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An improved algorithm in railway truss bridge optimization under stress, displacement and buckling constraints imposed on moving load

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Abstract. Railway truss bridges are amongst the essential structures in railway transportation. Minimization of the construction and maintenance costs of these trusses can effectively reduce investments in railway industries. In case of railway bridges, due to high ratio of the live load to the dead load, the moving load has considerable influence on the bridge dynamics. In this paper, optimization of the railway truss bridges under moving load is taken into consideration. The appropriate algorithm namely Hyper-sphere algorithm is used for this multifaceted problem. Through optimization the efficiency of the method successfully raised about 5 percent, compared with similar algorithms. The proposed optimization carried out on several typical railway trusses. The influences of buckling, deformation constraints, and the optimum height of each type of truss, assessed using a simple approximation method.

Keywords: railway truss bridge; moving load; numerical optimization; modified hyper-sphere algorithm; three-level technique

1. Introduction

Railway bridges vital to the transportation industries are among expensive and long life structures. These bridges endure heavy live loads with specific patterns, and are therefore in need of high precision accuracy in design. The live load is imposed by the passage of the moving load on the bridge deck. This makes the design more complicated. Bridge trusses are widely used in railway networks, all over the world. There is no stoppage on the efforts to minimize the construction and maintenance costs of the bridges. On the other hand, this should not jeopardize the structure integrity and its safety. Therefore, the industry owners and engineers always resort to paying considerable attention to the optimization techniques.

In Railway bridges, the ratio of the live load to the dead load is much higher compared with the highway bridges. Besides, the impact loads in railway bridges are more serious than the highway bridges. The effect of the moving load is highly important. The stresses imposed due to the live load are dependent on the position of the load on the bridge.

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There are many valuable documents about the truss optimization methods. However, most of the optimization based research in the past was concerned with the typical trusses under static point loads. Only few, have considered the truss optimization under the moving load. A highway bridge optimization under truck load is one of the rare articles that considered the moving load scenario. Continuous and discrete design variables were assumed. Many solution algorithms such as the genetic algorithms and SQP were used. Eventually, some types of trusses under HS20 moving load optimized (Toğan and Daloğlu 2009). Nevertheless, to the knowledge of the authors there is no distinct and recorded research concerned with the railway truss bridge optimization.

This research in concerned with the optimization of the railway truss bridge. Specifications of the moving load imposed on the bridge are compatible with the Eurocode loading pattern. The analysis of the combination of the railway bridge and the moving load is intrinsically a complicated task. Adding the truss optimization to this, adds to the complexity of the issue at hand. This requires a suitable and efficient algorithm in order to solve the problem. To serve the purpose, an efficient numerical optimization method is selected and the algorithm is modified to include the moving loads. The foremost aim of this research is to minimize the railway truss weight under the moving loads. The stress and buckling constraints are considered. The effects of some other parameters such as the displacement constraint and the optimum moving load step lengths, etc. are assessed.

2. The problem definition

This paper presents the optimization of the railway bridge trusses under the moving load. The objective function is the weight of the structure and the constraints are the stresses and the buckling. The railway bridge is assumed as single track. Therefore, half of the Eurocode load pattern is imposed on the structure. The dynamic effects of the load are considered by multiplying the Eurocode impact factor with the live load. The truss optimization analysis is carried out using finite element analysis with stiffness formulation.

Three types of railway trusses including WARREN, PRATT and PARKER with 36-meter length and three-meter bays were considered. In addition, as a sample, an indeterminate truss is optimized to obtain the symmetric optimal truss. The geometry of the selected railway trusses is presented in Fig. 1. It needs to be reminded that with the task already defined, the core objective of this research is to propose the appropriate solution algorithm. It is not intended to compare variety of trusses.

3. The research methodology

The solution procedure starts by developing a proper optimization algorithm. This is then followed by introducing some new ideas in order to enhance the algorithm performance. Applicability of the proposed method is examined by exercising the optimization of some commonly used trusses under the moving load. The results are compared with the results from some other existing methods. The next step involves applying a three-level algorithm in order to increase the convergence speed. This is followed by the determination of the proper starting point to solve the optimization problem. The research methodology is presented in a flowchart in Fig. 2.

