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Multi-objective optimization of foundation using global-local gravitational search algorithm

Mohammad Khajehzadeh[*1], Mohd Raihan Taha[2] and Mahdiyeh Eslami[3]

[1]Department of Civil Engineering, Anar Branch, Islamic Azad University, Anar, Iran
[2]Department of Civil and Structural Engineering, Universiti Kebangsaan Malaysia, Bangi, Selangor, Malaysia
[3]Department of Electrical Engineering, Science and Research Branch, Islamic Azad University, Kerman, Iran

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Abstract. This paper introduces a novel optimization technique based on gravitational search algorithm (GSA) for numerical optimization and multi-objective optimization of foundation. In the proposed method, a chaotic time varying system is applied into the position updating equation to increase the global exploration ability and accurate local exploitation of the original algorithm. The new algorithm called global-local GSA (GLGSA) is applied for optimization of some well-known mathematical benchmark functions as well as two design examples of spread foundation. In the foundation optimization, two objective functions include total cost and CO_2 emissions of the foundation subjected to geotechnical and structural requirements are considered. From environmental point of view, minimization of embedded CO_2 emissions that quantifies the total amount of carbon dioxide emissions resulting from the use of materials seems necessary to include in the design criteria. The experimental results demonstrate that, the proposed GLGSA remarkably improves the accuracy, stability and efficiency of the original algorithm.

Keywords: spread foundation; cost optimization; CO_2 emissions optimization; gravitational search algorithm

1. Introduction

Shallow foundations are one of the most common and utilized types of foundations and constitute an integral part of all structures. Spread foundations are by far the most common type of foundation, primarily because their low cost and ease of construction. These types of foundations required a minimum amount of equipment and skill for construction. Furthermore, the conditions of the spread foundations and the supporting soil can be readily examined. In the analysis and design of spread foundation, the structure must safely and reliably support the loads; it must have sufficient shear and moment capacities; the bearing capacity of the foundation cannot be exceeded or allowed to be in tensile stress; and the configuration of the steel reinforcement must me*et al*l building code requirements.

In addition to these design objectives, the structure should be optimized economically and environmentally. The traditional goals of engineers in the field of structural optimization design

∗Corresponding author, Ph.D., E-mail: mohammad.khajehzadeh@gmail.com

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were to minimize the objective function, which is usually the cost or the weight of the structure rather than environmental factors. Nowadays, the objective of structural design is to optimize the consumption of materials not only from an economic point of view, but also environmental impacts. It is worth to note that concrete is the most widely used material on Earth. Therefore, it seems vital to include design criteria to minimize the embedded CO_2 emissions in reinforced concrete (RC) structures. This study deals with the optimization of RC spread foundations, in terms of minimum cost and CO_2 emissions. The CO_2 objective function quantifies the total amount of carbon dioxide emissions resulting from the use of materials and minimization of embedded CO_2 emissions seems necessary to include design criteria.

In the last decades, several studies have been undertaken to implement different optimization approaches for solving structural engineering optimization problems. For instance, application of simulated annealing for the optimum design of reinforced concrete retaining structures (Ceranic *et al*. 2001), harmony search algorithm for optimization of truss structures (Lee and Geem 2004), ant colony optimization for optimum design of steel frames (Camp *et al.* 2005), genetic algorithm for structural optimization (Salajegheh and Gholizadeh 2005), harmony search algorithm for optimum design of steel structures (Degertekin 2008), particle swarm optimization for optimum design of spread footing and retaining wall (Khajehzadeh *et al*. 2011), artificial bee colony algorithm for optimum design of truss structures (Sonmez 2011), big bang-big crunch optimization for design of retaining walls (Camp and Akin 2012), etc. Gravitational search algorithm (GSA) is a novel and attractive swarm intelligence-based optimization technique inspired by the law of gravity and mass interactions (Rashedi *et al*. 2009). Due to its simplicity and ease of implementation, GSA has captured much attention and has been applied to solve many practical optimization problems. However, similar to other evolutionary algorithms, GSA suffers from some drawbacks such as slow convergence rate and premature convergence when solving complex optimization problems. Therefore, researchers tried to improve this algorithm by different ways to overcome its drawbacks (Sarafrazi *et al*. 2011, Yin *et al*. 2011, Khajehzadeh *et al*. 2012, Mirjalili *et al*. 2012, Khajehzadeh *et al*. 2013).

This paper develops a new version of GSA referred to as global-local gravitational search algorithm (GLGSA). In the proposed approach, a new chaotic decreasing operator is introduced and applied into the classical agents' updating position equation. In this way, the randomness, irregularity and the stochastic property of the new operator improve the global search ability of the algorithm and allow agents to escape from local minima when they are prematurely attracted to a local attractor. In addition, decreasing behavior of the new operator during the optimization procedure will increase the local exploitation ability of the algorithm in the later part of the optimization. To validate the efficiency of the proposed approach a set of six well-known benchmark functions is considered. Afterwards, the new algorithm is applied for multi-objective optimization of spread foundations to minimize the total cost and embedded CO_2 emissions of the structure simultaneously. The numerical simulation results in section 4 demonstrate that the proposed strategy has significantly better performance in terms of robustness and accuracy compared with the classical GSA.

