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Cost optimization of reinforced high strength concrete T-sections in flexure

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Abstract. This paper reports on the development of a minimum cost design model and its application for obtaining economic designs for reinforced High Strength Concrete (HSC) T-sections in bending under ultimate limit state conditions. Cost objective functions, behavior constraint including material nonlinearities of steel and HSC, conditions on strain compatibility in steel and concrete and geometric design variable constraints are derived and implemented within the Conjugate Gradient optimization algorithm. Particular attention is paid to problem formulation, solution behavior and economic considerations. A typical example problem is considered to illustrate the applicability of the minimum cost design model and solution methodology. Results are confronted to design solutions derived from conventional design office methods to evaluate the performance of the cost model and its sensitivity to a wide range of unit cost ratios of construction materials and various classes of HSC described in Eurocode2. It is shown, among others that optimal solutions achieved using the present approach can lead to substantial savings in the amount of construction materials to be used. In addition, the proposed approach is practically simple, reliable and computationally effective compared to standard design procedures used in current engineering practice.

Keywords: cost optimization; high strength concrete (HSC) T-sections; ultimate limit state (ULS); eurocode2 (EC-2); nonlinear programming; conjugate gradient algorithm

1. Introduction

Structural elements with T shaped-sections are economically more effective than rectangular elements and are frequently used in industrial construction (Fedghouche and Tiliouine 2012). They represent major components in various applications involving building and bridge structures. For repeated and large scale use of these components, as may be the case for precast reinforced High Strength Concrete (HSC) component production, special consideration should be devoted to their optimal design in order to make effective use of construction materials and ensure overall cost reduction of the project. From an economical perspective, it is desirable to integrate the numerous advantages of utilizing HSC in structural elements (Edward 1994, Kahleghi and Weigel 2005) within the optimal design procedure. By using HSC, the cross-section dimensions of the elements

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are reduced. Consequently, less concrete and less formwork are needed. At the same time, the amount of steel reinforcement can be reduced substantially. The net result is that the least expensive T-beam can be achieved with the smallest concrete cross-section, the least amount of reinforcement and the highest available concrete strength.

At the present time, the cost of HSC for concrete strength class C80/95 is about 1.50 higher than that of ordinary concrete of strength class C30/37. For HSC with higher classes such as C90/105, the overcost is of the order of 1.80 (Moreno 1998, Russell *et al*. 2002). However, this overcost is rather negligible as compared to the economic advantages achieved thanks to the reduction in the quantities of construction materials to be used. Furthermore, this reduction will result in weight reduction and hence lighter and less costly foundations.

Another important aspect in developing a cost effective design approach is the use of a particular suitable optimization algorithm. Various numerical methods have been used in engineering optimization (Nocedal and Wright 2006, Ozbay *et al*. 2010, Ozturk *et al*. 2012). Optimization techniques can be globally divided into three main categories: mathematical programming techniques (Edward 1994), methods based on optimality criteria (Adamu and Karihaloo 1994, Barros *et al*. 2005, Bonet *et al*. 2006, Ceranic and Fryer 2000, Bordignon and Kripka 2012, Choi *et al*. 2012) and heuristic search algorithms (Leps and Sejnoha 2003). Presently mathematical programming algorithms are the most commonly used in optimal design methods for solving constrained optimization problems based on transformation into unconstrained minimization problems (Raue 2006). Direct methods considering constraints directly as limiting surfaces in the space of design variables have been also developed. The latter class is a very active field of research in engineering optimization and many algorithms are available nowadays.

This paper reports on the development of a minimum cost design model and its application for obtaining economic designs for High Strength Concrete (HSC) T-sections in bending under Ultimate Limit State (ULS) conditions using the latest version of Eurocode 2 (EC-2). Although the developments reported in the present paper are based on the use a specific reinforced concrete design code, the methodology can be easily extended to design codes used in other countries without major alterations. The cost optimization is formulated as a nonlinear programming problem and the optimization process is developed through the use of the Conjugate Gradient algorithm (Nocedal and Wright 2006) in the space of only a reduced number of design variables.

For the cost optimization process, the global cost of construction materials, including costs of HSC, steel and formwork, represent the objective function. The set of constraints includes restrictions in behavior constraints, conditions on strain compatibility in steel and concrete, and geometric design variable constraints. Self weight of the T-beam which may contribute substantially to the ultimate bending moment capacity for long spans is considered variable both in the objective and the constraints functions.

