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Stiffness modeling of RC columns reinforced with plain rebars

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Abstract. Inaccurate predictions of effective stiffness for reinforced concrete (RC) columns having plain (undeformed) longitudinal rebars may lead to unsafe performance assessment and strengthening of existing deficient frames. Currently utilized effective stiffness models cover RC columns reinforced with deformed longitudinal rebars. A database of 47 RC columns (33 columns had continuous rebars and the remaining had spliced reinforcement) that were longitudinally reinforced with plain rebars was compiled from literature. The existing effective stiffness equations were found to overestimate the effective stiffness of columns with plain rebars for all levels of axial loads. A new approach that considers the contributions of flexure, shear and bond slip to column deflections prior to yielding was proposed. The new effective stiffness formulations were simplified without loss of generality for columns with and without lap-spliced plain rebars. In addition, the existing stiffness models for the columns with deformed rebars were improved while taking poor bond characteristics of plain rebars into account.

Keywords: reinforced concrete; columns; effective stiffness; plain rebars; guidelines

1. Introduction

Effective stiffness predictions in performance based seismic assessment of existing structures primarily affect the dynamic characteristics of structures via structural demand estimations (lateral deformations and shear) and consequently the cost of seismic retrofit. In this respect, the strong dependence of effective stiffness on column axial load level was recognized by various researchers (Hage *et al*. 1974, Paulay and Priestley 1992, Mehanny *et al*. 2001, Priestley 2003, Khuntia and Ghosh 2004, Elwood and Eberhard 2006, Elwood and Eberhard 2009, Kumar and Singh 2010). In addition, the influence of other parameters such as shear span to depth ratio, eccentricity and longitudinal reinforcement on effective stiffness was investigated and proposed stiffness formulations were modified accordingly (Mehanny *et al*. 2001, Khuntia and Ghosh 2004, Mirza 1990). Moreover, simplified design implications were imposed in various structural guidelines (ACI 318 2008, FEMA 356 2000, ASCE 2007b 2007, TEC-07 2007, EC-8 2005) using the axial load level as the main design parameter. The literature study conducted on the effective stiffness reveals that all effective stiffness formulations are for columns longitudinally reinforced with deformed rebars. ASCE/SEI 41 update report (Elwood *et al*. 2007) states that lower effective stiffness values might be attained for the structural components with plain rebars due to the

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reduced bond stress levels without further elaboration. Thus, it is expected that the effective stiffness recommended in assessment guidelines may show a tendency to overestimate the effective stiffness of vertical load bearing members having plain reinforcement. In addition, since the effective stiffness estimations substantially influence the anticipated seismic deformation demands, internal force distribution and dynamic response (Mehanny *et al*. 2001, Khuntia and Ghosh 2004, Elwood and Eberhard 2009, Kumar and Singh 2010), special attention should be paid especially for the performance based assessment of structures reinforced with plain rebars. Therefore, the aim herein is to propose sound column effective stiffness estimations for columns having plain rebars.

2. Column database with plain reinforcement

In order to examine the stiffness properties of RC columns with plain rebars, a database including 47 RC columns was formed (İlki *et al*. 2004, Arani *et al*. 2010, Ludovico *et al*. 2009, Lampropoulos and Dritsos 2011, Acun 2010, Ozcan *et al*. 2008, Ozcan *et al*. 2010, Verderame *et al*. 2008, Yalcin *et al*. 2008, Ozcan *et al*. 2010, Ozcan 2009, Marefat *et al*. 2009, Bousias *et al*. 2007, Bournas *et al*. 2009, Verderame *et al*. 2008) in which 33 of the columns were continuously reinforced and the remaining columns were spliced with various splice lengths. The database consisted of monotonic and cyclic lateral force - tip deflection responses of rectangular RC columns that were tested in single curvature. The axial load upper limit as taken from the database is 63% of the axial load carrying capacity with a shear span to depth ratio range of 2.8 to 9.2. In addition, longitudinal reinforcement ratios ranging between 0.6% and 2.5% were used with splice lengths from 15 to 40 bar diameters (Table 1). The monitored failure mechanisms for the columns in the database were flexure dominant (i.e., longitudinal rebar yielding in tension accompanied by other limit states such as concrete crushing, rebar buckling and rebar fracture) in addition to lap-splice failures for inadequate splice lengths. According to Fardis (2009), the insufficient bond performance of plain rebars hampered the mobilization of the tensile rebars and consequently the development of full sectional strength using a database of 40 columns. In this study, the development of the section strength for the database columns with continuous rebars was provided by the anchorage and penetration of longitudinal rebars by means of abundant extensions or end hooks into the specimen footings. As stated by Cosenza *et al*. (2006), and Ozcan *et al*. (2008), the presence of end hooks for the longitudinal rebars engenders the same behavior of embedded rebar extensions into the specimen footing such that both cases have the sufficient development length. Hence, the end hook and the straight rebar extension at the footing can be treated as a straight longitudinal rebar and the database calculations are carried out pursuant to this assumption.

2.1 Spliced steel model

Instead of a constitutive steel model that was used for continuous rebars (Eq. (1)), a spliced steel model was generated for plain rebars considering the failure mechanism as pull-out (Binici and Mosalam 2007, Talaat 2007) rather than splitting failure that is usually observed in columns with deformed reinforcing bars (Eligehausen *et al*. 1983, Zuo and Darwin 2000).

For the spliced steel model, the strain decomposition method was implemented in which the total strain (ε_{st}) was assumed to be decomposed into elongation (ε_s) and slip (ε_{ss}) components ($\varepsilon_{st}=\varepsilon_s+\varepsilon_{ss}=\varepsilon_s+u/L_d$). Herein, u/L_d term denotes the average slip (u) along the splice length (L_d). The

