Structural Engineering and Mechanics, *Vol. 49, No. 3 (2014) 285-296*
DOI: http://dx.doi.org/10.12989/sem.2014.49.3.285

Influence of some key factors on material damping of steel beams

Yuanfeng Wang*, Yuhua Pan[a], Jie Wen[b], Li Su[c] and Shengqi Mei[d]

School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, China

(Received May 12, 2012, Revised November 12, 2013, Accepted December 9, 2013)

Abstract. Material damping affects the dynamic behaviors of engineering structures considerably, but up to till now little research is maintained on influence factors of material damping. Based on the damping-stress function of steel, the material damping of steel beams is obtained by calculating the stress distribution of the beams with an analytical method. Some key influence factors of the material damping, such as boundary condition, amplitude and frequency of excitation, load position as well as the cross-sectional dimension of a steel beam are analyzed respectively. The calculated results show that even in elastic scope, material damping does not remain constant but varies with these influence factors. Although boundary condition affects material damping to some extent, such influence can be neglected when the maximum stress amplitude of the beam is less than the fatigue limit of steel. Exciting frequency, load position and cross-section dimension have great effects on the material damping of the beam which maintain the similar changing trend under different boundary conditions respectively.

Keywords: material damping; loss factor; stress distribution; influence factors; analytical method

1. Introduction

Damping, which dissipates energy, is important to dynamic structural analyses (Lazan 1968, Nashif *et al.* 1985, Osiński 1998). Existing of high damping means that more energy is dissipated in the system, thus the deformation and stress amplitude will be reduced effectively. In general, there are several types of damping that exist in structures (Rainieri *et al.* 2010), one comes from structures themselves (internal damping) and another from the added energy dissipation devices (Lin *et al.* 2003). Neglecting other types of damping, internal damping in material is an important property of structures and how to choose the damping value should be emphasized for proper structural dynamic analysis and design.

There are two kinds of damping theory widely adopted by researchers and engineers in structural dynamic analysis and design so far (Crandall 1970, Bert 1973). One is the viscous damping model which is linear and supposed to be determined only by the instantaneous

*Corresponding author, Professor, E-mail: cyfwang@bjtu.edu.cn
[a]Ph.D. Student, E-mail: 06121470@bjtu.edu.cn
[b]Ph.D. Student, E-mail: alacrity_wen@163.com
[c]Ph.D. Student, E-mail: su_bjtu@163.com
[d]Ph.D. Student, E-mail: woshimeishengqi@163.com

http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

generalized velocity. In the structural dynamic analysis, the damping matrix is assumed to be a linear combination of system's mass and stiffness matrices, which is the so-called 'proportional damping' or 'Rayleigh damping' introduced by Rayleigh (1877). Since the proportional damping matrix can be diagonalized simultaneously with the mass and stiffness matrices by real normal modes, a proportionally damped system can be decoupled into a set of principal single-degree-of -freedom (SDOF) systems. The linear differential equations of motion in viscous damping theory are easy to be solved. However, there is an inherent deviation existing in the viscous damping theory that the viscous damping model implies energy dissipation per cycle is linearly proportional to the frequency. It is a well known result that the energy dissipation per cycle of many materials is a consequence of internal friction, and a number of experiments indicate that it is essentially frequency independent (Lazan 1968). This observation led to the introduction of the complex damping model, proposed by Myklestad (1952), Bishop (1956a, b), which has alternative names such as hysteretic damping (Bishop 1956b, Inaudi *et al.* 1995, Lin *et al.* 2009, Chen *et al.* 2008, Maia 2009), structural damping (Gounaris 1999), and material damping (Lazan 1968, Bert 1973). In the complex damping theory, the damping force was proportional to restoring force and the phase was the same with that of velocity. In forced vibration to harmonic excitation of single-degree-of-freedom systems, damping force is given by

$$F_d = \mathrm{i}\eta KX \tag{1}$$

where η is hysteresis damping factor, K denotes system stiffness, X stands for system displacement and $\mathrm{i} = \sqrt{-1}$. Obviously the energy dissipation in hysteresis damping system is not associated with exciting frequency and agrees well with the experimental data.