2. Multi-objective optimization of foundation

Multiple objectives arise naturally in most real-world combinatorial optimization problems. Several principles and strategies have been developed and proposed in order to solve these problems. The aim is to find a vector of decision variables that satisfies constraints and optimizes (minimizes or maximizes) these functions. In a more precise mathematical way, formulation of a multi-objective problem includes a set of *n* design variables, a set of *m* objective functions and a set of *k* constraints and can be defined as follows

$$\begin{aligned} &\text{Find vector} && \mathbf{X} = [x_1, x_2, \ldots, x_n]^T \\ &\text{To minimize} && \mathbf{f}(\mathbf{X}) = [f_1(\mathbf{X}), f_2(\mathbf{X}), \ldots, f_m(\mathbf{X})] \\ &\text{Subject to} && g_j(\mathbf{X}) \le 0 \quad j = 1,2,\ldots,k \end{aligned} \tag{1}$$

where $f_1, f_2, \ldots, f_m$ denote the objective functions to be optimized simultaneously, $\mathbf{X}$ is the vector of decision variables and $g_j(\mathbf{X})$ denotes the inequality constraints.

For a multi-objective optimization, we can construct a new function called evaluation function to convert a multi-objective problem into a single-objective problem for simplification. There are many ways to construct an evaluation function. In this paper, in order to apply and solve the economic emissions foundation optimization problem using the GLGSA algorithm, we use the weighted aggregation method as an efficient way to combine and transform the two objective functions of the problem, i.e., the embedded CO_2 emissions and total cost of the structure, into one objective function. In this method, the problem is transformed into a single-objective function (U) by using weighting coefficients as follows

$$U = \sum_{i=1}^{m} w_i f_i(\mathbf{X}) \tag{2}$$

where w_i is a constant indicating the weight (and hence importance) assigned to f_i. By giving a relatively large value to w_i it is possible to favor f_i over other objective functions.

2.1 Objective functions

In this study, the multi-objective optimization of foundation consists of two objective functions; the embedded CO_2 emissions and total cost of the structure. Therefore, the optimization algorithm aims to minimize these objective functions simultaneously.

The first objective function measures the total amount of CO_2 emissions resulting from the use of materials, which involve emissions at the different stages of production and placement. The CO_2 emissions objective function can be presented mathematically in the following form

$$f_1(\mathbf{X}) = e_c V_c + e_e V_e + e_b V_b + e_f A_f + e_s W_s \tag{3}$$

where e_c, e_e, e_b, e_f and e_s are the CO_2 unit emissions of concrete, excavation, backfill, formwork, and reinforcement, respectively. In addition, V_c ,V_e and V_b denote the volume of concrete, excavation and backfill of the foundation, A_f is the area of formwork and W_s indicates the weight of steel of the structure. The CO_2 unit emissions considered for the optimization in the current study are given in Table 1 and are obtained from the study of (Yepes *et al*. 2012).

The second objective function quantifies the total cost of the structure. The cost minimization objective function includes the cost of the materials and costs associated with labor and installation. The cost function can be expressed in the following form

$$f_2(\mathbf{X}) = C_c V_c + C_e V_e + C_b V_b + C_f A_f + C_s W_s \tag{4}$$

Table 1 Spread foundation assembly unit cost and unit CO_2 emissions

Item	Unit	Unit Emission (Kg)	Unit Cost (US$)
Earth removal	m^3	13.16	11.41
Formwork	m^2	14.55	36.82
Reinforcement	kg	2.82	1.51
Concrete	m^3	224.94	104.51
Earth fill-in	m^3	27.20	38.1

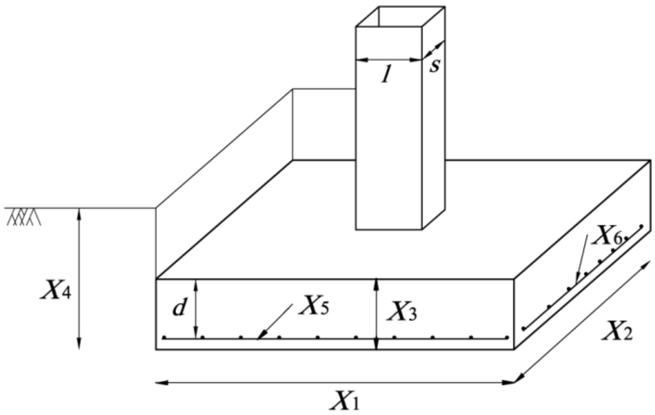

Fig. 1 Design variables of the foundation

where C_c, C_e, C_b, C_f and C_s are the unit cost of concrete, excavation, backfill, formwork, and reinforcement, respectively. The unit costs considered here are presented in Table 1 and are obtained from the study of (Yepes *et al.* 2012).

2.2 Design variables

Fig. 1 shows the design variables considered for the spread foundation model. The design variables are divided into two categories: those that describe the geometric dimensions and those that model the steel reinforcement. As it is shown in Fig. 1, there are four geometric design variables representing the dimensions of the foundation: X_1 is length of the foundation, X_2 is breadth of the foundation, X_3 is thickness of the foundation and X_4 is depth of embedment. There are two additional design variables related to the steel reinforcement: X_5 is the longitudinal reinforcement and X_6 is the transverse reinforcement.

In Fig. 1, d is the distance from compression surface to the centroid of tension steel, l is the long side of the column and s is the short side of the column.

2.3 Design constraints

Fig. 2 shows the general forces acting on the foundation. In this figure, E is Young's modulus, v is Poisson's ratio, c is cohesion, ϕ is effective friction angle, and γ is unit weight of soil. In addition, M is moment applied on the foundation, P is the axial load, and $q_{\min}$ and $q_{\max}$ are the minimum and maximum bearing stresses on the base of the foundation, respectively.

