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Effect of loading rate on softening behavior of low-rise structural walls

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Abstract. Cracked reinforced concrete in compression has been observed to exhibit lower strength and stiffness than uniaxially compressed concrete. The so-called compression softening effect responsible is thought to be related to the degree of transverse cracking and straining present. It significantly affects the strength, ductility and load-deformation response of a concrete element. A number of experimental investigations have been undertaken to determine the degree of softening that occurs, and the factors that affect it. At the same time, a number of diverse analytical models have been proposed by various researchers aimed at modeling this behavior. In this paper, the softened truss model thoery for low-rise structural shearwalls is employed using the principle of the stress and strain transformations. Using this theory the softening parameters for the concrete struts proposed by Hsu and Belarbi as well as by Vecchio and Collins are examined by 51 test shearwalls available in literature. It is found that the experimental shear strengths and ductilities of the walls under static loads are, in average, very close to the theoretical values; however, the experiment shear strengths and ductilities of the walls under dynamic loads with a low (0.2 Hz) frequency are generally less than the theoretical values.

Key words: softening behavior, structural wall, loading rate, truss model, shear strength, ductility.

1. Introduction

In recent years, it is generally agreed among researchers that the truss model theory is the most promising method to treat shear and torsion. This theory provides a unified concept for shear and torsion and is able to generate a unified design method for reinforced and prestressed concrete. When a truss model is employed, besides the equilibrium and compatibility equations the stress-strain relationship of concrete struts must be assumed before the post-cracking behavior of shear or torsional members can be predicted. The stress-strain curve obtained from the uniaxial compression test of standard concrete cylinders was used by several investigators without success. The nature of this difficulty was understood in 1964 when (Peter 1964) observed that a reinforced concrete panel subjected to compression was softened by tension in the transverse bars. However, he was unable to produce a softening parameter for the stress-strain curve, and suggested that a 15% reduction of the concrete compressive strength should be used (Vecchio and Collins 1981)

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tested seventeen 89 by 89 by 7 cm reinforced concrete panels. According to the test results they proposed a stress-strain curve incorporating a softening parameter. This softening parameter is a function of the ratio of the two principal strains. The softened stress-strain curve, proposed by (Vecchio and Collins in 1981) was used to predict the static behavior of reinforced concrete structural walls under shear (Hsu and Mo 1985, Mo 1988, Mo 1994) and box structures under torsion (Mo and Yang 1996).

The discovery and the quantification of this softening phenomenon have provided the major breakthrough in understanding of the shear problem in reinforced concrete. During the past 15 years, a number of diverse analytical models have been proposed according to the test results. The effect of these softening models on low-rise structural walls has been studied (Mo and Rothert 1995), and it is found that both the 1993 Vecchio and Collins' Model A and the 1991 Belarbi and Hsu's Model (Hsu 1993) provide the best results. However, these two models are based on static tests. Up to now, no softening models have been proposed for dynamic loads. Therefore, the purpose of this paper is to study the effect of loading rate on softening behavior of low-rise structural walls.

2. Softened truss model theory

Fig. 1 indicates a typical low-rise reinforced concrete structural wall. The softened truss model theory for such walls can be derived using the principle of stress and strain transformations (Hsu 1993). In this paper, the principal concepts are described below.

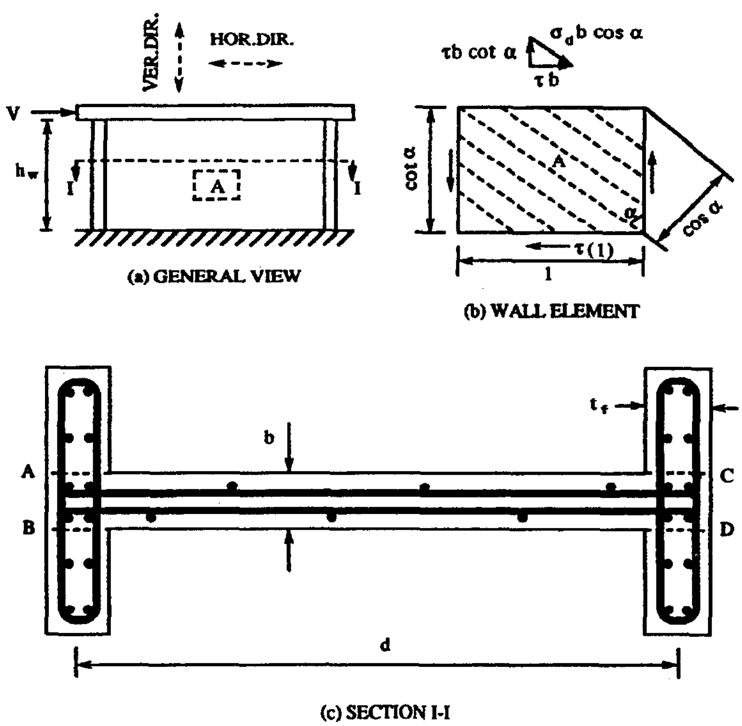
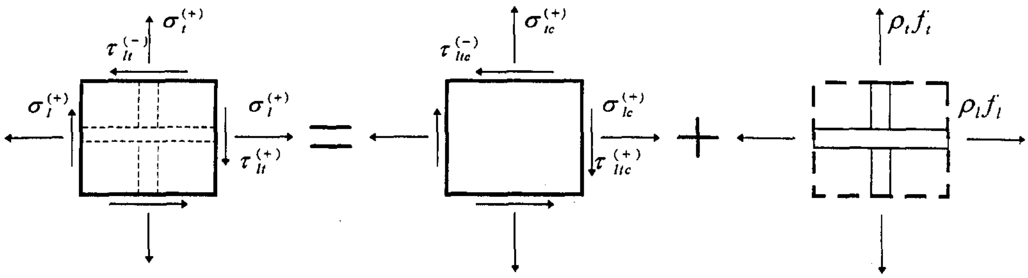


