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A direct damage detection method using Multiple Damage Localization Index Based on Mode Shapes criterion

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Abstract. A new method of multiple damage detection in beam like structures is introduced. The mode shapes of both healthy and damaged structures are used in damage detection process (DDP). Multiple Damage Localization Index Based on Mode Shapes (MDLIBMS) is presented as a criterion in detecting damaged elements. A finite element modeling of structures is used to calculate the mode shapes parameters. The main advantages of the proposed method are its simplicity, flexibility on the number of elements and so the accuracy of the damage(s) position(s), sensitivity to small damage extend, capability in prediction of required number of mode shapes and low sensitivity to noisy data. In fact, because of differential and comparative form of MDLIBMS, using noise polluted data doesn't have major effect on the results. This makes the proposed method a powerful one in damage detection according to measured mode shape data. Because of its flexibility, damage detection process in multi span bridge girders with non-prismatic sections can be done by this method. Numerical simulations used to demonstrate these advantages.

Keywords: structural damage detection; Multiple Damage Localization Index Based on Mode Shapes (MDLIBMS); Finite element modeling; Damage detection process (DDP)

1. Introduction

Damage occurrence in structures is being under consideration in recent years. Aging, fatigue, environmental effects, earthquakes and etc. are some inducements that cause damages in structures. Variation in mass, stiffness and damping ratio of the structures are the consequences and changing the responses of the structures is the following effect of damage presence in them. Several non-destructive test such as visual inspection, acoustic test and ultrasonic are used in damage detection process (DDP). These methods are more time consuming and cannot be utilized during the operation time of structures. Unlike these local methods, static and dynamic methods are used in DDP. Because of the easy performing in the field, dynamic methods are more attractive than the static ones (Guan and Karbhari 2008).

There are a lot of researches and published reports in the literature that deals with the dynamic field of damage detection in structures (Li and Chen 2013). The Multiple Damage Location Assurance Criterion (MDLAC) was introduced in order to estimate the size of defects in structures

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(Messina *et al*. 1998). According to the base of this criterion, measured frequencies of healthy and damaged structures are the main parameters of MDLAC correlation coefficient term. Sensitivity and statistical methods are also used in this method. Using incomplete mode shapes instead of modal frequency in MDLAC and performing the DDP is another method for localizing the damages (Shi *et al*. 2000).

Optimization methods were used in DDP. In these methods, an objective function is proposed and an optimization algorithm such as genetic, Particle Swarm Optimization (PSO) and etc. are used to find values that made the objective function optimum. Using objective function in the form of penalty function which is based on measured data (Friswell *et al*. 1998) is another method for damage detection in structures. After damage localization, a standard eigen-sensitivity method is used to optimize the damage extent. A combination of genetic algorithm and MDLAC based on correlation and sensitivity of modal data was used in long span, cable-stayed bridges (Koh and Dyke 2007). Developing MDLAC index by adding a frequency based part to it and using modified genetic algorithm (MGA) (Nobahari and Seyedpoor 2011) make a good performance in optimization based method. The algorithm modification was done by introducing two operators called health and simulator in order to accurately detect the location and extent of the eventual damage. Developing genetic algorithm with consideration of structural flexibility matrix was another method which was used in DDP in a shear building (Na *et al*. 2011). Modal Strain Energy Based Index (MSEBI) was used to identify the site and extent of damages in structures with a combination of PSO algorithm (Seyedpoor 2012).

Developing training approaches like neural network and using it in damage detection was studied in some published papers. Using Fuzzy Neural Networks (FNNs) and data fusion techniques (Jiang *et al*. 2011) is considered in structural health monitoring and damage detection of structures with enormous measured and undertrained data. Damage detection in pre stressed concrete beams (Jeyasehar and Sumangala, 2006) was done by using Artificial Neural Network (ANN) and comparing static and dynamic behavior of health and damaged ones. Identifying damage in a concrete cantilever beam according to Elman Neural Network (ENN) (Yu and Jianwei 2010) and detecting damage in structures by using fuzzy cognitive maps and Hebbian learning (Beena and Ganguli 2011) are some other studies in this field.

Frequency response function (FRF) based methods were also used in DDP. Using the FRF curvature (Sampaio *et al*. 1999), damage index method (Stubbs *et al*. 1995), considering the normalized imaginary part of FRF shapes (Liu *et al*. 2009), and FRF-based structural damage identification method (SDIM) for beam structures (Lee and Shin 2002) are some methods of damage detection in FRF field. Even combination of finite model updating method and frequency response function data of structures make a new damage detection method. Solution of sensitivity equations through the Least Square algorithm and weighting of these equations (Esfandiari *et al*. 2010) and using least-square algorithm method with appropriate normalization for solving the over-determined system of equations with noise-polluted data (Esfandiari *et al*. 2009) are some aspects of these works.

Using an evidence theory in DDP can be found in (Gou and Ling 2006). A combination of MDLAC based on frequencies and mode shapes, and frequency change damage detection method (FCDDM) is used to make the local decision and then, it would be sent to the fusion center in order to acquire a global decision by using evidence fusion technique. Performing sensitivity analysis of modal parameters for damage identification of a beam (Lakshmi *et al*. 1999), using a hybrid technique, consists of grey relation analysis to exclude the impossible damage locations and using genetic algorithm with simulated annealing and adaptive mechanisms for finding the actual damages (He and Hwang 2007) are other methods that are published in literature.

Some researchers considered wavelet and curvelet transform in damage detecting of structures. Continuous wavelet transformation of the structures forced dynamic response that represents the shape attributes of time series and enhance their delineation in the time-scale domain (Danai *et al*. 2012), experimental studies for crack detection of a beam structure under a static displacement with the spatial wavelet transform (Wu and Wang 2011) and using curvelet transform to assess damage location in plate structure (Bagheri *et al*. 2009) are successful researches in this field.

Considering modal data based method in damage diagnosis problems (Khorshidian and Esfandiari 2011) is another method that deals with modal response of structure and incomplete measured mode shapes and noise polluted data. Another method of damage detection in beam like structures, based on experimentally obtained modal parameters, was introduced that could detect fatigue damage occurrence in an aluminum cantilever beam (Radzien *et al*. 2011).

In current study, a new and simple dynamic method is proposed for damage detection in beam like structures. Using mode shapes of structures is the base of proposed method. Multiple Damage Localization Index Based on Mode Shapes (MDLIBMS) is presented as a criterion that combines modal parameters and localizes the damaged element(s) position. The main advantages of the proposed method are its simplicity, flexibility on the number of elements and so the accuracy of the damage(s) position(s), sensitivity to a small degrees of damage and capability of predicting the required number of mode shapes. Modeling procedure is done by using finite element modeling.

Damage usually causes a reduction in the local stiffness of the structures. One option is to model this as reduction in stiffness at the element by reduction in Modulus of Elasticity. This equivalent modeling approach is often sufficient for the identification of local damage using low frequency vibration measurements (Morassi and Vestroni 2008). So in this research, stiffness reduction by decreasing modulus of elasticity of structure elements is considered as damage presence in structure. Numerical simulations express the advantages of the proposed method in DDP. Flexibility property of proposed method makes it a usable method with a good performance in DDP of multi span bridge girders with non-prismatic sections. The following sections in this paper will be organized as follows:

Theoretical formulation is presented in section 2. A brief overview of mode shape computation according to finite element formulation, calculation of MDLIBMS criterion and applying noise to measured data are contents of this section in respect. In section 3, the proposed algorithm is being introduced. Prediction of the number of required modal parameters is discussed in this section. In section 4, the advantages of the proposed method are studied by three numerical simulations. Conclusion around the whole method is the final section of this paper.