A typical example problem is considered to illustrate the applicability of the minimum cost design model and solution methodology. Results are confronted to design solutions derived from conventional design office methods to evaluate the performance of the cost model and its sensitivity to a wide range of unit cost ratios of construction materials and various classes of HSC described in EC-2. It is demonstrated, among others, that optimal solutions achieved using the present approach can lead to substantial savings in the amount of the construction materials to be used. In addition, the proposed approach is shown to be practically simple, reliable and both computationally and economically effective compared to standard design procedures used in current engineering practice.

2. Ultimate limit state design of reinforced high strength concrete T-sections

Safe and economical design of a reinforced concrete structure under ultimate load conditions seldom depends on complex theoretical analysis. It can be achieved more by deciding on a proper choice for the construction site, a practical overall layout of the structure and its lateral resistant system, careful attention to construction detailing and sound construction practice. However, the overall cost reduction of the structure will strongly depend on an adequate choice of construction materials to be used including labor costs, as well as on the optimal design of the individual structural elements (as may be the case for large scale use in precast reinforced HSC component production). In addition, the optimal design of such components should be based on the nonlinear behavior of HSC and steel reinforcement in accordance with current concrete design codes.

In accordance with EC-2 (Eurocode2 2005), the hypotheses used at ULS for strain and stress distributions in the typical reinforced HSC T-beam cross section shown in Fig. 1(a) are respectively illustrated in Fig. 1(b) and Fig. 1(c).

The HSC cross section dimensions are defined by the following parameters:

b effective width of compressive flange
b_w web width
h total depth
h_f flange depth

The parameters d, d_s and A_s represent respectively:

d effective depth
d_s effective cover of reinforcement.
A_s area of reinforcing steel

In the linear strain diagram of Fig. 1(b), the symbols ε_c and ε_s designate concrete and steel deformations. The parameter α represents the relative depth of compressive concrete zone and the neutral axis is located at the distance αd from the upper fiber. In the assumed uniformly distributed stress diagram of Fig. 1(c), f_{cd}=0.85 f_{ck}/γ_c is the design value of concrete compressive strength, γ_c the partial safety factor for concrete and f_{ck} is the characteristic concrete strength. In accordance with EC-2, the possibility is offered to work with a rectangular stress distribution. This requires the introduction of a factor λ for the depth of the compression zone and a factor η for the design strength. The λ and η factors are both linearly dependent on the characteristic strength f_{ck} in accordance with the following equations

$$\lambda=0.8-(f_{ck}-50)/400 \tag{1}$$

$$\eta=1.0-(f_{ck}-50)/200 \tag{2}$$

with $50 \le f_{ck} \le 90$MPa. F_c and F_s denote the resultants of internal forces in HSC section and reinforcing steel respectively.

It should also be noted that the ultimate design of reinforced HSC cross sections requires the knowledge of the design stress-strain design curves (i.e., the nonlinear constitutive laws) which are described in the following subsections.

2.1 Nonlinear stress-strain curve of HSC for design of cross-sections

Various analytically based formulations of design stress-strain curves for HSC beams are available in specialized literature (Eurocode2 2005, Rashid and Mansur 2005). In accordance with Eurocode2 (EC-2), two alternative design stress-strain curves for concrete can be considered: one

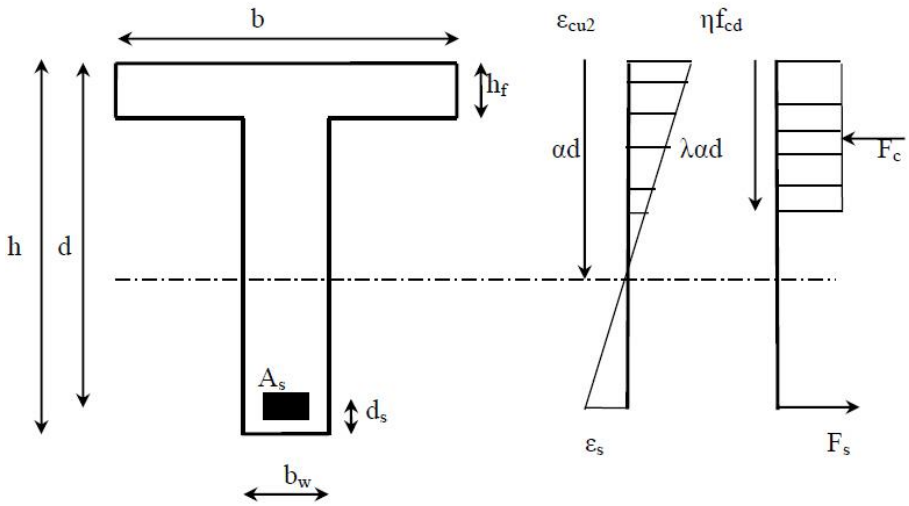

Fig. 1 (a) Typical T-Beam cross section (b) strains and (c) stresses

Table 1 Strength classes and properties for HSC

Class	C55/67	C60/75	C70/85	C80/95	C90/105
f_{ck}(MPa)	55	60	70	80	90
f_{ctm}(MPa)	4.2	4.4	4.6	4.8	5.0
ε_{c2}(‰)	2.2	2.3	2.4	2.5	2.6
ε_{cu2}(‰)	3.1	2.9	2.7	2.6	2.6
n	1.75	1.6	1.45	1.4	1.4
λ	0.7875	0.775	0.7500	0.7250	0.7000
η	0.975	0.950	0.900	0.850	0.800