Fig. 1 A framed shearwall.



(a) Reinforced concrete (b) Concrete struts (c) Steel reinforcement

Fig. 2 Stress condition in reinforced concrete.

2.1. Equilibrium equations

When a concrete element reinforced orthogonally with longitudinal and transverse steel bars, as shown in Fig. 2a, is studied, the three stress components σ_l, σ_t and τ_{lt} are the applied stresses on the reinforced concrete element viewed as a whole. The stresses on the concrete strust itself are denoted as σ_{lc}, σ_{tc} and τ_{ltc}, as shown in Fig. 2b. The longitudinal and transverse steel provide the smeared stresses of $\rho_l f_l$ and $\rho_t f_t$, as shown in Fig. 2c.

It is significant to recognize the difference between the two sets of stresses: σ_l, σ_t and τ_{lt} for the reinforced concrete element and σ_{lc}, σ_{tc} and τ_{ltc} for the concrete struts. Both sets of stresses (σ_l, σ_t, τ_{lt} and σ_{lc}, σ_{tc}, τ_{ltc}) satisfy the principle of stress transformation. In summing the concrete stresses and the steel stresses in the l and t directions, a fundamental assumption is made according to (Hsu 1993). It is assumed that the steel reinforcement can take only axial stresses. Any possible dowel action is neglected. Hence, the superposition principle for concrete and steel becomes valid and gives the general equilibrium equations for reinforced concrete.

$$\sigma_l = \sigma_d \cos^2 \alpha + \sigma_r \sin^2 \sigma + \rho_l f_l \tag{1}$$

$$\sigma_t = \sigma_d \sin^2 \alpha + \sigma_r \cos^2 \sigma + \rho_t f_t \tag{2}$$

$$\tau_{lt} = (-\sigma_d + \sigma_r) \sin\alpha \cos\alpha \tag{3}$$

where

σ_l = stress in l-axis
σ_d = stress in d-axis
σ_r = stress in r-axis
σ_t = stress in t-axis
τ_{lt} = shear stress in l-t axis
f_l = stress in longitudinal steel
f_t = stress in transverse steel
ρ_l = longitudinal steel ratio
ρ_t = transverse steel ratio
α = angle between two sets of coordinates

2.2. Compatibility equations

The same principle of transformation for stresses can be applied to strains. Therefore, the following compatibility equations can be derived (Hsu 1993).

$$\varepsilon_l = \varepsilon_d \cos^2\alpha + \varepsilon_r \sin^2\alpha \tag{4}$$

$$\varepsilon_t = \varepsilon_d \sin^2\alpha + \varepsilon_r \cos^2\alpha \tag{5}$$

$$\frac{\gamma_{lt}}{2} = (-\varepsilon_d + \varepsilon_r)\sin\alpha\cos\alpha \tag{6}$$

where

ε_l = strain in *l*-axis
ε_d = strain in *d*-axis
ε_r = strain in *r*-axis
ε_t = strain in *t*-axis
γ_{lt} = shear distortion in *l-t* axis

2.3. Constitutive laws

2.3.1. Softened compression stress-strain relationship of concrete

In the past three decades a number of diverse analytical models have been proposed by various researchers aimed at modeling the softening behavior of concrete. It was found by (Mo and Rothert 1995) that for reinforced concrete structural walls both the Model A of (Vecchio and Collins 1993) and the Model of (Belarbi and Hsu 1991) provide better results than the other models when compared to the 44 test specimens (Barda 1972, Barda, *et al.* 1977, Galletly 1952, Benjamin and Williams 1957, Mo and Chan 1996). Therefore, only these two models are described in this paper.