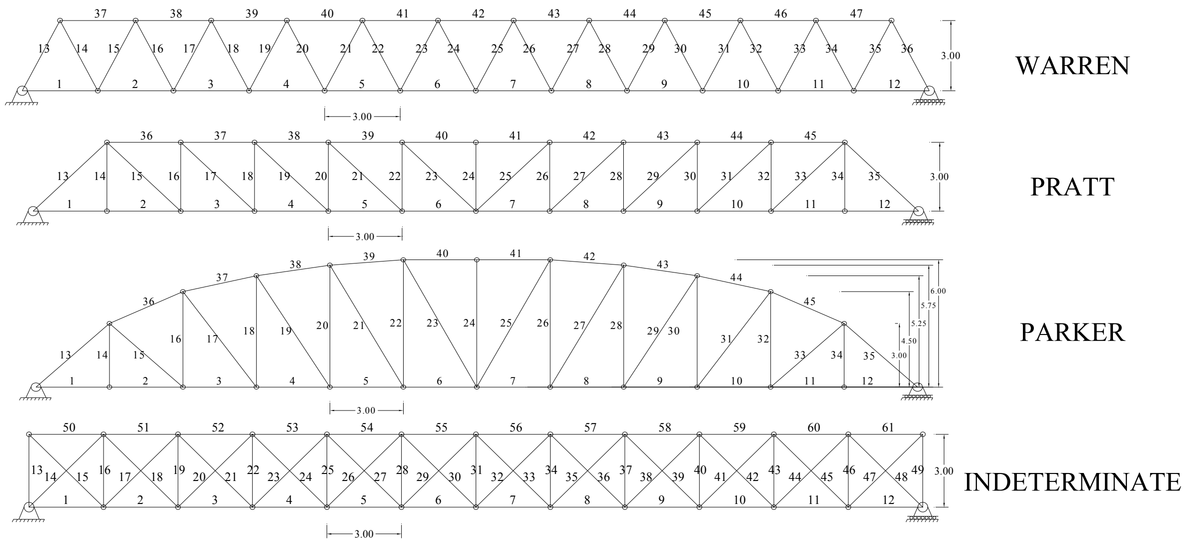


Fig. 1 The geometry of the selected railway trusses

4. The optimization algorithm

4.1 A brief review of the structural optimization literature

The subject of the structural optimization contains several well documented methods and strategies. Generally, such methods can be categorized as Mathematical programming techniques and Meta heuristic methods. Sequential linear programming is a significant mathematical programming technique while genetic algorithm is the most important among Meta heuristic methods. The references for some of these methods are categorically presented in Tables 1-2.

Table 1 The summarized references related to the Mathematical programming techniques

Applied algorithms	Definitions or explanations	References
Optimizations basics	Preliminary introduction of structure optimization	Schmit and Farshi (1974) Farshi and Schmit (1974) Vanderplaats (1982)
Sequential linear programming	Move limit concept SLP modified methods Method of center and hyper-sphere algorithm	Farshi and Alinia-ziazi 2010 John *et al.* (1987) Chen (1998) Pyrz and Zawidzka (2001) Pedersen and Nielsen (2003) Lamberti and Pappalettere (2000) Lamberti and Pappalettere (2003a,b,c) Lamberti and Pappalettere (2004) Lamberti and Pappalettere (2005) Gomes and Senne (2011)

Table 1 Continued

Sequential quadratic programming	Study the typical examples by combination of Forces method and SQP	Sedaghati (2005)
Other Mathematical programming techniques	-	Payten and Law (1998) Lee *et al.* (1998) Gil and Andreu (2001) Kelesoglu and Ulker (2005) Achtziger and Stolpe (2007)

Table 2 The summarized references related to the Meta heuristic methods

Applied algorithms	Definitions or explanations	References
Genetic algorithm	Genetic algorithm introduction Hybrid Genetic Algorithms with Fuzzy Hybrid Genetic Algorithms with Forces method Parallel genetic algorithm	Rajeev and Krishnamoorthy (1992) Erbatur *et al.* (2000) Sarma and Adeli (2000) Kaveh and Kalatjari (2002) Kaveh and Kalatjari (2003) Kaveh and Abdietehrani (2004) Kaveh and Kalatjari (2004) Kaveh and Rahami (2006a,b) Kelesoglu (2007) Rahami *et al.* (2008) Xu *et al.* (2010) Wei *et al.* (2011) Toğan and Daloğlu (2004) Zuo *et al.* (2011)
Ant Colony	-	Camp and Bichon (2004) Camp *et al.* (2005) Bland (2001)
Big bang	-	Erol and Eksin (2006) Camp (2007) Kaveh and Talatahari (2009b)
Particle Swarm	-	Perez and Behdinan (2007) Gomes (2011)
Simulated annealing	Simulated annealing introduction Modified methods	Sonmez (2007) Kolahan *et al.* (2007) Lamberti (2008)
Other methods	Fuzzy logic methods Artificial Bee Colony algorithm Comparison of the methods and hybrid methods	Lee and Geem (2004) Luo *et al.* (2006) Zhang (2007) Kaveh and Talatahari (2009a) Salajegheh *et al.* (2009) Toğan *et al.* (2011) Sonmez (2011) Fiouz *et al.* (2012) Toklu *et al.* (2013)

