Structural Engineering and Mechanics, *Vol. 48, No. 3 (2013) 291-308*
DOI: http://dx.doi.org/10.12989/sem.2013.48.3.291

Damage detection using the improved Kullback-Leibler divergence

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(Received July 11, 2013, Revised September 18, 2013, Accepted October 17, 2013)

Abstract. Structural health monitoring is crucial to maintain the structural performance safely. Moreover, the Kullback-Leibler divergence (KLD) is applied usually to asset the similarity between different probability density functions in the pattern recognition. In this study, the KLD is employed to detect the damage. However the asymmetry of the KLD is a shortcoming for the damage detection, to overcoming this shortcoming, two other divergences and one statistic distribution are proposed. Then the damage identification by the KLD and its three descriptions from the symmetric point of view is investigated. In order to improve the reliability and accuracy of the four divergences, the gapped smoothing method (GSM) is adopted. On the basis of the damage index approach, the new damage index (DI) for detect damage more accurately based on the four divergences is developed. In the last, the grey relational coefficient and hypothesis test (GRCHT) is utilized to obtain the more precise damage identification results. Finally, a clear remarkable improvement can be observed.

To demonstrate the feasibility and accuracy of the proposed method, examples of an isotropic beam with different damage scenarios are employed so as to check the present approaches numerically. The final results show that the developed approach successfully located the damaged region in all cases effect and accurately.

Keywords: KLD; symmetric description; GSM; DI; GRCHT

1. Introduction

According to the definition, the structural health monitoring is a scientific procedure comprising of several non-destructive processes including the identification of operational and environmental loads acting on the component, the recognition of the mechanical damage caused by that loading, and the observation of damage growth as the component operates. Finally, the structural health monitoring deals with assessing the future performance of the component as damage develops (Abdo 2012).

As evidenced by the vast literature in the damage detection, the structural health monitoring has

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

become an increasingly crucial issue. To data, significant efforts have been made by researchers in the damage identification; the presence of damage generally produces changes in the structural stiffness matrix. Meanwhile, these changes are accompanied with changes in modal parameters of the structure. This phenomenon has been widely noted and used for researchers in distinguishing the damage. However, using different modal parameters correlated with other relevant information in the damage identification may get very various results of various accuracies. For this reason, seeking a proper combination of dynamic parameters is an imperative purpose.

From the perspective of the damage detection, Sekhar (2008) provided an excellent review on research advances in damage detection areas over the twenty years. Fan and Qiao (2011) reviewed the vibration-based damage identification methods and gave a comparative study on the damage detection, and the strain-based damage index for the structural damage identification was reviewed by Li (2010). Yang *et al*. (2013) employed the modal parameter to detect damage, the recurrence quantification analysis has emerged as a useful tool for detecting subtle non-stationarities and changes in the time-series data, Nichols and Trickey (2006) extended the recurrence quantification analysis methods to multivariate observations for the damage detection. Yan and Golinval (2006) utilized the singular value decomposition to obtain characteristic subspaces, then the statistical process techniques is adopted for the damage detection of the aircraft wing. Wavelet (Xiang *et al*. 2012b, Wang *et al*. 2011a) is also employed to detect damage, on the basis of the wavelet, the wavelet finite element (Xiang *et al*. 2012a) is employed to detect damage. And other technology of the signal processing, for example envelope spectrum (Ming *et al*. 2011) and constrained independent component analysis (Wang *et al*. 2011b), are also employed to detect damage.

The KLD is a widely employed tool in the statistics and pattern recognition (Hershey and Olsen 2007), Moreno (2004) made use of the KLD as the based kernel of SVM (support vector machine); moreover, the SVM was applied to data classification in Multimedia. Eguchi and Copas (2006) interpreted the KLD with the Neyman-Pearson lemma, and overviewed the KLD in the information geometry. Smith *et al*. (2006) employed the KLD between true and candidate models to select the number of states and variables simultaneously in Markov-switching regression models. Whereas the KLD is not symmetrical, which is able to induce complexity and inconvenience for the damage detection, To overcoming these shortcomings, the J divergence (JD), Jeffrey divergence (JED) and χ^2 statistical distribution (SD) are proposed, which are three descriptions from the symmetric point of view. Mathiassen and Skavhaung (Mathiassen *et al*. 2002) employed the KLD and its symmetric descriptions to measure the texture similarity. Then Rubner and Puzicha (2001) also utilized them to evaluate the dissimilarity for the color empirical, and manifold subspace distance (Sun *et al*. 2013) is also employed to detect damage.

Ratcliffe (1997) proposed the GSM for a beam, which operates solely on the mode shape data of the damaged structure. In the first, only the mode shape data was employed to identify the local stiffness reduction in a notched steel beam. Additionally, Radcliffe applied this approach to locate the lamination in the composite beam successfully, which the mode curvature was used to establish the gapped smoothing damage index. Later, the frequency-dependent curvature shapes data was employed, which was obtained from the operating frequency response function data. The sensitivity of the identification of the structural variability was noticeably increased through this approach (Ratcliffe 2000). In order to calculate the structural irregularity index, Yoon and Heider *et al*. (2001) developed the global smoothing method with using the mathematical modal shape functions, furthermore, this approach showed substantial improved performance over the traditional GSM by eliminating the smearing effect on the edges of the damage and the noticeable noise at the boundary edges of the structure. In order to detect the damage in plate structure, Wu and Law (2004) employed the GSM to deal with the uniform load surface curvature of plate, nonetheless, Yoon *et al*. (2005) made use of the Student-t-distribution to construct the irregularity index of damage detection with respect to a level of significance. Recently, Baneen and Kinkaid (2012) detected the damage of a beam-type structure utilized the GSM and noise suppression method.