2. Theoretical formulation

2.1 Overview of mode shape computation according to finite element formulation

Beams are slender elements that are used to support transverse loading. Long horizontal members used in buildings and bridges are some examples of beams. The bending strain energy of an element of length *dx* in beams can be expressed as

$$dU = \frac{1}{2}\int_L \sigma(x).\varepsilon(x).dA dx = \frac{1}{2}\left(\frac{M^2(x)}{EI^2(x)}\int_A y^2 dA\right)dx = \frac{1}{2}\frac{M^2(x)}{EI(x)}dx \;\Rightarrow\; U = \frac{1}{2}\int_L \frac{M^2(x)}{EI(x)}dx \tag{1}$$

Where *M(x)* is the bending moment, *E* is the module of elasticity and *I(x)* is the section moment of inertia. According to the elementary beam theory, for a small deflection

$$\frac{d^2\psi(x)}{dx^2} = \frac{M(x)}{EI(x)} \tag{2}$$

Where *ψ(x)* is the deflection of the centroid axis at *x*. By substituting Eq. (2) in Eq. (1), the bending strain energy of a beam element structure can be expressed as

$$U_e = \frac{1}{2}\int_L EI(x)\left(\frac{d^2\psi(x)}{dx^2}\right)^2 dx \tag{3}$$

For a single beam element, with uniform stiffness *EI(x) =EI* and length l_e, Hermite cubic shape functions can be made and by substituting them in Eq. (3), the strain energy of a beam element yields

$$U_e = \frac{1}{2}\{u\}^T.k_e.\{u\} \tag{4}$$

Where {u} represents element nodal displacement vector and $[K_e]$ is the stiffness matrix of element which is

$$[K_e] = \frac{E_e.I_e}{l_e^3}\begin{bmatrix} 12 & 6.l_e & -12 & 6.l_e \\ 6.l_e & 4.l_e^2 & -6.l_e & 2.l_e^2 \\ -12 & -6.l_e & 12 & -6.l_e \\ 6.l_e & -2.l_e^2 & -6.l_e & 4.l_e^2 \end{bmatrix} \tag{5}$$

It is possible to evaluate the mass coefficients corresponding to the nodal coordinates of a beam element by a procedure similar to the determination of element stiffness coefficients. Consider a beam element with mass unit of volume ρ_e, cross area section of A_e and length of l_e. Using the Hermite cubic shape functions, the consistent mass matrix of element can be expressed as

$$m_e = \int_{le} \rho_e [\psi]^T [\psi] dV$$
$$[m_e] = \frac{\rho_e.A_e.l_e}{420}\begin{bmatrix} 156 & 22.l_e & 54 & -13.l_e \\ 22.l_e & 4.l_e^2 & 13.l_e & -3.l_e^2 \\ 54 & 13.l_e & 156 & -22.l \\ -13.l_e & -3.l_e^2 & -22.l_e & 4.l_e^2 \end{bmatrix} \tag{6}$$

According to the principals of dynamic analysis of structures, solving the differential equation of motion (Eq. (7)) yields the dynamic response of structure.

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{P(t)\} \tag{7}$$

Where [*M*], [*C*], [*K*] and {*P* (*t*)} are mass, damping, stiffness and assigned dynamic forces of structure. In order to compute structural mode shapes, calculation of eigenvector and eigenvalues can be done according to Eq. (8) and Eq. (9)

$$[[K]-\omega^2[M]]\{\varphi\}=0 \tag{8}$$

$$\left|[K]-\omega^2[M]\right|=0 \tag{9}$$

By solving Eq. (9), natural circular frequencies of the structure $\{\omega_i\}$ can be compute for each mode shape; calculation of natural mode shapes of structure can be done by substituting values of ω_i in Eq. (8).

After computing mass and stiffness matrices for the whole structure and considering boundary conditions, mode shapes of structure can be reached by substituting mass and stiffness matrices in Eq. (9) and Eq. (8). As all the required values are according to finite element analysis of the structures (healthy and damaged one) in this paper, stiffness reduction by decreasing modulus of elasticity of structure elements considered as damage presence.

2.2 Multiple Damage Localization Index Based on Mode Shapes (MDLIBMS) criterion

In this paper, the proposed method can find damaged elements by using a criterion that combines mode shapes of healthy and damaged structures. When damage is introduced in a structure, the bending stiffness at the location of the damage is reduced while at the same time the magnitude of the element rotation increases. The absolute rotation of an element can be measured by summing rotational mode shapes of element degrees of freedoms with rotation of element caused by transverse mode shapes. The second term is called first derivative of transverse mode shapes. Absolute rotation of a damaged element can be expressed as

$$Rot_i^D=\left(\frac{\delta_k-\delta_j}{l_e}\right)_D+\left(\theta_k+\theta_j\right)_D \tag{10}$$

Where δ and θ represent transverse and rotational mode shapes respectively, k and j represents initial and final node of element and D represent damaged element of structure. Comparing the absolute rotation of structural elements by subtracting the computed values of each neighbor elements, gives relative rotation of each element in compare to its neighbor

$$\Delta Rot_i^D=Rot_i^D-Rot_{i-1}^D \tag{11}$$

$$\Delta Rot_{i+1}^D=Rot_{i+1}^D-Rot_i^D \tag{12}$$

Eq. (11) represents the absolute rotation of ith element with its previous neighbor ($i-1$) and Eq. (12) represents the absolute rotation of ith element with its following neighbor ($i+1$).

Computing the same absolute rotation of health structure elements (Eq. (13) and Eq. (14) and Eq. (15)) and normalizing damaged values with health ones (Eq. (16) and Eq. (17)) represent MDLIBMS of structural elements and its one side neighbor

$$Rot_i^H=\left(\frac{\delta_k-\delta_j}{l_e}\right)_H+\left(\theta_k+\theta_j\right)_H \tag{13}$$

$$\Delta Rot_i^H = Rot_i^H - Rot_{i-1}^H \tag{14}$$

$$\Delta Rot_{i+1}^H = Rot_{i+1}^H - Rot_i^H \tag{15}$$

$$MDLIBMS_{i+1} = \frac{\Delta Rot_{i+1}^D}{\Delta Rot_{i+1}^H} \tag{16}$$

$$MDLIBMS_i = \frac{\Delta Rot_i^D}{\Delta Rot_i^H} \tag{17}$$

Drawing MDLIBMS values as a function of structural elements represent damage area without any specification of exact damaged elements. In order to overcome this deficiency, multiplying MDLIBMS *i*+1 and MDLIBMSi values gives a smooth diagram that shows exact damaged elements

$$MDLIBMS_{element} = MDLIBMS_{i+1} \times MDLIBMS_i \tag{18}$$

As it can be seen, the proposed method is very simple. Measuring mode shapes and replacing the values in equations will reach the user to damaged elements without any confusing calculations. This is one of the advantages of the proposed method.

2.3 Applying noise to measured data

In lots of researches, there are some random variables that affect the input data. Using these data causes error appearance in final results. In practical researches, because of measurement errors, the measured responses of structure are always containing errors which are called noises. These noises are produced randomly that follow by normal or Gaussian distribution. In order to get better results in practical fields of research, it is better to extend a method that has low sensitivity around noisy data.

As the main field of this research is based on simulated data, it is necessary to test the proposed method on both simulated and real data. Deficiency in real data made the authors to apply noise to simulated data and test the proposed algorithm by applying noise to measured mode shapes which follow by standard normal distribution. The general form of noise according to standard normal distribution can be expressed as

$$\varepsilon \sim \quad N(0, I) \tag{19}$$

If φ_D represents measured mode shapes of damaged structure according to simulated data, noise polluted data can be generated as

$$\varphi_D^{noisy} = \left(\left[I\right] + \xi \times \left[\varepsilon\right]\right) \times \varphi_D \tag{20}$$

Where [I] and ξ represent identity matrix and noise percentage in respect.