Until now no uniform theory or formula is applied in calculation of material damping in forced vibration because damping is an obscure property and the origin of energy dissipation cannot be defined clearly. It is found that the damping can be usually described by the ratio of dissipation energy to total strain energy and defined as the loss factor, that is, the hysteresis damping coefficient (Lazan 1968)

$$\eta = \frac{1}{2\pi}\frac{\Delta U}{U} \tag{2}$$

where U is the total strain energy and ΔU is the dissipation energy of damping. The key is how to determine the dissipation energy of damping. Based on experimental data of different materials including metal and polymer, etc, Lazan (1952, 1968) concluded that the dissipation energy of material damping is mainly influenced by three factors, that is, stress amplitude, load history and temperature. The influence of load history and temperature on different materials is so complicated that it is difficult to summarize them with a uniform equation, but under medium and high stress the energy dissipation is proportional to the logarithm of stress amplitude. Therefore, Lazan (1968) proposed the following function, that is, the damping-stress function

$$\Delta U(\sigma) = J\sigma^n \tag{3}$$

where $\Delta U(\sigma)$ is the dissipation energy of material damping, σ is stress amplitude and J and n are material constants. As n changes across the fatigue limit of steel, the following equation obtained by Lazan (1968) is adopted in this paper for the dissipation energy of material damping of steel

$$\Delta U(\sigma) = 6895(\sigma / \sigma_f)^{2.3} + 41360(\sigma / \sigma_f)^8 \tag{4}$$

where σ_f is the fatigue limit of steel. The material damping of structures can be calculated with the stress distribution in members and the damping-stress function. The stress distribution function of structural members only depends on the member dimension and cross section shape but independent of the material. Since the damping-stress function depends on material property but independent of member dimension and loading methods, the function can be obtained by simple experiments. Lazan (1952), Cochardt (1953, 1954), Yorgiadis (1954) and Hart (1975) calculated the damping value of materials with simple shapes. Kume (1982) derived the bending stress distribution function of cantilever beams on the basis of the deformation participated by single mode and then obtained loss factors of different modes without considering damping existing. By the finite element method, Gounaris (1999, 2007) divided structures into beam elements, assumed an initial loss factor in dynamic equilibrium and obtained the loss factor and structural response iteratively with stress distribution function and the damping-stress function. In addition, based on the study of material damping and stress amplitude relationship proposed by Lazan (1968), the damping-stress functions of reinforced concrete and concrete filled steel tube members were proposed by Wen (2008), Wang (2010). The solution method for the structural dynamic response with stress related complex damping was also discussed by Wang *et al.* (2008).

At present, few studies are maintained on the influence factors of material damping. There were limited experiments on the material properties including the damping-stress function and influence of temperature on material damping, etc., which are quite essential to the analysis of material damping. Moreover, the influence of system factors on material damping, such as boundary condition and loading methods has not been analyzed thoroughly. Although Lazan (1952), Kume (1982) presented the material damping of cantilever beams, other boundary conditions were not included. In addition, some influence factors such as loading methods, beam's cross-sectional dimension and shape, were merely considered in maximum stress amplitude roughly and the changing law has not been discussed.

In this paper, by adopting the damping-stress function proposed by Lazan (1968), the influence factors on the material damping for a steel beam under harmonic excitation are further studied. The dynamic equations of stress-dependent damped beams with four types of boundary conditions under vibration are presented, the analytical solutions are obtained to determine the stress distributions and loss factors of the beams. In addition, the influence of various boundary conditions, exciting frequencies, load positions and beam's cross-sectional dimensions on the loss factors are also analyzed.

2. Theory

Total strain energy U of a member can be defined as

$$U = \frac{1}{2}\int_V \frac{\sigma^2}{E} dV$$

$$= \frac{1}{2E}\int_0^1 \sigma^2 d(V / V_m)$$

$$= \frac{1}{2E}\int_0^1 \sigma^2 f(\sigma/\sigma_m) d(\sigma/\sigma_m)$$

$$= \frac{1}{2E}\int_0^{\sigma_m} \sigma^2 f(\sigma/\sigma_m) d(\sigma) \tag{5}$$

where E is the Young's modulus of the member, σ is the stress amplitude, σ_m the maximum stress amplitude in the member, V is the volume of the beam and V_m is the bulk volume of the beam.