on the basis of a power law (see Fig. 2) and the other based on a bilinear relationship. In the present work, only the design stress-strain curve based on the power law will be retained. In this case, the design stress-strain relationship can be written as

$$\sigma = f_{cd}[1-(1-\varepsilon_c/\varepsilon_{c2})^n] \qquad \text{for } \varepsilon_c \leq \varepsilon_{c2} \tag{3-1}$$

$$\sigma = f_{cd} \qquad \text{for } \varepsilon_{c2} \leq \varepsilon_c \leq \varepsilon_{cu2} \tag{3-2}$$

where the parameters n, ε_{c2} and ε_{cu2} can be obtained from the following equations

$$n=1.4+23.4[(90-f_{ck})/100]^4 \tag{4}$$

$$\varepsilon_{c2}(‰)=2.0+0.0085(f_{ck}-50)^{0.53} \tag{5}$$

$$\varepsilon_{cu2}(‰)=2.6+35[(90-f_{ck})/100]^4 \tag{6}$$

Table 1 summarizes the values of the characteristic properties for HSC classes as prescribed by EC-2 provisions.

In this table, the following definitions are used:

ε_{c2} Strain at the maximum stress for the (σ_c-ε_c) power law

ε_{cu2} Ultimate strain of the compressive concrete for (σ_c-ε_c) power law

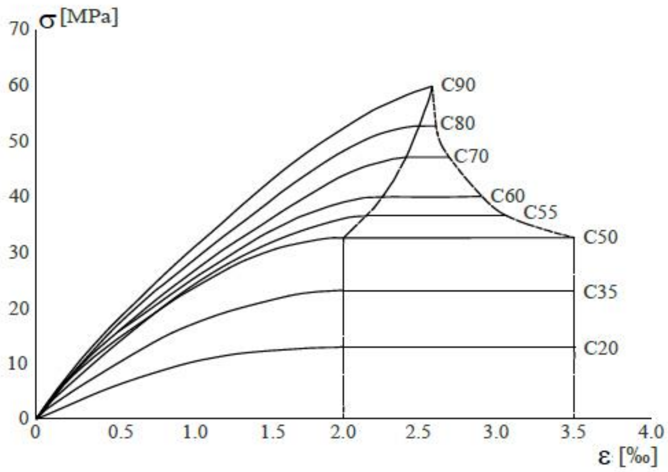

Fig. 2 Design stress-strain curves for ordinary concrete and HSC

n Factor

f_{ctm} Tensile strength of concrete

Note that the above rectangular relation for power law reduces to a parabola rectangular relation for ordinary concrete classes (i.e., for n=2 and f_{ck}≤50MPa).

It should also be noted that HSC shows more brittle behavior as reflected by shorter horizontal branches for values of characteristic compressive strength 50≤f_{ck}≤90MPa.

The limit design means that rupture is considered to crush at ε_{cu2}(‰). In accordance with ultimate limit state design concrete does not stand for tensile stresses.

In the present work, for the sake of simplification the resultant forces and moment acting on a T-beam HSC section were computed using a uniformly distribution stress diagram as indicated in Fig. 1(c) and as recommended by the French annex provisions to EC-2 in bending for partially compressed sections in the case of Pivot B.

2.2 Nonlinear stress-strain curve of reinforcing steel

The representative short-term design stress-strain curve for steel reinforcement is given by the simplified behavior curve shown in Fig. 3. The behavior of the steel is of elastic-perfectly plastic type being linear in the elastic range up to the design yield strength of steel reinforcement $f_{yd}=f_{yk}/\gamma_s$ where, f_{yk} is the characteristic elastic limit of steeland γ_s is the partial safety factor. It should be noted that EC-2 permits the use of an alternative design stress-strain curve to that shown in Fig. 3 with an inclined top branch and the maximum strain limited to a value which is dependent on the class of reinforcement. However the more commonly used curve shown in Fig. 3 will be used in this paper.