Table 1 Column properties in the database

Research	**Type**	**Specimen**	b *mm*	h *mm*	L *mm*	f_c' *MPa*	cc *mm*	f_y *MPa*	n	ρ_l	$\frac{L_d}{d_b}$
İlki *et al*. 2004	**Continuous**	C-0-1	200	300	1200	9.0	15	336	0.47	0.01026	0
Arani *et al*. 2010		WOS-M	250	250	850	23.9	15	370	0.15	0.00724	0
		WOS-C	250	250	850	22.9	15	370	0.15	0.00724	0
Ludovico *et al*. 2009		S300P-M	300	300	1800	18.9	20	330	0.20	0.01005	0
		S300P-C	300	300	1800	18.9	20	330	0.20	0.01005	0
		R300P-C	500	300	1800	18.9	20	330	0.10	0.00905	0
		R500P-C	300	500	1800	18.9	20	330	0.10	0.00905	0
Lampropoulos and Dritsos 2011		C	250	250	1600	27.0	15	425	0.41	0.00985	0
Acun 2010		1P2	350	350	2000	13.5	20	315	0.20	0.01005	0
		2P3	350	350	2000	12.2	20	315	0.20	0.01005	0
		3P3_NO4	350	350	2000	13.1	20	315	0.40	0.01005	0
		4P4	350	350	2000	12.4	20	315	0.20	0.01005	0
		5P5	350	350	2000	11.4	20	315	0.20	0.01005	0
		6PV1	350	350	2000	12.5	20	315	0.20	0.01005	0
		7P3_U	350	350	2000	13.2	20	315	0.20	0.01005	0
Ozcan *et al*. 2008		S-NL-0-34	350	350	2000	14.0	30	287	0.41	0.01662	0
Ozcan *et al*. 2010		S1	200	400	2000	12.0	30	287	0.51	0.02545	0
Verderame *et al*. 2008		M-270B1	300	300	1570	25.0	20	355	0.12	0.00754	0
		M-270B2	300	300	1570	25.0	20	355	0.12	0.00754	0
		M-540B1	300	300	1570	25.0	20	355	0.24	0.00754	0
Yalcin *et al*. 2008		L0C0	200	400	1610	16.0	15	320	0.25	0.01155	0
Ozcan *et al*. 2010		S-H-0-00	350	350	2000	20.0	30	287	0.33	0.02482	0
Ozcan 2009		C1	350	350	2000	20.0	30	287	0.29	0.01662	0
Marefat *et al*. 2009		PN_CS3	150	300	825	23.0	20	356	0.00	0.00559	0
		PN_CS4	150	300	825	27.8	20	356	0.00	0.00894	0
		PC_C2	200	200	1375	25.0	20	356	0.00	0.01979	0
		PC_M2	200	200	1375	29.0	20	356	0.00	0.01979	0
		PN_C1	200	150	1375	25.0	20	356	0.00	0.02262	0
Bousias *et al*. 2007		Q_0	250	250	1600	27.0	15	313	0.44	0.00985	0
Bournas *et al*. 2009		L0S_C	250	250	1600	29.8	15	372	0.28	0.00985	0
Verderame *et al*. 2008		C-270B1	300	300	1570	25.0	15	355	0.12	0.00754	0
		C-540B1	300	300	1570	25.0	15	355	0.24	0.00754	0
		C-540B2	300	300	1570	25.0	15	355	0.24	0.00754	0
İlki *et al*. 2004	**Lap-Spliced**	LS-0-1	200	300	1200	9.4	15	336	0.45	0.01026	40
Arani *et al*. 2010		SOS-C	250	250	850	24.0	15	370	0.15	0.00724	40
Verderame *et al*. 2008		M-270A1	300	300	1570	25.0	20	355	0.12	0.00754	40
		M-270A2	300	300	1570	25.0	20	355	0.12	0.00754	40
		M-540A1	300	300	1570	25.0	20	355	0.24	0.00754	40
Verderame *et al*. 2008		C-270A1	300	300	1570	25.0	15	355	0.12	0.00754	40
		C-270A2	300	300	1570	25.0	15	355	0.12	0.00754	40
		C-540A1	300	300	1570	25.0	15	355	0.24	0.00754	40
Yalcin *et al*. 2008		L50C0	200	400	1610	17.0	15	320	0.25	0.01155	36
Bousias *et al*. 2007		Q_0L2	250	250	1600	30.3	15	313	0.42	0.00985	25
		Q_0L2a	250	250	1600	28.1	15	313	0.57	0.00985	25
Arani *et al*. 2010		HOS-C	250	250	850	24.8	15	370	0.15	0.00724	20
Bousias *et al*. 2007		Q_0L1	250	250	1600	30.3	15	313	0.41	0.00985	15
		Q_0L1a	250	250	1600	28.1	15	313	0.63	0.00985	15

strain components were calculated iteratively while satisfying equilibrium (Eq. (2)) and bond stress equations (Eqs. (3a)-(3d)) concurrently (Binici and Mosalam 2007, Talaat 2007).

$$f_s = \begin{cases} E_s \varepsilon_s & \varepsilon_s \le \varepsilon_{sy} \\ f_y & \varepsilon_{sy} < \varepsilon_s \le \varepsilon_{sh} \\ f_{su} - \left(f_{su} - f_y\right)\left(\dfrac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sh}}\right)^2 & \varepsilon_{sh} \le \varepsilon_s \end{cases} \tag{1}$$

where E_s, ε_s and f_s denote steel elasticity modulus, instantaneous strain and stress. The parameters of ε_{sh}, ε_{su}, f_y, and f_{su} indicate hardening strain, ultimate strain, yield stress and ultimate stress, respectively. Concerning the equilibrium of forces at splice location, Eq. (2) indicates the steel stress (f_s) that can be transferred by bond stresses (τ_m) along the splice length (L_d) of a rebar having a diameter of d_b. The adopted constitutive relationship of bond stress (τ_m) vs. slip (u) (Xiao and Ma 1997) depends on maximum bond stress ($\tau_{\max}$) and maximum slip ($u_{\max}$) in MPa and mm, respectively. The constant r was used as 1.5 for Grade 40 steel and for the Eqs. (3a)-(3d), f'_c and σ_3 denote concrete compressive strength and confining stresses in MPa, respectively.

$$f_s = 4\tau_m L_d / d_b \tag{2}$$

$$\tau_m = \frac{\tau_{\max} r (u/u_{\max})}{r - 1 + (u/u_{\max})^r} \tag{3a}$$

$$\tau_{\max} = 20\sqrt{f_c'}/d_b + 1.4\sigma_3 \tag{3b}$$

$$u_{\max} = 0.25(1 + 75\sigma_3 / f_c') \tag{3c}$$

$$r = r_0 - 13\sigma_3 / f_c' \tag{3d}$$

Since the confining stresses primarily affect the bond stresses and corresponding slips for deformed reinforcement pertinent to splitting type of bond failure (Eligehausen *et al*. 1983, Zuo and Darwin 2000), the confining stresses (σ_3) included in Eqs. (3b)-(3d) were taken as zero as it would merely affect the bond behavior of plain rebars. In addition, the maximum bond stress expression given in Eq. (3b) was modified for plain rebars using the lap-spliced beam database that was obtained from literature (Hassan 2011). The acquired database comprised of 15 lap-spliced RC beams that were tested under 4-point bending and reinforced with plain rebars having bar diameters (d_b) between 19.0 to 31.8 mm (Table 2).

The splices were located at the bottom tension reinforcement with various splice lengths ($12.8d_b$ – $32.4d_b$) in the middle of the beams. Analytical calculations were based on standard section analysis carried out by the constitutive models of concrete (Mander *et al*. 1988) and spliced steel. In the analyses, the analytically obtained beam moment capacities ($M_{n,a}$) were compared with experimental moment capacities ($M_{n,\exp}$) in order to acquire the maximum bond stress expression for plain rebars that can be used instead of Eq. (3b). Hereby, the maximum bond stress equation was determined as $\tau_{\max} = 0.5\sqrt{f_c'}$ along with the constants of $u_{\max}$=0.25 and r=1.5 while providing the analytical to experimental moment capacity ratio close to unity (Table 2).