The stress distribution function $f(\sigma/\sigma_m)$ is then defined by

$$f(\sigma/\sigma_m) = d(V/V_m)/d(\sigma/\sigma_m) \tag{6}$$

When only one mode of vibration is considered, the deflection of a beam under harmonic excitation becomes

$$w(x,t) = \frac{F_0 \left[\phi(x)\right]_{x=l_1} \phi(x)}{\rho A(\omega^2 - p^2)\int_0^l \phi(x)^2 dx} \sin\ pt \tag{7}$$

where $w(x,t)$ is the deflection of the beam, F_0 is the amplitude of the exciting force, x denotes the co-ordinate along the longitudinal axis of the beam, $x=l_1$ is the exciting point, p is the exciting frequency, ρ and A stand for the density and area of the cross section of the beam respectively, ω and l is the natural frequency and the total length of the beam alternatively, $\phi(x)$ is the modal function.

To simplify the analysis, it is assumed that the material is isotropic and Hook's law remains valid; the tensile Young's modulus is of the same value as the compressive; the cross section remains plane; the stress is uniform across the width of the beam; the material damping associated with shear stress will be neglected.

According to the material mechanics, the relation of bending stress ($\sigma_t(x,y,t)$) and deflection $w(x,t)$ of a flexural beam can be written as

$$\sigma_t(x,y,t) = -Ey\partial^2 w(x,t)/\partial x^2 \tag{8}$$

where y is the co-ordinate along the thickness of the beam.

Substituting Eq. (7) into Eq. (8) gives

$$\sigma_t(x,y,t) = C_f \xi(x,y) \sin\ pt \tag{9}$$

$$C_f = \frac{F_0 \left[\phi(x)\right]_{x=l_1}}{\rho A(\omega^2 - p^2)\int_0^l \phi(x)^2 dx} \tag{10}$$

$$\xi(x,y) = -Ey\frac{d^2\phi(x)}{dx^2} \tag{11}$$

Then the stress amplitude becomes

$$\sigma(x,y) = C_f \xi(x,y) \tag{12}$$

σ_m is the maximum stress amplitude found to be at the point (x_m, y_m)

$$\sigma_m = -Ey_m C_f \left[d^2\phi(x)/dx^2 \right]_{x=x_m} \tag{13}$$

From Eqs. (12) and (13), the equation of the constant stress contour line is

$$y(\sigma/\sigma_m, x) = y_m \left[d^2\phi(x)/dx^2 \right]_{x=x_m} (\sigma/\sigma_m)/\left[d^2\phi(x)/dx^2 \right] \tag{14}$$

In order to obtain the stress distribution function, the volume of the beam in which stress is less than σ is calculated, that is the volume stress function

$$\frac{V}{V_m} = \frac{2b}{V_m}\int_0^l y(\sigma/\sigma_m, x)dx \tag{15}$$

where V_m is the total volume of the beam and b is the width of the beam.
The stress distribution function is

$$f(\sigma/\sigma_m) = d(V/V_m)/d(\sigma/\sigma_m) \tag{16}$$

Similarly, dissipation energy ΔU can be defined as

$$\Delta U = \int_V \Delta U(\sigma)dV$$

$$= \frac{1}{2E}\int_0^{\sigma_m} \Delta U(\sigma) f(\sigma/\sigma_m)d(\sigma) \tag{17}$$

Using Eqs. (2), (5) and (17), the loss factor is obtained as

$$\eta = \frac{E}{\pi}\frac{\int_0^{\sigma_m} \sigma^2 f(\sigma/\sigma_m)d(\sigma)}{\int_0^{\sigma_m} \Delta U(\sigma) f(\sigma/\sigma_m)d(\sigma)} \tag{18}$$

3. Influence of different factors on material damping of steel beams

The model beam in reference (Gounaris 1999) is adopted for calculation and the data for analyzing different factors on material damping of steel beams are shown in Table 1.