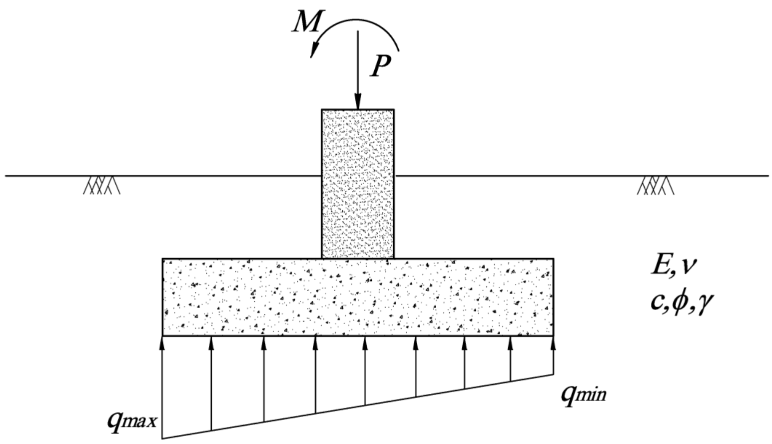

Fig. 2 Forces acting on the foundation

Table 2 Bearing capacity factors, shape factors and depth factors (Vesic 1975)

Bearing capacity factors	Depth factors	Shape factors
$N_q = \exp(\pi\tan\phi)\tan^2(45+\frac{\phi}{2})$	$d_q = \begin{cases} 1+2\tan\phi(1-\sin\phi)^2(X_4/X_2), & X_4/X_2 \le 1 \\ 1+2\tan\phi(1-\sin\phi)^2\tan^{-1}(X_4/X_2), & X_4/X_2 > 1 \end{cases}$	$s_q = 1+(X_2/X_1)\tan\phi$
$N_c = (N_q - 1)\cot\phi$	$d_c = \begin{cases} 1+0.4(X_4/X_2), & X_4/X_2 \le 1 \\ 1+0.4\tan^{-1}(X_4/X_2), & X_4/X_2 > 1 \end{cases}$	$s_c = 1+(X_2/X_1)(N_q/N_c)$
$N_\gamma = 2(N_q+1)\tan\phi$		$s_\gamma = 1-0.4(X_2/X_1)$
	$d_\gamma = 1$	

The typical design philosophy of a spread foundation seeks designs that provide safety and stability against failure modes and comply with concrete building code requirements. These requirements may be classified into four general groups of design constraints: stability, capacity, reinforcement configuration, and geometric limitations. The various design constraints to be considered in the optimization of the spread foundation are presented in detail in the following sections.

2.3.1 Bearing capacity failure mode

The bearing capacity of the foundation must be large enough to resist the stresses acting along the base. To have a safe design the imposed stress should be less than the safe bearing capacity of soil as follows

$$q_{\max} \le \frac{q_{ult}}{FS} \tag{5}$$

where q_{ult} is the ultimate bearing capacity of the foundation soil, FS is the factor of safety and $q_{\max}$ is the maximum contact pressure at the interface between the bottom of a foundation and the underlying soil. In this study, q_{ult} is evaluated using Vesic's method (Vesic 1975), according to

$$q_{ult} = cN_c d_c s_c + qN_q d_q s_q + 0.5\gamma N_\gamma d_\gamma s_\gamma \tag{6}$$

where q is the effective vertical stress at the footing base level; N_c, N_q and N_γ are bearing capacity factors, d_c, d_q and d_γ are depth factors and s_c, s_q and s_γ are shape factors. The equations for bearing capacity, shape and depth factors are summarized in Table 2.

The minimum and maximum applied bearing stresses on the base of the foundation are evaluated by

$$q_{\substack{\min\\ \max}} = \frac{P}{X_1 X_2}\left(1 \mp \frac{6e}{X_1}\right) \tag{7}$$

where e is the eccentricity which is the ratio of the uniaxial moment to the axial forces.

2.3.2 Eccentricity failure mode

In order to prevent tensile stresses at the bottom that tend to uplift the foundation, the following conditions must be satisfied (Gunaratne 2006)

$$e \le \frac{X_1}{6} \tag{8}$$

2.3.3 Settlement of foundation

Settlement of foundation should be within a permissible limit according to the following inequality

$$\delta \le \delta_{all} \tag{9}$$

where δ_{all} is allowable settlement and δ is the settlement of foundation. According to the elastic solution suggested in Poulos and Davis (1974), the settlement can be calculated as follows

$$\delta = \frac{P(1-\nu^2)}{\kappa_z E \sqrt{X_1 X_2}} \tag{10}$$

where κ_z is the shape factor. The shape factor suggested in Wang and Kulhawy (2008) is adopted in this study according to

$$\kappa_z = -0.0017(X_1/X_2)^2 + 0.0597(X_1/X_2) + 0.9843 \tag{11}$$

2.3.4 One way (wide beam) shear failure mode

For one way shear, the foundation must be considered as a wide beam and the ultimate shear force (V_u) should be less than nominal shear strength of concrete according to (ACI 2005)

$$V_u \le \frac{1}{6}\phi_V \sqrt{f'_c}\, X_2 d \tag{12}$$

$$V_u \le \frac{1}{6}\phi_V \sqrt{f'_c}\, X_1 d \tag{13}$$

where ϕ_V is the shear strength reduction factor equal to 0.75 (ACI 2005) and f'_c is the compression strength of concrete. The ultimate shear is taken along a vertical plane extending the full width of the base (X_1 or X_2) located at distance d from face of column.