1) 1991 Belarbi and Hsu's Model

Using a softening parameter, β, the proposed model involved modifying the Hognestad parabola (Fig. 3a), which was employed as the base curve describing the uniaxial compressive response of concrete. For the ascending portion of the curve, the equation is

$$\sigma_d = f_c'\left[2\left(\frac{\varepsilon_d}{\varepsilon_o}\right) - \frac{1}{\beta}\left(\frac{\varepsilon_d}{\varepsilon_o}\right)^2\right] \tag{7}$$

For the descending portion of the curve, the equations is

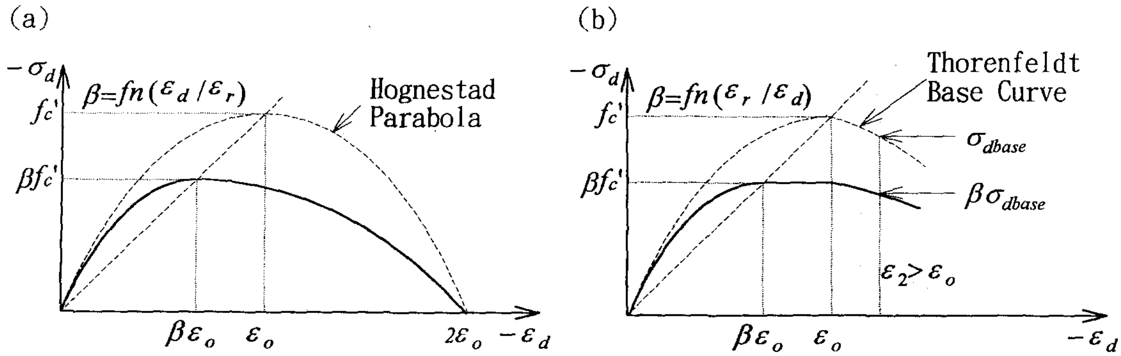
$$\sigma_d = \beta f_c'\left[1 - \left(\frac{\varepsilon_d - \varepsilon_p}{2\varepsilon_o - \varepsilon_p}\right)^2\right] \tag{8}$$

where $\varepsilon_p = \beta\varepsilon_o$. The softening parameter β was suggested to be

$$\beta = \frac{0.9}{\sqrt{1+600\varepsilon_r}} \tag{9}$$

2) 1993 Vechio and Collins' Model A

(Vecchio and Collins 1993, Vecchio, *et al.* 1994) found that the Hognestad parabola, used as the base curve for the softening models, does not provide a good representation of the response of high-strength concrete, and the Thorenfeldt, *et al.* (1987) curve (Fig. 3b) resulted in the best



(a) Hognestad curve is used; (b) Thorenfeldt curve is used.

Fig. 3 Compression softening models.

correlations for the full range of concrete strengths represented in the database. The Thorenfeldt base curve was calibrated (Collins and Porasz 1989) as follows:

$$\sigma_{d\,base} = -f_p \frac{n \cdot \left(\frac{\varepsilon_d}{\varepsilon_p}\right)}{(n-1) + \left(\frac{-\varepsilon_d}{\varepsilon_p}\right)^{nk}} \tag{10}$$

where

$$n = 0.80 + \frac{f_p(\text{MPa})}{17} \tag{11}$$

$$\text{If} \quad -\varepsilon_p < \varepsilon_d < 0, \quad k = 1.0 \tag{12}$$

$$\text{If} \quad \varepsilon_d < -\varepsilon_p, \quad k = 0.67 + \frac{f_p(\text{MPa})}{62} \tag{13}$$

ε_p = corresponding strain at maximum obtainable compressive stress in softened concrete. And softening parameter β is

$$\beta = \frac{1}{1.0 + K_c \cdot K_f} \tag{14}$$

where

$$K_c = 0.35 \cdot \left(\frac{-\varepsilon_r}{\varepsilon_d} - 0.28\right)^{0.80} \geq 1.0 \tag{15}$$

$$K_f = 0.1825\sqrt{f_c'}\,(\text{MPa}) \geq 1.0 \tag{16}$$

If $0 < -\varepsilon_d < -\beta\varepsilon_o$, σ_d is determined by Eq. (10) with
$f_p = -\beta \cdot f_c'$, and $\varepsilon_p = -\beta \cdot \varepsilon_o$
If $-\beta\varepsilon_o < -\varepsilon_d < -\varepsilon_o$, $\sigma_d = \beta \cdot f_c'$
If $-\varepsilon_o < -\varepsilon_d$, $\sigma_d = \beta \cdot \sigma_{d\,base}$ and $f_p = -f_c'$, $\varepsilon_p = -\varepsilon_o$
Note that Eqs. (10) to (13) are used with $f_p = \beta \cdot f_c'$ and $\varepsilon_p = \varepsilon_o$