Excitation amplitude and frequency are selected as 4000N and 1000rad/sec respectively in order to remain the stress amplitude of the beam less than the fatigue limit. Besides, for the convenience of comparison of different boundary conditions, the load is located on the mid-span of the steel beams.

3.1 Boundary conditions

Four types of boundary condition in real engineering are selected: fixed ends, fixed-free ends, fixed-hinged ends, hinged ends. Calculated by the above method, the eigen functions, coordinates of points with maximum stress and expressions of loss factor under four types of boundary

Table 1 data for material damping calculation

	Symbol	Value	Unit
Beam height	h	0.04	m
Beam width	b	0.1	m
Beam length	l	0.4	m
Beam area	A	0.004	m^2
Young's modulus	E	2.06×10^{11}	pa
Fatigue limit	σ_f	1.86×10^{8}	pa
Excitation amplitude	F	4000	N
Excitation frequency	p	1000	rad/sec
Load position	l_1	0.2	m
Density	ρ	7.8×10^{3}	kg/m3

Table 2 Mode functions of four types of boundary condition

Boundary condition	Eigen function	Mode function
Fixed ends	$\cos\beta l\cdot\cosh\beta l-1=0$	$\varphi(x)=(\sin\beta l-\sinh\beta l)(\cos\beta x-\cosh\beta x)$ $-(\cos\beta l-\cosh\beta l)(\sin\beta x-\sinh\beta x)$
Fixed-free ends	$\cos\beta l\cdot\cosh\beta l+1=0$	$\varphi(x)=(\sin\beta l+\sinh\beta l)(\cos\beta x-\cosh\beta x)$ $-(\cos\beta l+\cosh\beta l)(\sin\beta x-\sinh\beta x)$
Fixed-hinged ends	$\cos\beta l-th\beta l1=0$	$\varphi(x)=(\sin\beta l-\sinh\beta l)(\cos\beta x-\cosh\beta x)$ $-(\cos\beta l-\cosh\beta l)(\sin\beta x-\sinh\beta x)$
Hinged ends	$\sin\beta l=0$	$\phi(x)=\sin\beta x$

Table 3 Coordinate of point with maximum stress under four types of boundary condition

Boundary condition	First mode		Second mode		Third mode	
	Coordinate	Eigenvalue	Coordinate	Eigenvalue	Coordinate	Eigenvalue
Fixed ends	x=0, y=h/2	4.730041	x=0, y=h/2	7.853205	x=0, y=h/2	10.995608
Fixed-free ends	x=0, y=h/2	1.875104	x=0, y=h/2	4.694091	x=0, y=h/2	7.854757
Fixed-hinged ends	x=0, y=h/2	3.926602	x=0, y=h/2	7.068583	x=0, y=h/2	10.210176
Hinged ends	x=l/2, y=h/2	3.141593	x=l/4	6.283105	x=l/6	9.424778

Table 4 loss factor expressions under four types of boundary condition

Boundary condition	Loss factor expression
fixed ends	$\eta=3.52\text{e-}5\cdot\sigma_m^{0.3}+2.88\text{e-}0.52\cdot\sigma_m^{6}$
fixed-free ends	$\eta=3.73\text{e-}005\cdot\sigma_m^{0.3}+3.45\text{e-}052\cdot\sigma_m^{6}$
fixed-hinged ends	$\eta=3.48\text{e-}005\cdot\sigma_m^{0.3}+2.10\text{e-}052\cdot\sigma_m^{6}$
hinged ends	$\eta=3.79\text{e-}005\cdot\sigma_m^{0.3}+3.52\text{e-}052\cdot\sigma_m^{6}$

condition with an analytical methods are shown in Table 2, Table 3 and Table 4 respectively.

In the Table 2, βl is the relevant eigenvalue of the beam under different boundary condistion; in the Table 4, σ_m is the maximum stress amplitude in the steel beams.