3. Damage detection

3.1 Damage detection procedure using mode shapes and MDLIBMS criterion

A new method of damage detection of beam like structures is being proposed in this paper. As damage presence in structure has effects on dynamic response of it, mode shapes are also being affected. So by using MDLIBMS criterion, which was introduced in section 2, and following the subsequent algorithm, damaged element(s) position can be detected. The proposed algorithm can be expressed as:

Step1: Compute first mode shape vectors for healthy and damaged structures.

Step2: Separate transvers and rotational mode shapes of each structure.

Note1: Because of their difficulty in measurement, rotational mode shapes are not measured in experimental works (Pandey and Samman 1991), and they can be computed in analytical studies. So, only transvers mode shapes can be considered for damaged structures in practical works. In contrast, because the responses of healthy structures are usually computed by simulated analysis, both transvers and rotational mode shapes can be calculated and considered for healthy structures.

Step3: Draw transverse mode shapes of healthy structure as a function of element nodes and specify turning points.

Step4: Draw rotational mode shapes of healthy structure as a function of element nodes and specify proportional nodes of relative maximum or minimum values.

Note2: Let's call the node that satisfies the conditions in steps 3 or 4, Conditional Node (CN).

Step5: Compute MDLIBMS criterion and draw it as a function of element number.

Note3: Considering transverse or both transvers and rotational mode shapes of damaged and healthy structure in computing MDLIBMS criterion depends on whether rotational mode shapes of damaged structure are measured in step2 or not. Measuring transvers mode shapes of damaged structure only, omits rotational terms from Eq. (13) and Eq. (14). So, only transvers mode shapes of healthy and damaged structure build MDLIBMS criterion.

Step6: Referring to the MDLIBMS diagram, for all the structure nodes that are not CNs (in the first mode), if there is any distortion in the diagram, mark proportional element number as damaged one.

Step7: If there is any node (according to the first mode) that is CN, increase mode number one unit and go back to step 2. Do this procedure till all the specified elements number (according to the first mode), and their 1 or 2 unit neighborhood, being free from the conditions of steps 3 and 4.

Step8: Referring to the MDLIBMS diagram(s), damaged elements collection can be reached by collecting all the marked elements number.

The flowchart of the proposed algorithm is shown in Fig. 1.

3.2 Predicting the number of required parameters

As it was mentioned before, the proposed method has the ability of predicting the number of required parameters (here is mode shapes). Contrary to some other methods (like MDLAC), knowing about the requirement data will decrease the number of measurements. Referring to steps 3 and 4, in the first mode shape diagrams of healthy structure, if there is any node that appear as turning or relative maximum or minimum, it would be difficult to judge about the health or damaged condition of the elements, proportional to these nodes according to MDLIBMS diagram of the first mode shape (see sections 4.2 and 4.3). So it needs to use MDLIBMS criterion of upper mode shapes in order to clear the real condition of these elements. To make a prediction about the required number of mode shapes, one can extract the next modal vector of health structure by using simulated analysis and draw the separated diagrams of mode shapes and specify conditional nodes (steps 3 and 4). Omitting coincident nodes between the new diagrams and the later ones, if there is any coincident

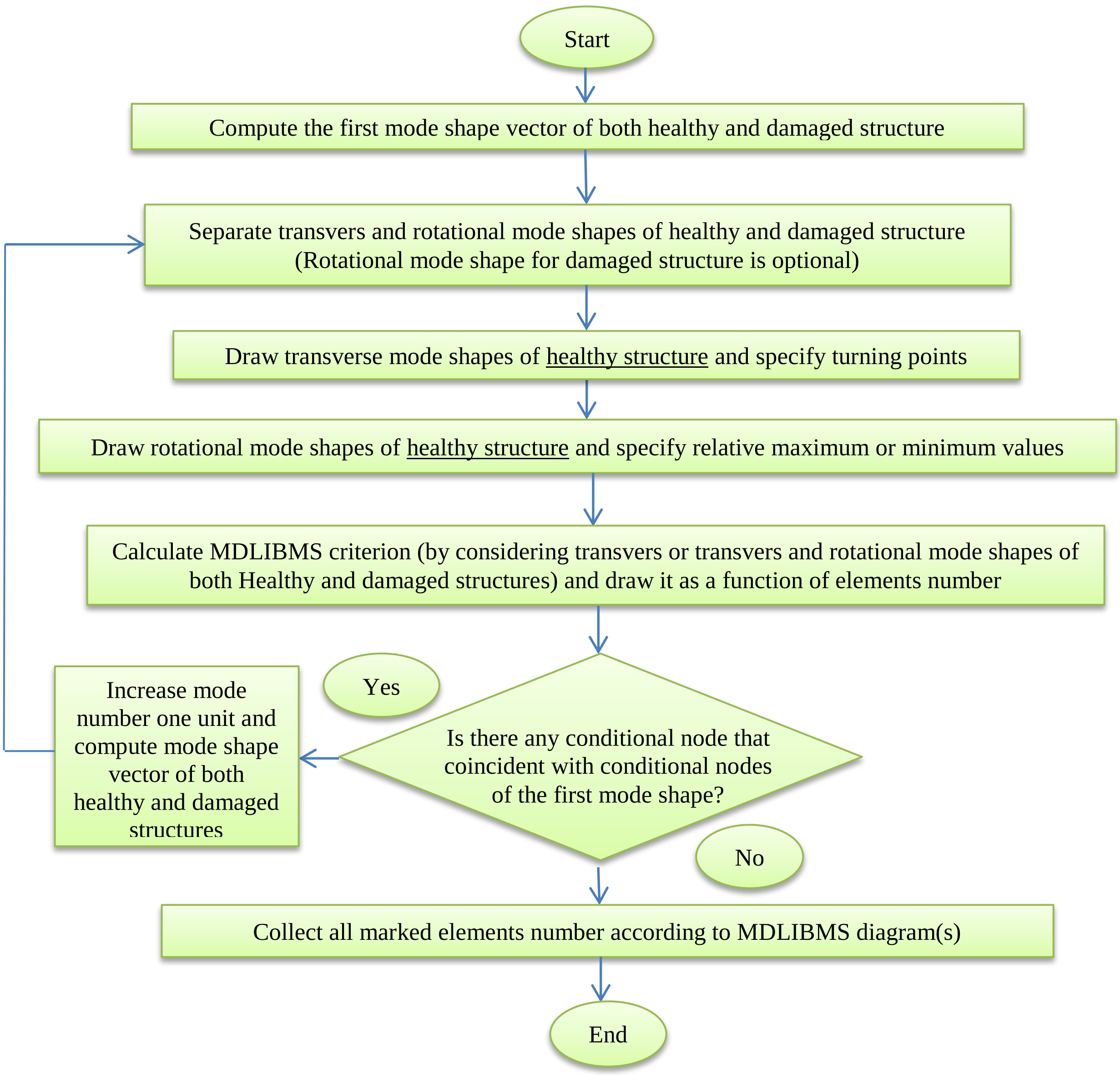

Fig. 1 The flowchart of proposed algorithm

Table 1 Beam section properties

Model No.	Description	Modulus of elasticity E (Gpa)	Mass unit of volume ρ_e (Kg/cm^3)	Section height h (cm)	Section width b (cm)	Element Length l_e (cm)
1	Cantilever beam	210	7.85E-3	2.50	2.50	1.00
2	One span Concrete beam	25	2.5E-3	50	30	5.00
3	Bridge girder	210	7.85E-3	Var.	70	Var.

between the remain conditional nodes of new diagram and the ones belong to the first mode, it needs to increase mode number one unit and do the above procedure for the next mode. Having no coincident around conditional nodes means that there is no need to compute upper mode shapes and the required number of mode numbers are given. Numerical simulations in the following section will show how this procedure can be done in practical problems.