In the present work, the stress in the reinforcing steel was kept equal to f_{yd}. The elastic phase shown in Fig. 3 is represented for completeness and only for the purpose of indicating the lower limit of the plastic domain. The later corresponds to the best use of steel reinforcement at the ultimate limit state. In addition the steel strain is considered unlimited as shown in Fig. 3 in accordance with to Eurocode2 provisions.

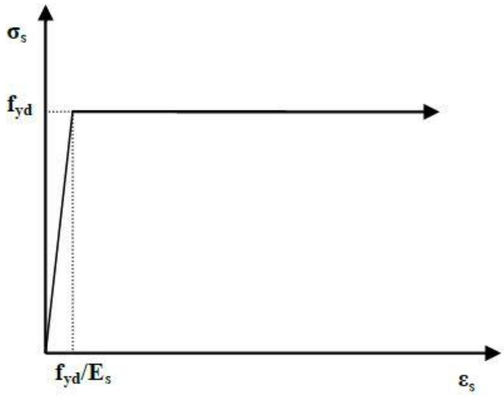

Fig. 3 Design stress-strain curve of reinforcing steel

In this paper, for an optimal use of steel, the strain must be always be greater or equal to elastic limit strain $\varepsilon_{yd}=f_{yd}/E_s$ where E_s represents the elasticity modulus for steel.

3. Minimum cost design model of HSC T-beams

3.1 Statement problem

The design variables of the model are the geometrical dimensions of the T-beam cross section: b, b_w, d, h_f, the amount of steel A_s, and the relative depth of compressive concrete zone α.

To obtain the design variables b, b_w, d, h_f, A_s and α

Given that:

Beam span: L

Ultimate bending moment capacity including selfweight: M_{Ed}

Ultimate shear capacity including selfweight: V_{Ed}

Characteristic compressive cylinder strength of HSC at 28 days: f_{ck}

Design strength factor: η

Compressive zone depth factor: λ

Characteristic elastic limit for steel reinforcement: f_{yk}

Young's elastic modulus: E_s

Minimum steel percentage: $p_{\min}$

Maximum steel percentage: $p_{\max}$

Total cost per unit length of HSC T-beam: C

Unit cost of reinforcing steel: C_s

Unit cost of HSC concrete: C_c

Unit cost of formwork: C_f

3.2 Cost optimization of HSC T-sections and implementation of minimum cost design model

Consider now the HSC T-beam cross section shown in Fig. 1(a), and let C be the objective function representing the total cost per unit length of the beam. This function can be defined as

$$C = C_c(b_w\, d+(b-b_w)\, h_f)+C_s A_s+C_f[b+2(d_s+d)] \tag{7}$$

where unit cost ratios C_c, C_s and C_f as previously defined.

In this work, cost components of construction parameters such as concrete formwork, construction detailing, steel forming and all relevant demands comprising supply, installation and implementation are implicitly included in the objective function as appropriate percentages of the unit costs of concrete and steel respectively. Further practical requirements involving other design codes and constraints as well as more sophisticated cost objective functions and other cross sections geometry can be implemented within the present cost optimization model without major alterations.

In developing a minimum cost model, it is necessary to include in the model, design constraints. In general, the behavior constraints are based on design codes which may differ from one country to another. For illustrative purposes, the design constraints will be herein defined in accordance with the design code specifications of the French Annex to EC-2.

Thus and without loss of generality, the formulation of the minimum cost design of HSC T-beams under ultimate loads can be mathematically stated as follows:

Find the design variables b, b_w, d, h_f, A_s, and α that minimize total cost of construction material per unit length of HSC T-beam such that

$$C/\, C_c=b_w d+(b-b_w)h_f+(C_s/C_c)As+C_f/C_c[(b+2(d_s+d)] \tag{8}$$

Subject to the following constraints

(a) Behavior constraints

$$M_{Ed}\le\eta f_{cd}(b-b_w)h_f(d-0,\ 50h_f)+\eta\lambda f_{cd}.b_w.d^2\alpha(1–0,\ 5\lambda\alpha) \tag{9}$$

(External moment including selfweight ≤ Resisting moment of the cross section)

$$\alpha=(f_{yd}/f_{cd})(A_s/\eta\lambda b_w.d)-(b-b_w)h_f/\ \lambda b_w.d \quad \text{(Internal force equilibrium)} \tag{10}$$

$$A_s\,/b_w d\ge p_{\min} \text{ (Minimum steel percentage)} \tag{11}$$

$$A_s/(b_w\, h+(b-b_w)h_f)\ \le p_{\max} \quad \text{(Maximum steel percentage)} \tag{12}$$

In Eqs. (9) and (10) above, it is assumed that the neutral axis position is under the beam flange which ensures that the section is behaving as the T-beam section shown in Fig. 1(a).