2.3.2. Tensile stress-strain relationship of concrete

From the tests of panels subjected to shear, it was clear that the tensile stress of concrete, σ_r, is not zero as assumed in the simple truss model (Vecchio and Collins 1981, Belarbi and Hsu 1991). Based on the tests of 35 full-size panels (Belarbi and Hsu 1991, Pang and Hsu 1992), a set of formulas were recommended as follows:

$$\text{If } \varepsilon_r \leq \varepsilon_{cr}, \qquad \sigma_r = E_c \cdot \varepsilon_r \tag{17}$$

$$\text{If } \varepsilon_r > \varepsilon_{cr}, \qquad \sigma_r = f_{cr} \cdot \left(\frac{\varepsilon_{cr}}{\varepsilon_r} \right)^{0.4} \tag{18}$$

where

$E_c = 47000\sqrt{f_c'}$, where both f_c' and $\sqrt{f_c'}$ are in pounds per square inch
ε_{cr} = strain cracking of concrete = 0.00008
$f_{cr} = 3.75\sqrt{f_c'}$

2.3.3. Stress-strain relationship of steel

The stress-strain curve of a steel bar in concrete relates the average stress to the average strain of a large length of bar crossing several cracks, whereas the stress-strain curve of a bare bar relates the stress to the strain at a local point. In other words, a steel bar in concrete is stiffened by tensile stress of concrete. If the tensile strength of concrete is neglected, as assumed in the most of truss models, the following equations are used.

$$\text{If } \varepsilon_l \leq \varepsilon_{ly}, \quad f_l = E_s \varepsilon_l \tag{19}$$

$$\text{If } \varepsilon_l = \varepsilon_{ly}, \quad f_l = f_{ly} \tag{20}$$

where E_s = modulus of elasticity of steel bars
f_{ly} = yield stress of longitudinal steel bars
ε_{ly} = yield strain of longitudinal steel bars

2.4. Solution procedures

As discussed by (Hsu and Mo 1985), in the design of low-rise structural walls the boundary elements are reinforced to resist the applied bending moment, while the webs are designed to resist the applied shear force. Due to the restriction of the boundary elements, the strain of transverse steel in low-rise framed shearwalls can be neglected, as verified by the PCA tests (Barda 1972, Barda, *et al.* 1977), i.e., $\varepsilon_t = 0$. Therefore, adding Eq. (4) and Eq. (5) gives

$$\varepsilon_r = \varepsilon_l - \varepsilon_d \tag{21}$$

Inserting $\varepsilon_r \sin^2\alpha = \varepsilon_r - \varepsilon_r \cos^2\alpha$ into Eq. (4) gives

$$\cos^2\alpha = \frac{\varepsilon_r - \varepsilon_l}{\varepsilon_r - \varepsilon_d} \tag{22}$$

Substituting Eqs. (21) and (22) into Eq. (1) results in

$$f_l = \frac{1}{\rho_l}\left[\sigma_l - \sigma_d \cdot \frac{(-\varepsilon_d)}{(\varepsilon_l - 2\varepsilon_d)} - \sigma_r \frac{(\varepsilon_l - \varepsilon_d)}{(\varepsilon_l - 2\varepsilon_d)}\right] \tag{23}$$

Neglecting the tensile strength of concrete, i.e., $\sigma_r = 0$, gives

$$f_l = \frac{1}{\rho_l}\left[\sigma_l - \sigma_d \cdot \frac{(-\varepsilon_d)}{(\varepsilon_l - 2\varepsilon_d)}\right] \tag{24}$$

For low-rise structural walls, the average shear stress τ on the horizontal cross section is defined as

$$\tau = \frac{V}{bd} \tag{25}$$

where d is the effective depth, which will be defined later; b is the width of the web; V is the horizontal shear force. The deflection at the top of the shearwall, δ, is determined by

$$\delta = \gamma h \tag{26}$$

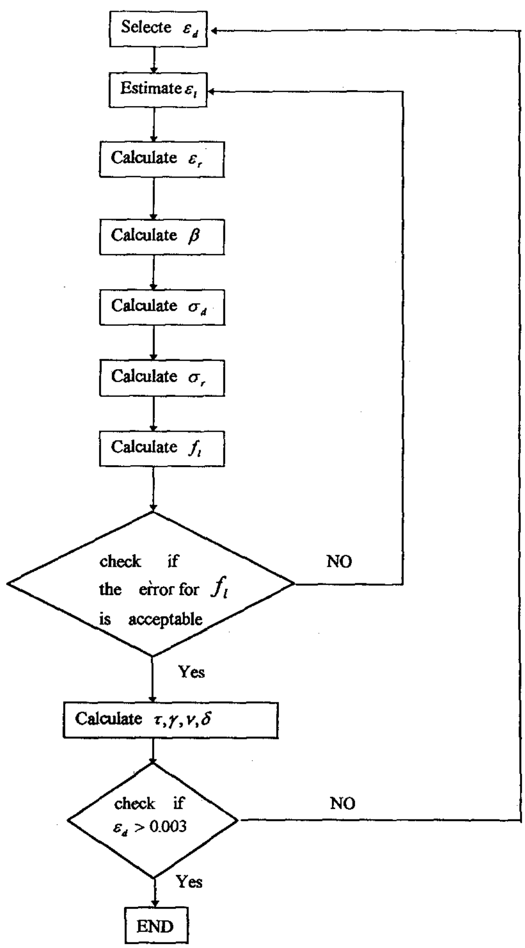


Fig. 4 Algorithm for framed shearwall analysis.

