Structural Engineering and Mechanics, Vol. 5, No. 1 (1997) 105-113
DOI: http://dx.doi.org/10.12989/sem.1997.5.1.105

Simulation of concrete shrinkage taking into account aggregate restraint

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Abstract. This paper proposes a model for simulating concrete shrinkage taking into account aggregate restraint. In the model, concrete is regarded as a two-phase material based on shrinkage property. One is paste phase which undergoes shrinkage. Another is aggregate phase which is much more volumetrically stable. In the concrete, the aggregate phase is considered to restrain the paste shrinkage by particle interaction. Strain compatibility was derived under the assumption that there is no relative macroscopic displacement between both phases. Stresses on both phases were derived based on the shrinking stress of the paste phase and the resisting stress of the aggregate phase. Constitutive relation of paste phase was adopted from the study of Yomeyama, K. *et al.*, and that of the aggregate phase was adopted from the author's particle contact density model. The equation for calculating concrete shrinkage considering aggregate restraint was derived from the equilibrium of the two phases. The concrete shrinkage was found to be affected by the free shrinkage of the paste phase, aggregate content and the stiffness of both phases. The model was then verified to be effective for simulating concrete shrinkage by comparing the predicted results with the autogeneous and drying shrinkage test results on mortar and concrete specimens.

Key words: modeling; concrete; shrinkage; aggregate restraint; multi-phase material.

1. Introduction

Durability design has the merit of relating materials, method of construction and maintenance to the design of concrete structures. Shrinkage is one of the subjects concerning not only with the durability but also with the mechanical properties of the concrete. Shrinkage can be divided into two types based on mechanisms. One is drying shrinkage which is the result of volume reduction due to the contraction of the calcium silicate gel in the hardened cement-waters paste when the moisture content of the gel is decreased by losing to the environment. Another is autogeneous shrinkage which does not involve exchange of moisture between the concrete and its ambient environment.

There are various models proposed for simulating concrete shrinkage, for examples models

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by Shimomura, *et al.* and by Hobbs. However, some of those models were not applicable to simulate autogeneous shrinkage and did not involve the aggregate restraint, also some were not applicable for all range of aggregate content especially when the aggregate content is very large.

The objective of the model proposed in this paper is to provide a tool for a mix design aiming for high durability. The model, when completed, can be used to design a concrete with minimum shrinkage and also to combine with other models for designing highly durable concrete.

2. Concrete shrinkage model

Concrete is regarded as a two-phase material comprising of paste phase and aggregate phase. The two phases are divided based on their differences in shrinkage properties. Paste phase, being the part to undergo shrinkage, consists of all cementitious and powder materials, water, all kinds of mineral and chemical admixtures and air voids. The aggregate phase, considered much more stable in volume, consists of coarse and fine aggregates. The important assumptions in this model are as follows:

1) Aggregates, behaving as a particle assembly, are uniformly dispersed in the paste phase and therefore in the concrete. It implies that the paste phase is also uniformly distributed in the concrete.
2) Both phases develop a full bond. So, the two phases are assumed to be perfectly interacted with no relative average displacement. This assumption may not be accurate when shrinkage of paste phase is very high such as in the case of very high water to cement ratio.

2.1. Concrete as a two-phase material

Fig. 1 shows the conceptual illustration of concrete regarded as a two-phase material. One can defined the volume ratio of paste phase (n_p) and aggregate phase (n_a) as follows

$$n_p = V_p / V_{conc} \tag{1}$$

$$n_a = V_a / V_{conc} \tag{2}$$

where V_p is the volume of paste phase in the concrete, V_a is the volume of aggregate phase

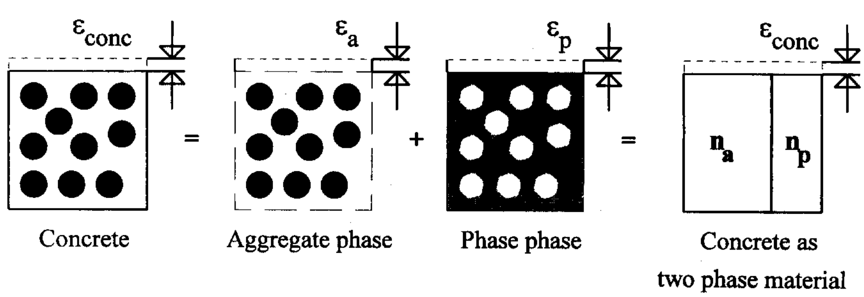

Fig. 1 Illustration of concrete as a two-phase material.

in the concrete and V_{conc} is the total volume of the concrete.