4. Numerical simulations

In this section, the performance of the proposed method in damage detection is going to be studied. To calculate modal parameters, finite element modeling is used. In Table 1, beam section properties are shown. In order to show the advantages of the proposed method, three different numerical simulations are presented. As it was mentioned before, in order to test the proposed method with real data, noise polluted data are considered in some examples. In the first example, a cantilever beam is going to be studied without applying noise to measured mode shapes of damaged structure. As this example is a popular one in simulated damage detection researches, both transition and rotational modal parameters are considered. In the second example, a one span concrete beam is going to be studied. In this example, noise polluted data of both transition and rotational modal parameters are considered. The last simulation deals with a bridge girder with non-prismatic section. In this example, in order to simulate the conditions of a real damage detection problem, noise polluted data with only transition mode shapes are used in damage detection process. It should be mentioned here that in the last two examples, 5% normal distributed noise is applied to the measured data. The following examples represent the advantages of the proposed method.

4.1 Cantilever beam with 100 elements

A cantilever beam of 100 elements with uniform cross section is considered as the first numerical simulation. This model is a popular one in damage detection literature [4- 6, and 8]. Most of the authors considered a cantilever beam of 15 to 20 elements and maximum 2 damaged ones [5, 6, and 8] and proposed optimization algorithm to detect damaged elements. These methods would be affected by the size of the problem and get into instability and time consuming trouble as the size of the problem increase. In general, most of the numerical based methods face time consuming problem. In other word, increasing the number of elements in a finite element modeling would reached to an accurate answer, but duration of problem solving and convergence of the algorithm rise another problems for the user.

The first model that is going to be discussed is a cantilever beam as shown in Fig. 3. The main

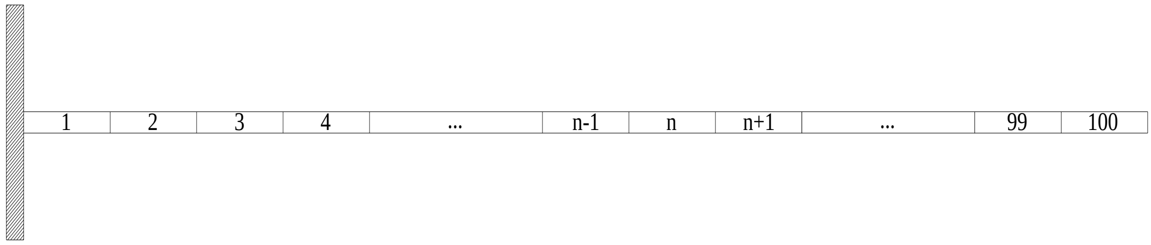

Fig. 2 A cantilever beam with 100 elements

Table 2 Damage scenario of the 100 elements cantilever beam

Damaged elements No.	Damage ratio
3	15%
11	20%
25	10%
33	5%
54	25%
89	30%

purpose of presenting this example is to emphasis the advantage of the proposed method in flexibility about number of total elements and number of damaged ones. Damage scenario for this beam is presented in Table 2. DDP is being done according to the proposed method.

After 1st mode shapes computation of healthy and damaged structures, (Step 1), transverse (Fig. 3(a)) and rotational (Fig. 3(b)) mode shapes are being drawn (Steps 2 to 4). In order to make a prediction about required parameters, these two diagrams were checked and no CN was found. So, only first mode shapes can be used in DDP in a cantilever beam and there is no need to calculate upper mode shapes number. As it can be seen from MDLIBMS diagram (Fig. 3(c)), the distortions represent damage presence in structural elements (Step 5 to 8).

It should be mentioned that by referring to Table 2, the maximum damage ration is belongs to element with number 89 and elements with numbers 54, 11, 3, 25 and 33 carrying higher damage ratio respectively. Referring to Fig. 3(c) it can be seen that there is a proportion between the values of MDLIBMS criterion for damaged elements. This is another capability of the proposed method that let the user to compare damage extent of elements with each other. As it was mentioned before, the proposed method has a great sensitivity to a small amount of damage. This fact can be seen from element No.33 with 5% damage ratio.

4.2 A one span concrete beam

The second numerical simulation is a one span concrete beam with fixed-pinned boundary conditions at each end. The main porous of this example is to make the reader familiar with the proposed algorithm and how to deals with the CNs. Applying noise to measured mode shapes is also considered in this example. As it is shown in Table 1, physical properties of this beam are similar to concrete with $f_c \approx 250$ Kg/cm^2. The moment of inertia of the beam section was multiplied by 0.35 in order to consider cracks in sections. This beam is modeled by 100 elements, similar to