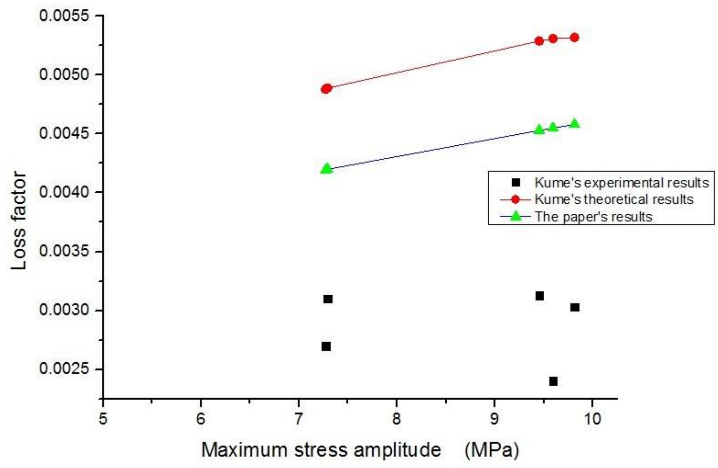

Fig. 1 Comparative results of loss factors(the first mode of vibration)

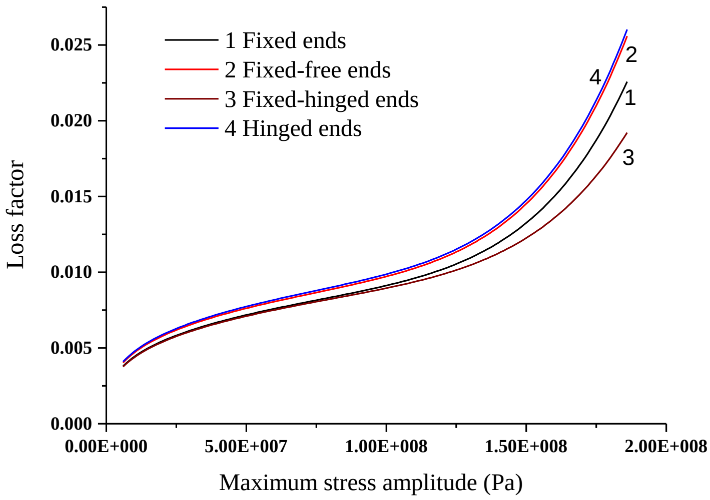

Fig. 2 Comparison of loss factor under varied types of boundary condition

To prove the validity of the proposed theoretical method, a comparative study was conducted between Kume's theoretical and experimental data (Kume 1982) and our analytical results. Only a cantilever beam has been studied in Kume's paper, the loss factors of a beam under various maximum stress amplitudes were provided, we calculated the corresponding loss factors under the same maximum stress amplitudes. As shown in Fig. 1, the comparative results indicate that the loss factors of the beam calculated by these two theoretical methods both increase with the increase of maximum stress amplitudes, and the proposed results were closer to Kume's experimental results.

The loss factors of the beams under varied types of boundary condition are calculated, the results are shown in Fig. 2. It can be seen that the material damping of steel beams increases nonlinearly with the increase of the maximum stress amplitude of the beams because of energy dissipation due to micro-plastic- deformation and crystal lattice dislocation. For example, when the maximum stress amplitude changes from 1Mpa to 102Mpa, the loss factor of the beams with hinged ends changes from 2.93×10^{-3} to 1.00×10^{-2}, and when the maximum stress amplitude changes to 186MPa, the loss factor reaches 2.60×10^{-2}. The loss factors of the beam under varied types of boundary condition are different as seen in Fig. 2. The loss factor of the beam with hinged ends is maximum and that of the beam with fixed-hinged ends is minimum, because the stress distribution in steel beam has great effect on loss factor besides material property. As for the same maximum stress amplitude, the volume of high stress in the steel beam with hinged ends is larger than that of the beam with fixed-hinged ends. As a result the dissipation energy and loss factor of the steel beam with hinged ends is more than that of the beam with fixed-hinged ends correspondingly. The difference under varied types of boundary condition is slight in low stress scope. When the maximum stress amplitude is less than 1.00×10^{8} Pa, the difference in loss factor between steel beams under varied types of boundary condition is around 7%~8%. The greater the maximum stress amplitude is, the more obvious the difference becomes. If the maximum stress amplitude reaches1.86×10^{8} Pa, that is the fatigue limit of steel, the difference in loss factors between the beam with hinged ends and that with fixed-hinged ends reaches 15.3%, which is induced by different stress distribution functions too. With the increase of maximum stress amplitude, the micro-plastic-deformation accumulates and the dissipation energy increases. For the steel beam with hinged ends, its proportion under a high-level stress is more than that in the steel beam with fixed-hinged ends, accordingly leading to that the proportion with micro-plastic-deformation and the energy dissipation in the steel beam with hinged ends are more as well. When the maximum stress amplitude becomes higher, the difference in the proportion with high-level stress and micro-plastic-deformation also becomes greater between the steel beams with these two types of boundary condition, finally resulting in a more distinct difference in the energy dissipation and loss factor. However, the absolute difference between the loss factors of the steel beam under varied types of boundary conditions is little. At the fatigue limit, the loss factor of the beam with hinged ends is only 3.45×10^{-3} more than that with fixed-hinged ends. Therefore, with the same maximum stress amplitude, varied types of boundary condition have influence on material damping of steel beams, but when the maximum stress amplitude is less than fatigue limit, the influence can be neglected.