where

$$h = \text{height of wall}$$

Based on the softened truss model theory presented above, the algorithm is shown in Fig. 4 and is explained below

1) Select a given ε_d.
2) Assume a value of ε_l.
3) Calculate ε_r from Eq. (21).
4) Calculate β using Eqs. (7) to (16).
5) Calculate σ_d from Eq. (10).
6) Calculate σ_r from Eq. (7) or (8).
7) Calculate f_l from Eq. (23) or (24).
8) Check f_l using Eqs. (9) and (20).

9a) If the calculated value for f_l determined in step 8 is not sufficiently close to the value shown in step 7, repeat steps 2 through 7.

9b) If the calculated value for f_l determined in Step 8 is sufficiently close to the value shown in Step 7, proceed to calculate τ(or V) and γ(or δ) from Eq. (3) (or Eq. (25)) and Eq. (6) (or Eq. (26)), respectively. This will provide one set of the solutions.

10) Select other values of ε_d and repeat steps 2 through 9 for each ε_d. This will provide a number of sets of quantities. From these sets of quantities the shear stress versus distortion curve (or shear force versus deflection curve), the longitudinal steel strain versus deflection curve, and the longitudinal steel strain versus concrete strain curve can be plotted. In this study the maximum ε_d value was chosen as 0.003 with an increment of 0.00005.

3. Comparison of theory with tests

3.1. Shear force-deflection relationships

Two specimens are chosen for discussion: Specimen B6-4 with a concrete strength of 21.2 MPa tested at PCA (Barda, Hanson and Corley 1976) and Specimen LM4-2 with a concrete strength of 66.0 MPa tested at National Cheng Kung University (NCKU) (Mo and Chan 1996). The shear force-deflection curves for these two specimens are plotted in Figs. 5 and 6. Each figure includes one experimental curve and two theoretical curves computed from the 2 softening models discussed previously. Examination of the two theoretical curves reveals the following observations:

1) The curve using the 1993 Vecchio and Collins' Model A for softened concrete is represented by a curve with solid circles. These theoretical shear force-deflection curves are very close to the corresponding test curve in all aspects: (1) the predicted maximum shear strengths are only 4.6 and 5.6 percent greater than the test values for Specimens B6-4 and LM4-2, respectively; (2) each shear force-deflection curve exhibits a descending branch; (3) the deflection at maximum shear strengths are very close to the test values; and (4) the ascending portion of the theoretical shear force-deflection curves are reasonably close to the test curves.
2) The curve using the 1991 Belardi and Hsu's Model for softened concrete is represented by a curve with solid triangles. It can be seen that the deflections at maximum shear strengths are much less than the test values, that in turn affect the ascending portion of

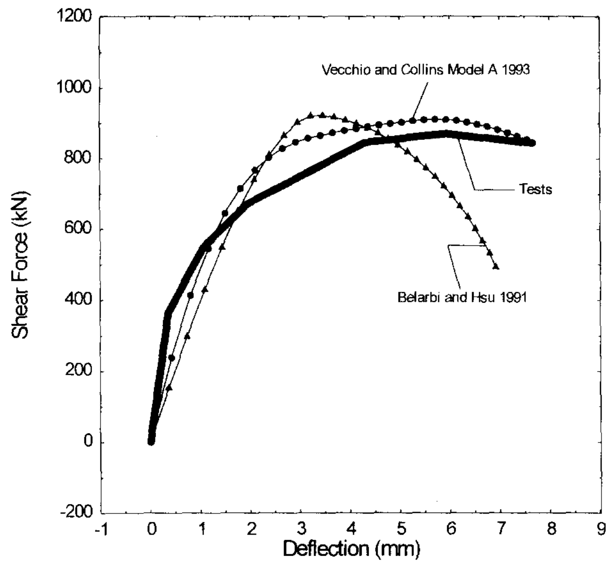


Fig. 5 Shear force-deflection curves of wall B6-4.