On the basis of the theory of the modal stiffness, Stubbs and Kim (1995) proposed the damage detection index of a beam, which operates the mode shape data of the intact and damaged beam, later, Stubbs and Kim (1996) employed the damage index by the hypothesis test in the statistical sense. Then the work presented in (Farrar and Jauregui 1998a, b) shows an example of damage detection by the damage index for a real *I*-to bridge Rio Grande in Albuquerque NM USA experimentally and numerically, respectively. Furthermore, the damage index has been successfully utilized to damage localization in the isotropic (Sampaio *et al*. 1999, Maia *et al*. 2003), composite (Hamey *et al*. 2004, Lestari *et al*. 2007), concrete (Kim *et al*. 2006) beams, recently, the damage index was used to process the time-domain response (Choi and Stubbs 2004) of the isotropic beam in damage identification. In this study, the new damage index is constructed to process the KLD and its three symmetric descriptions with the intension of obtaining a more precise result.

Grey relational coefficient (GRC) method is a data process, which is always applied to sort out the correlation extent of effect factors with uncertain information. Thus, the purpose of the grey relational coefficient analysis is look for the regularity of variable by dealing with the raw data. Furthermore, the grey relational coefficient analysis is expressed as the quantity analysis to the developing trend, and the more similar of developing trends, the greater of the relational extent. The comparison of the relational extent among the raw data is called as the grey relational coefficient analysis. Fu *et al*. (2001) made use of GRC to analyze the corrosion failure of oil tubes, Mohamed (Abdo 2012) employed the GRC to detect the damage on the basis of the static response, whereas the error of the damage detection is relative large. In this study, the hypothesis test is adopted to obtain the less error and more precise result of the damage detection. Hypothesis testing (HT) is a crucial question in the statistical inference. With the absolutely absent knowledge of the entire distribution function, the intension of the hypothesis testing is to remove some hypothesis of the entire. Thus it is able to obtain some knowledge for the entire distribution function, which can be helpful to make a decision about whether the structure is damaged. On the basis of the interpolation algorithm, Limongelli (2010) employed the hypothesis testing to deal with the sensitivity of the frequency response function under changing environment for the damage detection, the damage detection (Limongelli 2011) for frames under seismic excitation is completed.

Due to the flexible measurement and relative low cost, the application of modal analysis for detect damage has been increasingly. Thereby the modal parameter is adopted for the damage in this study. First of all, a new damage index named the KLD is proposed, and three symmetric descriptions are employed to detect the damage with the purpose of overcoming the shortcoming of the KLD. In the first, the GSM is employed to improve the result of the damage detection, in addition, the damage index based on the KLD and its three symmetric descriptions is constructed to reduce the error of the damage detection, and the GRCHT is applied to obtain a satisfied error of the damage detection. Finally, the numerical example invalidates the robustness and effectiveness of the proposed method.

2. Theories

2.1 The KLD

The KLD is a fundamental concept about the expected log-likelihood ratio in statistics, which is a widely used tool in the statistics and pattern recognition. Assuming the $P(x)$ and $Q(x)$ be two probability distributions with probability density functions $p(x)$ and $q(x)$, respectively, thereby the K-L divergence (Rubner *et al.* 2001) from the $P(x)$ to the $Q(x)$ is defined as

$$D(P,Q)=\int p(x)\log\frac{p(x)}{q(x)}dx \tag{1}$$

Considering a vibration problem of a beam, the $p(x)$ and $q(x)$ can be replacement by the lth mode shape $\phi^l(x)$ and $\phi^{dl}(x)$ of the intact and damaged beam, after normalization of the mode shape, the corresponding KLD of the beam is given as follows

$$KLD_l=\int_0^L \phi^{dl}(x)\log\frac{\phi^{dl}(x)}{\phi^l(x)}\mathrm{d}x \tag{2}$$

Supposed that the beam structure is divided in n elements, the KLD_i^l of the ith element is defined as follows

$$KLD_i^l=\int_{a_i}^{a_{i+1}} \phi_i^{dl}(x)\log\frac{\phi_i^{dl}(x)}{\phi_i^l(x)}\mathrm{d}x \tag{3}$$

To account for all available modes, the KLD_i along the beam employed for the damage detection can be expressed as

$$KLD_i=\sum_{l=1}^{NM}\int_{a_i}^{a_{i+1}} \phi_i^{dl}(x)\log\frac{\phi_i^{dl}(x)}{\phi_i^l(x)}\mathrm{d}x \tag{4}$$

2.2 The JD

The JD (Mitra *et al.* 2002) is used to measure the distance between two probabilistic distributions and the similarity of two vectors, which also can be used to establish and recognize the sample of the working status. Furthermore, the JD is given as

$$J\left(F_1,F_2\right)=\frac{1}{2N}\sum_{i=1}^{N}\left(\frac{f_{1i}}{f_{2i}}+\frac{f_{2i}}{f_{1i}}\right)-1 \tag{5}$$

In which f_{1i} and f_{2i} are elements of sets F_1 and F_2 of the sample, and N is the number of the sample of the sample set.

Considering a vibration problem of a beam, the analogous equation can be obtained as follows:

The JD of the lth mode shape is calculated as follows

$$JD_l=\sum_{l=1}^{NM}\left[\frac{1}{2}\left(\frac{\phi^l}{\phi^{dl}}+\frac{\phi^{dl}}{\phi^l}\right)-1\right] \tag{6}$$

Thus the JD of the *i*th element for the *l*th mode shape in a beam can be given as

$$JD_i^l = \frac{1}{2}\left(\frac{\phi_i^l}{\phi_i^{dl}} + \frac{\phi_i^{dl}}{\phi_i^l}\right) - 1 \tag{7}$$

To account for all available modes, the JD along the beam employed for the damage detection can be expressed as

$$JD_i = \sum_{l=1}^{NM}\left[\frac{1}{2}\left(\frac{\phi_i^l}{\phi_i^{dl}} + \frac{\phi_i^{dl}}{\phi_i^l}\right) - 1\right] \tag{8}$$