3.2 Exciting frequency

Fig. 3 shows the influence of exciting force frequency on loss factor. When p/w changes from 0.1 to 2.0, influence of exciting frequency under four types of boundary condition on loss factor is presented in Fig. 3.

The trend of loss factor varied with exciting frequency under different boundary conditions is similar. For a single mode vibration, when the p/w is less than 0.7, the loss factor is remains constant, and as the p/w is more than 0.7, the loss factor will increase slowly. But if the exciting force frequency is close to the natural frequency of the beams, the loss factor will increase sharply, because the inertia force induced by exciting force increases quickly when the excitation frequency is close to the natural frequency and it leads to high stress and great energy dissipation in the beams. It is shown that material damping is important to reduce forced vibration response and the stress amplitude near resonance frequency. Since the stress distribution with no damping is applied to the calculated the loss factor in this paper, the stress amplitude and loss factor at the resonance frequency will be infinite. In reality, the stress amplitude and material damping are coupling and will reach critical values finally which can be obtained by the iterative method (Gounaris 1999).

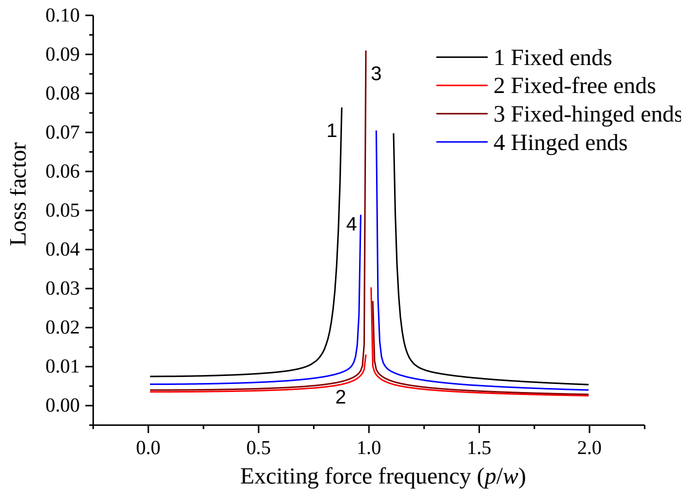

Fig. 3 Influence of exciting force frequency on loss factor

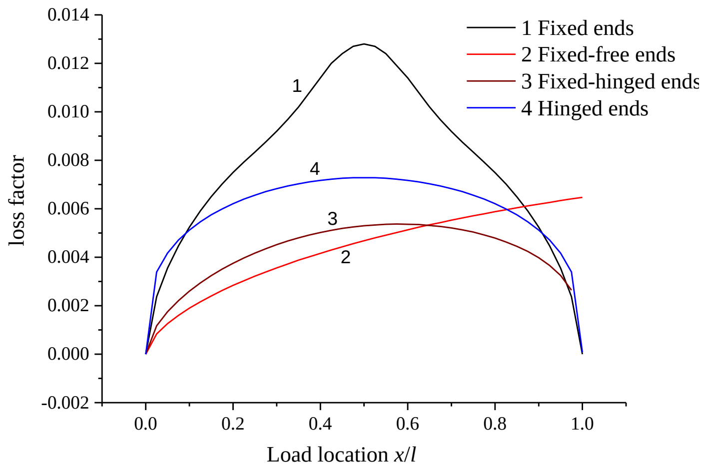

Fig. 4 Influence of load locations on loss factor

3.3 Load locations

The influence of load locations under four types of boundary condition of the beam on loss factor is shown in Fig. 4.