Conditions on strain compatibility in steel and concrete

$$\varepsilon_{cu2}((1/\alpha)-1\,)\ge f_{yd}\,/\,E_s \tag{13}$$

(Optimal use of steel requires that strains in steel must be limited to plastic region at the ULS)

$$\lambda\alpha(1–0,\ 5\lambda\alpha)\le\mu_{limit} \text{ (Compression reinforcement is not required)} \tag{14}$$

(b) Shear strength constraint

$$V_{Ed}\le V_{Rd,\max} = v_1 f_{cd} b_w z/(tg(\theta)+\mathrm{cot}g(\theta)) \tag{15}$$

(c) Geometric design variables constraints including pre-design rules of current practice

$$h\ge L\,/\,16 \tag{16}$$

$$d\,/\,h = 0.90 \tag{17}$$

$$0.20 \le b_w / d \le 0.40 \tag{18}$$

$$(b-b_w) / 2 \le L / 10 \tag{19}$$

$$b/h_f \le 8 \tag{20}$$

$$h_f \ge h_{\min} \tag{21}$$

where:

μ_{limit} limit value of reduced moment

θ is the angle between concrete compression struts and the main chord.

v_1 a non dimensionnel coefficient.

$v_1 = 0.60(1 - f_{ck}/250)$

z lever arm, $z = 0.9d$

$h_{\min}$ minimum depth of flange

It should be noted that the unit cost ratios appearing in Eq. (8) and more specifically the cost ratio C_s/C_c varies from one country to another and may eventually depend from one region to another for certain countries. The values of these cost ratios can be estimated on the basis of data given in applicable unit price books of construction materials (Davis 2011, Pratt 2011).

3.3 Solution methodology

The objective function Eq. (8) and the constraints equations, Eq. (9) through Eq. (21), together form a nonlinear optimization problem. The reasons for the nonlinearity of this optimization problem are essentially due to the expressions for the cross sectional area, bending moment capacity and other constraints equations as well as the requirement to update iteratively the self weight of the T-beam, both in the constraints functions and the objective function. Both the objective function and the constraint functions are nonlinear in terms of the design variables.

In order to solve this nonlinear optimization problem, the Conjugate Gradient Method (Nocedal and Wright 2006) is used as it is widely recognized as an efficient method for solving a relatively wide class of nonlinear unconstrained optimization problems. The significance of this class of problems stems from the fact that some of the most powerful and convenient methods of solving constrained problems are based on transformation of the problem to one of unconstrained minimization. An excellent survey of development of different versions of nonlinear conjugate gradient methods, with special attention to global convergences properties, is presented by Hager and Zhang (2006).

All the conjugate gradient algorithms start out by searching in the steepest descent direction (negative of the gradient) on the first iteration.

It turns out that, although the function decreases most rapidly along the negative of the gradient, this does not necessarily produce the fastest convergence.

The general procedure for solving a problem by the conjugate gradient algorithm can be summarized as follows:

Choose an initial point $\{X_0\}$.

Compute

$$\{G_0\} = \{\nabla F_0\} \tag{22}$$

where $F_0 = F_0\{X_0\}$ and $G_0 = \nabla F_0$ are the objective function and its gradient vector calculated at point $\{X_0\}$.

Determine

$$\{S_0\} = \{-G_0\} \tag{23}$$

For each iteration step, *q*, *q*=1,…, *n*

Find the step size α_q^* to minimize

$$F\left(\{X_q\} + \alpha_q \{S_q\}\right) \tag{24}$$

Calculate

$$\{X_{q+1}\} = \{X_q\} + \alpha_q^* \{S_q\} \tag{25}$$

where

$$\{S_q\} = \{-G_q\} + \frac{\{G_q\}^T \{G_q\}}{\{G_{q-1}\}^T \{G_{q-1}\}} \{S_{q-1}\} \tag{26}$$

And

$$\{\mathrm{G}_q\} = \{\nabla \mathrm{F}_q\} \tag{27}$$

Since

$\{S_q\}$ *is a linear combination of* $\{S_0\}, \{S_1\}, \ldots, \{S_{q-1}\}$ *and* $\{G_q\}$ *it is also a linear combination of* $\{G_0\}, \{G_1\}, \ldots, \{G_q\}$. It can be shown that for the case of a quadratic function, the conjugate gradient method generates a set of mutually directions $\{S_q\}$. Thus, theoretically the process should converge in n or fewer steps.

A more detailed description of the conjugate gradient algorithm can be found in (Andrei 2013).

4. Numerical results and discussion

A typical example problem is now considered. The step by step application of the HSC T-beam minimum cost design model is presented, followed first by a comparison between the optimal cost design solution and the standard design solution using HSC, and finally by a study of the behaviorof minimum cost design solution. Particular attention is paid to the sensitivity of minimum cost design solutions to various classes of HSC and a wide range of unit cost ratios of construction materials.