Table 2 Specimen properties with results for spliced beam database (Hassan 2011)

Specimen	d_b mm	$\frac{L_d}{d_b}$	f_c' MPa	f_y MPa	$M_{n,\exp}$ kNm	$M_{n,a}$ kNm	$\frac{M_{n,a}}{M_{n,\exp}}$
19-305	19.0	16.0	17.4	326	24.29	24.66	1.015
19-410	19.0	21.6	17.4	326	26.12	32.71	1.252
19-510	19.0	26.8	18.7	326	28.38	41.34	1.457
19-610	19.0	32.1	21.0	326	55.88	50.94	0.912
25-410	25.3	16.4	23.7	346	54.02	50.91	0.942
25-510	25.2	20.4	24.0	346	61.75	62.00	1.004
25-610	25.3	24.4	22.8	346	67.38	70.01	1.039
25-810	25.3	32.4	19.8	346	90.53	77.04	0.851
25-410I	25.3	16.4	21.5	346	46.06	48.34	1.050
25-510I	25.3	20.4	20.8	346	55.61	57.68	1.037
25-610I	25.3	24.4	21.5	346	48.91	67.94	1.389
32-410	31.7	12.8	19.8	348	47.55	55.46	1.166
32-610	31.7	19.1	15.8	348	68.34	71.17	1.041
32-810	31.7	25.3	19.7	348	96.68	92.35	0.955
32-910	31.8	28.4	19.2	348	103.55	97.92	0.946
						Mean	1.071
						St.Dev.	0.170

2.2 Stiffness approximations and code comparisons

The experimental effective stiffness calculations for the columns were based on estimated yield deflections ($\Delta_{y,\exp}$) according to the yield forces (F_{fy}) that were determined by either standard section analysis or load deflection curves due to the unavailability of the measured strain data for longitudinal reinforcement. Thus, the incipient yield deflections (Δ_{fy}) were determined initially as corresponding to the preceding lateral force (F_{fy}) at which the tensile reinforcement yielded or concrete compressive strain reached 0.002 by using standard section analysis with the constitutive models for concrete (Mander *et al*. 1988), steel and spliced steel (Fig. 1(a)). In accordance with Elwood and Eberhard (2009), for the cases where the experimental lateral strength ($F_{\max,\exp}$) did not exceed the calculated yield force (F_{fy}) by a minimum 7%, the yield force was assumed as 80% of the experimental lateral strength concerning the influence of axial load and shear induced failure mechanisms (Fig. 1(b)). Herein, the inhibition of tensile rebars to reach yielding was speculated to be a consequence of either the high axial load level that prevailed concrete to reach crushing strain or the shear interaction due to low shear span to depth ratios or inadequate transverse reinforcement that surpassed the flexural response of columns. The experimental yield deflections ($\Delta_{y,\exp}$) were acquired by extrapolating the attained incipient yield deflections (Δ_{fy}) in proportion to the ratio of the lateral force at which the strain at extreme concrete fiber reached 0.004 ($F_{0.004}$) to the incipient yield force (F_{fy}) (Eq. (4a)). The yield curvatures (κ_y) were determined similarly as shown in Figs. 1(c)-(d) (Eq. 4b). The extrapolation is essential due to the compatibility requirement for the bi-linearization of structural response that is majorly implemented in performance based assessment of structures while reflecting the definition of yield as compatible with the previous studies (Elwood and Eberhard 2006, 2009). Thus, the experimental effective

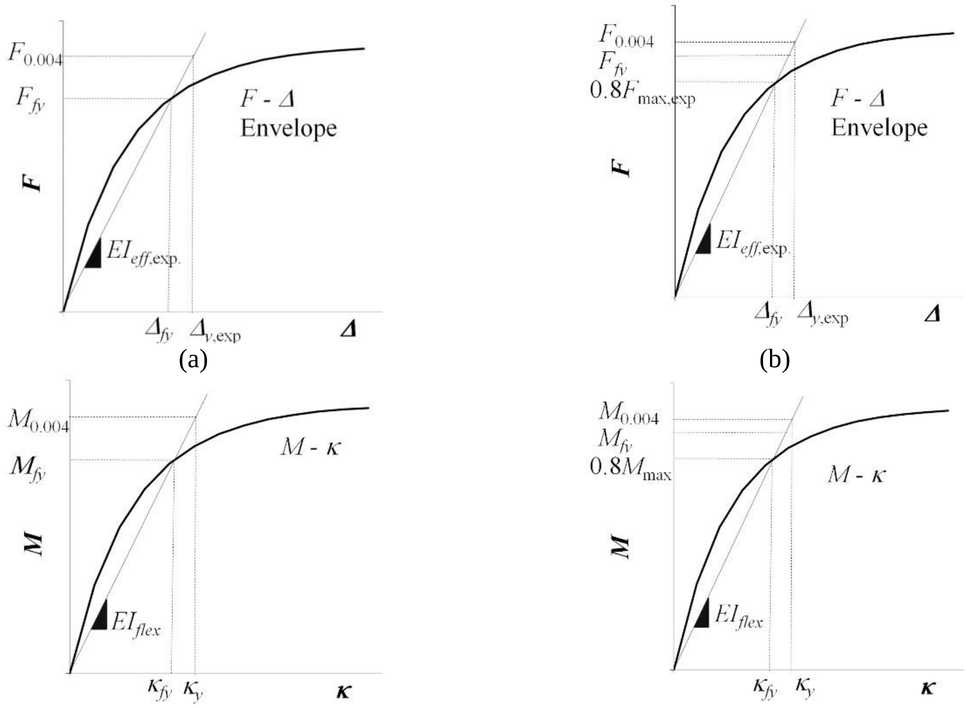


Fig. 1 Determination of effective stiffness for columns that (a) yield, (b) do not yield by force deflection envelopes and flexural stiffness for columns that (c) yield and (d) do not yield by moment - curvature plot

stiffness ($EI_{eff,\exp}$) is defined in Eq. (5a) assuming a linear curvature distribution over the column shear span (L) and the flexural stiffness (EI_{flex}) is calculated as shown in Eq. (5b).

$$\Delta_{y,\exp} = \Delta_{fy} \, F_{0.004} / F_{fy} \tag{4a}$$

$$\kappa_y = \kappa_{fy} \, M_{0.004} / M_{fy} \tag{4b}$$

$$EI_{eff,\exp} = F_{0.004} L^3 / 3\Delta_{y,\exp} \tag{5a}$$

$$EI_{flex} = M_{fy} / \kappa_{fy} = M_{0.004} / \kappa_y \tag{5b}$$

Herein, 11 columns out of the database were identified as not yielding while the experimental lateral strength was not monitored to exceed the calculated yield force at least by 7%. Herein, 2 beams and 7 columns were not identified to yield due to the relatively low shear span to depth ratio of 2.8 and high axial load ratio beyond 30% that introduced the effect of shear and axial load induced failure mechanisms, respectively. The remaining 2 columns (1 lap-spliced column and 1 column with relatively low axial load ratio of 12%) were observed to exceed the calculated yield force by approximately 5%. In reference to the measured effective stiffnesses for continuous and lap-spliced columns, the influences of key parameters ranging in investigation limits are presented in Figs. 2(a)-(i).