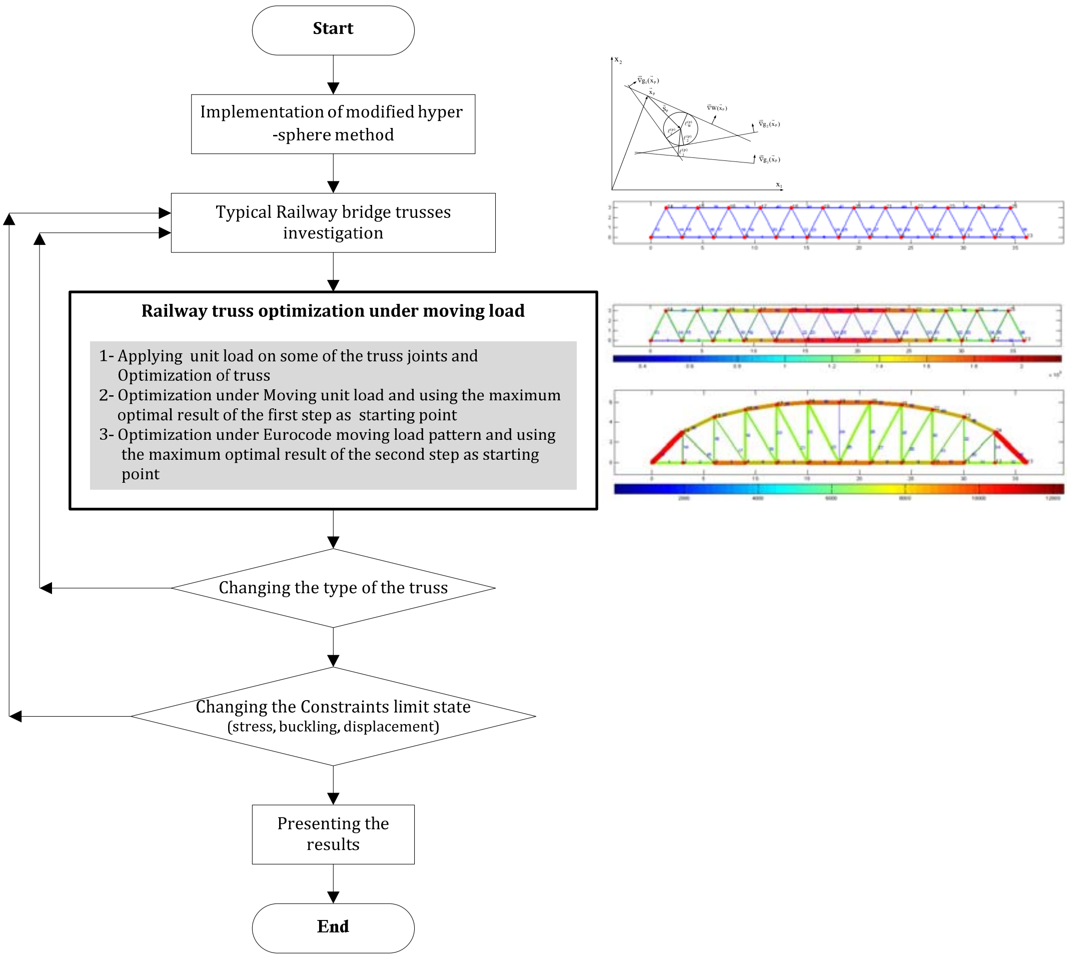


Fig. 2 The research methodology

4.2 The applied algorithm

Considering the moving load in the analysis, leads to the increase in the number of the problem constraints. This in turn necessitates a solution procedure that is highly capable of taking in all the required input data and provides proper convergence speed. Consequently, the hyper-sphere algorithm was selected due to the following reasons:

1. A novel and newly presented method.
2. Efficient (Farshi and Alinia-ziazi 2010) (the variation of the objective function in each cycle can be stated as high efficiency)
3. Only requires first order derivatives. (unlike to the SQP method that requires second order derivatives and *Hessian matrix*, the hyper-sphere algorithm include simpler and faster calculations)

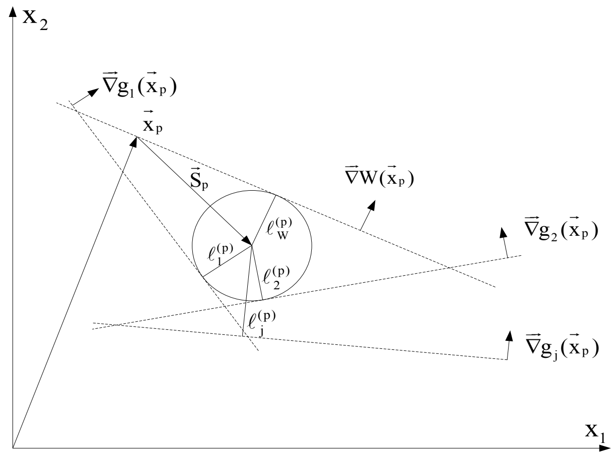


Fig. 3 The classical hyper-sphere method

4. Much faster comparing to the past ordinary methods. (these methods such as the genetic algorithm, ant colony algorithm and simulated annealing, are time consuming algorithms)
Furthermore, in the cases of simulation with some benchmark samples, the results obtained with this algorithm are more accurate. This is attributed to the modifications in the algorithm.

4.2.1 The hyper-sphere algorithm

The basis of the algorithm is presented in Fig. 3. The algorithm is completely explained in references (Schmit and Farshi 1974, Farshi and Alinia-ziazi 2010) and a summary is presented, hereunder.

The linear approximations (g_1, g_2 and g_j), their gradients and the objective function (W) are clearly depicted in Fig. 3. These linear approximations are estimated in X_p design point. Some constraints that are far apart from the design point (g_j) are deleted and the largest circle is found between the remaining constraints and the linear objective function (r_p is the radius of the circle). The centre of the drawn circle is the new design point for the following repetitive steps. The circle lies in two dimensions and the sphere in three dimensions. This makes the hyper-sphere an n-dimensional problem hence the method is called “hyper sphere”. Fig. 4 shows the optimization algorithm of the classical hyper-sphere method. The finite differences are used in order to find the derivatives of the constraints.

4.2.2 The algorithm modification

To solve the problem, the following modifications performed on the algorithm (as indicated in Figs. 4-5):

- The area to the maximum area ratios are calculated and presented in a matrix. After a certain number of iterations of the optimization algorithm, if the ratio is less than the determined initial value (e.g., 0.1) the value of the random variable is deleted and the variable is shifted to the initial value.

The aforementioned procedure reduces the number of design parameters and increases the convergence speed. Constraint related variables are sufficiently increased to achieve the required condition, if reduction of the parameters leads to constraints violation. In addition, the value obtained from the last iteration of the main cycle is assigned as new area, when an area increase is required for the truss member. New increased cross sectional area is constantly used till the optimization is ended.

- Scaling of the variables

The constraints that may be violated are stress or displacement constraints. If violated constraints are stress constraints, its related members are increased according to the amount of the violation. If the displacement constraints are violated, the maximum cross sectional area of the connected members is raised to an amount equal to the size of the violation.