Because of the symmetry of the beams with fixed ends or with hinged ends, the curve of loss factor is symmetric too. When the load moves from the end to the mid-span of the beam, the loss factor increases gradually, till at the mid-span, that is $x/l=0.5$, the maximum loss factor is obtained, because the maximum moment is generated and the maximum stress distribution appears. The trend is a little different for the beam with fixed-hinged ends, where the loss factor will increase nonlinearly as the load moving from the fixed end to the hinged end. However, the maximum loss factor appears at $x/l\approx 0.56$, where x is the length between the load position and the fixed end, and then the loss factor decreases gradually and will be zero at the hinged end.

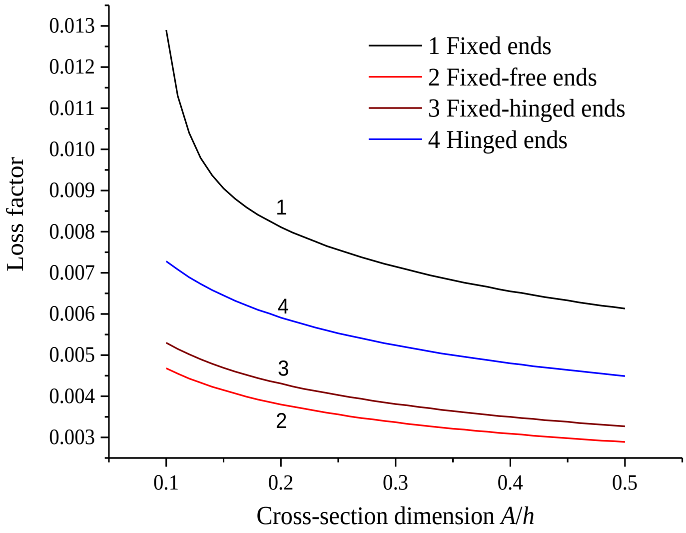

Fig. 5 Influence of cross-sectional dimension on loss factor

For the beam with fixed-free ends, the increase of moment and stress in beam leads to the nonlinear increase of the loss factor as the load moving from the fixed end to the free end. At the free end, that is x/l=1, the maximum loss factor is obtained.

It is noted that with the same excitation amplitude, the material damping value of the steel beam with fixed ends is not always the maximum. When the load is located in x/l =0~0.09, the material damping of the steel beam with fixed ends is more than that with hinged ends. But the material damping value of the steel beam with fixed-free ends is maximum when the load is located in x/l =0.88~1.0. It is indicated that under the different conditions, different influence factors will operate in the change of material damping.

3.4 Beam's cross-sectional dimension

When the A/h changes from 0.1 to 0.5, the influence of the cross-sectional dimension of the steel beam under four types of boundary condition on the loss factor is similar as shown in Fig. 5. With increase of the A/h, the loss factor decreases. For example, when the A/h changes from 0.1 to 0.5, the loss factor of the beam with fixed ends decreases by 27%. Since the decrease of cross-section height means that the distance of points on cross section to neutral axis is reduced, lower stress will appear in the beam and less the loss factor is obtained under the same moment.

4. Conclusions

As analyzed above, the material damping of steel beams does not remain constant even in elastic scope under forced vibration. Although boundary condition has effect on material damping to some extent, such influence can be neglected when the maximum stress amplitude is less than the fatigue limit of steel. Exciting frequency, load position and cross-section dimension have great effect on material damping which maintains the similar changing trend under different boundary conditions respectively. Therefore, as a complicated character of dynamic systems, material damping should be studied more comprehensively and profoundly further.