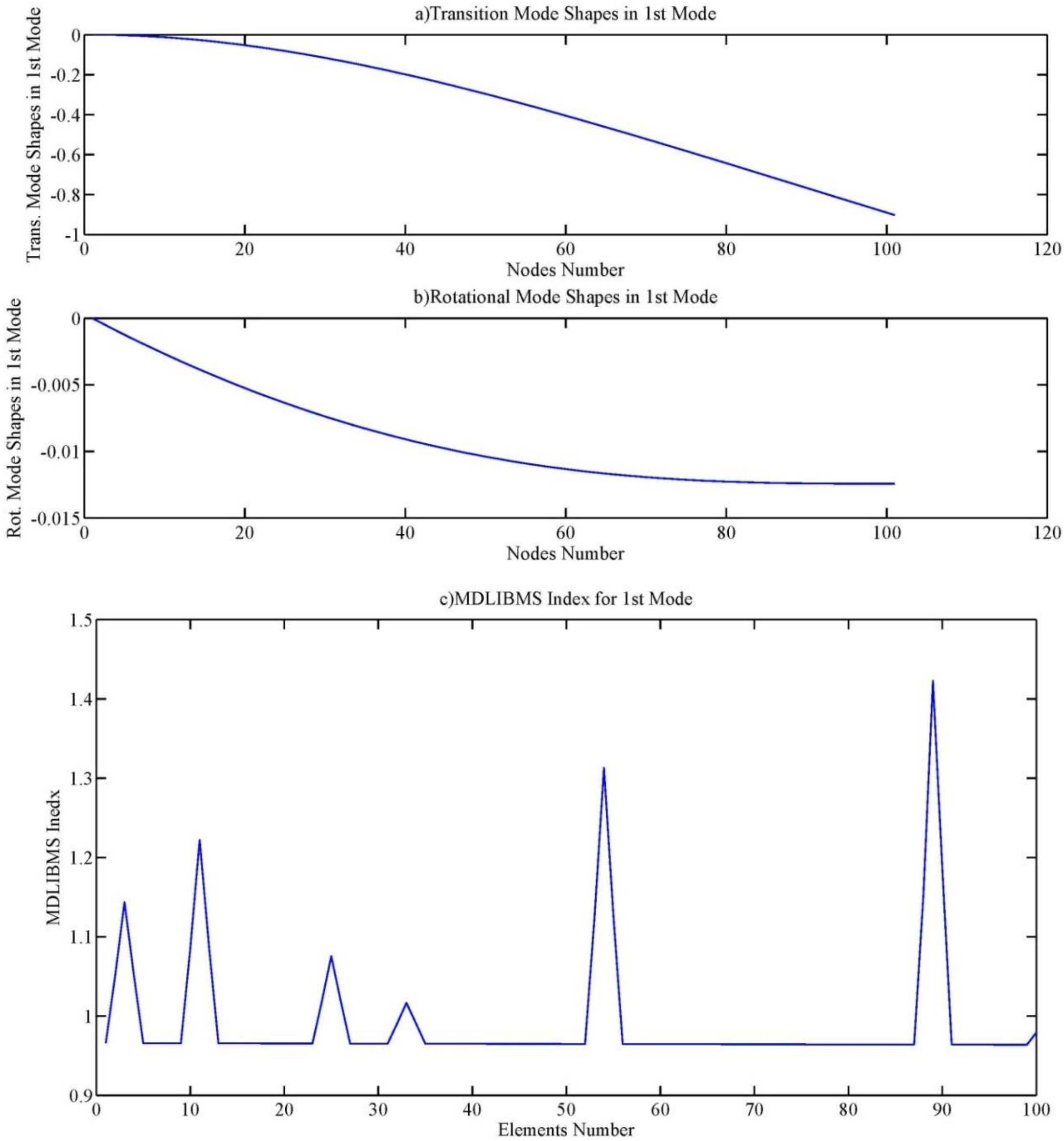

Fig. 3 100 elements cantilever beam diagrams (a) Transverse mode shapes of the first mode number (b) Rotational mode shapes of the first mode number (c) MDLIBMS criterion in the first mode

the previous example but the total length of the beam reaches to 5.00 meter (Fig. 5). Damage scenario for this beam is presented in Table 3. As it was mentioned previously, 5% noise considered in measured data. According to Steps 1 to 5, related diagrams are drawn in Fig. 5. It can be seen from Fig. 5(a) that there is a turning point between nodes 26 and 27 in the transverse mode shapes diagram. These nodes are proportional to 52 and 54 degrees of freedom (DOFs). Referring to Fig. 5(b), relative maximum happen between nodes 26 and 27 either. These nodes are

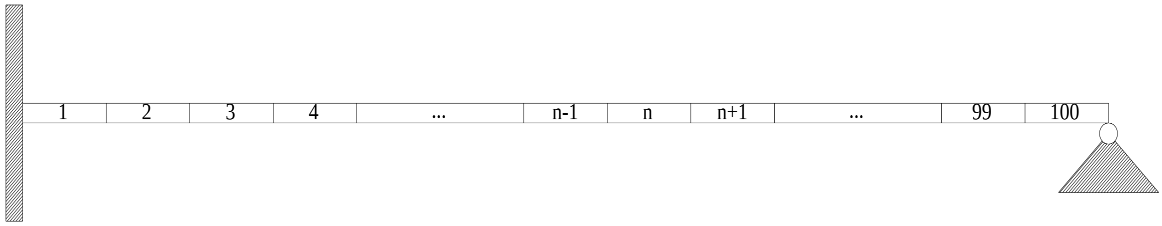

Fig. 4 A fixed-pinned concrete beam

Table 3 Damage scenario of the 100 elements fixed-pinned concrete beam

Damaged elements No.	Damage ratio
5	10%
13	25%
29	10%
40	15%
55	30%
69	30%
74	20%
93	20%

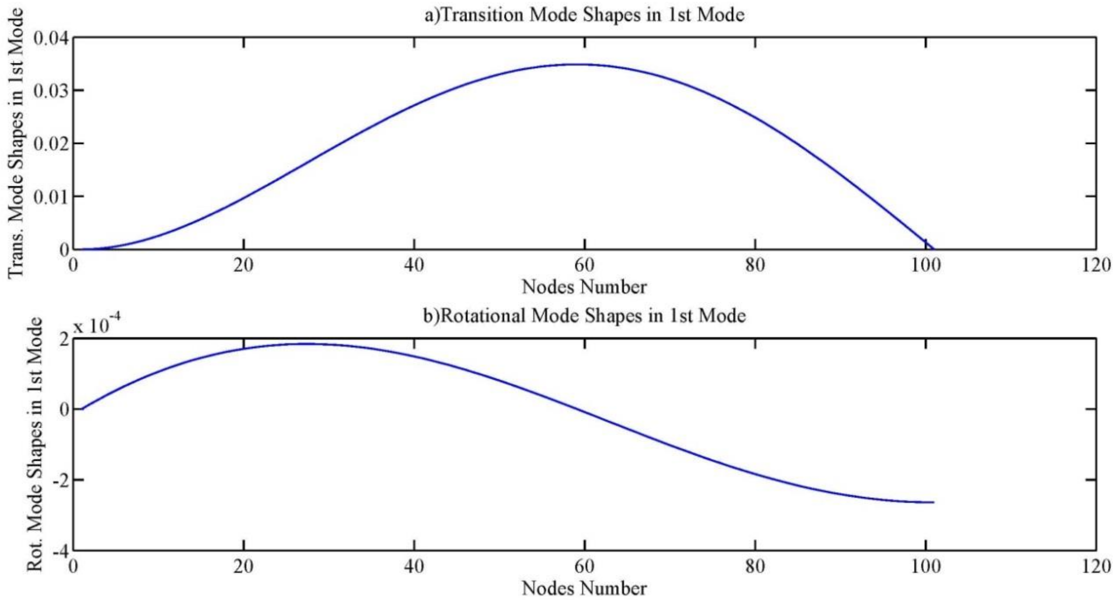

Fig. 5 100 elements fixed-pinned concrete beam diagrams (considering 5% noise) (a) Transverse mode shapes of the first mode number (b) Rotational mode shapes of the first mode number

proportional to 51 and 53 DOFs. According to step 7, the same analysis should be performed for the second mode shapes and relative diagrams are drawn in Fig. 6. Transverse mode shapes