The most significant dependence can be observed for axial load level ($P/A_g f_c'$) in which P and

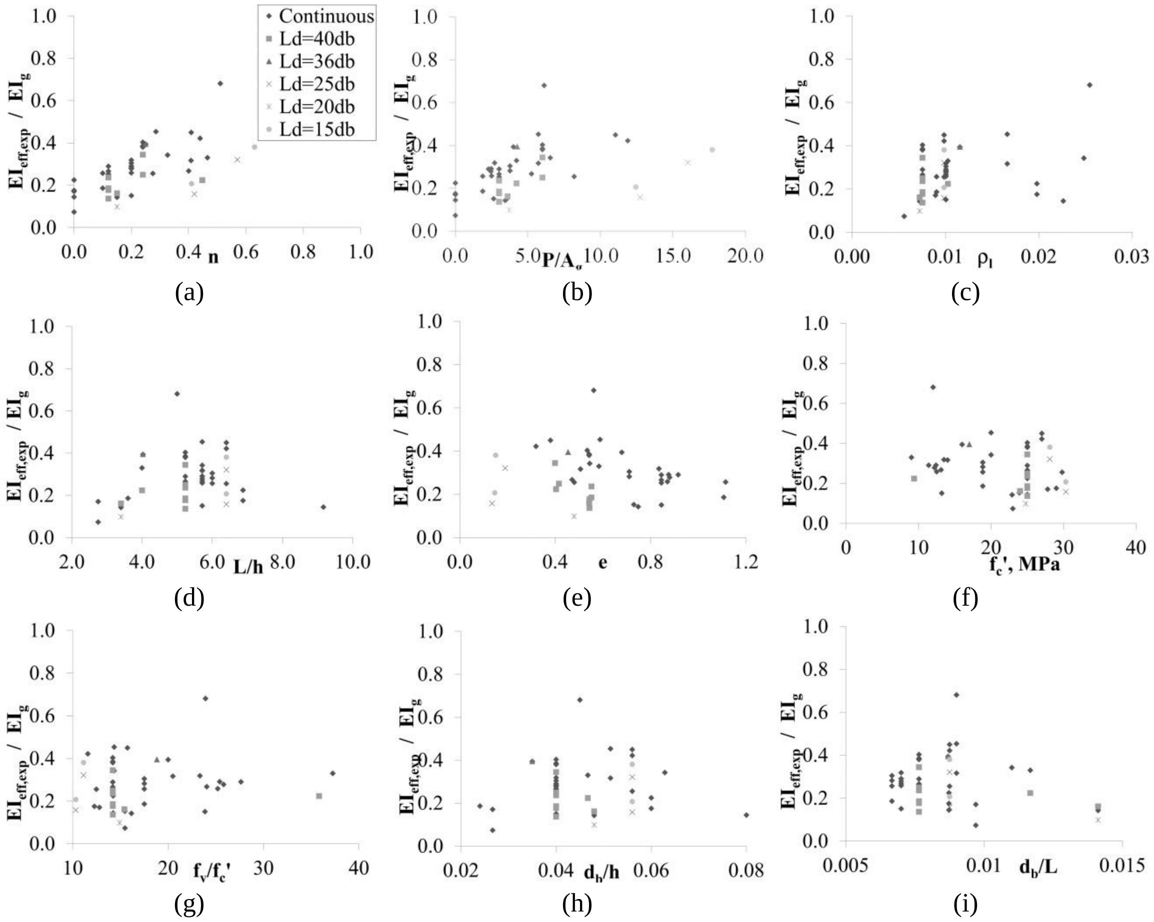


Fig. 2 Experimental effective stiffness variations according to key parameters.

A_g specify the axial load and gross cross-section area, respectively (Fig. 2(a)). However, due to the cluster of data below 7 MPa and increasing scatter beyond that region impeded the interpretation of the dependence of effective stiffness on mean axial stress (P/A_g) (Fig. 2(b)), additional tests to investigate the column performances beyond 7 MPa are needed for more reliable evaluation. Since the stiffening effect of axial compression on effective stiffness was verified among many researchers in the literature (Mehanny *et al*. 2001, Khuntia and Ghosh 2004, Elwood and Eberhard 2006, 2009, Kumar and Singh 2010), a similar response was observed for the columns longitudinally reinforced with plain rebars (Fig. 2(a) and 2(b)). The direct correlation between longitudinal reinforcement ratio (ρ_l) and effective stiffness can be attributed to the comparative increase in moment capacity rather than yield deflection with increasing reinforcement ratio (Fig. 2c) (Khuntia and Ghosh 2004, Elwood and Eberhard 2006, 2009, Kumar and Singh 2010) however additional tests for longitudinal reinforcement ratios beyond 1% can provide better interpretation.

In addition, a slight influence of shear span to depth ratio (L/h) (Elwood and Eberhard 2006, Elwood and Eberhard 2009, Mirza 1990) and eccentricity ratio ($e=M_{0.004}/Ph$) (Khuntia and Ghosh 2004, Mirza 1990) on the effective stiffness can be observed (Figs. 2(d)-(e)). Since the eccentricity

Table 3 The guideline expressions for effective stiffness

Guideline	Expression (EI_{eff}/EI_g)
ACI 318 (2008)	$\begin{cases} 0.35, n < 0.1 \\ 0.70, n \geq 0.1 \end{cases}$
FEMA 356 (2000)	$0.5 \leq n + 0.2 \leq 0.7$
ASCE 41 (2007)	$0.3 \leq n + 0.2 \leq 0.7$
TEC-07 (2007)	$0.4 \leq 4/3n + 0.8/3 \leq 0.8$
EC-8 (2005)	$\dfrac{M_y L}{3EI_g\theta_y}, \theta_y = \dfrac{\kappa_y L}{3} + 0.013\left(1 + 1.5\dfrac{h}{L}\right) + 0.13\kappa_y \dfrac{d_b f_y}{\sqrt{f_c'}}$
Elwood and Eberhard (2006)	$0.2 \leq 5/3n + 0.4/3 \leq 0.7$
Biskinis and Fardis (2010)*	$a\left(0.8 + \ln\left(\max\left(\dfrac{L}{h}, 0.6\right)\right)\right)\left(1 + 0.048\min\left(50MPa, \dfrac{P}{A_g}\right)\right)$

*a is 0.081 for columns and 0.10 for beams.

ratio is infinite for the beams, the cases with no axial load were not included in Fig. 2(e). The weakest interrelation with the column effective stiffness was examined for concrete compressive strength (f'_c), steel yield stress to concrete compressive strength ratio (f_y/f'_c), bar diameter to depth (d_b/h) and shear span ratios (d_b/L) as similar symptoms were attained in literature (Figs. 2(f)-(i)) (Elwood and Eberhard 2006, Elwood and Eberhard 2009). In line with the experimental data obtained from the column database, the stiffness approximations for columns with deformed rebars given in structural assessment guidelines (Table 3) were evaluated regarding the Figs. 3(a)-(b). Since all guidelines (ACI 318 2008, FEMA 356 2000, ASCE 2007b 2007, TEC-07 2007, EC-8 2005) predicted effective stiffness of the columns presuming deformed rebars as longitudinal reinforcement, a significant overestimation in effective stiffnesses was monitored for the columns longitudinally reinforced with plain rebars for all levels of axial load as shown in Fig. 3a. Herein, FEMA 356 (FEMA 356 2000) and TEC-07 (Turkish Earthquake Code - 2007) (TEC-07 2007) gave the almost upper bound predictions of effective stiffness for low and high axial load ratios that represent the actual cases for beams and columns, respectively.