2.3 The JED

The JED (Mathiassen *et al.* 2002) is also an description of the KLD from the symmetry point of view, which is more numerically stable for empirical distributions (Puzicha *et al.* 1997). Furthermore, the JED is defined as

$$D(q,p) = \sum_{j=1}^{N}\sum_{l=0}^{L-1}\left[q_j(l)\log\frac{q_j(l)}{p_j(l)} + p_j(l)\log\frac{p_j(l)}{q_j(l)}\right] \tag{9}$$

In the same way, the JED along the beam employed for the damage detection can be expressed as

$$JED_i = \sum_{l=1}^{NM}\left(\phi_i^l \log\frac{\phi_i^l}{\phi_i^{dl}} + \phi_i^{dl}\log\frac{\phi_i^{dl}}{\phi_i^l}\right) \tag{10}$$

2.4 χ^2 Statistic distribution

The χ^2 statistic distribution (Rubner *et al.* 2001) is also a description of the Kullback-Leibler divergence from the symmetric and statistic point of view, and it is expressed as

$$D_{\chi^2}(q,p) = \sum_{j=1}^{N}\sum_{l=1}^{L-1}\frac{\left[q_j(l) - q_j(l)\right]^2}{q_j(l) + q_j(l)} \tag{11}$$

In the same way, the χ^2 distribution along the beam employed for the damage detection can be expressed as

$$SD_i = \sum_{l=1}^{NM}\frac{\left(\phi_i^{dl} - \phi_i^l\right)^2}{\phi_i^{dl} + \phi_i^l} \tag{12}$$

2.5 Gapped smoothing method

The GSM is also called modified Laplacian operator (MLO) (Radzienski *et al.* 2011), and it doesn't require the baseline data set of the intact structure for comparison, which is different from the most method of the damage detection.

The GSM assumes that the mode shape of a structure without the presence of damage is

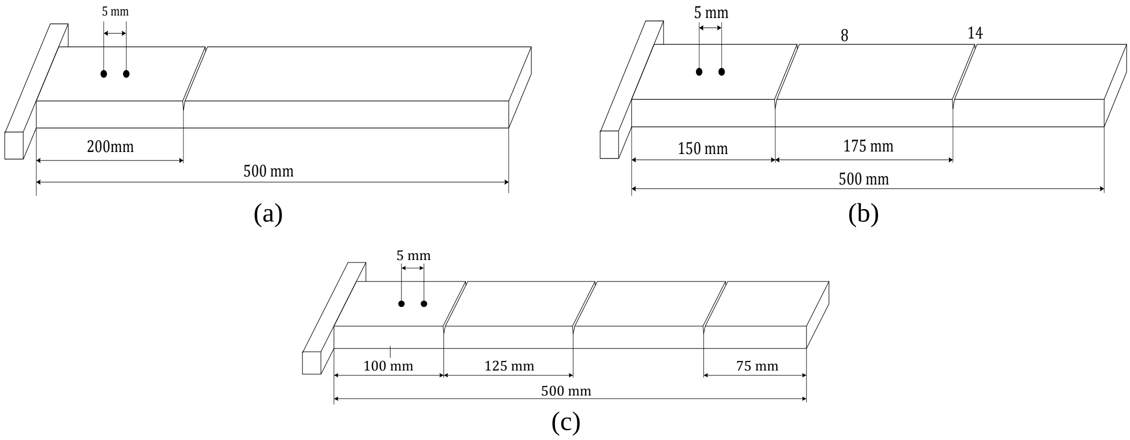


Fig. 1 damage scenarios of the beam: (a) the single damage, (b) two damages, (c) three damages

continuous and smooth; furthermore, this smoothness can be approximated by a polynomial. The basic idea of the GSM is that the mode shape has a smooth surface without the presence of any damage, whereas the small change will appear in modal parameters due to damage. To extract and capture this small change, the locally fitting a gapped cubic polynomial to the mode shape is adopted. Hence even a small amount of damage will trigger the substantial features in the damage index.

The one-dimensional gapped smoothing method was utilized to give the location of variations in structural stiffness. This process is to fit a gapped cubic polynomial of one variable to the mode shape curve, and then the damage detection index is calculated as the difference between the measured mode shape and the calculated polynomial.

The Fig. 1 shows how to calculate the damage detection index using the one-dimensional gapped smoothing method, the open circular mark represents the gapped point and the black circular mark represents the employed data points for the mode shape curve smoothing, moreover, the continuous line shows the measured mode shape, and the line of dashes denotes the cubic polynomial function at the *i*th element, furthermore, the gapped cubic polynomial $\tilde{\phi}(x_i)$ for the *i*th element of the mode shape ϕ at position x_i along the beam is defined as

$$\tilde{\phi}_i(x_i) = a_0 + a_1 x_i + a_2 x_i^2 + a_3 x_i^3 \tag{13}$$

In which x_i is the coordinate of the ϕ_i , which is equal to the distance between the *i*th measured point and the end of the one-dimensional structure, and a_0, a_1, a_2, a_3 are the coefficients of the ϕ_i, which is determined by the ϕ_{i-2}, ϕ_{i-1}, ϕ_{i+1}, ϕ_{i+2}. For the first and last two measured point, the fitting mode shapes of the cubic polynomial are calculated using four-point backward or forward looking with the aim of maintaining a Bachman-Landau order of magnitude of $O(h^2)$ (Cornwell *et al.* 1999). Then the formulation of the damage index δ_i is written as follows

$$\delta(x_i) = \left|\phi_i(x_i) - \tilde{\phi}_i(x_i)\right| \tag{14}$$

2.6 The damage index method

2.6.1 The damage index based on the KLD

Stubbs and Kim (1995) developed the damage index method for a beam, which operates on the mode data of the beam before and after the presence of the damage. On the basis of the study of the Stubbs and Kim, the damage index based on the KLD is proposed.