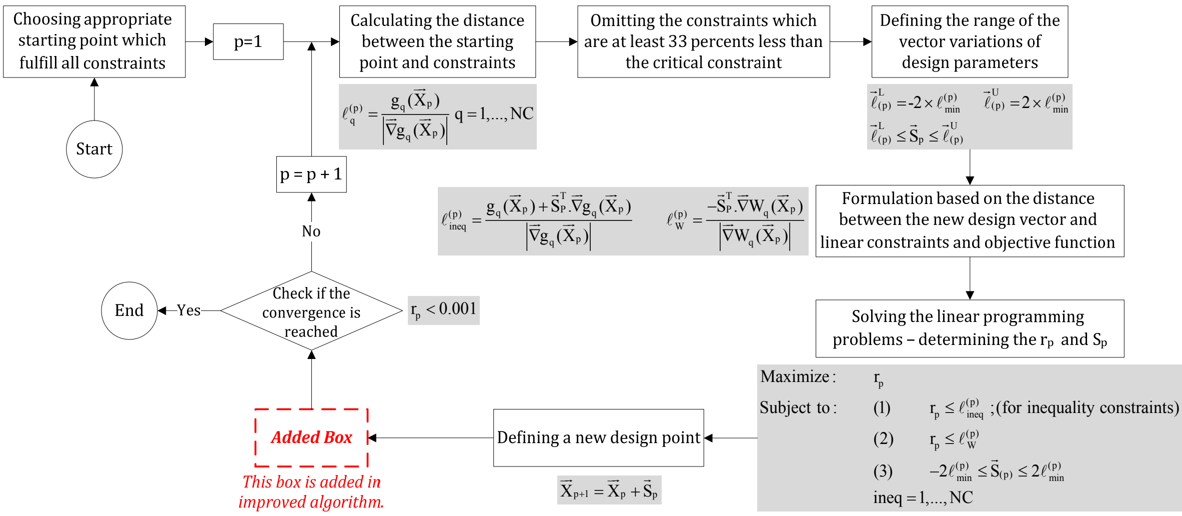


Fig. 4 The optimization methodology within the classical hyper-sphere method

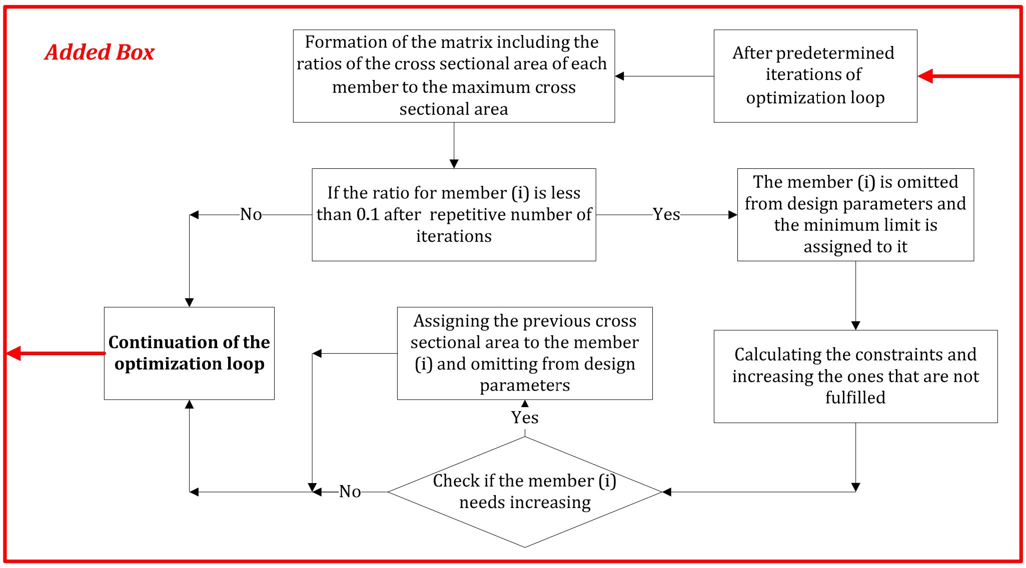


Fig. 5 The algorithm modification

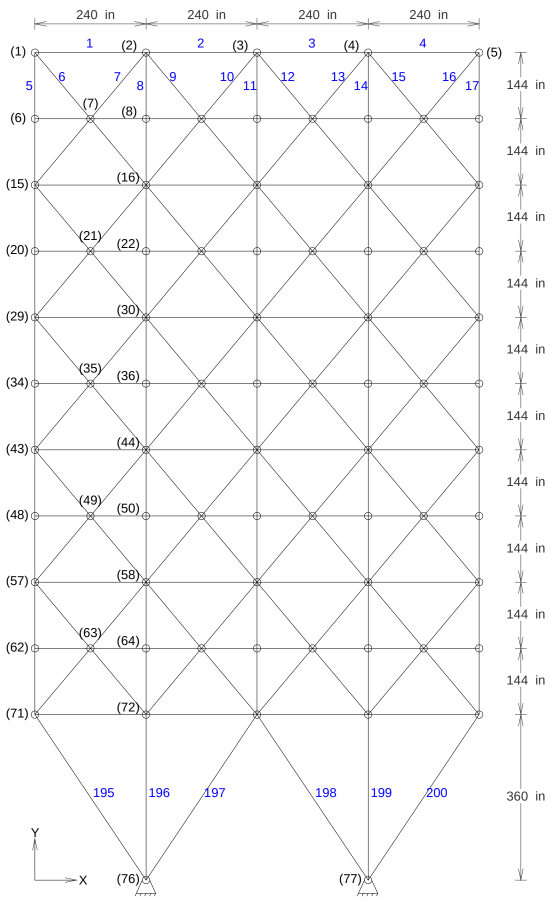


Fig. 6 The planar 200-bar truss

Example 1: 200-bar truss

To examine the algorithm efficiency, benchmarking with a sample that was prevalently noticed in several papers is presented. The 200-member plane truss that is shown in Fig. 6 includes 77 joints and its members, are categorized in 29 groups. The material properties and the allowable stress are as follows:

$$\mathrm{E} = 206{,}842 \ \text{MPa} \ (3\times10^{4} \ \text{ksi}) \qquad \rho = 7833 \ \text{kg/m}^{3} \ (0.283 \ \text{lb/in}^{3})$$

$$\mathrm{A}^{\mathrm{L}} = 0.6451 \ \text{cm}^{2} \quad (0.1 \ \text{in}^{2}) \qquad \sigma_{\text{all}} = \pm 68.95 \ \text{MPa} \qquad (10 \ \text{ksi})$$