For Eurocode 8 (EC-8 2005), the moments and curvatures at yield were computed according to the guideline provisions and compared with the aforementioned results of standard section analysis as shown in Figs. 4(a)-(b). Herein, the overestimated yield moments were observed to have an increasing dispersion with the axial load as similar to the underestimated yield curvatures. Since the lower bond stress levels of plain rebars and poor bond slip performance enhanced the slip component of the yield deflections as indicated in the splice model and in the previous research (Ozcan et. al. 2008, Ozcan et. al. 2010, Hassan 2011), the effective stiffnesses were monitored to be overestimated particularly for low axial load levels. However, for the columns under high axial load ratio, the development of slip induced deflections was not permitted as much as the columns under low axial load and the slip contribution can be expected to be higher for lap-spliced columns regarding the lower bond stress levels.

The outcomes of the previous research (Fardis 2009, Biskinis and Fardis 2010, Fardis 2013) claimed the influence of poor bond performance of plain rebars on the yield deflection, yield

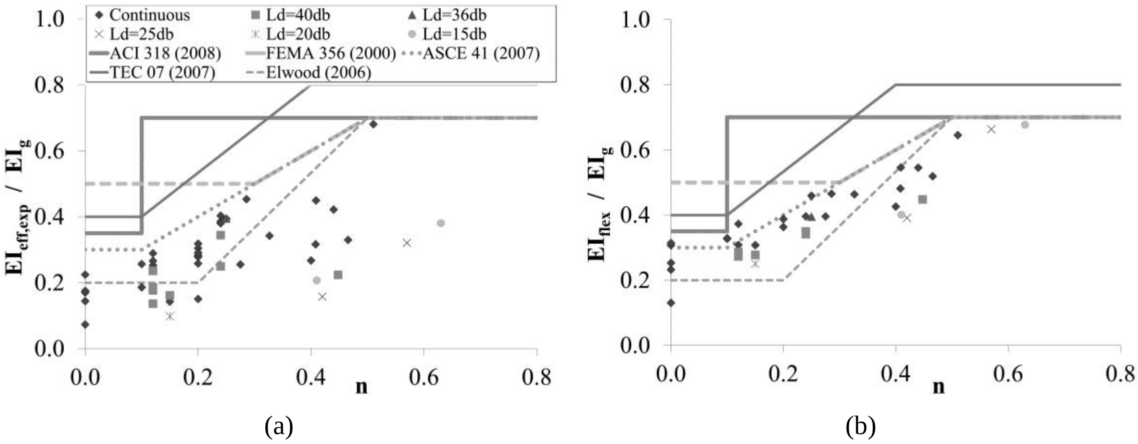
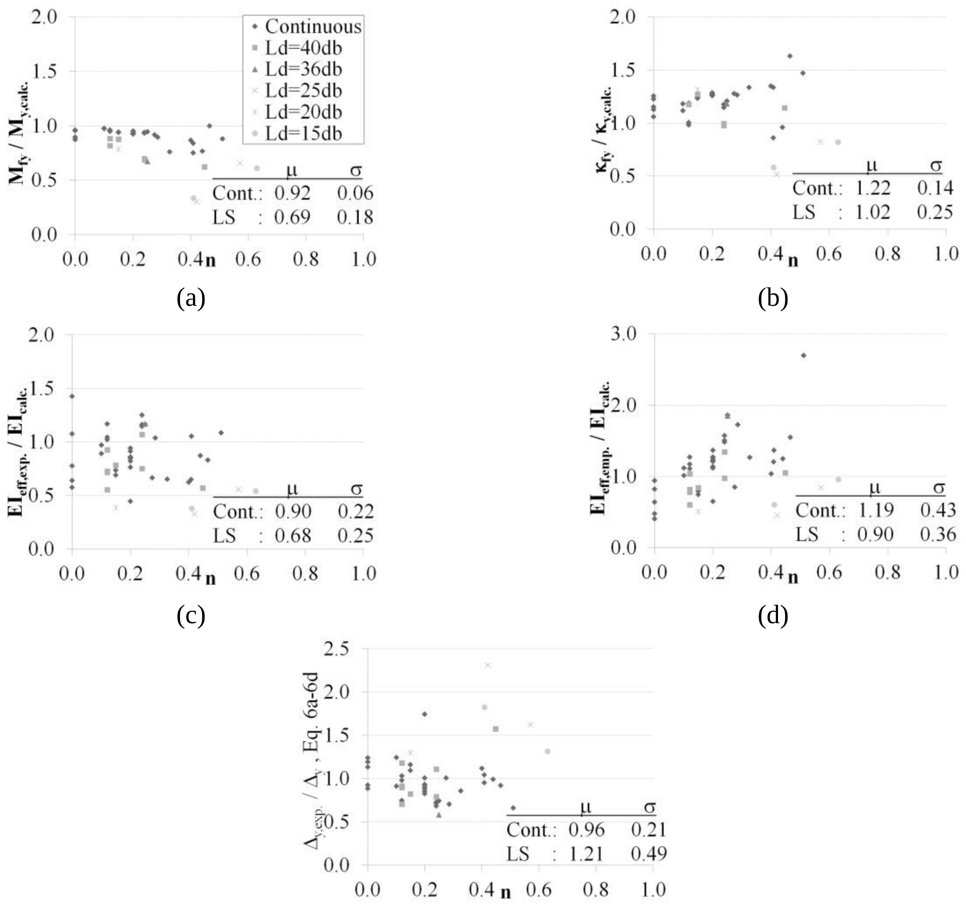


Fig. 3 The normalized (a) experimental effective stiffness and (b) flexural stiffness variations according to code models and previous studies

strength and effective stiffness of RC columns to be insignificant considering a database including 20 columns reinforced with plain rebars. Considering the inadequate bond strength along plain rebars that inhibited the full mobilization of yield strength, the yield moments were overestimated with an experimental to prediction ratio of 0.95 (Fardis 2009) and it can be anticipated that the development of yield strength can be inhibited further for lap-spliced columns as compared to the columns with continuous rebars. Thus, this phenomenon was found to be responsible for the reduced mean levels for yield moments (0.92 and 0.69 for continuous and lap-spliced, respectively) with a low scatter as shown in Fig. 4(a). Further, the suggested empirical formulation by Biskinis and Fardis (2010) (Table 3) was observed to underestimate the effective stiffnesses as compared to EC8-3 while satisfying slightly higher mean and standard deviation (Figs. 4(c)-(d)). However, the effective stiffness and yield properties of the columns should be evaluated while preventing shear force underestimations or displacement demand overestimations for low effective stiffnesses (Elwood and Eberhard 2009, Fardis 2009).