For a one-dimensional beam structure, the modal stiffness K^l associated with the lth mode shape is given as

$$K^l = \int_0^L EI(x)\left[\left(\phi^l(x)\right)''\right]^2 dx \tag{15}$$

In which L denotes the length of the beam; $\phi^l(x)$ indicates the lth mode shape of the beam; and EI is the bending stiffness of the beam.

Supposed that the one-dimensional beam structure is divided into n elements, the modal stiffness K_i^l of the ith element associated with the lth mode shape of the beam is expressed as

$$K_i^l = \int_{a_i}^{a_{i+1}} EI_i(x)\left[\left(\phi_i^l(x)\right)''\right]^2 dx \tag{16}$$

Where a_i and a_{i+1} represent the distance between the ith node and the i+1th node to the beam end, respectively; and EI_i denotes the bending stiffness of the ith element, furthermore, the $\phi_i^l(x)$ indicates the lth mode shape of the beam for the ith element.

Thus the element sensitivity F_i^l of the ith element about the lth mode shape of the beam is defined the fraction of the modal stiffness K_i^l of the ith element to the modal stiffness K^l associated with the lth mode shape, namely

$$F_i^l = \frac{K_i^l}{K^l} \tag{17}$$

Owing to the one-dimensional beam structure is divided into n elements, we can get

$$\sum_{i=1}^{n} F_i^l = 1 \tag{18}$$

By the same token, the damage function D^l based on the KLD associated with the lth mode shape is defined as

$$D^l = \int_0^L EI^d(x)\left(KLD^l\right)^2 dx \tag{19}$$

In which the KLD^l denotes the KLD of the one-dimensional beam structure calculated by the lth mode shape, furthermore, the damage function D_i^l and the element sensitivity of the damage function DF_i^l of the lth element are defined as follows similarly

$$D_i^l = \int_{a_i}^{a_{i+1}} EI_i^d(x)\left(KLD_i^l\right)^2 dx \tag{20}$$

$$DF_i^l = \frac{D_i^l}{D^l} \tag{21}$$

$$\sum_{i=1}^{n} DF_i^l = 1 \tag{22}$$

In view of the Eqs. (28) and (32), the contribution of the K_i^l and DF_i^l to the K^l and D^l is so small that $K_i^l = 1 \quad DF_i^l = 1$, when the beam is divided into fifty elements. Hence, we can get

$$1 + F_i^l = 1 + DF_i^l \tag{23}$$

Substituting the Eqs. (28) and (32) into the Eq. (34) yields

$$1 = \frac{\left(D_i^l + D^l\right) K^l}{\left(K_i^l + K^l\right) D^l} \tag{24}$$

Substituting the Eqs. (26), (27), (30) and (31) into the Eq. (35) yields

$$1 = \frac{\left(\int_{a_i}^{a_{i+1}} EI_i^d\left(x\right)\left(KLD_i^l\right)^2 dx + \int_0^L EI^d\left(x\right)\left(KLD^l\right)^2 dx\right)\int_0^L EI\left(x\right)\left[\left(\phi^l(x)\right)''\right]^2 dx}{\left(\int_{a_i}^{a_{i+1}} EI_i\left(x\right)\left[\left(\phi_i^l(x)\right)''\right]^2 dx + \int_0^L EI\left(x\right)\left[\left(\phi^l(x)\right)''\right]^2 dx\right)\int_0^L EI^d\left(x\right)\left(KLD^l\right)^2 dx} \tag{25}$$

Assume that the bending stiffness of the beam is almost constant; the above equation can be simplified to

$$1 = \frac{EI_i^d\left(\int_{a_i}^{a_{i+1}}\left(KLD_i^l\right)^2 dx + \int_0^L\left(KLD^l\right)^2 dx\right)\int_0^L\left[\left(\phi^l(x)\right)''\right]^2 dx}{EI_i\left(\int_{a_i}^{a_{i+1}}\left[\left(\phi_i^l(x)\right)''\right]^2 dx + \int_0^L\left[\left(\phi^l(x)\right)''\right]^2 dx\right)\int_0^L\left(KLD^l\right)^2 dx} \tag{26}$$

Furthermore, the Eq. (37) can be simplified to

$$\frac{EI_i}{EI_i^d} = \frac{\left(\int_{a_i}^{a_{i+1}}\left(KLD_i^l\right)^2 dx + \int_0^L\left(KLD^l\right)^2 dx\right)\int_0^L\left[\left(\phi^l(x)\right)''\right]^2 dx}{\left(\int_{a_i}^{a_{i+1}}\left[\left(\phi_i^l(x)\right)''\right]^2 dx + \int_0^L\left[\left(\phi^l(x)\right)''\right]^2 dx\right)\int_0^L\left(KLD^l\right)^2 dx} \tag{27}$$

To account for all available modes, the damage index $KLDDI_i$ based on the K-L divergence along the beam can be defined as

$$KLDDI_i = \frac{\sum_{i=1}^{NM}\left(\left(\int_{a_i}^{a_{i+1}}\left(KLD_i^l\right)^2 dx + \int_0^L\left(KLD^l\right)^2 dx\right)\int_0^L\left[\left(\phi^l(x)\right)''\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left[\left(\phi^l(x)\right)''\right]^2 dx}{\sum_{i=1}^{NM}\left(\int_{a_i}^{a_{i+1}}\left[\left(\phi_i^l(x)\right)''\right]^2 dx + \int_0^L\left[\left(\phi^l(x)\right)''\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left(KLD^l\right)^2 dx} \tag{28}$$

In which NM denotes the number of the modes.