2.3.5 Two way (punching) shear failure mode

Punching shear indicates the tendency of the column to punch through the foundation slab. To avoid such a failure, the upward ultimate shearing force (V_u) must be lower than the nominal punching shear strength according to (ACI 2005).

$$V_u \le min\left\{(1+\frac{2}{\beta_c})/6, \left(\frac{\alpha_s d}{b_0}+2\right)/12, \frac{1}{3}\right\}\phi_V \sqrt{f'_c}\, b_0 d \tag{14}$$

where b_0 is the perimeter of critical section taken at $d/2$ from face of column ($[l+d]\times[s+d]$), β_c is the ratio of long side to short side of column section (l / s) and α_s is the 40 for interior columns.

2.3.6 Bending moment failure mode

The moment capacity of the foundation should be less than the nominal flexural strength according to (ACI 2005)

$$M_u \le \phi_M X_5 f_y \left(d - \frac{X_5 f_y}{1.7 X_2 f_c'}\right) \tag{15}$$

$$M_u \le \phi_M X_6 f_y \left(d - \frac{X_6 f_y}{1.7 X_1 f_c'}\right) \tag{16}$$

where M_u is the bending moment of the reaction forces due to the applied load at the face of the column (for foundation supporting a reinforced concrete column), ϕ_M is the flexure strength reduction factor equal to 0.9 (ACI 2005) and f_y is the yield strength of steel.

2.3.7 Minimum and maximum reinforcements

The amount of steel reinforcement in each direction of the foundation must satisfy minimum and maximum reinforcement area limits required by building codes (ACI 2005) according to

$$\rho_{\min}.d.X_2 \le X_5 \le \rho_{\max}.d.X_2 \tag{17}$$

$$\rho_{\min}.d.X_1 \le X_6 \le \rho_{\max}.d.X_1 \tag{18}$$

where ρ_{min} and ρ_{max} are the minimum and maximum reinforcement ratio based on the following equations (ACI 2005)

$$\rho_{\min} = \max\left\{\frac{1.4}{f_y}, 0.25\frac{\sqrt{f_c'}}{f_y}\right\} \tag{19}$$

$$\rho_{\max} = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{600}{600+f_y}\right) \tag{20}$$

Table 3 Constraints for optimum design of spread footing

Constraint number	Failure mode	Constraint
$g_1(X)$	Bearing capacity	$q_{\max} - \frac{q_{ult}}{FS} \le 0$
$g_2(X)$	Eccentricity failure	$e - \frac{X_1}{6} \le 0$
$g_3(X)$	Settlement of footing	$\delta - \delta_{all} \le 0$
$g_{4\text{-}5}(X)$	One way shear failure	$V_u - \frac{1}{6}\phi_V \sqrt{f'_c} X_2 d \le 0$ $V_u - \frac{1}{6}\phi_V \sqrt{f'_c} X_1 d \le 0$
$g_6(X)$	Punching shear failure	$V_u - min\left\{\frac{1+\frac{2}{\beta_c}}{6}, \left(\frac{\alpha_s d}{b_0} + 2\right)/12, \frac{1}{3}\right\}\phi_V \sqrt{f'_c}\, b_0 d \le 0$
$g_{7\text{-}8}(X)$	Bending moment failure	$M_u - \phi_M X_5 f_y \left(d - \frac{X_5 f_y}{1.7 X_2 f_c'}\right) \le 0$ $M_u \le \phi_M X_6 f_y \left(d - \frac{X_6 f_y}{1.7 X_1 f_c'}\right) \le 0$
$g_9\ (X)$	Minimum depth of embedment	$0.5 - X_4 \le 0$
$g_{10}(X)$	Maximum depth of embedment	$X_4 - 2 \le 0$
$g_{11\text{-}12}(X)$	Minimum steel area	$\rho_{\min}.d.X_2 - X_5 \le 0$ $\rho_{\min}.d.X_1 - X_6 \le 0$
$g_{13\text{-}14}(X)$	Maximum steel area	$X_5 - \rho_{\max}.d.X_2 \le 0$ $X_6 - \rho_{\max}.d.X_1 \le 0$

2.3.8 Limitation of depth of embedment

Finally, the depth of embedment (X_4) should be greater than a minimum depth to prevent frost damage and should be limited to a maximum depth to minimize disturbance to adjacent structures. Therefore

$$0.5 \le X_4 \le 2 \tag{21}$$

Finally, the inequality constraints for multi-objective optimization of spread footing can be summarized as shown in Table 3.

3. Global-Local gravitational search algorithm

Gravitational search algorithm (GSA) is a newly developed stochastic search algorithm presented originally by Rashedi *et al*. (2009). The GSA could be considered as a small artificial world of masses obeying the Newtonian laws of gravitation and motion (Rashedi *et al*. 2009). In this approach, all the individuals (search agents) can be viewed as objects and their performances are evaluated by their masses. All these objects attract each other by a gravity force, and this force causes the movement of all objects globally towards objects with heavier masses. The heavy masses correspond to good solutions of the problem. The position of the agent represents a potential solution of the problem, and its mass is determined using a fitness function. Over time, masses are attracted by the heaviest mass, which is probably close to the optimum solution in the search space.

In order to explain GSA, consider a system with N agents (masses) in which the position of the agent i is represented by

$$X_i=(x_i^1,\ldots,x_i^d,\ldots,x_i^n) \qquad \text{for } i = 1, 2, \ldots, N \tag{22}$$

where x_i^d is the position of agent i in dimension d and n is the search space dimension.