Three different load cases are considered as follows:

Case 1: 4449.741 N (or 1000 lb) load, in positive direction of the X axis applied in joints: 1,6,15,20,29,34,43,48,57, 62 and 71

Case 2: 44497.412 N (or 1000 lb) load, in negative direction of the Y axis applied in joints: 1,2,3,4,5,6,8,10,12,14,15,16,17,18,19,20,22,24,26,28,29,30,31,32,33,34,36,38,40,42,43,44 ,45, 46,47,48,50,52,54,56,57,58,59,60,61,62,64,66,68,70,71,72,73,74 and 75

Case 3: Simultaneous application of the first and the second loading.

The assumed constraints in this problem are the stress constraints (Eq. 1). The algorithm used is such that the constraints obtained from Eq. (1), are less than zero.

$$c_i = \frac{\sigma_i}{\sigma_{all}} - 1 \qquad i = 1, 2, \ldots, 200 \tag{1}$$

It needs to be reminded that this truss is almost the most complicated one surveyed and reported in several articles (Farshi and Alinia-ziazi 2010, Lamberti 2008, Lee and Geem 2004). The convergence procedure is indicated in Fig. 7. From the results it is clear that the weight variation is negligible after 60 iterations.

The results from the implemented algorithm indicate that its accuracy is 5.55 percents higher than the classic hyper-sphere algorithm. It is nearly 5.51 percents higher than the approach presented by the Lamberti (2008), using the simulated annealing algorithm. Besides, unlike the results obtained from the other methods, no constraint has been violated in the proposed algorithm (see Table 3).

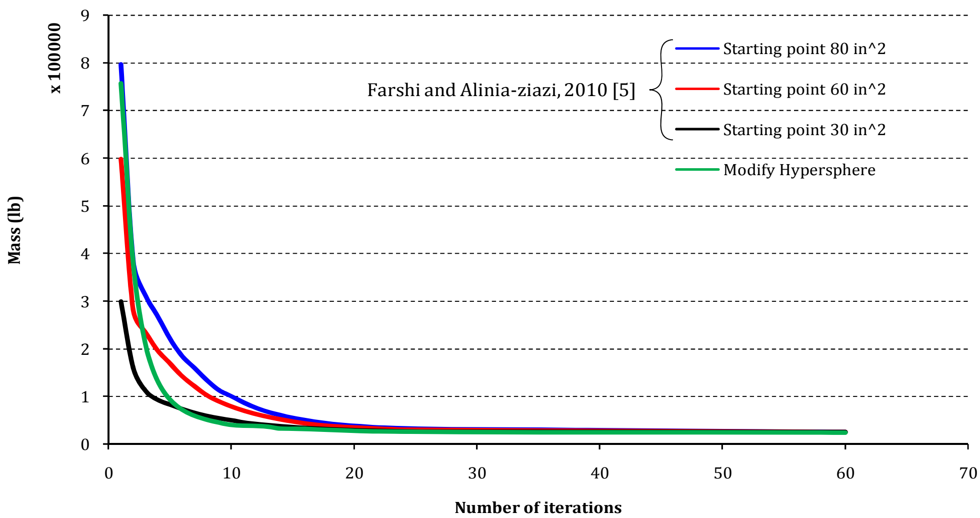


Fig. 7 The convergence procedure in 200-bar truss optimization

Table 3 Comparison of the results for 200-bar truss

Group number	Members	Lee and Geem (2004)	Lamberti (2008)	Farshi and Alinia-ziazi (2010)	The applied algorithm
		area (in^2)	area (in^2)	area (in^2)	area (in^2)
1	1,2,3,4	0.1253	0.1467	0.1470	0.1522
2	5,8,11,14,17	1.0157	0.9400	0.9450	0.9443
3	19,20,21,22,23,24	0.1069	0.1000	0.1000	0.1000
4	18,25,56,63,94,101,132,139,170,177	0.1096	0.1000	0.1000	0.1000
5	26,29,32,35,38	1.9396	1.9400	1.9451	1.9446
6	6,7,9,10,12,13,15,16,27,28,30,31,33,34,36,37	0.2686	0.2962	0.2969	0.2988
7	39,40,41,42	0.1042	0.1000	0.1000	0.1000
8	43,46,49,52,55	2.9731	3.1040	3.1062	3.1193