2.6.2 The damage index based on the JED

By the same token, the damage index $JEDDI_i$ based on the Jeffrey divergence along the beam can be defined as

$$JEDDI_i = \frac{\sum_{i=1}^{NM}\left(\left(\int_{a_i}^{a_{i+1}}\left(JED_i^l\right)^2 dx + \int_0^L\left(JED^l\right)^2 dx\right)\int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx}{\sum_{i=1}^{NM}\left(\int_{a_i}^{a_{i+1}}\left[\left(\phi_i^l(x)\right)^{''}\right]^2 dx + \int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left(JED^l\right)^2 dx} \tag{29}$$

2.6.3 The damage index based on the JD

By the same token, the damage index $JDDI_i$ based on the J divergence along the beam can be defined as

$$JDDI_i = \frac{\sum_{i=1}^{NM}\left(\left(\int_{a_i}^{a_{i+1}}\left(JD_i^l\right)^2 dx + \int_0^L\left(JD^l\right)^2 dx\right)\int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx}{\sum_{i=1}^{NM}\left(\int_{a_i}^{a_{i+1}}\left[\left(\phi_i^l(x)\right)^{''}\right]^2 dx + \int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left(JD^l\right)^2 dx} \tag{30}$$

2.6.4 The damage index based on the χ^2 statistic distribution

By the same token, the damage index $SDDI_i$ based on the χ^2 statistic distribution along the beam can be defined as

$$SDDI_i = \frac{\sum_{i=1}^{NM}\left(\left(\int_{a_i}^{a_{i+1}}\left(\chi^2 D_i^l\right)^2 dx + \int_0^L\left(\chi^2 D^l\right)^2 dx\right)\int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx}{\sum_{i=1}^{NM}\left(\int_{a_i}^{a_{i+1}}\left[\left(\phi_i^l(x)\right)^{''}\right]^2 dx + \int_0^L\left[\left(\phi^l(x)\right)^{''}\right]^2 dx\right)\sum_{i=1}^{NM}\int_0^L\left(\chi^2 D^l\right)^2 dx} \tag{31}$$

2.7 The hypothesis testing

In the real world, owing to several sources of the error in the recorded mode shape, the value of the mode shape can change even if on damage occurs. This can lead to false or missing damage identification

In order to take this aspect into account, a statistical classification criterion based on the hypothesis testing has been utilized.

The value of the damage index can be considered as a realization of a random variable described by a probability distribution. In the benchmark, the damage index is characterized a certain probability distribution. Furthermore, the probability distribution of the damage index is usually known, and the reason for this is that it is possible to obtain data on the structure and to estimate the probability distribution based on the collected data.

From the content above, the damage index is a continuous transformation of the sum of independent terms; as a result, the distribution of the damage index is well approximated by a Gaussian Law for the Central Limit Theorem. This assumption will be adopted in this paper for the damage index based on all divergences. Moreover, for the application reported in the following sections, a normality test has been employed in order to further this assumption.

With the aid of this assumption of normality, the distribution of the damage index can be got once its statistic parameters (mean μ_x and variance σ_x^2) are calculated. Whereas in the real state, only mode shapes recorded during a limited number of locations are available, thereby a single value of the damage index rather than its probability distribution is available at each measured location. Based on this content, a decision about the single recorded point should be taken about the possible existence of the damage at the measured location x_i .

This and the following sections are carried out by a statistic hypothesis test, generally speaking, the "null" hypothesis H_0 denotes that the structure is undamaged at a given location, and the alternate hypothesis H_1 represents that the structure is damaged at that location, moreover; the hypothesis H_0 is tested against the alternate hypothesis H_1.

Supposing that the variance $\sigma^2(x)$ is constant (Limongelli 2011, 2010), the previous hypothesis testing problem can be regarded as a standard one-side test of the unknown mean value $\mu(x)$ of the damage, and the basic hypothesizes are expressed as follows

(1) H_0: $z_i < z_\eta$ null hypothesis: the structure is intact on the location x

(2) H_1: $z_i \geq z_\eta$ alternate hypothesis: the structure is damaged on the location x

Furthermore, the null hypothesis H_0 is rejected the alternate hypothesis H_1 at the α critical level of significance (Koo *et al*. 2010) if

$$X = \frac{x_i - E(x)}{\sigma(x)} > Z_\alpha \tag{32}$$

Where Z_α denotes the α percentile of the standard normal distribution.

Thereby the condition of the presence of the damage at the α level of significance can be expressed as

$$DI(x) = \frac{x_i - E(x)}{\sigma(x)} - Z_\alpha \tag{33}$$

On the basis of the acceptation of the alternative hypothesis H_1 in a statistical sense, this criteria assigns the tag "damaged" to the location x if $DI(x) > 0$; in the contrary, this criteria gives the tag "intact" to the location x.

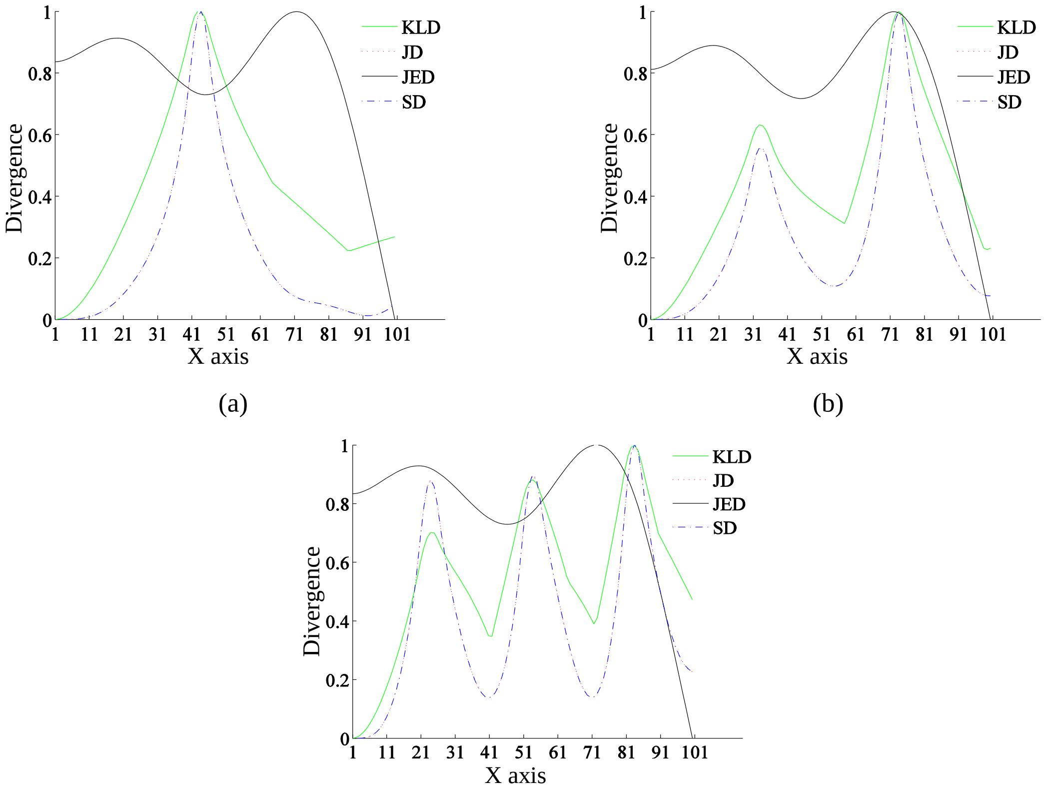
In view of the above research, the value of the Z_α is a tradeoff between the damaged state and the intact state assigned of the location x, thus it is crucial to quantify the value of the Z_α.