Considering a unit volume of concrete, the relation between n_p and n_a can be written as

$$n_p+n_a=1.0 \tag{3}$$

2.2. Strain compatibility

According to the two assumptions written earlier and since the strain of paste and aggregate phases are defined based on the total volume of concrete, the deformational strain of both phases can be assumed to be equal so that the strain compatibility is derived as

$$\varepsilon_a=\varepsilon_p=\varepsilon_{conc} \tag{4}$$

where ε_a is the strain of the aggregate phase in the concrete, ε_p is the strain of the paste phase in the concrete and ε_{conc} is the strain of the concrete.

2.3. Stress on aggregate phase from paste phase

By assuming that the paste phase is uniformly distributed in concrete, the strain of the paste phase is the averaged spatial strain based on a unit volume of the concrete. Define ε_{p0} as the strain due to shrinkage of the paste (concrete with $n_a=0$). If the aggregate phase does not resist the shrinkage of the paste phase, shrinkage of the paste phase in concrete with aggregate volume ratio n_a (ε_p) will be proportional to the volume ratio of the paste phase as

$$\varepsilon_p=\varepsilon_{p0}\cdot(1-n_a) \tag{5}$$

However, in actual, the shrinkage can be restrained by particle interaction of the aggregate phase. Assuming that the shrinkage of the paste phase considering aggregate restraint is equal to ε_{par}, a portion of shrinkage which is restrained by aggregate phase can be determined as

$$\varepsilon_{(p-a)}=\varepsilon_p-\varepsilon_{par}=\varepsilon_{p0}\cdot(1-n_a)-\varepsilon_{par} \tag{6}$$

Stress on aggregate phase ($\sigma_{(p-a)}$) is resulted form the restrained shrinkage of the paste phase by the aggregate phase, consequently $\sigma_{(p-a)}$ can be derived from

$$\sigma_{(p-a)}=E_p\cdot\varepsilon_{(p-a)}=E_p\cdot\varepsilon_{p0}\cdot(1-n_a)-E_p\cdot\varepsilon_{par} \tag{7}$$

where E_p is the paste stiffness considered to be a time dependent and nonlinear material constant.

2.4. Stress on paste phase from aggregate phase

Aggregate phase is regarded as a particle assembly with dispersed particles in the paste phase with many pairs or groups of local contacts. The strain of the aggregate phase is considered as an macroscopic average value. In the study of Pickett, aggregate phase was considered to deform due to the deformation of the particles itself. In this study, the deformational strain of the aggregate phase is due to the local displacement of all pairs of contact particles. The displacement of particle assembly mainly consists of two types providing that crushing of the particles does not occur. One is the displacement due to particle slip and rearrangement (ε_{av}), the other is the displacement due to particle deformation (ε_{ad}).

$$\varepsilon_a = \varepsilon_a(\varepsilon_{av}, \varepsilon_{ad}) \tag{8}$$

where ε_a is the total strain of the aggregate phase, ε_{av} is the displacement due to particle slip and rearrangement and ε_{ad} is the displacement due to particle deformation.

Accordingly, the resisting stress of the aggregate phase ($\sigma_{(a-p)}$) can be calculated from

$$\sigma_{(a-p)} = E_a \cdot \varepsilon_a \tag{9}$$

Where E_a is the stiffness of the aggregate assembly and can be derived from the particle contact density model by Deesawangnade, *T.*

2.5. Equilibrium condition of the concrete undergoing shrinkage

The equilibrium condition can be written in stress form because all stress are defined as the corresponding forces per unit area of the concrete. Because there is only internal stress in the process of shrinkage, so the equilibrium condition can be written as

$$\sum \sigma_i = \sigma_{(p-a)} + \sigma_{(p-a)} \tag{10}$$

where $\sum \sigma_i$ is the summation of the internal stresses, $\sigma_{(p-a)}$ is the stress on aggregate phase from the paste phase and $\sigma_{(a-p)}$ is the stress on paste phase from the aggregate phase.