4.1 Design example

The objectives of this application testare:

(i) to evaluate the performance of the minimum cost design model and the solution methodology.

(ii) to examine the characteristics of the solution in order to identifythe binding and the nonbinding constraints,

(iii) to provide minimum cost design solutions that can be used as a basis for comparison in future investigations.

As previously mentioned, the design constraints are defined in accordance with the code design specifications of the French Annex to EC-2. The optimal solutions are compared to the standard design solutions obtained in accordance with EC-2 design code. To further illustrate the variability of the optimal solutions (including self weight effects) with unit costsofmaterials, the optimal solutions are computed for given unit cost ratios. The results in terms of the corresponding gains are presented graphically. Increases in cost saving due to the requirement to update the cross section dimensions with new self weight of the optimized beams are also investigated.

The study concrete T-beam with pined supports corresponds to a T-beam belonging to a high performance concrete bridge deck, simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

The corresponding pre-assigned parameters are defined as follows:

- Beam span: L = 29m
- Ultimate bending moment capacity: M_{Ed}= 1.35M_G+ 1.5M_Q = 8MNm

where M_G and M_Q designate maximum design moments under dead and live loads respectively.

- Ultimate design shear capacity: V_{Ed} = 1.35V_G+ 1.5V_Q= 3MN

where V_G and V_Q designate maximum design shears under dead and live loads respectively.

Input data for HSC characteristics:

- Strength class of concrete: C80/95.
- Characteristic compressive cylinder strength of concrete at 28 days: f_{ck} = 80MPa.
- Partial safety factor for concrete: γ_c = 1.5
- Allowable compressive stress: f_{cd} = 45.33MPa

λ = 0.725

η = 0.850

Input data for steel characteristics:

- Steel class: S500
- Elastic limit: f_{yk} = 500MPa
- Partial safety factor for steel: γ_s = 1.15
- Allowable tensile stress: f_{yd}= f_{yk}/γ_s = 435MPa
- Young's elastic modulus: E_s=2×10^{5}MPa
- Minimum steel percentage: $p_{\min}$= 0.26f_{ctm}/f_{yk}= 0.002496
- Maximum steel percentage: $p_{\max}$= 4%
- f_{ctm}= 4.4MPa

Input data for units costs ratios of construction materials:

C_s / C_c = 25

C_f / C_c = 0

4.2 Step by step application of minimum cost design model for HSC

The preceding minimum cost design model is now applied to the design of the reinforced HSC T-beam cross section for which a classical design solution based on the EC-2 concrete design provisions is briefly presented in the appendix. Following the problem formulation developed in the preceding section and considering unit length of the reinforced HSC T-beam, the minimum cost design problem can be mathematically stated as:

Given the above pre-assigned parameters, construction materials characteristics and unit cost ratios,

Find the design variables b, b_w, d, h_f, A_s and α that minimize construction materials cost per unit length of HSC T-beam subject to:

(a) Behavior constraint:

Table 2 Comparison between HSC classical design solution and optimal cost design solutions excluding or including selfweight effects

HSC Classical	solution	HSC Optimal solution	HSC Optimal solution including self weight effects
b(m)	1.00	0.80	0.80
b_w(m)	0.40	0.32	0.30
h(m)	1.70	1.80	1.80
d(m)	1.53	1.62	1.62
h_f(m)	0.12	0.10	0.10
A_S(m^2)	126×10^{-4}	122×10^{-4}	114×10^{-4}
α	0.159	0.236	0.214
C/C_c	0.999773	0.876514	0.858287

(b) Shear strength constraint and

(c) Geometric design variables constraints (including pre-design rules of current practice) described in subsection 3.2. with a minimum flange depth: $h_{\min}$= 0.10m.

It should be noted that the solution vector of the above problem cannot be considered as the final solution of the minimum cost design problem. As a matter of fact, because of the requirement to update the geometric dimensions of the section with the new self weight of the optimized beam, the degree of nonlinearity of the resulting optimization problem enhances further. The final optimal solution is thus obtained in two phases:

Phase 1 is concerned with the determination of the optimal solution using the initial loading parameters (i.e., with initial self weight corresponding to the starting solution).

Phase 2 is concerned with the requirement to update the self weight of the beam (both in the constraints functions and the objective function) with the geometric dimensions of the optimized section obtained inphase 1. The modified forces due to the new self weight are computed, the new dimensions of the beam are optimized and theprocess continued until convergence is achieved.In the present example, the optimal solution vector is reached after 3 cycles of iteration only.