Table 3 Continued

9	57,58,59,60,61,62	0.1309	0.1000	0.1000	0.1000
10	64,67,70,73,76	4.1831	4.1040	4.1052	4.1196
11	44,45,47,48,50,51,53,54,65,66,68,69,71,72,74,75	0.3967	0.4034	0.4039	0.4070
12	77,78,79,80	0.4416	0.1922	0.1934	0.1611
13	81,84,87,90,93	5.1873	5.4282	5.4289	5.4587
14	95,96,97,98,99,100	0.1912	0.1000	0.1000	0.1000
15	102, 105, 108, 111, 114	6.2410	6.4282	6.4289	6.4591
16	82,83,85,86,88,89,91,92,103,104,106,107,109,110,112,113	0.6994	0.5738	0.5745	0.5666
17	115, 116, 117, 118	0.1158	0.1325	0.1339	0.1614
18	119, 122, 125, 128, 131	7.7643	7.9726	7.9737	7.9872
19	133, 134, 135, 136, 137, 138	0.1000	0.1000	0.1000	0.2331
20	140, 143, 146, 149, 152	8.8279	8.9726	8.9737	8.9875
21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151	0.6986	0.7048	0.7053	0.8249
22	153, 154, 155, 156	1.5563	0.4202	0.4215	0.7120
23	157, 160, 163, 166, 169	10.9806	10.8666	10.8675	11.3003
24	171, 172, 173, 174, 175, 176	0.1317	0.1000	0.1000	0.1823
25	178, 181, 184, 187, 190	12.1492	11.8666	11.8674	11.9773
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189	1.6373	1.0344	1.0349	1.4133
27	191, 192, 193, 194	5.0032	6.6838	6.6849	4.0896
28	195, 197, 198, 200	9.3545	10.8083	10.8101	8.2151
29	196, 199	15.0919	13.8339	13.8379	13.2923
	Weight (lb)	25447.1000	25446.3316	25456.5700	24044.2209
	Maximum amount of constraint	0.0203	3.23E-04	3.23E-04	-1.75E-08

5. Railway truss bridges optimization under moving load

In this section, the optimization is carried out using the aforementioned algorithm. The moving load is imposed based on Eurocode pattern LM71 (Fig. 8). According to the assumption that two trusses exist in each bridge, half of the load (LM71 pattern), together with the impact factor, is imposed to the bridge deck. For the bridge with standard maintenance, the impact factor is calculated using the following equation (Final Draft prEN 2002, UIC Code 776-1 2006)

$$\Phi_3 = \frac{2.16}{\sqrt{L} - 0.2} + 0.73 \qquad 1.00 \le \Phi_3 \le 2.00 \tag{2}$$

The mid-span deformation (δ_{mid}) under the moving load (see Fig. 8) is controlled using the Eq. (3) (EN1990–Annex A2 2005).

$$\frac{L}{\delta_{mid}} > 600 \tag{3}$$

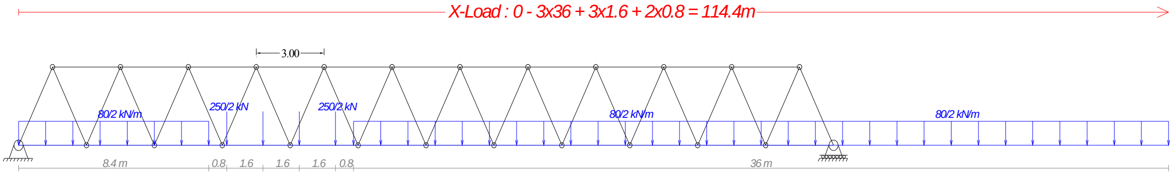


Fig. 8 The moving load pattern

The beginning of the first distributed load is outlined by X-Load. The moving of the load continues until the second distributed load is placed completely on the bridge. Hence, for 36-meters truss, the beginning of the first distributed load started from zero and continues to 114 meters (3 × 36 + 2 × 0.8 +3 × 1.6 = 114.4). To consider the effect of the moving load, the load is moved at very short steps along the bridge, and the structure is analyzed in each step. The results are considered lately as a constraint in the optimization problem.

The objective function, structure weight and the constraints are separately applied in three different problems. Various formulations of the problem are presented in Eq. (4) and Table 4.

$$\begin{aligned} &\min: \quad M=\sum_{i=1}^{NM}\rho A_i L_i \\ &\underset{\text{Table 4}}{\text{sub to:}}\ c=\begin{Bmatrix} cs \\ cd \end{Bmatrix}=\begin{cases} cs_{ik}=\dfrac{\sigma_{ik}}{\sigma_{all}}-1 & i=1,2,\dots,NM \\ cd_{jk}=\dfrac{\delta_{jk}}{\Delta_{all}}-1 & j=middle\ span \end{cases} \quad k=1,2,\dots,NL \end{aligned} \tag{4}$$

Table 4 The problem formulation

	Constraints	Tension and Compression $\sigma_{all}=-144\ MPa$ $cs_{ik}=\dfrac{\sigma_{ik}}{\sigma_{all}}-1$	Buckling(1) $\sigma_{all}=-\dfrac{\pi^2 E_i}{(k^*\times L_i/r_i)^2}$ $cs_{ik}=\dfrac{\sigma_{ik}}{\sigma_{all}}-1$	Displacement $\Delta_{all}=\dfrac{L}{600}$ $cd_{jk}=\dfrac{\delta_{jk}}{\Delta_{all}}-1$
1	• Allowable stresses in tension and compression	✓	-	-
2	• Allowable stress in tension • The lowest between allowable stress in compression or critical buckling stress	✓	✓	-
3	• Allowable stress in tension • The lowest between allowable stress in compression or critical buckling stress • The displacement in mid-span	✓	✓	✓

(1) $k^*=1,\quad r_i=\dfrac{1}{2}\sqrt{\dfrac{A_i}{\pi}}$ (Radius of gyration)

5.1 Development of the three-step algorithm

A three-level algorithm is applied in the optimization, Fig. 9. In the first step, the unit load is applied to some of the truss joints and the optimization is carried out for each separate conditions. In the next step, the maximum values from the first step, are applied as the starting points. Then, the structure is optimized under the condition that the unit moving load is applied to the joints.
In the last step, according to the member stress ratio (stress when unit load applied / stress when main load applied), the results of the previous step is multiplied and used as the new starting point for the optimization problem. In hyper-sphere optimization method, the starting point should fulfill all constraints. Therefore, the scaling method should be used if not all constraints are satisfied. Comparison of the three-level algorithm to the classical algorithm is presented in Fig. 10.