From Eqs. (4), (7), (9) and (10), we obtain the equation for computing shrinkage strain of concrete as

$$\varepsilon_{conc} = \frac{\varepsilon_{p0} \cdot E_p(1-n_a)}{(E_p + E_a)} \tag{11}$$

2.6. Constitutive relations

2.6.1. Stiffness of the paste phase

Paste stiffness is adopted from the study on effective tensile young's modulus with no historically sustained tension or compression proposed by Yomeyama, K. *et al.*. In their model, the paste stiffness at any age is considered to be a sole function of compressive strength as following

$$E_p = 1.05 \times 10^4 \times (f_c)^{0.474} \tag{12}$$

where E_p is the paste stiffness in tension and f_c is the compressive strength of the paste.

2.6.2. Stiffness of the aggregate phase

A microscopic aggregate stiffness model was developed by the author from the concept of particle contact density and contact stresses. It was derived from the developed model that the stiffness of aggregate assembly, E_a, is the function of aggregate concentrations (n_a) and strain of aggregate phase (ε_a). The aggregate phase is assumed to consist of a number of local contact points. If the aggregate concentration increases, number of aggregate contact points also increases, resulting in higher stiffness of the aggregate assembly. Also, when the aggregate phase deforms in compression, in other word the compressive strain increases, the interparticle contact stresses at the aggregate contact both from the particle deformation and friction arise, then leading to increase in stiffness of the aggregate assembly. In this paper, the equations for aggregate stiffness are derived from the developed micro model as follows.

For aggregate phase containing fine aggregate only, the stiffness of the aggregate was derived as

$$E_{sm}=1.71\times10^{8}\times\varepsilon_{s}\times(n_{a})^{0.70}+4.45\times10^{5}\times(n_{a})^{0.77} \tag{13}$$

where E_{am} is the stiffness of fine aggregate (kgf/cm^2), ε_s is the strain of aggregate phase (with only fine aggregate in the phase) and n_a is the volume concentration of fine aggregate.

For aggregate phase containing coarse aggregate only, the stiffness of the aggregate was derived as

$$E_{gm}=4.48\times10^{8}\times\varepsilon_{g}\times(n_{a})^{0.17}+8.18\times10^{5}\times(n_{a})^{0.17} \tag{14}$$

where E_{gm} is the stiffness of coarse aggregate (kgf/cm^2), ε_g is the strain of aggregate phase (with only coarse aggregate in the phase) and n_a is the volume concentration of coarse aggregate.

2.7. Free shrinkage of paste phase

In this study, the free shrinkage of paste phase (ε_{p0}) was obtained from the shrinkage test on paste mixtures. However, a model for predicting free shrinkage of paste phase is needed in the future.

The paste stiffness model, model of free shrinkage of paste and aggregate stiffness model were used to simulate autogeneous and drying shrinkage of mortar and no-fine concrete specimens.

3. Experimental investigation

3.1. Materials and mix proportions

Ordinary Portland cement type *I*, natural river sand passing sieve No. 4 and crushed limestone with a maximum size of 20 mm were utilized. Properties of the materials used in the experiments are given in Table 1. The tested mix proportions are listed in Table 2 and Table 3.

3.2. Testing procedure

Strain was measured on two sides of each prism, and three specimens were used to obtain

Table 1 Physical properties of materials used in the test

Material	Max. size (mm)	Specific gravity (g/cm^3)	Blaine finess (cm^2/g)	Absorption (%)	Void content (%)
Gravel	.25	2.678	–	0.61	45.3
Sand	5	2.564	–	0.90	33.0
Cement	–	3.15	3467	–	

Table 2 Mix proportion of autogeneous shrinkage specimens

Mix no.	W/C (%)	Cement (kg/m^3)	Sand (kg/m^3)	Gravel (kg/m^3)	$\frac{n_a}{n_{a,max}}$	n_a
1	30	1,548	–	–	–	–
2	30	630	1,457	–	0.85	0.570
3	30	792	1,201	–	0.70	0.469
4	30	900	1,029	–	0.60	0.402
5	30	1,008	858	–	0.50	0.335
6	30	1,116	686	–	0.40	0.268
11	30	661	–	1,485	1.0	0.550
12	30	838	–	1,188	0.80	0.440
13	30	891	–	1,016	0.60	0.330
14	30	594	–	1,194	0.40	0.220

Table 3 Mix proportion of cement paste and mortar specimens for drying shrinkage

Mix no.	W/C (%)	Cement (kg/m^3)	Sand (kg/m^3)	$\frac{n_a}{n_{a,max}}$	n_a
15	50	517	1,372	0.80	0.536
16	50	640	1,115	0.65	0.436
17	50	762	858	0.50	0.335

an average value. Shrinkage strains were measured using a mechanical gauge with gauge length of 4 inch. Measurement was started at 1 day after casting. Cement paste and mortar specimens have dimensions of 4×16×1.5 cm while those with dimensions of 4×16×4 cm were applied to no-fine concrete. The control room conformed to ASTM Test Method C157-89. The condition in the room was maintained at a temperature of 25± 1℃ and a relative humidity of 60± 2%.