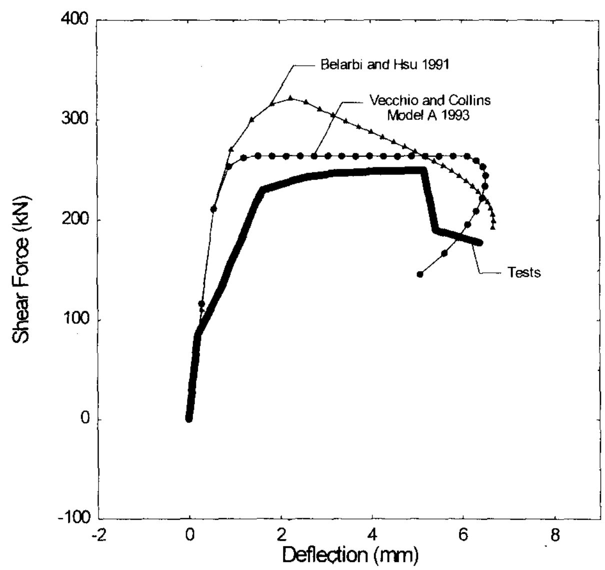


Fig. 6 Shear force-deflection curves of wall LM4-2.

the theoretical shear force-deflection curves.

3.2. Shear strength

The experimental specimens of fifty-one low-rise framed shearwalls were found and their detailed dimensions and material properties are summarized in (Mo and Rothert 1995, Mo 1996). The specimens tested at Stanford University (SU) (Benjamin and Williams 1957) and MIT (Galletly 1952) were subjected to monotonic loads; the specimens tested at PCA (Barda, Hanson and

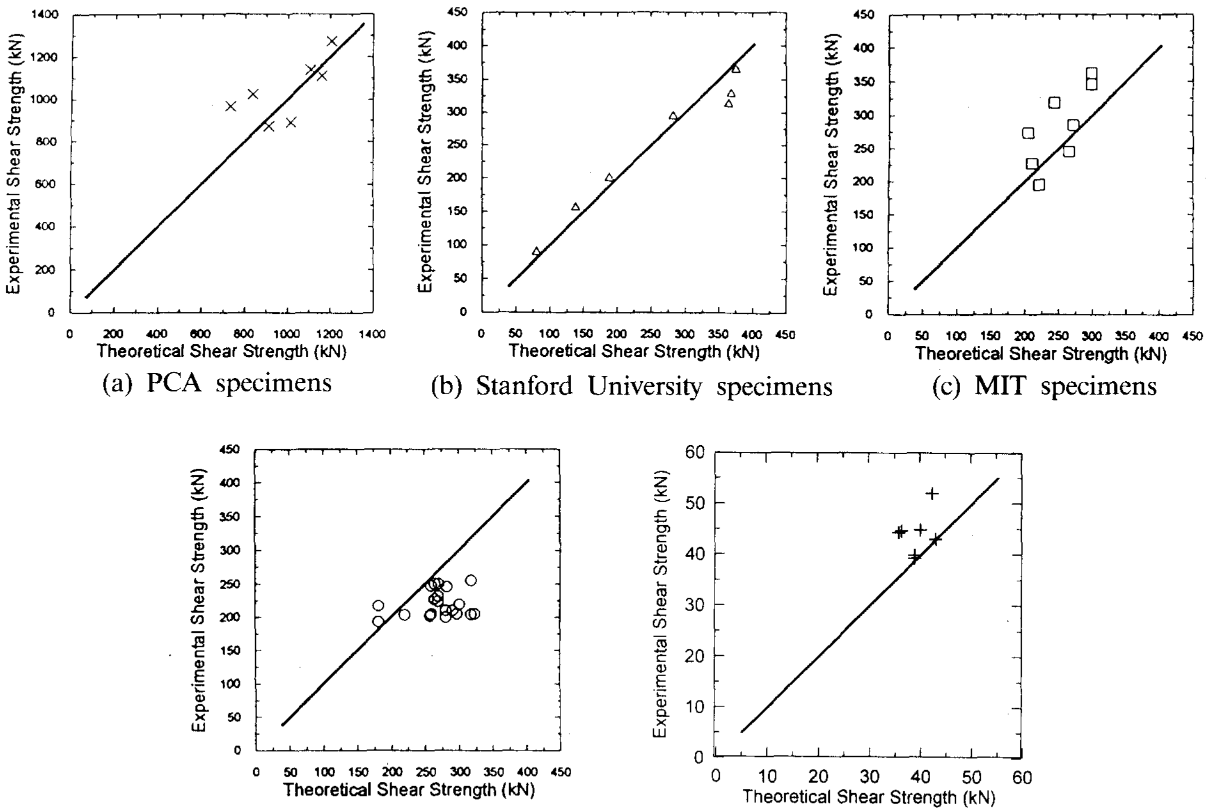
Table 1 Effect of softening models on shear strengths