After evaluating the current population fitness, the mass of each agent is calculated as follows

$$M_i(t)=\frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{23}$$

where

$$m_i(t)=\frac{\mathrm{fit}_i(t)-\mathrm{worst}(t)}{\mathrm{best}(t)-\mathrm{worst}(t)} \tag{24}$$

where $\mathrm{fit}_i(t)$ represent the fitness value of the agent i at time t. best(t) and worst(t) is the best and worst penalized fitness values of all agents (for a minimization problem) at time t and defined as follows

$$\mathrm{best}(t)=\min_{j\in\{1,\ldots,N\}} \mathrm{fit}_j(t) \tag{25}$$

$$\mathrm{worst}(t)=\max_{j\in\{1,\ldots,N\}} \mathrm{fit}_j(t) \tag{26}$$

To evaluate the acceleration of an agent, total forces from a set of heavier masses applied on it should be considered based on a combination of the law of gravity according to

$$F_i^d(t)=\sum_{j\in kbest,\, j\neq i} rand_j\, G(t)\frac{M_j(t)\times M_i(t)}{R_{i,j}(t)+\varepsilon}(x_j^d(t)-x_i^d(t)) \tag{27}$$

where $rand_j$ is a random number in the interval [0, 1], $G(t)$ is the gravitational constant at time t, M_i and M_j are masses of agents i and j, ε is a small value and $R_{i,j}(t)$ is the Euclidean distance between two agents, i and j. One way to perform a good compromise between exploration and exploitation is to reduce the number of agents with a lapse of time in Eq. (27). Hence, in this algorithm, only a set of agents with a bigger mass apply their force to the other. However, this property may reduce the exploration ability and increase the exploitation capability. To improve the performance of the GSA by controlling exploration and exploitation ability, the *kbest* agents will attract the others. *kbest* is a function of time, whose initial value is K_0 at the beginning and decreases with time. Therefore, all agents apply the force at the beginning, and as time passes, the term *kbest* decreases linearly. At the end, there will be just one agent applying force to the others.

By the law of motion, the acceleration of the agent i at time t, and in direction d, $a_i^d(t)$, is given as follows

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} = \sum_{j \in kbest, j \neq i} rand_j G(t) \frac{M_j(t)}{\left\| X_i(t), X_j(t) \right\|_2 + \varepsilon} (x_j^d(t) - x_i^d(t)) \tag{28}$$

The searching strategy on this concept can be described by the following equations.

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{29}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{30}$$

where $rand_i$ is a uniform random variable in the interval [0, 1]. This random number is applied to give a randomized characteristic to the search. x_i^d represents the position of agent i in dimension d, v_i^d is the velocity and a_i^d is the acceleration.

It must be pointed out that the gravitational constant $G(t)$ is important in determining the performance of GSA and is defined as a function of time t (Rashedi *et al*. 2009)

$$G(t) = G_0 \times \exp\left(-\beta \times \frac{t}{t_{\max}} \right) \tag{31}$$

where G_0 is the initial value of gravitational constant, β is a constant, t is the current iterations and t_{max} is the maximum iteration number.

In this algorithm, each agent attracts every other agents with the gravitational force that is directly proportional to the product of their masses and inversely proportional to the distance between them. As these masses absorb every other agent, there will not be any recovery for the algorithm if premature convergence happens. In order to overcome this problem and increase the flexibility and efficiency of the algorithm, a new operator is added into the standard GSA. In the current study, we propose a new version of the algorithm, namely global-local gravitational search algorithm (GLGSA) by introducing and applying a new global-local (*GL*) operator during the updating stage of agents' position. In the proposed GLGSA, the position of each solution (agent) will change based on the following equation instead of Eq. (30)

$$x_i^d(t+1) = GL(t) \times \left[x_i^d(t) + v_i^d(t+1) \right] \tag{32}$$

where *GL* is a chaotic decreasing function according to

$$GL(t) = \lambda(t+1) \times \alpha(t) \tag{33}$$

where $\lambda(t)$ is a chaotic map with randomness, irregularity and the stochastic property and it is used to improve the global search ability of the algorithm. In this study, the well-known Logistic map (May 1976, Caponetto *et al*. 2003) is used to generate chaotic variables.

$$\lambda(t+1) = 4 \times \lambda(t) \times (1 - \lambda(t)), \qquad \lambda(1) \in (0,1),\ \lambda(1) \neq 0.25, 0.5, 0.75 \tag{34}$$

In addition, $\alpha(t)$ in Eq. (33) is a nonlinear decreasing function of time according to

$$\alpha(t) = \exp[-4 \times (t / t_{\max})^2] \tag{35}$$

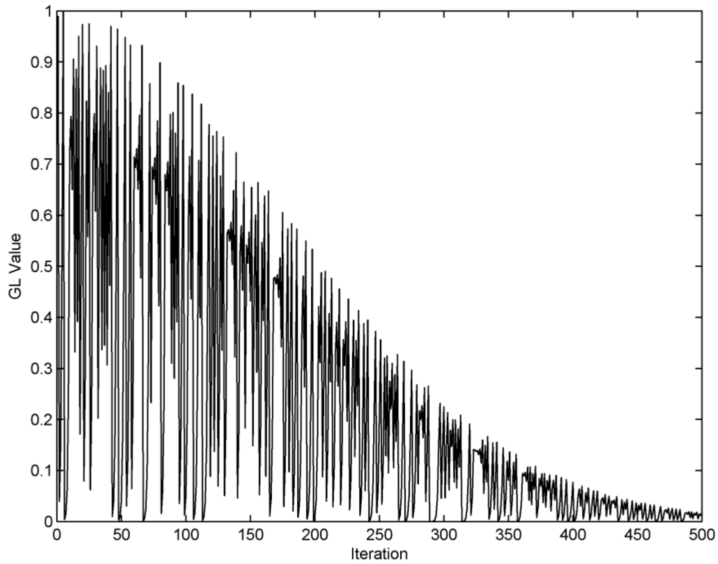

Fig. 3 Variation of *GL* during the iterations

Initialize *N* random positions of the agents
$t = 1$
while $t < t_{\max}$
    Evaluate fitness of each agent
    for $i = 1$ to N
        Determine the best and the worst of the population
        Calculate the mass of agent *i* using Eq. (23)
        Calculate the acceleration of agent *i* using Eq. (28)
        Update the agent's velocity using Eq. (29)
        Evaluate *GL* based on Eq. (33)
        Update the agent's position using Eq. (32)
    end
$t = t + 1$
end while
Output the best solution

Fig. 4 The framework of the GLGSA algorithm

Fig. 3 shows the variation of the *GL* operator during the iteration when $\lambda(1)=0.55$.