Acknowledgements

The authors would like to gratefully acknowledge the financial support of National Science Foundation of China (grant no.: 51278038).

References

Bert, C.W. (1973), "Material damping: an introductory review of mathematic measures and experimental technique", *J. Sound Vib.*, **29**(2), 129-153.

Bishop, R.E.D. (1956a), "The behavior of damped linear system in steady oscillation", *Aeronaut. Quart.*, **7**, 156-168.

Bishop, R.E.D. (1956b), "The general theory of hysteretic damping", *Aeronaut. Quart.*, **7**, 60-70.

Chen, K.F. and Zhang, S.W. (2008), "On the impulse response precursor of an ideal linear hysteretic damper", *J. Sound Vib.*, **312**(4), 576-583.

Cochardt, A.W. (1954), "A method for determining the internal damping of machine members", *J. Appl. Mech.*, **22**, 257-262.

Crandall, S.H. (1970), "The role of damping in vibration theory", *J. Sound Vib.*, **11**(1), 3-IN1.

Gounaris, G.D. and Anifantis, N.K. (1999), "Structural damping determination by finite element approach", *Comput. Struct.*, **73**(1), 445-452.

Gounaris, G.D., Antonakakis, E. and Papadopoulos, C.A. (2007), "Hysteretic damping of structures vibrating at resonance: an iterative complex eigensolution method based on damping-stress relation", *Comput. Struct.*, **85**(23), 1858-1868.

Hart, G.C. and Vasudevan, R. (1975), "Earthquake design of buildings: damping", *J. Struct. Div.*, **101**(1), 11-30.

Inaudi, J.A. and Kelly, J.M. (1995), "Linear hysteretic damping and the Hilbert transform", *J. Eng. Mech.*, **121**(5), 626-632.

Kume, Y., Hashimoto, F. and Maeda, S. (1982), "Material damping of cantilever beams", *J. Sound Vib.*, **80**(1), 1-10.

Lazan, B.J. (1952), *Effect of damping constants and stress distribution on the resonance response of members*, Minnesota University, Minneapolis.

Lazan, B.J. (1968), *Damping of Material and Members in Structural Mechanics*, Pergamon Press, London.

Lin, R.M. and Zhu, J. (2009), "On the relationship between viscous and hysteretic damping models and the importance of correct interpretation for system identification", *J. Sound Vib.*, **325**(1), 14-33.

Lin, Y.Y. and Chang, K.C. (2003), "Study on damping reduction factor for buildings under earthquake ground motions", *J. Struct. Eng.*, **129**(2), 206-214.

Maia, N. (2009), "Reflections on the hysteretic damping model", *Shock Vib.*, **16**(5), 529-542.

Myklestad, N.O. (1952), "The concept of complex damping", *Appl. Mech.*, **19**, 284-286.

Nashif, A.D., Jones, D.I. and Henderson, J.P. (1985), *Vibration Damping*, John Wiley & Sons, New York.

Osiński, Z. (1998), *Damping of Vibrations*, A.A. Balkema, Netherlands.

Rainieri, C., Fabbrocino, G. and Cosenza, E. (2010), "Some remarks on experimental estimation of damping for seismic design of civil constructions", *Shock Vib.*, **17**(4), 383-395.

Rayleigh, L. (1877), *Theory of Sound*, Two Volumes, Dover Publications, NewYork. (Re-issued 1945)

Wang, Y.F. and Li, P. (2008), "Analysis of influence of material damping on the dynamic response of reinforced concrete frame structures", *China Civil Eng. J.*, **41**(11), 39-43. (in Chinese)

Wang, Y.F. and Wen, J. (2010), "Computation and formula for material damping of concrete-filled steel tube components under axial cycle load", *J. Vib. Shock*, **29**(4), 12-16. (in Chinese)

Wen, J. and Wang, Y.F. (2008), "Calculation of material damping of reinforced concrete cantilever beams", *China Civil Eng. J.*, **41**(2), 77-80. (in Chinese)

Yorgiadis, A. (1954), "Damping capacity of materials", *Product Eng.*, **32**, 164-170.