Source	No. of Specimens	Statistics	$V_{u,test}/V_{u,calc.}$ Belarbi and Hsu 1991	$V_{u,test}/V_{u,calc.}$ Vecchio and Collins Model A 1993
PCA	7	mean	1.022	1.060
		cov.	0.122	0.157
Stanford University	7	mean	0.968	1.010
		cov.	0.121	0.105
MIT	8	mean	1.041	1.121
		cov.	0.152	0.168
NCKU (0.2Hz)	20	mean	0.793	0.834
		cov.	0.140	0.141
NCKU (4Hz)	7	mean	1.03	1.11
		cov.	0.103	0.102
All specs.	49	mean	0.925	0.977
		cov.	0.163	0.173

Corley 1976) were statically loaded under cyclic reversals; the specimens tested at NCKU by the first author were dynamically loaded under cyclic reversals (Mo and Chan 1996) and shake table tests (Mo 1996) with a loading frequency of 0.2Hz and 4Hz, respectively. The shear strength of all the test specimens are compared to their theoretical values in Table 1 using the softening models described previously. Of all these specimens, two are ineligible for comparison.

After excluding these two specimens, the average $V_{u,test}/V_{u,calc}$ values calculated from the two softening models for all forty-nine walls are listed in Table 1. It can be seen from Table 1 that using the 1993 Vecchio and Collins' Model A and the 1991 Belarbi and Hsu's Model the average $V_{u,test}/V_{u,calc}$ values are 0.977 and 0.925, respectively, and the corresponding standard deviations are 0.173 and 0.163. The calculated shear forces are often close to the experimental values for static tests (such as: PCA, SU and MIT specimens) while they are generally somewhat greater than the experimental values for dynamic tests with a low frequency of load (such as: NCKU (0.2Hz) specimens). The following reasons may have been involved. 1. Walls under dynamic loads will have more serious softening of concrete struts than those under static loads, and the two proposed softening models are based on static tests. 2. Although walls are subjected to dynamic loads, material strengths (concrete and steel) do not increase because of the low frequency of loads (0.2Hz).

The experimental and theoretical shear strengths for the five groups of walls are compared in Fig. 7. In general, using the 1993 Vecchio and Collins' Model A the theoretical shear strengths for the walls under static loads (such as: PCA, SU and MIT specimens) are in excellent agreement with the experimental observations. In contrast, the theoretical shear strengths for the walls under dynamic loads (such as: NCKU (0.2Hz) specimens) are often greater than the experimental



(a) PCA specimens (b) Stanford University specimens (c) MIT specimens

(d) NCKU (0.2Hz) specimens (e) NCKU (4Hz) specimens

Fig. 7 Comparison of theoretical and experimental shear strengths.

Table 2 Ductilities

Source	No. of Specimens	μ_{test}/μ_{calc}	
		mean	cov.
PCA	1	0.979	0
Stanford Univ.	6	0.797	0.088
MIT	6	1.207	0.311
NCKU (0.2Hz)	20	0.576	0.227
NCKU (4Hz)	7	0.886	0.205
All specs.	40	0.768	0.306

values. However, when the loading frequency is increased to 4Hz, the theoretical shear strengths are close to the experimental values again. In other words, low-rise structural walls subjected to dynamic load with low frequency have more serious softening of concrete.

3.3. Ductility factors

As discussed previously, in general the 1993 Vecchio and Collins' Model A provide better results, such as: maximum shear force, deflection at the maximum shear force and shear force-deflection curve. Therefore, in this section only the 1993 Vecchio and Collins' Model A was used to calculate the theoretical ductility factors. The theoretical ductility factors, μ_{calc}, is defined as the theoretical deflection at the maximum shear force, $\delta_{u,calc}$, divided by the theoretical yield deflection, $\delta_{y,calc}$. The experimental ductility, μ_{test}, is defined by the experimental deflection at the maximum shear force, $\delta_{u,test}$, divided by the theoretical yield deflection. $\delta_{y,calc}$. The results are shown in Table 2. The specimens without yielding are excluded in the comparison, since no theoretical yield deflections can be found. After the exclusion, the average μ_{test}/μ_{calc} value for all 40 specimens is 0.768 and the standard deviations is 0.306. The average μ_{test}/μ_{calc} values and the standard deviations are also given for those five groups of tests. It can be seen that the average μ_{test}/μ_{calc} values for the specimens under static loads (such as: PCA, SU and MIT specimens) is close to unity, and the average μ_{test}/μ_{calc} values for the specimens under dynamic loads, such as: NCKU (0.2Hz) and NCKU (4Hz) specimens are 0.576 and 0.886, respectively. In other words, the specimens under dynmaic loads will have less ductility factors than those under static loads. All in all, the shear strengths and ductilities of low-rise structural walls are influenced by the loading types (static or dynamic loads). It can also be seen from Table 2 that the average μ_{test}/μ_{calc} value increases when the loading frequency is changed from 0.2Hz to 4Hz.