As it is presented in Fig. 3, the new operator oscillates and decreases simultaneously during the iterations. The oscillation and chaotic manner of the *GL* during the optimization can increase the global exploration of the algorithm and enhance the ability of escaping from local minima when the agents are prematurely converged to local optima. In addition, the gradually reduction of the proposed operator during the iteration can improve the local search ability and exploitation of the algorithm in the later part of the optimization. In this way, the new algorithm may find an optimum more quickly and accurately. The procedure of the proposed GLGSA algorithm is presented in Fig. 4.

Table 4 Standard benchmark functions

Function Name	Function	Dimension (n)	Range
Quadric	$F_1(X)=\sum_{i=1}^{n}\left(\sum_{j=1}^{i}x_j\right)^2$	30	$[-100,100]^n$
Schwefel	$F_2(X)=\max_i\left\{\lvert x_i\rvert, 1\le i\le n\right\}$	30	$[-100,100]^n$
Quartic	$F_3(X)=\sum_{i=1}^{n}ix_i^4+random[0,1)$	30	$[-1.28,1.28]^n$
Rastrigin	$F_4(X)=\sum_{i=1}^{n}(x_i^2-10\cos(2\pi x_i)+10)$	30	$[-5.12,5.12]^n$
Ackley	$F_5(x)=-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i\right)+20+e$	30	$[-32,32]^n$
Griewank	$F_6(X)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	30	$[-600,600]^n$

4. Experiments

We validated the efficiency and robustness of the proposed GLGSA compared with the original algorithm in two sets of experiments: numerical and foundation optimization applications. In all experiments, the algorithms' parameters are set as follows: population size (N) is 50; maximum iteration number ($t_{\max}$) is 500; G_0 and β are 100 and 20, respectively. In addition, in GLGSA, $\lambda(1)$ is equal to 0.55. These parameters are selected based on the authors' experience and the general recommendations given in the literature (Rashedi *et al*. 2009, Sarafrazi *et al*. 2011). The following subsections describe the experimental methodology.

4.1 Numerical applications

The aim of this experiment is to evaluate and compare the performance of the new method with the original algorithm for solving numerical optimization. Well-defined benchmark problems can be used as objective functions to measure and test the performance of optimization methods. A set of six unimodal and multimodal benchmark functions are used in this experiment. The first three functions are unimodal functions whereas others are multimodal optimization problems with a considerable amount of local minima. All the functions are to be minimized. Table 4 shows the main properties of the selected benchmark functions.

The presented benchmark functions in Table 4 are solved using both GSA and GLGSA algorithms. Every experiment is repeated 30 times independently each starting from a different random population and statistical analyses are presented. The results are shown in Table 5 in terms of the best, worst, median, mean and standard deviation of the solutions obtained in the 30 independent runs by each algorithm. In addition, Fig. 5 graphically presents the comparison of the two algorithms in terms of convergence characteristics in solving the six different problems.

As shown in Table 5, the proposed GLGSA algorithm is able to reach the global optimum for the Rastrigin and Griewank functions. Moreover, the new algorithm could provide a significantly better solution for all other functions based on mean and best fitness values achieved by each method. In terms of standard deviation, the results obtained by the GLGSA in 30 independent runs

Table 5 Minimization result of benchmark functions

Function	Method	Best	Mean	Median	Worst	Standard deviation
F_1	GSA	216.94	469.48	440.28	893.66	156.75
	GLGSA	1.26e-24	1.02e-23	8.05e-24	3.44e-23	7.63e-24
F_2	GSA	1.28	3.41	3.49	6.13	1.19
	GLGSA	3.58e-13	9.32e-13	8.67e-13	1.45e-12	3.33e-13
F_3	GSA	0.012	0.0267	0.0232	0.0734	0.014
	GLGSA	1.49e-6	5.48e-5	4.36e-5	1.93e-4	4.37e-5
F_4	GSA	8.955	16.152	15.919	26.864	5.078
	GLGSA	0.00	0.00	0.00	0.00	0.00
F_5	GSA	3.03e-9	4.78e-9	4.63e-9	6.84e-9	9.14e-10
	GLGSA	4.95e-13	1.25e-12	1.27e-12	2.7e-12	5.51e-13
F_6	GSA	9.17	17.37	17.44	26.08	4.69
	GLGSA	0.00	0.00	0.00	0.00	0.00