4.3 Comparison between the optimal cost design solutions and the standard design approach using HSC

The vector of design variables including the geometric dimensions of the T-beam cross section and the area of tension reinforcement as obtained from the standard design approach solution and the optimal cost design solution using the proposed approach, are shown in Table 2.

From the above results, it is clearly seen that the relative depth of the compressive concrete zone associated with the optimal solution is 48% larger than that given by the classical solution, thus leading to a much better use of the concrete. It is also seen from the values of the relative costs C/C_c associated with the classical and optimal solutions, that a significant cost saving of the order of 14% can be obtained by using the proposed design formulation.Note also that this cost saving can be increased up to 16% when comparing the standard design solution to the HSC optimal design solution including selfweight effects.

4.4 Behavior of minimum cost design solutions

A study of the inequality constraints indicated that the design constraints of the beam were all non binding except for the behavior constraints associated with ultimate bending moment capacity Eq. (9); the geometrical design constraints Eq. (16); Eq. (18); Eq. (20); and Eq. (21). The values of the geometric design variables b_w (web width), h_f (flange depth) and h (total depth) were found to be all on the specified lower limit values.

In order to further illustrate the variability of optimal solution with the unit cost ratio C_s/C_c, the optimal solution has also been computed for various ratios C_s/C_c = 13; 25; 36; 70; 100; 130; 160; 200.

The overall cost reduction achieved on the T-beam for a given unit cost ratio C_s/C_c, can be measured as of the corresponding relative gain (in percent) defined as follows

$$\text{Gain in percent (\%)} = ((C_{classical} - C_{optimal})/C_{classical}) \times 100 \tag{28}$$

The relative gains can be determined for the various values of the unit cost ratios. The corresponding results are reported in Table 3 and illustrated graphically in Fig. 4 for HSC class C80/95.

It can be observed from Fig. 4, that the relative gain decreases rapidly for increasing values of the unit cost ratio, stabilizes around an average value approximately equal to 10% for values of $70 \leq C_s/C_c \leq 100$ and then increases significantly beyond this average value.

Furthermore, the performance and sensitivity of present HSC minimum cost design model to various classes of HSC and material stress ratio prescribed in EC-2 have been examined. The results are reported in tabular form Table 4, below, for C_s/C_c=25. It is clearly seen that for the example problem considered herein, the gain percentages are rather insensitive to changes in HSC classes and material stress ratios. This insensitivity can be interpreted the result of the compatible changes of mechanical properties of both steel and concrete coupled with a rational design of the studied HSC- T sections in flexure. Significant cost savings up to 14% (16% when including selfweight effects which may important for long beam spans) can be achieved for this design example.

Table 3 Variation of relative gain in percent (%) versus unit cost ratio C_s/C_c of construction materials

C_s/C_c	13	25	36	70	100	130	160	200
Gain in percent (%)	16	14	13	10	10	12	14	18

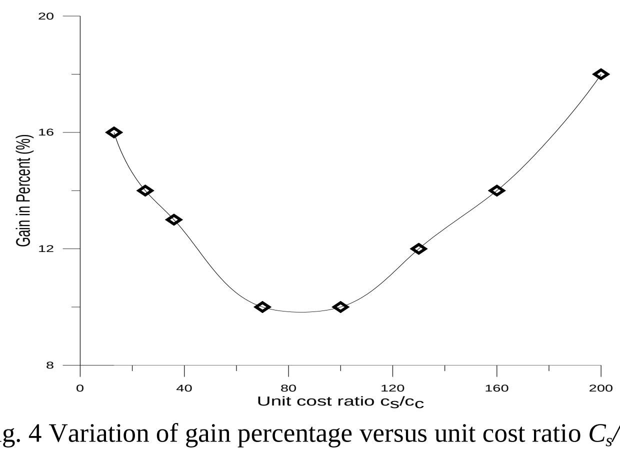

Fig. 4 Variation of gain percentage versus unit cost ratio C_s/C_c

Table 4 Performance of HSC minimum cost design model versus HSC class and material stress ratio

Class of HSC	C55/67	C60/75	C70/85	C80/95	C90/105
f_{yd}/f_{cd}	14	13	11	9	8
Gain in percent (%)	13	14	14	14	14

4. Conclusions

A minimum cost design model is presented for the optimal design of reinforced HSC T-beams in bending under ultimate limit state conditions considering design stress-strain relationships used in Eurocode2 provisions. Cost objective functions, behavior constraints including nonlinearities of steel and HSC, conditions on strain compatibility in steel and concrete and geometric design variable constraints are derived and implemented within the Conjugate Gradient optimization algorithm. The optimal solution vector includes the following design variables i)-optimal dimensions of HSC T cross section $(b, b_w, h, h_f)_{opt}$; ii)-optimal area of tensile reinforcement $(A_s)_{opt.;}$ iii)-optimal relative depth of compressive HSC zone $(\alpha)_{opt}$, from which the optimal values of any related quantity can be determined. Particular attention was paid to problem formulation and solution methodology. The present model is applied to five different classes of HSC: C55/67; C60/75; C70/85; C80/95; C90/105, described in

EC-2. In order to further illustrate the variability of optimal solution with costs of construction materials, the optimal solutions have been also computed for a wide range of unit cost ratios C_s/C_c=13; 25; 36; 70; 100; 130; 160; 200.