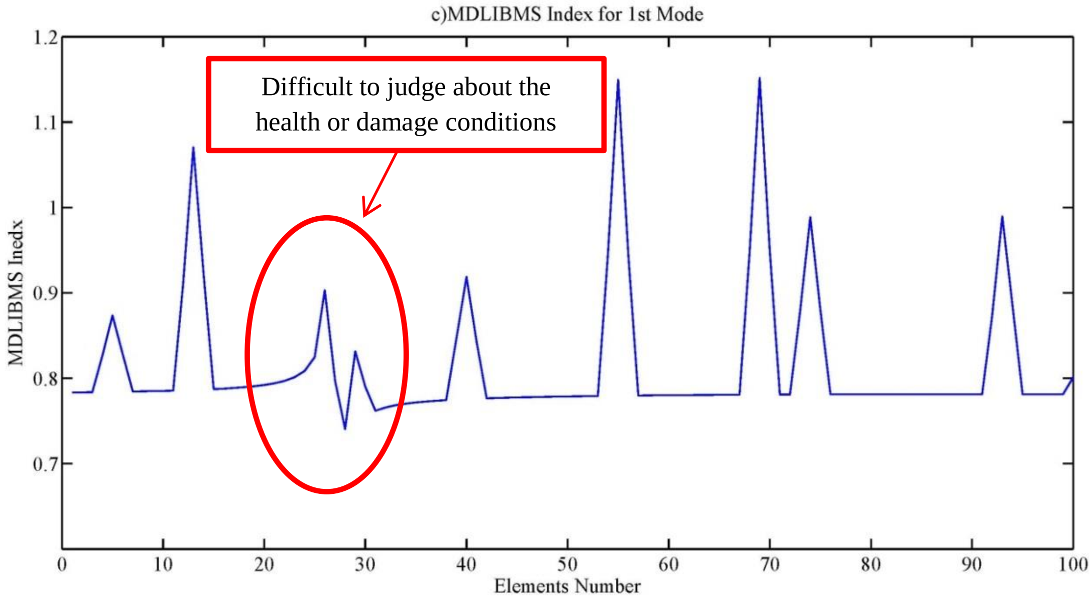

Fig. 5 Continued (c) MDLIBMS criterion in the first mode

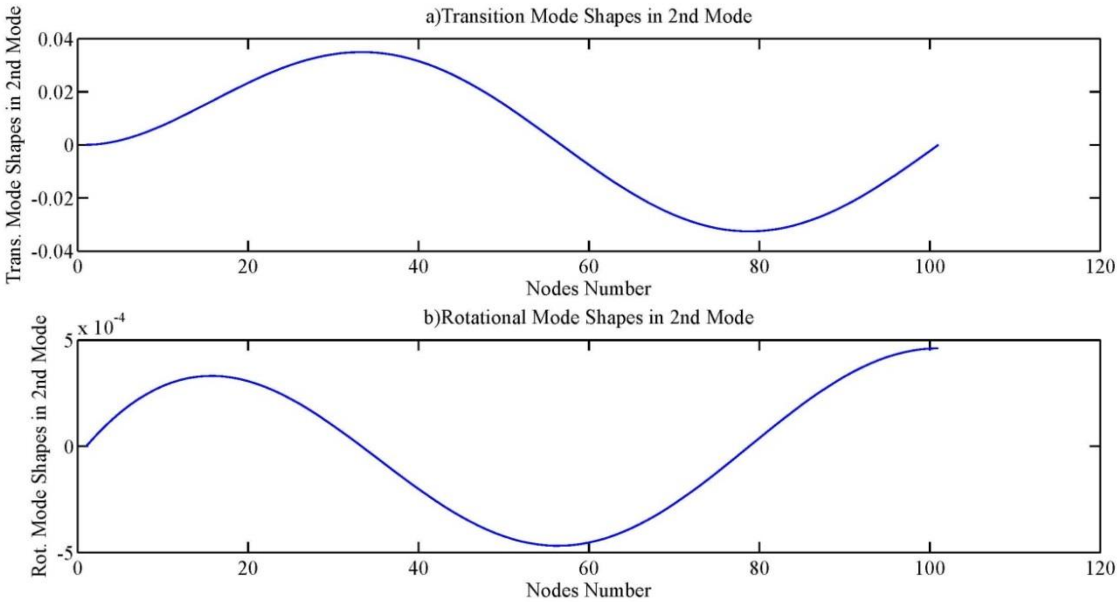

Fig. 6 100 elements fixed-pinned concrete beam diagrams (considering 5% noise) (a) Transverse mode shapes of the second mode number (b) Rotational mode shapes of the second mode number

diagram (Fig. 6(a)) represent turning points nodes between nodes 14-15 and 55-56. These nodes are proportional to 27-29 and 109-111 DOFs respectively. Referring to rotational mode shapes diagram (Fig. 6(b)), it can be seen that between nodes 15-16 and 55-56 (proportional to 30-32 and

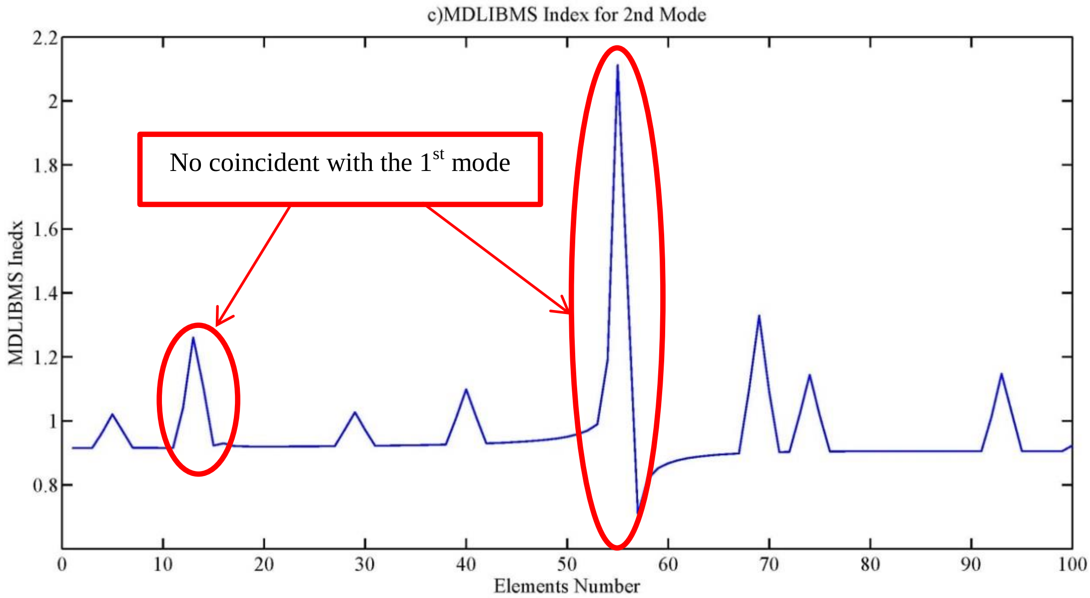

Fig. 6 Continued (c) MDLIBMS criterion in the second mode

110-112 DOFs respectively), relative maximum and minimum happen respectively. Comparing the above values with the ones from the first mode shapes, it appears that none of the DOFs from the firs mode shapes coincident with the ones from the second mode shapes.

Therefore, damaged elements can be detected by collecting the marked elements proportion to distorted areas in MDLIBMS diagrams (Fig. 5(c) and Fig. 6(c)).

First mode results: $A_1 : \{5, 40, 13, 74, 93, 55, 69\}$

Second mode results: $A_2 : \{5, 29, 40, 13, 74, 93, 69\}$

Collecting results: $A_1 \cup A_2 : \{5, 29, 40, 13, 74, 93, 55, 69\}$

4.3 Bridge girder with non-prismatic section

The third numerical simulation is a three span continuous steel bridge girder (Fig. 7). The main purpose of this example is to test the flexibility of the proposed method for complex structures. In this structure, beam sections varied in height. In order to use a finite element modeling of current bridge beams, the following elements length considered:

For varied sections (Sec. A): $l_e = 2$ cm.

For uniform sections (Sec. B): $l_e = 5$ cm.

Total number of elements reaches to 1120 elements. Damaged scenario for current structure is shown is Table 4. After dynamic analysis of the beam, as shown in Fig. 8, transverse and rotational mode shapes of health structure are drawn for the first mode number. Table 5 represents CNs according to these diagrams. Because of CNs appearance in the first mode, for better judgment, 2nd mode shapes of the beam are calculated. The results are shown in Fig. 9. CNs of this mode are summarized in Table 6. As it can be seen, there is no similarity between CNs according to Tables