The chord rotations at yield (θ_y) were computed accordingly with the components of flexure, shear and bond slip (Table 3) in order to determine the effective stiffnesses that were overestimated beyond axial load ratios of approximately 0.2 (Fig. 4(c)) while ignoring the effect of plain rebars. The closest and most reliable estimations were obtained by Elwood *et al*. (2006) for low axial load levels up to the ratio of approximately 0.3, above which the effective stiffness was still overestimated. For all the investigated guidelines, effective stiffness overestimations can be attributed to the consideration of deformed rebars. Regarding the normalized flexural stiffness (EI_{flex}/EI_g), the most consistent predictions were obtained by ASCE 41 (ASCE 2007b 2007) and Elwood *et al*. (2006) as shown in Fig. 3b. Since the other tip deflection contributors of shear and bond slip were not accounted, overestimations were majorly observed for the other guidelines (ACI 318, FEMA 356 and TEC-07) for all levels of axial load. Thus, the flexural stiffness estimations were reduced using the three component approach in order to provide more reliable predictions for the effective stiffness of the columns that were longitudinally reinforced with plain rebars.



(e)

Fig. 4 The comparison of experimental to calculated (a) yield moments, (b) yield curvatures, (c) effective stiffnesses according to Eurocode 8 (EC-8, 2005), (d) Biskinis and Fardis (2010) and (e) yield deflections

3. Effective stiffness modeling

The effective stiffness modeling estimations were based on the idea of reducing the flexural stiffness that was computed by standard section analysis pursuant to the adverse effects of shear and bond slip deformations. Since this phenomenon was a consequence of the three component approach (Sozen 1974, Lehman and Moehle 1998) in order to estimate column tip deflections, the same idea was utilized in effective stiffness computations for columns (Khuntia and Ghosh 2004, Elwood and Eberhard 2009a, b, Kumar and Singh 2010). The flexural component (Δ_{flex}) of the column yield deflection (Δ_y) (Eq. (6a)) was computed as shown in Eq. (6b) by integrating the curvature distribution which was assumed to be linear over the column shear span (L). Eq. (6c) enables the calculation of the shear term (Δ_{shear}) of the yield deflection while being comparatively lower regarding the other sources of deformation such as flexure and bond slip. The effective shear area (A_v=0.83of gross area, A_g for rectangular columns) (Elwood and Eberhard 2009, Kumar and Singh 2010, Sezen 2002) and the effective shear modulus (G_{eff}=G/2=E_c/4.8) (Elwood and Eberhard

2009) terms were concerned in order to reflect the deteriorating impact of shear cracking on column stiffness. Bond slip deflections (Δ_{slip}) were computed considering the rigid body rotation of the column as a result of elongation and slip of longitudinal rebars at the interface of column base and footing (Elwood and Eberhard 2009). Thereby, as shown in Eq. (6d), the bond slip component of yield deflection was calculated using the yield curvature (κ_y), rebar diameter (d_b) and column shear span (L). The bond strength for plain rebars was calculated regarding the beam database and previous studies (Ozcan *et al*. 2008, Hassan 2011) by recalibrating Eq. (3b) as ($\tau_{max} = 0.5\sqrt{f_c'}$). The experimental and analytical yield deflections are compared in Fig. 4(e) while providing close estimations except five lap-spliced columns for which the yield deflections were underestimated by a factor of 1.58 to 2.31.

$$\Delta_y = \Delta_{flex} + \Delta_{shear} + \Delta_{slip} \tag{6a}$$

$$\Delta_{flex} = \kappa_y \frac{L^2}{3} = \frac{M_{0.004}}{3EI_{flex}} \tag{6b}$$

$$\Delta_{shear} = \frac{M_{0.004}}{A_v G_{eff}} = \frac{4.8M_{0.004}}{A_v E_c} \tag{6c}$$

$$\Delta_{slip} = \kappa_y \frac{f_s d_b L}{8\tau_{max}} = \kappa_y \frac{d_b L}{4} \frac{f_y}{\sqrt{f_c'}} \frac{f_s}{f_y} \tag{6d}$$

For all columns in the database, the flexural component of yield deflection increased proportionally with the axial load as compatible with previous observations (Elwood and Eberhard 2006, Elwood and Eberhard 2009). However, this increase was more significant for lap-spliced columns (Figs. 5(a)-(b)) since the ratio of steel stress to yield stress in tension rebars (f_s/f_y) and consequently the bond slip deflections tended to diminish more rapidly for lap-spliced columns while the suppressing effect of axial load prevents the development of tensile stresses in longitudinal rebars.

For flexure dominated columns in which the tip deflection is majorly comprised of flexural deflections, the axial load level was observed to primarily designate the contribution of slip induced deflections which were not observed to exceed flexural component at all levels of axial load (Ozcan *et al*. 2008 and 2010). The development of slip induced deflections was hampered by the increase in axial load that diminished the slip contribution to yield deflections (Fig. 5(a) and 5(b)). In addition, the curvatures and consequent tensile steel stresses for lap-spliced columns could not be developed as much as continuous columns for increasing axial load levels (Fig. 6). For increasing longitudinal reinforcement ratios, no significant trend was observed for both continuous and lap-spliced columns (Figs. 5(a)-(b)). It was demonstrated that as the columns became more slender, the flexural deflection components increased and thus a flexure dominant response occurred for all columns. Higher contribution of bond slip component was induced by either increasing bar diameter to shear span ratio or splice length due to enhancing the tensile stresses that can be developed in longitudinal rebars. Further, Eq. (5a) was rewritten in terms of dimensionless parameters encapsulating Eq. (5b) and Eqs. (6a)-(6d) that yielded Eq. (7) where $\alpha=EI_{flex}/EI_g$ denotes normalized flexural stiffness and $r_v^2=I_g/A_g$ is the radius of gyration in loading direction. I_g and A_g denote the gross moment of inertia and gross cross-section area, respectively.