4. Conclusions

Using the principle of transformation, the softened truss model theory can be used for low-rise reinforced concrete structural walls. the theory can predict not only the shear strength but also the shear distortion, the steel strains, and the concrete strains. The softening models for concrete struts, proposed by Hsu and Belarbi as well as by Vecchio and Collins have been examined by 51 test walls with concrete strength from 16.3 MPa to 66.0 MPa available in literature. It is found that the 1993 Vecchio and Collins' Model A provides better results.

The experimental shear strengths and ductilities of the walls under static loads are, on average, very close to the theoretical values; however, the experimental shear strengths and ductilities of the walls under dynamic loads with a low (0.2Hz) frequency are generally less than the theoretical values. In other words, further research in the area of dynamic compression softening of reinforced concrete is needed.

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References

Barda, F. (1972), "Shear strength of low-rise walls with boundary elements", Ph. D thesis, Lehigh University, Bethlehem, 265.

Barda, F., Hanson, J.M. and Corley, W.G. (1976), "Shear strength of low-rise walls with boundary elements", *Portland Cement Association publication RD043D*, 20.

Belarbi, A. and Hsu, T.T.C. (1991), "Constitutive laws of reinforced concrete in biaxial tension-compression", *Research Report UHCEE 91-2*, Department of Civil Engineering, University of Houston, Houston, Texas, U.S.A.

Benjamin, J.R. and Williams, H.A. (1957), "The behavior of one-story reinforced concrete shear walls", *Journal of the Structural Division, ASCE*, **83**(ST3), May, 1254.

Collins, M.P. and Porasz, A. (1989), "Shear strength for high strength concrete", *Bull. No. 193-Design Aspects of High Strength Concrete, Comite Euro-International du Beton (CEB)*, 1989, 75-83.

Galletly, G.D. (1952), "Behavior of reinforced concrete shear walls under static load", MIT Department of Civil and Sanitary Engineering, Cambridge, Mass., August.

Hsu, T.T.C. (1993), *Unified Theory of Reinforced Concrete*, CRC Press, Boca Ration, Florida, 313 pp.

Hsu, T.T.C. and Mo, Y.L. (1985), "Softening of concrete in low-rise shear walls", *Journal of the American Concrete Institute*, **82**(6), November-December, 883-889.

Mo. Y.L. (1988), "Analysis and design of low-rise structural walls under dynamically applied shear forces", *ACI Structural Journal*, **85**(2), March-April, 180-189.

Mo, Y.L. (1994), *Dynamic Behavior of Concrete Structures*, Elsevier Science Publishers B. V., Amsterdam, Netherlands, June, 424.

Mo, Y.L. (1996), "Effect of boundary elements on low-rise structural walls," (in Chinese), *National Science Council Project Report*, No. NSC 85-2621-P006-028, July, Taiwan.

Mo, Y.L. and Chan, J. (1996), "Behavior of reinforced concrete framed shear walls", *Nuclear Engineering and Design*, **166**, 55-68.

Mo, Y.L. and Rothert, H. (1995), "Effect of softening models on behavior of reinforced concrete framed shearwalls", *Research Report*, Institute für Statik, Universität Hannover, Hannover, Germany, October.

Mo, Y.L. and Yang, R.Y. (1996), "Dynamic response of box tubes to combined shear and torsion", *Jouranl of Structural Engineering, ASCE*, **122**(1), January, 47-54.

Pang, X.B. and Hsu, T.T.C. (1992), "Constitutive laws of reinforced concrete in shear", *Research Report UHCEE92-1*, Department of Civil and Environmental Engineering, University of Houston, Houston, Texas.

Peter, J. (1964), "Zur bewehrug von dcheiben und schalen für Hauptspannungen schiefwinklig zur Bewehrungsrichtung", Dissertation, Lehrstuhl für Massivbau, Technische Hochschule Stuttgart, Germany.

Thorenfeldt, E., Tomaszewicz, A. and Jensen, J.J. (1987), "Mechanical properties of high-strength concrete and application in design", *Proc. Symposium Utilization of High-Strength Concrete*, Stavanger, Norway, Tapir Trondheim.

Vecchio, F.J. and Collins, M.P. (1981), "Stress-strain characteristics of reinforced concrete in pure shear", *IABSE Colloquim on Advanced Mechanics of Reinforced Concrete*, Delft, Final Report, 211-225.

Vecchio, F.J. and Collins, M.P. (1993), "Compression response of cracked reinforced concrete", *Journal of Structural Engineering, ASCE*, **119**(12), December, 3590-3610.

Vecchio, F.J., Collins, M.P. and Aspiotis, J. (1994), "High-strength concrete elements subjected to shear", *ACI Structural Journal*, **91**(4), July-August, 423-433.