- The results obtained in this work demonstrated that the Conjugate Gradient optimization technique can be successfully applied to the minimum cost design of High Strength Concrete T-sections in flexure at Ultimate Limit State offering an approach that can be used without prior knowledge of mathematical optimization. Comparisons with the standard design approach have clearly shown that the optimal solutions achieved using the present cost model, will indeed reach the minimum quantities of construction materials to be used.
- The optimal solutions are very sensitive to cross section initial configuration and the relative cost ratio C_s/C_c of construction materials to be used.
- The gain percentage achieved using the present minimum cost design formulation isfound to be rather insensitive to changes in HSC classes and material stress ratios. Significant cost savings up to 14% (or 16% if selfweight effects are included, which may be important for long beam spans) can be achieved regardless of the adopted HSC grades.
- The proposed approach is practically simple, reliable and computationally effective compared to standard design procedures used in current engineering practice.
- Further practical requirements involving other design codes and manufacture constraints as well as more sophisticated cost objective functions and cross section geometry can be implemented within the present cost optimization model without major alterations.

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Appendix. Classical design solution for design example

The study concrete T-beam with pined supports corresponds to a T-beam belonging to a high performance concrete bridge deck, simply supported at its ends and pre-designed in accordance with provisions of EC-2 design code.

Determine the area of steel reinforcement for the reinforced HSC T-beam with the cross section dimensions indicated below:

b= 100cm; b_w= 40cm; h= 170cm; d= 153cm; h_f= 12cm as obtained from a preminairly design of the study HSC T-beam.

Given the following data:

Beam span: L = 29m
Ultimate bending moment capacity including selfweight: M_{Ed} = 8MNm
Ultimate shear capacity including selfweight: V_{Ed} = 3MN
Strength class of concrete: C80/95.
Characteristic compressive cylinder strength of concrete at 28 days: f_{ck} = 80MPa.
Partial safety factor for concrete: γ_c =1.5
Allowable compressive stress: f_{cd} = 45.33MPa
Design strength factor: η = 0.850
Compressive zone depth factor: λ = 0.725
Steel class: S500
Elastic limit: f_{yk} = 500MPa
Partial safety factor for steel: γ_s = 1.15
Allowable tensile stress: $f_{yd}= f_{yk}/\gamma_s$ = 435MPa
Unit cost ratios: C_s / C_c = 25; C_f / C_c= 0

Moment of resistanceof the flange, M_f:

$M_f = \eta b h_f f_{cd}(d\text{-}0.5h_f)$
M_f = 6.7968MNm
$M_f < M_{Ed}$, the stress block must extend below the flange
$\mu_{lim}=0.725\alpha_{lim}(1\text{-}0.3625\alpha_{lim})$, limit value of reduced moment
α_{lim} = 0.5445
μ_{lim} = 0.317
$\mu= (M_{Ed}\text{-}M_f((b\text{-}b_w)/b)/b_w d^2\eta f_{cd}<0.317$, compression reinforcement is not required
$\mu=(M_{Ed}\text{-}M_f((b\text{-}b_w)/b)/b_w d^2\eta f_{cd}$ = 0.109
$\alpha = [1\text{-} (1\text{-}2\mu)^{0.5}]/\lambda$ = 0.159

Lever arm:

$z=d(1\text{-}0.5\lambda\alpha)$, z = 1.44m

The area of tension steel A_s:

$A_s= [M_{Ed}\text{-}M_f((b\text{-}b_w)/b)/zf_{yd}]+(b\text{-}b_w)h_f\eta f_{cd}/f_{yd}$
A_s=[8-6.7968((1-0.4)/1)/1.44*435)]+(1-0.4)0.14*0.85*45.33/435
A_s=126cm^2,
Check: $A_{s,\min}<A_s<A_{s,\max}$
$A_{s,\min}$=15cm^2, $A_{s,\max}$ = 245cm^2

$15 < 126 < 245$ OK

Total cost per unit length of HSC T-beam:

$C = 0.999773 C_c$