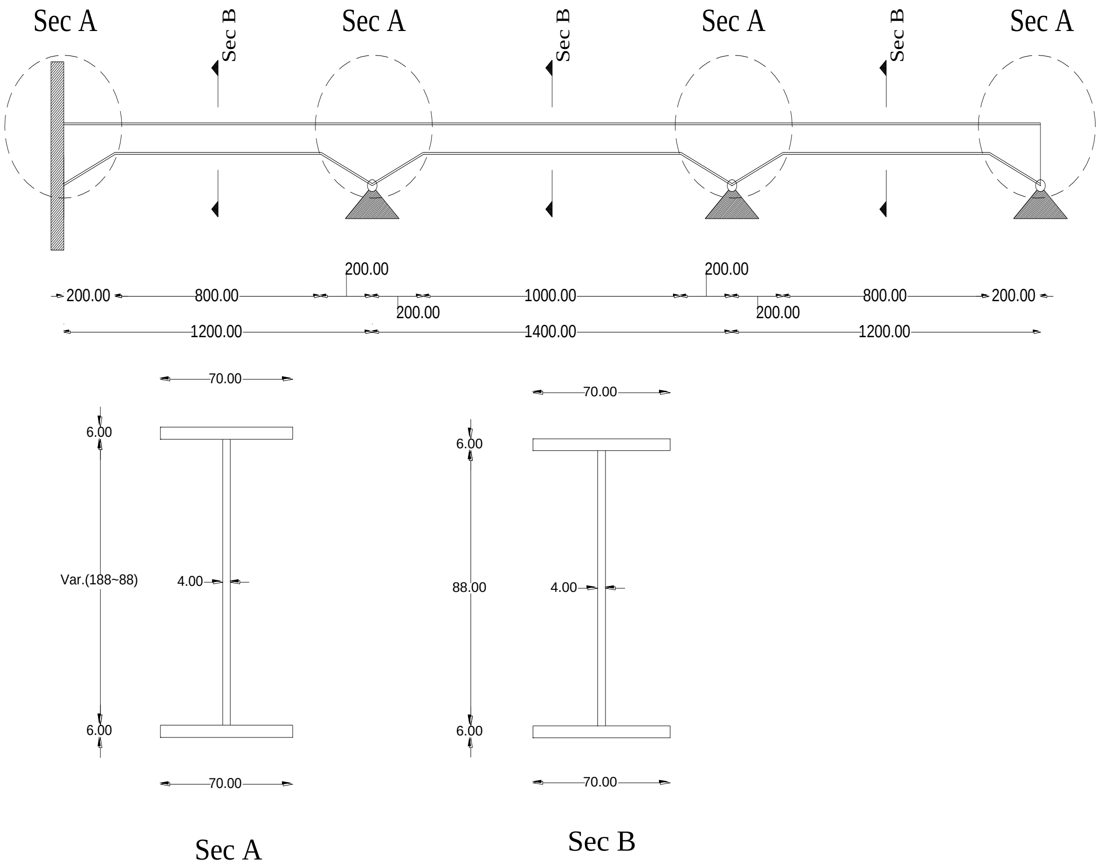

Fig. 7 A 3 spans steel bridge girder

Table 4 Damage scenario of the 3 spans steel bridge girder

Damaged elements No.	Damage ratio
53	10%
102	15%
142	10%
233	5%
362	15%
604	20%
687	10%
706	25%
888	15%
1012	20%

5 and 6. So, after computing MDLIBMS criterion for both modes, the results are shown in two separate diagrams (Fig. 8(c) and Fig. 9(c)). Referring to these diagrams, there might be a doubt about damage presence in node ranges 140 to 150. Zooming the MDLIBMS diagram of second

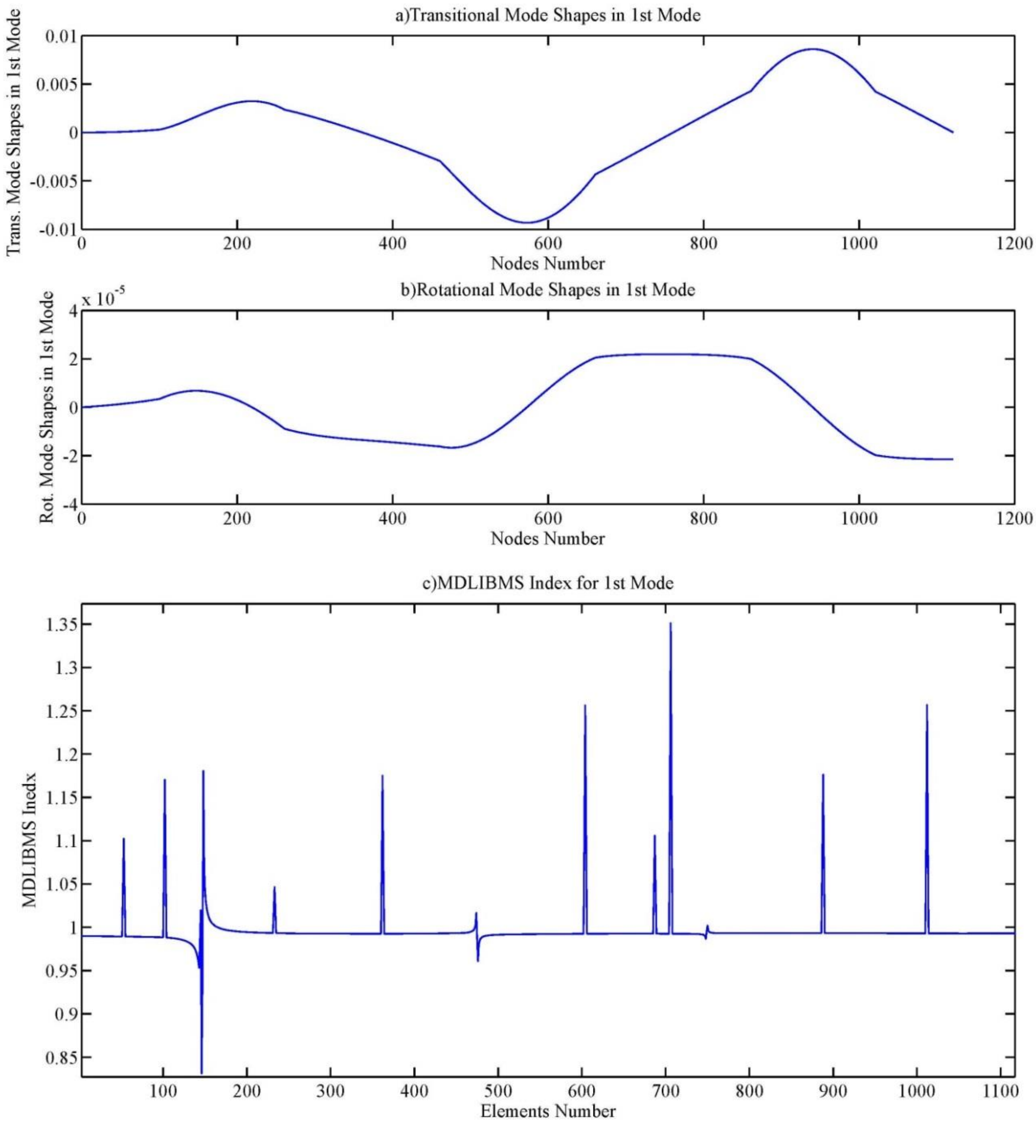

Fig. 8 3 spans steel bridge girder diagrams (considering 5% noise) (a) Transverse mode shapes of the first mode number (b) Rotational mode shapes of the first mode number (c) MDLIBMS criterion in the first mode (considering transverse modes only)

mode around the doubtable range, it would be obvious that element 145 is a damaged one. Nevertheless, to make a better decision, the third mode analysis were done and related MDLIBMS diagram about element No.145 were drawn in Fig. 10. So damaged elements can be detected by collecting distorted elements of MDLIBMS diagrams:

Table 5 CNs of the first mode shapes of bridge girder

CNs of transverse mode shapes		CNs of rotational mode shapes	
Node	DOF	Node	DOF
146-147	291-293	146-147	292-294
474-475	947-949	475-476	950-952
748-749	1495-1497	749-750	1498-1500