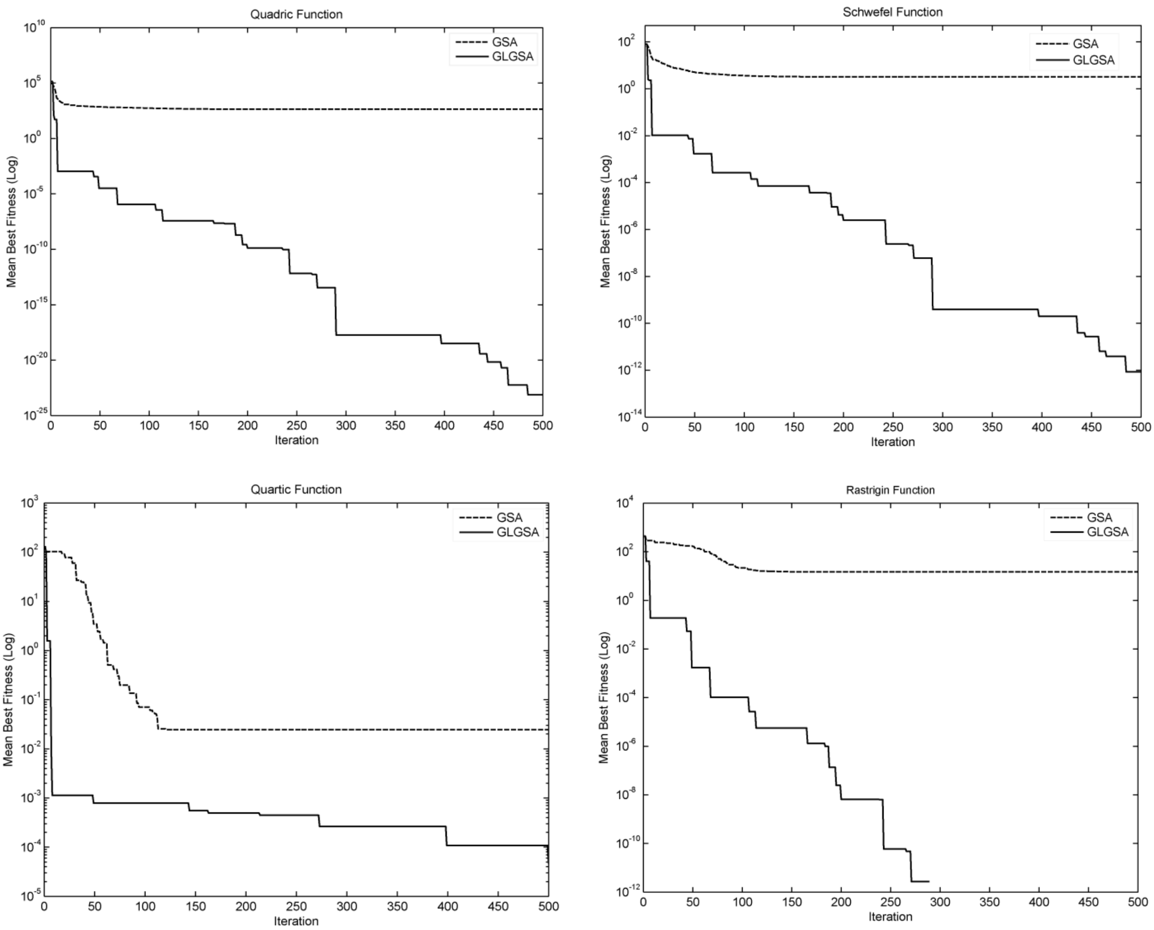

Fig. 5 Convergence performance of GSA and GLGSA on the six test functions

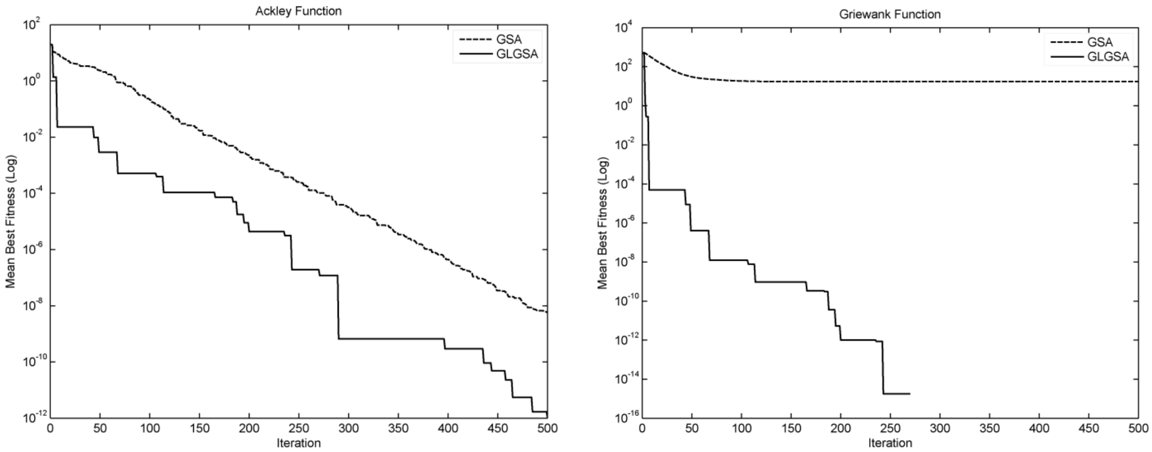

Fig. 5 Continued

are much smaller than those computed by GSA for all the functions, which indicates the higher stability of the new method. In addition, the convergence performance comparison of the algorithms in Fig. 5 shows that the fitness values obtained by GLGSA descend much faster to a lower level than those of GSA. In addition, the evolutionary behavior of GLGSA has an inflexion and greatly outperforms GSA which falls into local trap very quickly during the evolutionary process. As shown in Fig. 5, for all test functions except Ackley function, the resulting history converges very quickly by GSA within the first 100 iterations but does not improve after the initial convergence. In other words, after becoming converged, the GSA loses its ability to explore and then becomes inactive. However, the new algorithm is more successful in exploring the search space. From the above results, it can be concluded that the GLGSA algorithm possesses superior performance in terms of accuracy, convergence speed, stability and robustness when compared to the standard algorithms.