In this study, $\alpha = 0.05$ and the corresponding $Z_\alpha = 1.645$ is adopted (Choi and Stubbs 2004).

3. Case study: beam or numerical example

3.1 Material properties

To certify the proposed methodology, a one-side clamped beam is considered. The material properties are taken from steel where the elastic modulus $E = 2.06\times10^{11} Pa$, the Poisson's ration $\nu = 0.3$ and the density $\rho = 7800 kg/m^3$. The geometrical dimension is given as follows: the length of the beam is 5m, the area of cross-section is $0.25\times0.2m^2$, and the geometrical moment of inertia for the beam cross-section is the $1.67\times10^{-4} m^4$.



(c)

Fig. 2 results of the damage detection by the K-L divergence: (a) the single damage, (b) two damages, (c) three damages

3.2 Damage scenarios

It is suppose that the occurrence of the structural damage only causes a reduction in the local stiffness, furthermore, the mass matrix remains unchanged. Thus the stiffness reduction is employed to simulate the damage. The beam is divided into 100 elements, and the elastic modulus of elements from 41 to 45 is reduced to half in order to simulate the single damage. In order to test the ability of the proposed method in detecting multiple damages, the elastic modulus of elements from 31 to 35 and from 71 to75 is reduced to half for simulating two damages. Finally, *t* the elastic modulus of elements from 21 to 25, from 51 to 55 and from 81 to 85 is reduced to half with the intension of simulating triple damages, furthermore, damage scenarios are shown in Fig. 3.

According to the one-dimensional finite element method, the modal analysis of the beam is carried out. Furthermore, first four modes of the intact and damaged beam are obtained.

3.3 Results

According to the calculation formula of the KLD and its three descriptions from the symmetric point of view, corresponding results of the damage detection for the beam are shown in the Fig. 2,

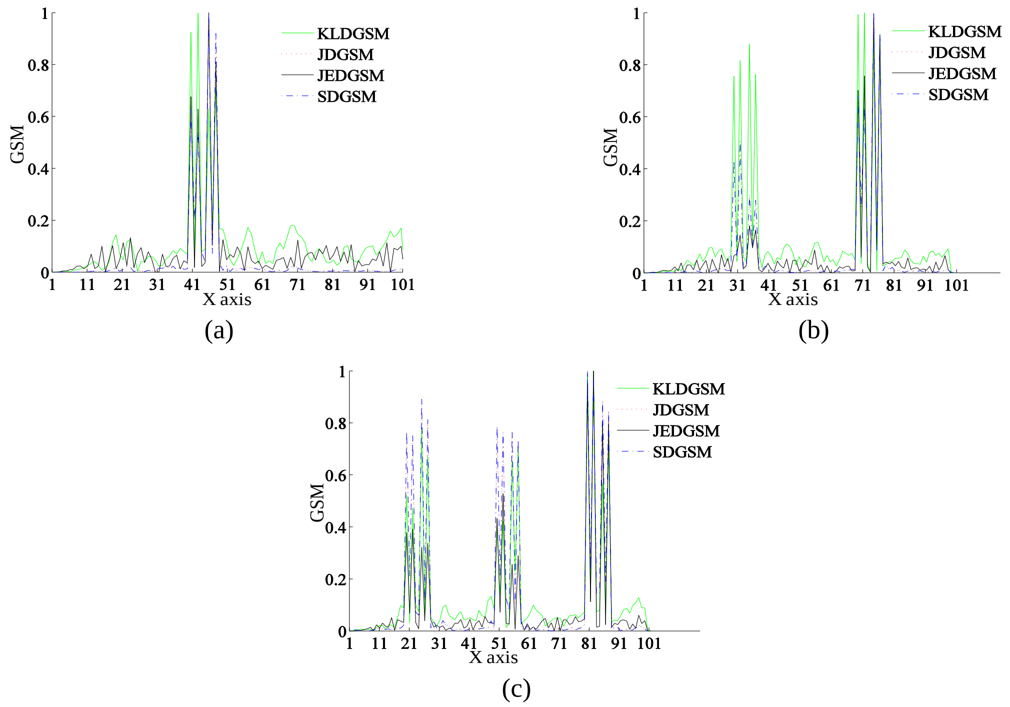


Fig. 3 results of the damage detection by the GSM: (a) the single damage, (b) two damages, (c) three damages

