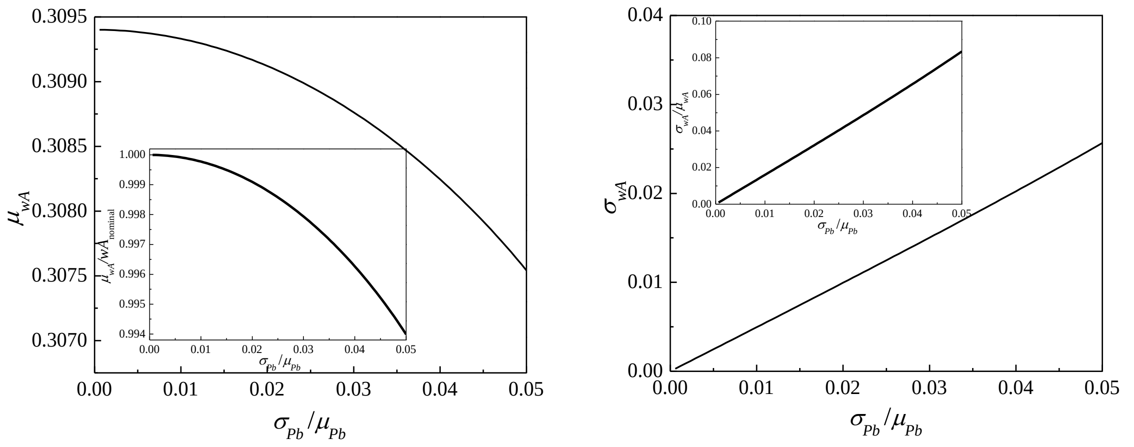
(a) Mean value ($\mu_{\varepsilon c}$) (b) Standard deviation ($\sigma_{\varepsilon c}$)

Fig. 8 Mean value and standard deviation of the buckling critical strain varying with the ratio (σ_E/μ_E) of standard deviation to mean value of the elastic modulus (E)

the basis of the above analysis, the probabilistic design method can be directly established for the buckling deformation problem. For the buckling strength problem, the axial strain of the thin film includes the compressive strain and the bending strain, and the maximal strain can be derived through the asymptotic solution in Eq. (8) and the critical strain in Eq. (16) as

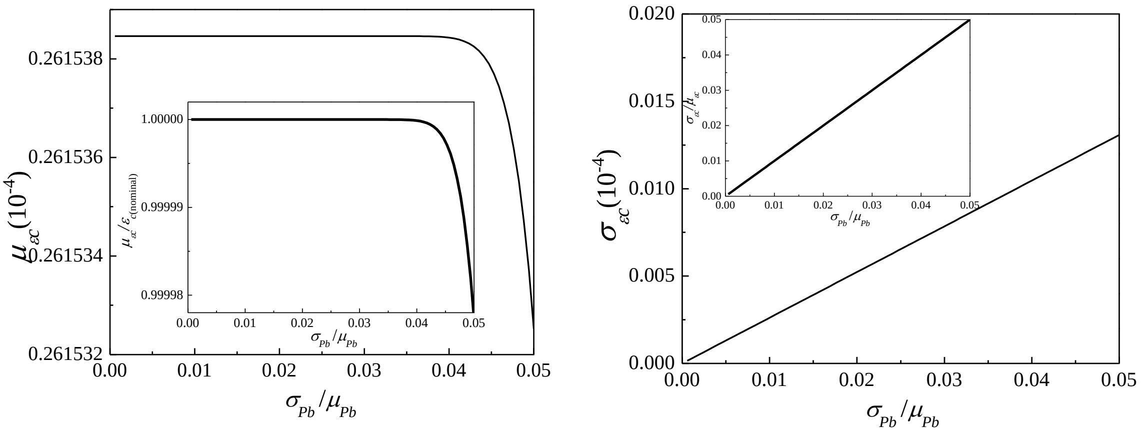
$$\varepsilon_{\max} = 2\pi^2\left(4B + 9B_1 + 25B_2 + \cdots\right) + 4\pi^2\left(B + 9B_1 + 25B_2 + \cdots\right)\Big/\left[\lambda^2\left(B + B_1 + B_2 + \cdots\right)\right] \quad (25)$$

The statistics of the maximal strain induced by the uncertain parameters are evaluated through the probabilistic analysis similar to the above procedure. And then, the probabilistic reliability method can be adopted to select the film parameters to guarantee the strength requirement. The



(a) Mean value (μ_{wA}) (b) Standard deviation (σ_{wA})

Fig. 9 Mean value and standard deviation of the buckled amplitude varying with the ratio (σ_{Pb}/μ_{Pb}) of standard deviation to mean value of the compressive force (P_b)



(a) Mean value ($\mu_{\varepsilon c}$) (b) Standard deviation ($\sigma_{\varepsilon c}$)

Fig. 10 Mean value and standard deviation of the buckling critical strain varying with the ratio (σ_{Pb}/μ_{Pb}) of standard deviation to mean value of the compressive force (P_b)

proposed design method for micro-films can be regarded as the probabilistic version of the classical design based on the strength of material (Song *et al* 2009).

5. Conclusions

The probabilistic nonlinear buckling analysis of micro-films with uncertain parameters has been developed. The nonlinear differential equation and its asymptotic solution for the buckling deformation of a micro-film with nominal parameters have been given. The dependences of the buckled film shape and amplitude on pre-tension strain and the critical strain on buckling amplitude have been determined analytically. The influences of the uncertainty of the film parameters such as span length, thickness, elastic modulus and compressive force on the buckling amplitude and critical strain have been estimated based on the probabilistic analysis and illustrated with numerical results in terms of mean values, standard deviations and variation coefficients. The analysis and results obtained are useful for accurate buckling film design in terms of probability, and it can be regarded as a useful supplement to the classical design method based on the strength of material. Moreover, the proposed analysis method is applicable to the micro-film buckling for various boundary conditions under small and middling pre-strains.

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