4.2 Foundation optimization application

In this section, the efficiency and robustness of the proposed algorithm for multi-objective optimization of foundation will be investigated. In order to demonstrate, compare and analyze the effectiveness and performance of the new method, two illustrative examples of spread foundation optimization will be presented.

In the following experiments, we use w_1=w_2=0.5 as a weighting factor for each objective function, since both of the objectives (i.e., cost and CO_2 emissions) have equal importance in the given problem, and therefore, they have to contribute equally to the formulation of the objective function. In addition, a penalty function method is utilized to handle the constraints and convert a constrained optimization to an unconstrained one. Now, the problem can be formulated as follows

$$F(\boldsymbol{X}) = 0.5 \times f_1(\boldsymbol{X}) + 0.5 \times f_2(\boldsymbol{X}) + 1000 \times \sum_{i=1}^{k} \max\left\{0, g_i(\boldsymbol{X})\right\}^2 \tag{36}$$

where, $f_1(\boldsymbol{X})$ and $f_2(\boldsymbol{X})$ are CO_2 emissions and cost objective functions defined in Eqs. (3) and (4),

Table 6 Input parameters for design examples 1 and 2

Parameter	Unit	Value for example 1	Value for example 2
Vertical load (*P*)	kN	3000	3000
Moment (*M*)	kN-m	0.0	1000
Load factor	−	1.4	1.4
Effective friction angle of base soil	degree	35	30
Unit weight of base soil	kN/m^3	18.5	18
Young's modulus	MPa	50	35
Poisson's ratio	−	0.3	0.3
Concrete cover	cm	7.0	7.0
Yield strength of reinforcing steel	MPa	400	400
Compressive strength of concrete	MPa	28	30
Long side of column	m	0.5	0.5
Short side of column	m	0.5	0.5
Factor of safety for bearing capacity	−	3.0	3.0
Allowable settlement of foundation	m	0.04	0.04

Table 7 Optimization result for design example 1

Design variable	Unit	CO_2 emissions (Kg)	Cost (Euros)	Multi-objective
Length of the foundation (X_1)	cm	312.72	301.36	308.86
Breadth of the foundation (X_2)	cm	52.41	56.19	53.63
Thickness of the foundation (X_3)	cm	51.72	51.72	51.73
Depth of embedment (X_4)	cm	200	200	200
Longitudinal reinforcement (X_5)	cm^2	8	8	8
Transverse reinforcement (X_6)	cm^2	84.2	76	81.2
Objective function value (GLGSA)		633.8	582.6	608.5
Objective function value (GSA)		646.4	587.3	623.8

respectively. The last term in Eq. (36) is a penalty term and added to the objective function to penalize constraint violations. $g_j(\boldsymbol{X})$ are inequality constraints summarized in Table 3.
The examples are solved using both GSA and GLGSA algorithms and the results are compared.

4.2.1 Design example 1

The first example is concern with the optimum design of an interior spread foundation in dry sand to carry a vertical load. Other input parameters for this example are given in Table 6.

The problem is solved using both the GSA and GLGSA algorithm and the results of the analyses are presented in Table 7. The third and the fourth columns of this table show the values of design variables when the CO_2 emission and cost objectives defined in Eqs. (3) and (4) have been considered separately. In addition, in the last column of Table 7, the results of the single-objective function of the problem as defined in Eq. (36) are presented which considered both cost and emission objective functions simultaneously.

Table 7 shows that the fitness values evaluated by the proposed GLGSA for all objective functions are lower than those computed by GSA and the new method could provide a better solution.

Table 8 Optimization result for design example 2

Design variable	Unit	CO_2 emissions (Kg)	Cost (Euros)	Multi-objective
length of the foundation (X_1)	cm	403.32	382.9	394.06
breadth of the foundation (X_2)	cm	113.6	120.67	116.62
thickness of the foundation (X_3)	cm	51.6	51.67	51.62
depth of embedment (X_4)	cm	200	200	200
longitudinal reinforcement (X_5)	cm^2	10.1	11.75	10.81
transverse reinforcement (X_6)	cm^2	101.2	92.45	97.17
Objective function value (GLGSA)		1466.2	1199.2	1333.3
Objective function value (GSA)		1492.6	1212.1	1388.5

4.2.2 Design example 2

Optimum design of a reinforced spread foundation under an eccentric load in dry sand is investigated in the second example. Other input parameters for this example are given in Table 6. This example is solved using both algorithms to minimize the total cost, CO_2 emissions and combination of both objectives. The results of the analyses are presented in Table 8.

As shown in Table 8, for all objective functions, the optimum values obtained by the proposed GLGSA are lower than those calculated by classical GSA.

5. Conclusions

This article introduced a new version of gravitational search algorithm (GSA) by application of an effective chaotic decreasing function of time referred to as the global-local GSA (GLGSA). The new algorithm has been applied to a series of some mathematical benchmark functions and to the multi-objective optimization of spread foundation. For the optimization of foundation, two objective functions, namely the cost and the amount of embedded CO_2 emissions have been considered simultaneously. The performance of the proposed algorithm as a global optimization technique is investigated using a set of six well-known unimodal/multimodal benchmark functions. In comparison with the results obtained from the classical GSA, the GLGSA algorithm has been verified to possess excellent performance in terms of accuracy, convergence rate, stability and robustness. In addition, in the foundation optimization, the results comparison between presented method and classical GSA demonstrated better performance of the GLGSA in terms of efficiency and robustness.

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