Example 2: Highway bridge truss under moving load

To verify the efficiency of the proposed optimization algorithm in solving the issues of the moving load, two trusses with different geometries are optimized, Fig. 11. The moving load pattern is compatible with HS20 (Toğan and Daloğlu 2009). The impact factors for the spans of the two trusses are calculated based on ASHTOO recommendations. The allowable stress equations presented in AISC, are used. Table 5 presents the constant values for the calculation of the radius of gyration.

$$\rho = 7850\ kg/m^3 \qquad E = 210\times10^3\ \ MPa \qquad \sigma_{yield} = 212\ MPa \qquad \sigma_{all}^{t} = 0.6\sigma_{yield}$$

$$\begin{cases} \sigma_{all}^{c} = \dfrac{\left(1-\dfrac{(\frac{k\times L}{R})^2}{2C_c^2}\right)}{\dfrac{5}{3}+\dfrac{3(\frac{k\times L}{R})}{8C_c}-\dfrac{(\frac{k\times L}{R})^3}{8C_c^3}}\sigma_{yield} & if \quad \dfrac{k\times L}{R} < C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_{yield}}} \\ \sigma_{all}^{c} = \dfrac{12}{23}\dfrac{\pi^2 E}{(\frac{k\times L}{R})^2} & else \end{cases}$$

$$k = 1 \qquad R = aA^b\ (a,b \quad Table\,5)$$

Table 5 The constant values for the calculation of the radius of gyration

	Angle	Pipe	Tee	Double Angle
a	0.8338	0.4993	0.2905	0.584
b	0.5266	0.6777	0.8042	0.524

5.2 Solving a numerical example (36- meter truss)

The mechanical parameters of the truss material and the other assumptions are presented hereunder:

$$E = 205\ GPa \qquad \rho = 7800\ kg/m^3 \qquad \sigma_{all} = \pm 144\ MPa \qquad A^L = 100\ mm^2 \quad eps = 0.001$$