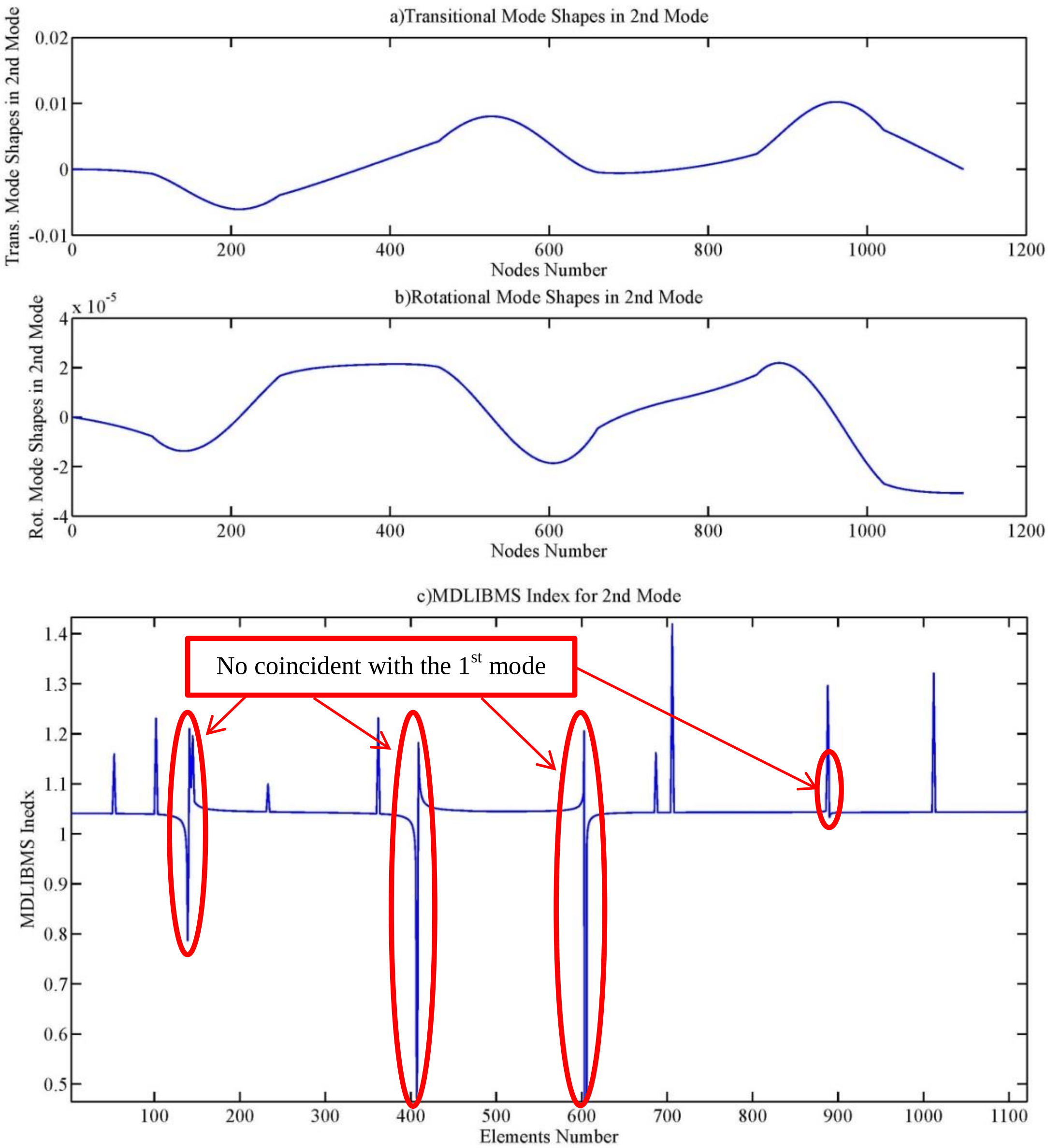

Fig. 9 3 spans steel bridge girder diagrams (considering 5% noise) (a) Transverse mode shapes of the second mode number (b) Rotational mode shapes of the second mode number (c) MDLIBMS criterion in the second mode (considering transverse modes only)

Table 6 CNs of the second mode shapes of bridge girder

CNs of transverse mode shapes		CNs of rotational mode shapes	
Node	DOF	Node	DOF
139-140	277-279	139-140	278-280
407-408	813-815	407-408	814-816
603-604	1205-1207	604-605	1208-1210
888-889	1775-1777	888-889	1776-1778

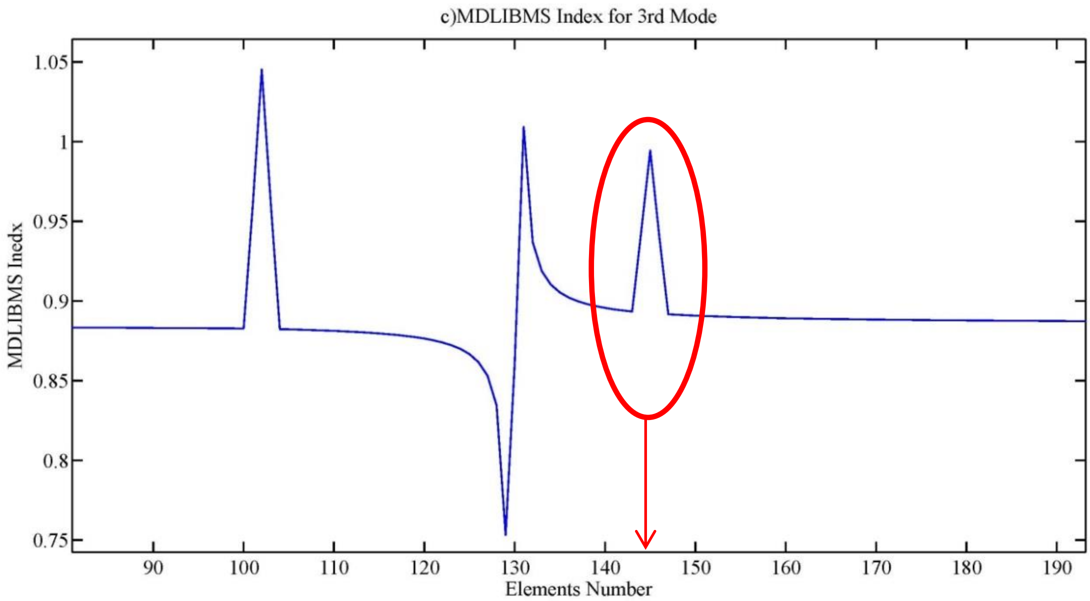

Fig. 10 3 spans steel bridge girder diagrams of MDLIBMS about element No.145 in third mode number

First mode results: $A_1 : \{53, 102, 233, 362, 604, 687, 706, 888, 1012\}$

Second mode results: $A_2 : \{53, 102, 145, 233, 362, 687, 706, 888, 1012\}$

Collecting results: $A_1 \cup A_2 : \{53, 102, 145, 233, 362, 604, 687, 706, 888, 1012\}$

5. Conclusions

The main subject of this paper is multiple damage detection in beam like structures. MDLIBMS criterion is introduced as an operator to detect damaged elements according to mode shapes. Three different numerical examples were studied in order to show the advantages of proposed method. Referring to the results of these examples, the usefulness of the proposed method in detecting and locating state of damage is demonstrated. Increasing element rotations in damaged locations is used to detect damaged elements. Although the proposed criterion, based on absolute rotations of structural elements, but in case of practical works, rotational mode shapes can be neglected and only first derivative of transvers mode shapes (related rotation to transvers mode shapes) considered in computations.

Simplicity and flexibility of the proposed method was also discussed through this research. These advantages will make it a popular method between the users. Considering noise polluted data in the last two examples shows that the proposed criterion has low sensitivity around noisy data. This makes the MDLIBMS criterion compatible to be used in practical works.

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