4. Test results and verification of shrinkage model

4.1. Verification of shrinkage model on autogeneous shrinkage of mortar

All mixes were made with the total water to cement ratio of 0.30 and under sealed condition. The tested mortar specimens have the volume concentrations of fine aggregate (n_a) equal to 0.570, 0.469, 0.402, 0.335 and 0.268, respectively. Cement paste was tested to obtained free shrinkage (ε_{p0}) and to calculate the stiffness of paste phase. The stiffness of fine aggregate in Eq. (13) was used in the prediction equation for mortar strain (ε_{conc}) and the results are given in Fig. 2 to Fig. 3.

4.2. Verification of shrinkage model on autogeneous shrinkage of no-fine concrete

The verification were conducted on no-fine concrete specimens with the ratio between volume concentration of coarse aggregate to its dry rodded volume concentration ($n_a/n_{a,max}$) equal to

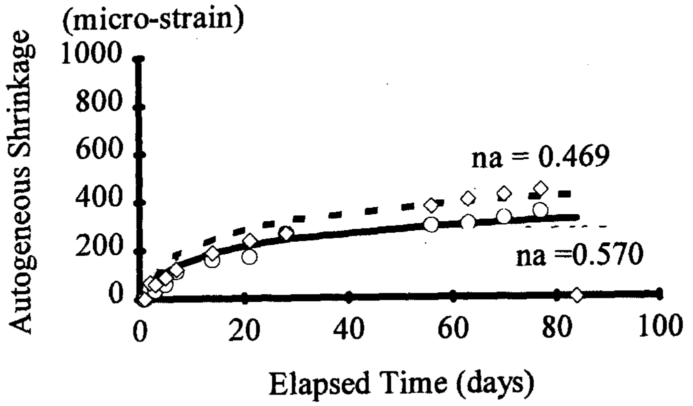

Fig. 2 Test and analytical results of autogeneous shrinkage of mortar specimens.

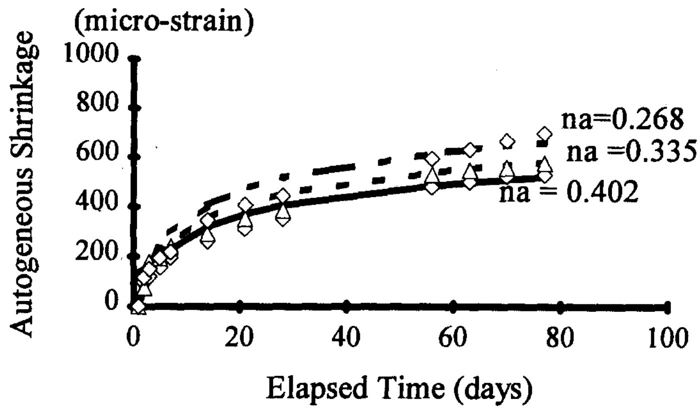

Fig. 3 Test and analytical results of autogeneous shrinkage of mortar specimens.

1.0, 0.80, 0.60 and 0.40 or coarse aggregate volume ratio (n_a) equal to 0.55, 0.44, 0.33 and 0.22, respectively. The stiffness of coarse aggregate in Eq. (14) was used in the prediction equation of no-fine concrete strain (ε_{conc}) and the results are shown in Fig. 4 and Fig. 5.

4.3. Verification of the shrinkage model on drying shrinkage of mortar

To verify the versatility of concrete shrinkage model, drying shrinkage of mortars was tested. Drying shrinkage strain of cement paste and stiffness of paste were obtained from test results of cement paste with water to cement ratio equal to 0.50. The mortar specimens was designed to have volume concentration (n_a) equal to 0.536, 0.436 and 0.335. Stiffness of fine aggregate in Eq. (13) was used in prediction of concrete strain and the results are given in Fig. 6.