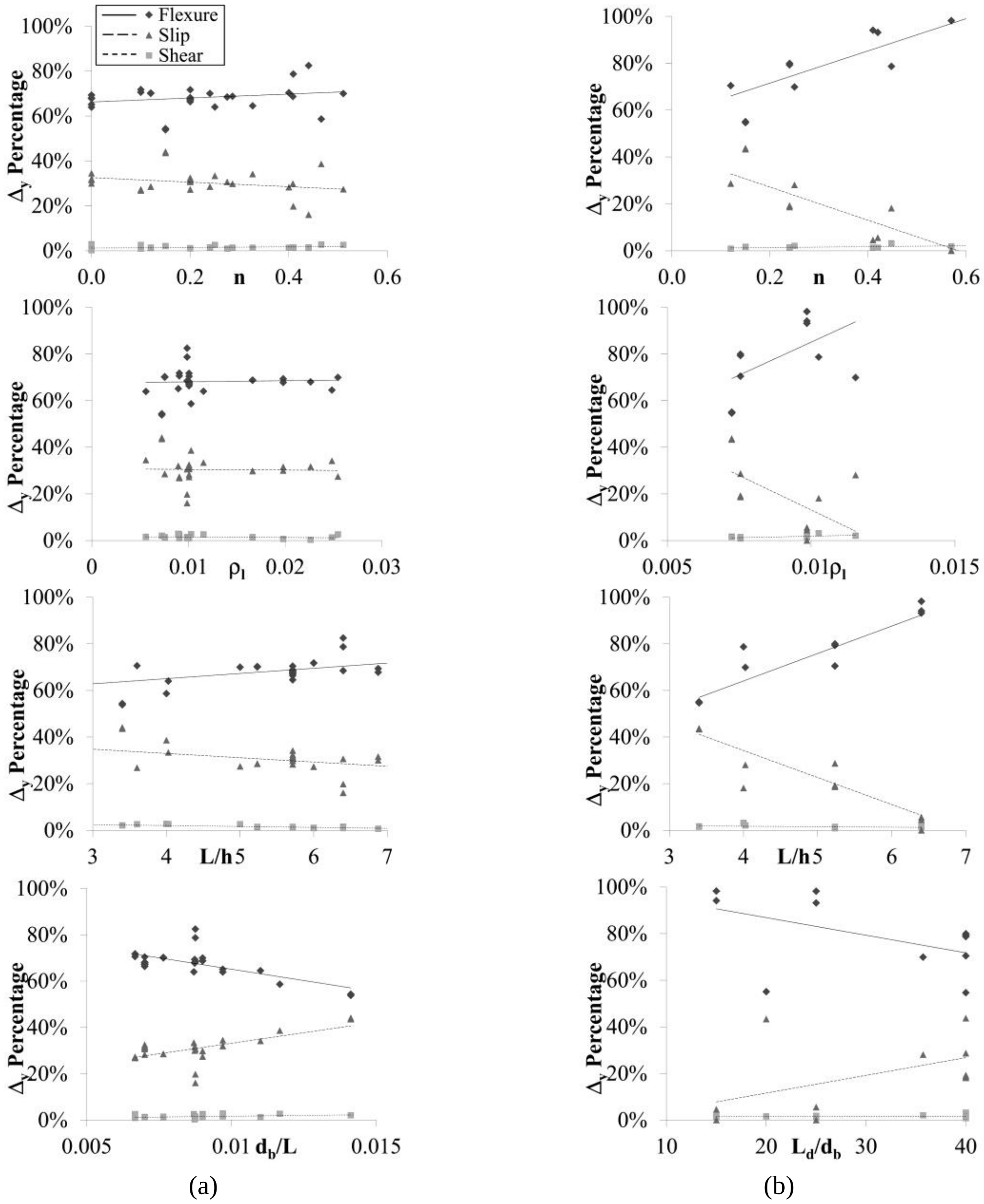


Fig. 5 Tip deflection components of (a) continuous and (b) lap-spliced columns

The experimental and calculated effective stiffnesses are compared in Table 4.

$$\frac{EI_{eff,calc}}{EI_g} = \frac{\alpha}{1 + 17.28\alpha\left(\frac{r_v}{L}\right)^2 + 0.75\frac{f_s}{f_y}\frac{f_y}{\sqrt{f_c'}}\frac{d_b}{L}} \tag{7}$$

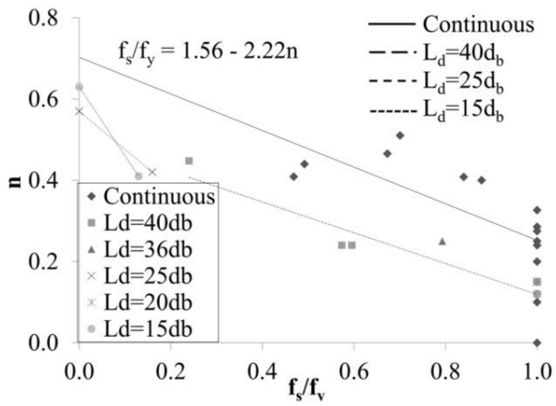


Fig. 6 The variation of steel stress with axial load.

4. Stiffness models

The aforementioned stiffness model acquired by Eq. (7) was further simplified in order to eliminate the need of standard section analysis. Considering design simplifications, the parameters of α and f_s/f_y were estimated in terms of only axial load (n) (Eqs. (8a)-(8b)) for continuous (*cont*) columns as shown in Fig. 6. However, for lap-spliced columns, the parameters of splice length (L_d/d_b) and shear span to depth ratio (L/h) were embodied in addition to the axial load as presented in Eqs. (9a) and (9b).

For Eqs. (9a)-(9b), since the database did not include lap-spliced beams, the equations were valid within database limitations for axial loads in the range between 12 to 63% for lap-spliced columns and between 0 and 51% for the columns with longitudinal rebars. Table 4 demonstrates that reliable estimations could be obtained for the effective stiffnesses using aforementioned predictions of α and f_s/f_y. As can be observed in Table 4, the introduced design formulations for continuous and lap-spliced columns had a good correlation with the analytical results. Furthermore, the predicted yield curvatures ($\kappa_{y,cont}$) (Priestley 2003) by Eq. (10a) were monitored to correlate well with the continuously reinforced columns.

$$\alpha_{cont} = 0.26 + 0.65n \tag{8a}$$

$$\left(\frac{f_s}{f_y}\right)_{cont} = 1.56 - 2.22n \le 1.0 \tag{8b}$$

$$\alpha_{LS} = 0.39n^{0.5}\left(\frac{L_d}{d_b}\right)^{0.05}\left(\frac{L}{h}\right)^{0.25} \tag{9a}$$

$$\left(\frac{f_s}{f_y}\right)_{LS} = 0.18n^{-1}\left(\frac{L_d}{d_b}\right)^{0.25}\left(\frac{L}{h}\right)^{-0.8} \le 1.0 \tag{9b}$$

$$\kappa_{y,cont} = 2.1\frac{\varepsilon_y}{d} \tag{10a}$$

$$\kappa_{y,LS} = \kappa_{y,cont}\left(0.75n^{-0.06}\left(\frac{L_d}{d_b}\right)^{0.16}\left(\frac{L}{h}\right)^{-0.37}\right) = 1.575\frac{\varepsilon_y}{d}^{-0.06}\left(\frac{L_d}{d_b}\right)^{0.16}\left(\frac{L}{h}\right)^{-0.37} \tag{10b}$$

However, since this formulation tended to yield overestimated results for lap-spliced columns, Eq. (10a) was modified for lap-spliced columns ($\kappa_{y,LS}$) with regard to axial load, splice length and shear span to depth ratio (Eq. (10b)). For the equations, ε_y and d identify the steel yield strain and column effective depth, respectively. In addition, Eq. (7) can be further simplified to yield Eq. (11) by ignoring the shear term in the denominator since it constitutes at most 3% of the tip deflection as shown in Figs. 5(a)-(b). In Eq. (11), α can be obtained by Eqs. (8a) and (9a) and the β constant has a value of 50 for plain reinforcement. Thus, Eq. (11) gave more realistic estimations (Table 4) as compared to the effective stiffness predictions of the previous studies that were related to the RC columns reinforced with deformed rebars (Elwood and Eberhard 2006, 2009). The term that denoted the ratio of the flexural stiffness to gross stiffness was defined for continuous and lap-spliced longitudinal reinforcement separately as α_{cont} and α_{LS}, respectively. For the Eq. (11), the difference between two cases was ignored and a more general formula was suggested for the sake of simplicity.