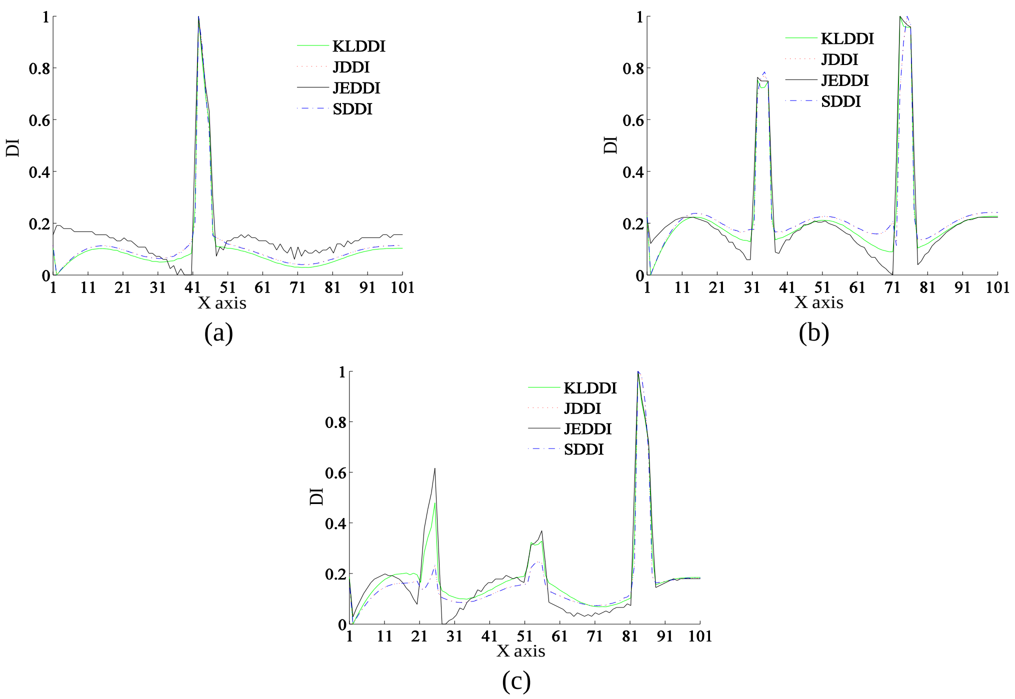


Fig. 4 results of the damage detection by the DI: (a) the single damage, (b) two damages, (c) three damages

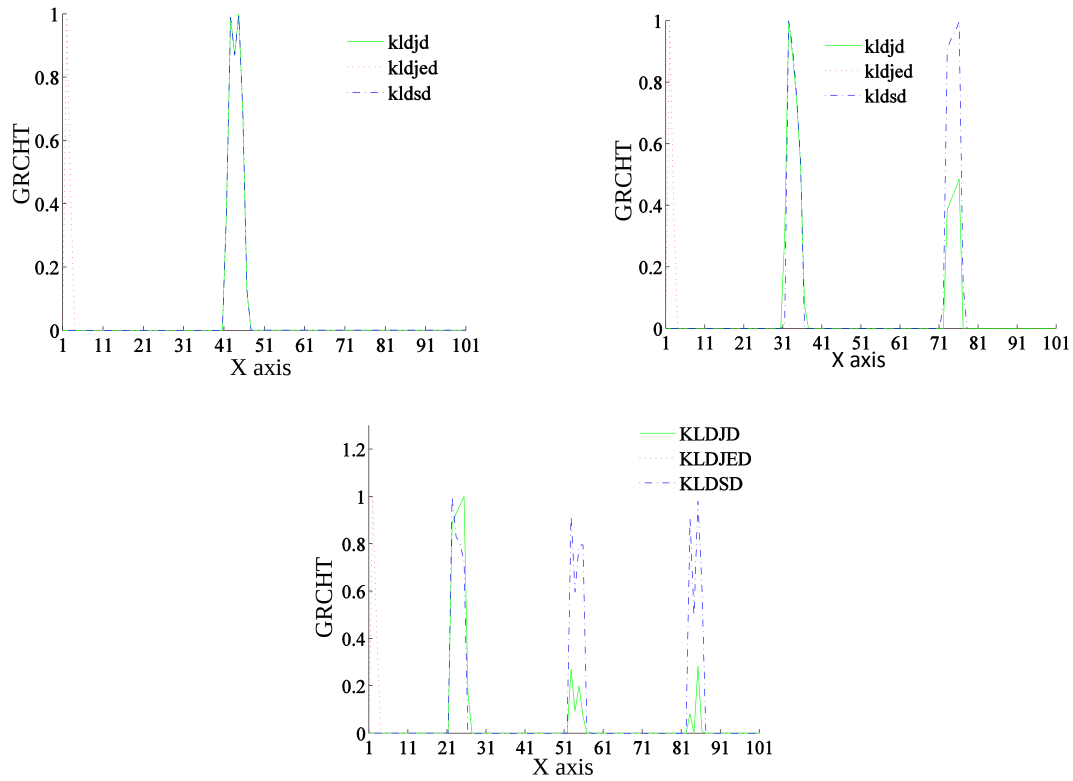


Fig. 5 results of the damage detection by the GRCHT: (a) the single damage, (b) two damages, (c) three damages

then the GSM is adopted to improve the result of the damage detection, and the corresponding result based on the KLD and its three description from the symmetric point of view is shown in the Fig. 3. Similarly, the damage index based on the KLD and its three description from the symmetric point of view is shown in the Fig. 4. In the last, the grey relational coefficient and hypothesis test is applied to obtain the satisfied result of the damage detection, which is shown in the Fig. 5.

3.4 Discussions

3.4.1 Discussions on the KLD and its three symmetric descriptions

In view of the results obtained, the sudden anomaly in the corresponding curve denotes the accurate location of the damage in beams for all scenarios, except the Jeffrey divergence. From the Fig. 2 it can be know that the JD for all damage scenarios are almost identifiable, and it is not able to detect the damage for the beam. Thus the ability of the JD to describe the KLD from the symmetric point of view is worst of three symmetric descriptions.