5. Conclusions

A model for predicting shrinkage of concrete by considering effect of aggregate restraint was

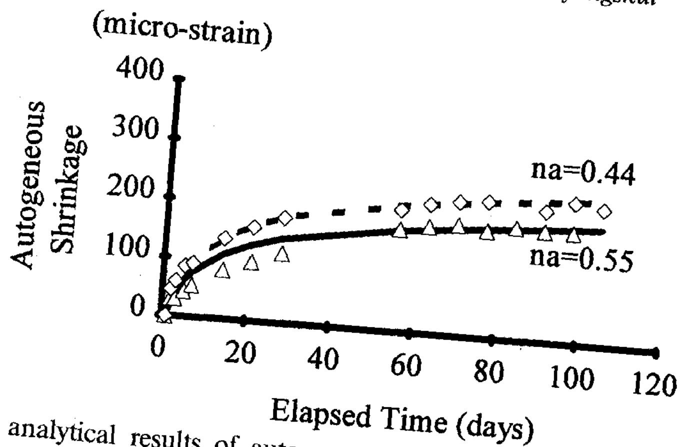

Fig. 4 Test and analytical results of autogeneous shrinkage of no-fine concrete specimens.

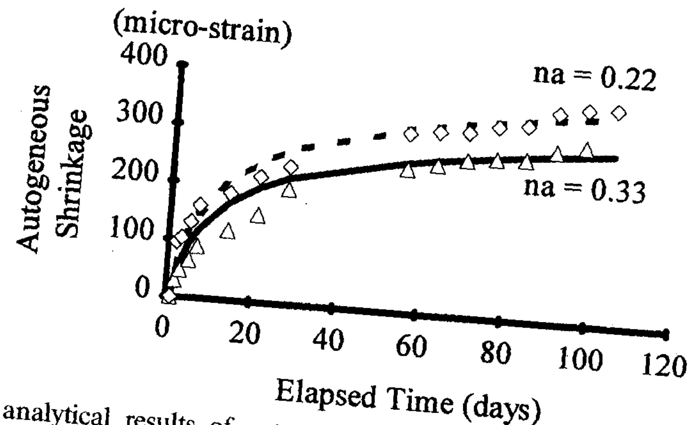

Fig. 5 Test and analytical results of autogeneous shrinkage of no-fine concrete specimens.

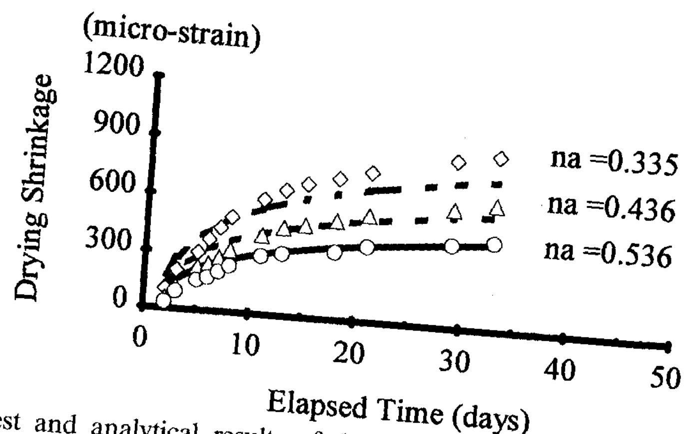

Fig. 6 Test and analytical results of drying shrinkage of mortar specimens.

derived based on a two-phase material concept. Concrete was considered to be composed of paste and aggregate phases.

Aggregate was assumed to be uniformly distributed in the paste phase, and both aggregate and paste phases underwent no relative average displacement. Strain compatibility was derived based on the assumptions.

Stress acting on the aggregate phase is the result of shrinkage of the paste phase which is the function of the paste shrinkage strain and the paste stiffness and the stress resisted by the aggregate phase is the function of the strain of the aggregate phase and its stiffness. Equilibrium condition can then be obtained by the balance of these two stresses.

The stiffness of paste phase was adopted from Yomeyama's study. The stiffness equation of the aggregate phase was derived from the particle contact density model by the authors.

Free shrinkage of paste phase in this study was tested, however, a quantitative model is needed.

The model was verified to be effective by comparing the test results of autogeneous shrinkage and drying shrinkage on mortar and no-fine concrete with the analytical results.

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