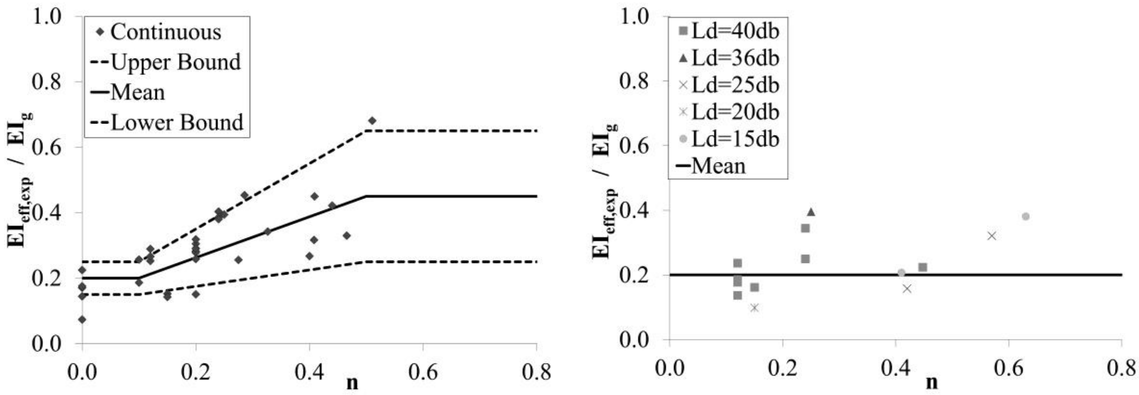
$$\frac{EI_{eff,calc}}{EI_g} = \frac{\alpha}{1+\beta\dfrac{d_b}{L}} \tag{11}$$

Herein, the suggested values for α=0.45+2.5n and β=110 in literature (Elwood and Eberhard 2009) were shown to overestimate the effective stiffness of RC columns reinforced with plain

Table 4 Mean and standard deviation variations

Column Type	*	$\frac{EI_{eff,calc}}{EI_{eff,\exp}}$					$\frac{\Delta_{y,\exp}}{\Delta_y}$	$\frac{\alpha_{calc}}{\alpha_{pred}}$	$\frac{\kappa_{y,calc}}{\kappa_{y,pred}}$
		EC-8 (2005)	*a*	*b* SSA	*c* Eq. (7)	*d* Eq. (11)	*e*	*f*	*g*
Continuous (Cont.)		0.90	1.19	1.08	1.04	1.02	0.96	0.97	1.02
		0.22	0.43	0.21	0.25	0.26	0.21	0.12	0.11
Lap-Spliced (LS)		0.68	0.90	0.95	0.91	0.98	1.21	1.06	1.02
		0.25	0.36	0.36	0.35	0.32	0.49	0.12	0.09
Average		0.83	1.10	1.04	1.00	1.01	1.04	1.00	1.02
		0.25	0.43	0.27	0.28	0.28	0.33	0.13	0.10

*μ and σ: Mean and standard deviations for experimental to calculated ratios.
a. Biskinis and Fardis (2010)
b. SSA: Standard section analysis
c. α, f_s/f_y: Cont: Eqs. (8a)-(8b), LS: Eqs. (9a)-(9b)
d. α: Cont: Eq. (8a), LS: Eq. (9a), β=50
e. Eq. (4a), Eq. (6a)-(6d)
f. Cont: Eq. (8a), LS: Eq. (9a)
g. Cont: Eq. (10a), LS: Eq. (10b)



(a) (b)

Fig. 7 The effective stiffness recommendations for (a) continuous and (b) lap-spliced columns with plain rebars

rebars since the formulations were suggested primarily for RC columns with deformed rebars. In comparison with Eq. (8a), it can be inferred that the use of plain longitudinal rebars induced a lower rate of increase of flexural stiffness with axial load as compared to the deformed longitudinal rebars. All design formulations were compared in Table 4 and good estimations were provided since the terms of Δ_y, α and κ_y could be predicted contiguous to experimental measurements. The recommended effective stiffness predictions for continuous columns were defined for upper and lower bounds for force based and displacement based analysis in case of either underestimating shear demands or overestimating displacement demands, respectively (Elwood and Eberhard 2009). Considering 90% of confidence level, the upper and lower bounds for effective stiffness were defined as shown in Eqs. (12a)-(12c) (Fig. 7(a)). For lap-spliced columns, the ratio of effective stiffness to gross stiffness (EI_{eff}/EI_g) was recommended to have a constant value of 0.2, since no significant trend was observed for varying splice lengths used in the database (Fig. 7(b)).

$$\left(\frac{EI_{eff}}{EI_g}\right)_{lowerbound} = 0.15 \le \frac{1}{4}n + \frac{1}{8} \le 0.25 \tag{12a}$$

$$\left(\frac{EI_{eff}}{EI_g}\right)_{mean} = 0.20 \le \frac{5}{8}n + \frac{11}{80} \le 0.45 \tag{12b}$$

$$\left(\frac{EI_{eff}}{EI_g}\right)_{upperbound} = 0.25 \le n + \frac{3}{20} \le 0.65 \tag{12c}$$

As a final remark, in common practice, the circular and rectangular cross sections generally pertain to RC bridge and building columns with low to high axial load level, respectively. Thus, circular columns are liable to possess lower axial load and d_b/L values with shear spans generally longer than ordinary building columns. Therefore, although all the formulations implemented in this research were based on a database of rectangular columns, the suggested formulations can also be utilized for circular columns regarding the range of variables under investigation.

5. Conclusions

Predicting the effective stiffness for RC members is crucial in assessment and strengthening since the seismic characteristics and consequent response of RC structures is majorly influenced by cracking under gravity and lateral forces. The current guideline recommendations on effective stiffness of RC columns may mislead for plain reinforcement due to the major postulation of deformed rebars as longitudinal reinforcement. Thus, a RC column database was constituted by monotonic and reversed cyclic tests of 47 columns in which 33 columns were continuously reinforced and the remaining were spliced with various splice lengths. In the analysis of lap-spliced columns, the adopted spliced steel model from literature was modified using a lap-spliced beam database reflecting the failure mechanism of plain rebars. The analyses approved the dependence of effective stiffness on axial load and longitudinal reinforcement ratio as compatible with previous researches on deformed rebars. In this sense, it was ascertained that due to the shortcomings of the current structural guidelines upon utilization of longitudinal plain rebars, column effective stiffnesses were overestimated. Therefore, more reliable effective stiffness predictions were obtained by reducing the flexural stiffness of the columns regarding the deflection constituents of shear and bond slip on the basis of the three component approach while taking plain rebars into account. In addition, upper and lower bound effective stiffness expressions were recommended that can be utilized for force and displacement based assessment of RC structures, respectively.

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