It can also be know that the KLD, JD and χ^2 distribution can detect the single and multiple damages, though the error of the damage detection is relative large. Moreover values of the JD and the χ^2 distribution are nearly identical, therefore, they have the almost error of the damage detection, generally speaking, the error of the damage detection for the KLD is larger than the JD and χ^2 distribution, which can be obtained from the Fig. 2.

For the multiple damages detection, the value of the peak in the damage location increases along the X axis except the triple damage detection by the JD, hence it can come to the conclusion that the detection precision of the damage with different locations is various, which can be also seen from the Fig. 2.

The distribution range of the KLD is wider than the JD and χ^2 distribution, hence it demonstrates the localized ability of the KLD in the vicinity in the location of the damage is stranger than the JD and χ^2 distribution, and the KLD is less sensitive to the damage than the JD and χ^2 distribution.

3.4.2 Discussions on the improved method-GSM

The GSM index can reduce the error and improve the result of the damage detection in comparison with the KLD and its three symmetric descriptions. Furthermore, the distribution range of the GSM index is much narrower than the single KLD and its three descriptions from the symmetric point of view and it demonstrates that the GSM index is more sensitive to the damage.

No damage can be detected by the JED; however, the corresponding JEDGSM based on the JED is able to detect all scenarios of the damage. First of all, it can come to the conclusion that the JED contains the information needed for the damage detection. In addition, the GSM index is a robustness and effectiveness method, which can extract the useful information and accomplish the damage detection.

From the Fig. 3 we also can be know that the error of the damage detection can be given as from large to small: KLDGSM, JEDGSM, JDGSM, SDGSM, and KLDGSM denotes the index obtained through processing the KLD by the GSM, furthermore, the other can be got in the same token.

From the Fig. (b) it can be know, the value of the peak in the first damage location is relative too small. Thus, the characteristic of the damage is not intense, furthermore, the error of the damage detection is relative large. It is still need to improve the ability of the GSM to fulfill the damage detection.

3.4.3 Discussions on the improved method-DI

The DI method can reduce the error and improve the result of the damage detection in comparison with the KLD and its three symmetric descriptions. Furthermore, the distribution range of the DI index is much narrower than the single KLD and its three descriptions from the symmetric point of view and it demonstrates that the DI index is more sensitive to the damage.

Therefore, the DI index is a robustness and effective method of the damage detection.

No damage can be detected by the JED; however, the corresponding JEDDI based on the JED is able to detect all scenarios of the damage. First of all, it can come to the conclusion that the JED contains the information needed for the damage detection. In addition, the DI index is a robustness and effectiveness method, which can extract the useful information and accomplish the damage detection.

From the Fig. 4 we also can be know that the error of the single damage detection can be given as from large to small: JEDDI, JDDI, SDDI, KLDDI, furthermore, for the multiple damages detection, the order from large to small is JDDI, SDDI, KLDDI, JEDDI. Hence, the error of the damage detection is different as the number of the damage varies. As the location number of the damage rises, the error of the damage increases. Moreover, when the damage is near the clamped end of the one-dimensional beam, the peak value is relative small.

From the Fig. 3 and the Fig. 4, following observations can be obtained:

They both can improve the damage detection and reduce the error significantly; furthermore, they both are more sensitive to the damage than the single KLD and its three symmetric descriptions.

The error of the double damage detection by the GSM and the triple damage detection by the DI both on the basis of the KLD and its approximation are both relative large, thereby, they both still have marked room for improvement.

The GSM index has individual peaks but the DI index has a whole peak on the location of the damage, the reason for this is that they focus on different things in the process of the damage detection. Furthermore, on the basis of the interpolation, the GSM index focuses on the error between the interpolation and the raw value of every individual "measured" point, whereas the DI index focuses on the whole difference between the KLD and its three descriptions from the symmetric point of view.

3.4.4 Discussions on the improved method-GRCHT

The method of the GRCHT is applied to measure the similarity between the KLD and its three descriptions from the symmetric point of view. From the Fig. (a)) in the Fig. 5 it can be know that the GRCHT between the KLD and JED is not able to detect the damage in all scenarios, which can be attributed to the widely different geometry shape of the KLD and JED.

Additionally, the GRCHT between the KLD and JD is able to detect the single and multiple damages, whereas values of the GRCHT on different locations of the damage varies greatly, which can reduce the accuracy and enlarge the error of the damage detection.

In the last place, the GRCHT between the KLD and χ^2 statistic distribution got a satisfied result of the single and multiple damages, furthermore, the error of the damage detection is relative small and values of the GRCHT on different locations of the damage only have marginal difference. Thus, a satisfied tool of the damage is obtained.

4. Conclusions

Numerical simulations are performed to verify the identification capability of proposed indices, and guidance on selecting the proper parameters for the damage detection is given. Moreover the following conclusions can be drawn from the above study:

(1) The KLD is employed as a new tool for the damage detection, to overcome the shortcoming of asymmetry for the KLD which may lead to complexity and inconvenience for the damage detection, the JED, JD and the χ^2 statistic distribution are adopted. Finally, the KLD and its three symmetric descriptions all can detect the damage except the JED.

(2) In order to obtain the more accurate result and less error for the damage detection, methods of GSM, DI, and GRCHT are applied to improve the damage detection result by the KLD and its three symmetric descriptions. In addition, a significant improvement can be observed for the detection of the single and multiple damages. Thus, the proposed method is a robustness and effectiveness method

(3) The applicability of the proposed method to different types of structures and different sizes of damages is under experimentally validating and it will be illustrated in the future work.

(4) Results of the present work refer to the beam, whereas they can be easily extended to more complex structures and more complicated boundary conditions.

All in all, the present consequences provide a foundation of using the KLD and its improved method as an efficient tool for the damage detection in the beam. Furthermore, further work is needed to enhance the reliability and precision of the proposed method.

Acknowledgments

The research described in this paper has been supported by the National Natural Science Foundation of China (Grant No. 51035007), the National Natural Science Foundation of China (Grant No. 51175401) and the research fund for the Doctoral Program of Higher Education of China (No:20110201130001)

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