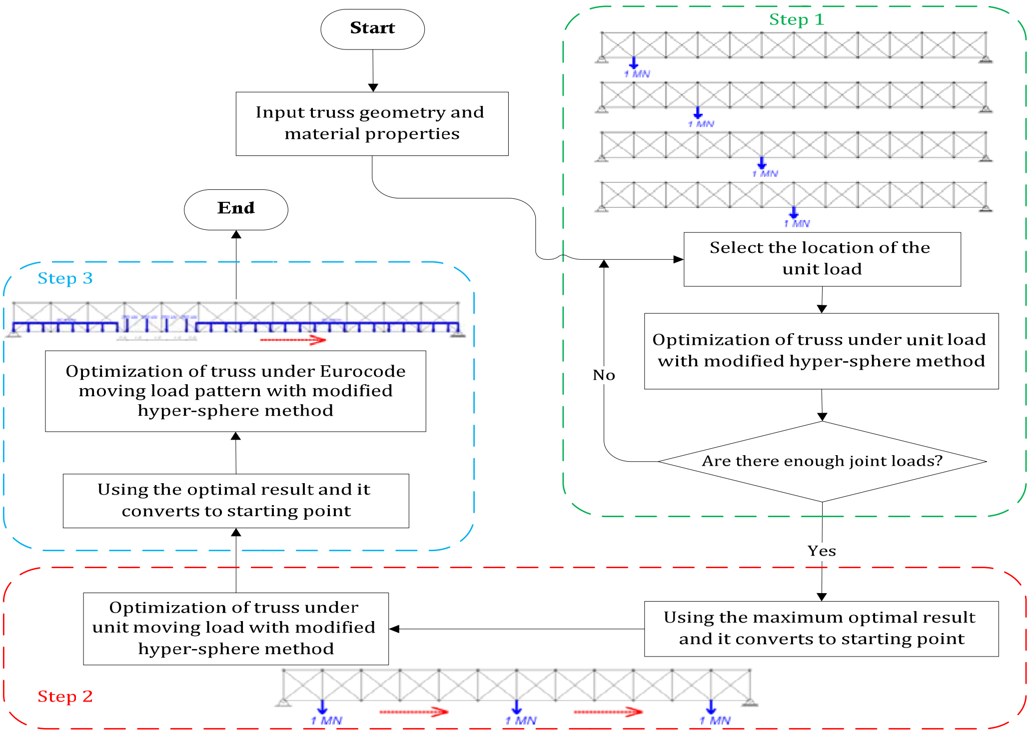


Fig. 9 The optimization of the railway truss bridges with three-step algorithm

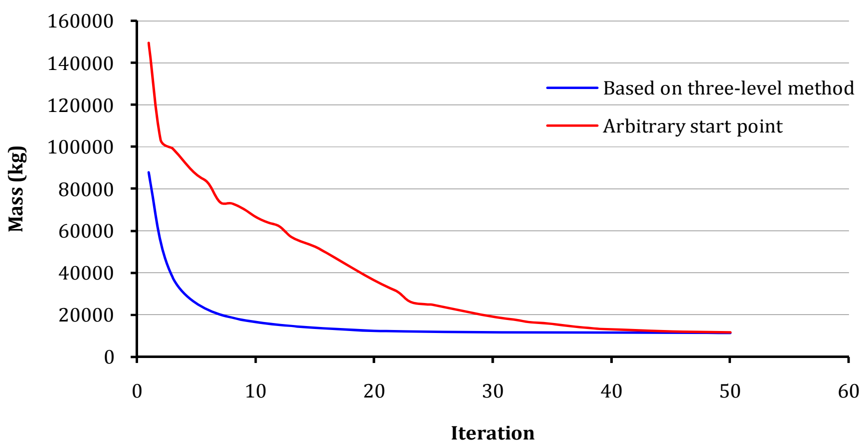
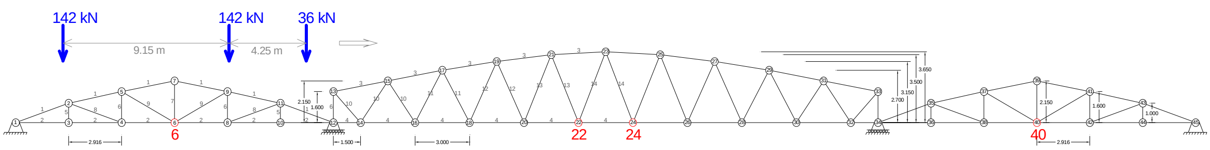
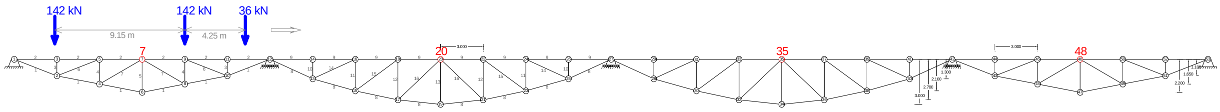


Fig. 10 Comparison of the three-level algorithm to the classical algorithm



(a) 85-bar highway truss
Fig. 11 Geometry of highway trusses



(b) 100-bar highway truss
Fig. 11 Continued

Table 6 Impact factor values and the allowable deformations

	Impact factor	Allowable deformation
85-bar truss	$Span1,3 \quad L=17.496m \quad I_{1,3}=\frac{50}{17.496\times 3.2808+125}=0.2741$ $Span2 \quad L=30m \quad I_{2}=\frac{50}{30\times 3.2808+125}=0.2238$	Joint 6,40 $\Delta_{all}=22mm$ Joint 22,24 $\Delta_{all}=38mm$
100-bar truss	$Span1,4 \quad L=18m \quad I_{1,4}=\frac{50}{18\times 3.2808+125}=0.2717$ $Span2,3 \quad L=24m \quad I_{2,3}=\frac{50}{24\times 3.2808+125}=0.2454$	Joint 7,48 $\Delta_{all}=23mm$ Joint 20,35 $\Delta_{all}=30mm$

Table 7 The optimum cross sectional area of the selected trusses (85 and 100-bar highway trusses)

85-bar bridge truss		A (mm^2) Toğan and Daloğlu (2009)			A (mm^2)	100-bar bridge truss		A (mm^2) Toğan and Daloğlu (2009)			A (mm^2)
Group	Section	SCP	SQP	EVOL	SLP_MHSM	Group	Section	SCP	SQP	EVOL	SLP_MHSM
1	Double angle	5235	5309	5423	5877.75	1	Tee	5379	5831	5638	4662.13
2	Double angle	4840	4831	4722	3779.64	2	Tee	5140	4904	4911	3554.07
3	Tee	5587	5312	5682	4795.2	3	Angle	550	749	619	1391.79
4	Tee	3542	3810	3201	2645.38	4	Angle	550	631	567	1161.83
5	Pipe	837	1138	905	1412.61	5	Angle	1902	1319	1813	1410.3
6	Pipe	1167	1429	1509	2368.74	6	Angle	955	1265	70	1421.35
7	Pipe	1713	1350	1717	1233.57	7	Angle	1024	1636	1047	1583.32
8	Pipe	762	931	772	1982.78	8	Double angle	5465	5624	6034	4869.85
9	Pipe	759	762	765	1912.4	9	Double angle	4287	4614	3789	4436.54
10	Pipe	2271	2354	2516	2680.24	10	Angle	564	749	596	1386.17
11	Pipe	1640	1599	1517	1427.28	11	Angle	601	717	631	1249.78
12	Pipe	871	1357	998	1119.71	12	Angle	712	712	773	1330.48
13	Pipe	936	1140	1005	937.46	13	Angle	1595	1015	941	1094.94
14	Pipe	1222	913	1221	1008.45	14	Angle	1237	1524	1290	1308.45
Weight (kN)		60.16	61.31	60.48	58.73	15	Angle	1059	1413	1078	1151.6
						16	Angle	1206	1419	1212	1221.27
						Weight (kN)		77.05	81.38	77.41	71.35

$$\frac{\delta_{mid}}{L} < \frac{1}{600} \xrightarrow{L=36m} \delta_{mid} < 60mm$$

The optimization carried out using the modified algorithm, for four different types of 36-meters trusses, and the results are presented in Tables 8-9. It is evident that:

• The algorithm efficiency is independent of the truss shape

• The algorithm indicates reasonable convergence and stability.

• The algorithm reaches the optimum solution after about 90 iterations (see Fig. 13)

• The optimum solution is not dependent on the starting point in three-level algorithm, compared to the classical algorithm.

Referring to Table 9, the Parker truss indicates more reasonable results and the Pratt truss has the maximum weight. The point that should be mentioned is that the Parker truss is actually the Pratt truss, with variable height at different points. The aforementioned point proves the importance of the truss geometry and its optimization.

Table 8 The optimization results for 36-meter span trusses (Warren and Indeterminate trusses)

Indeterminate truss				Warren truss			
Group	Stress constraints	Stress and Buckling constraints	Stress, Buckling and deformation constraints	Group	Stress constraints	Stress and Buckling constraints	Stress, Buckling and deformation constraints
	A (mm^2)	A (mm^2)	A (mm^2)		A (mm^2)	A (mm^2)	A (mm^2)
1, 12	5997.61	3033.28	2976.34	1, 12	3319.07	3319.07	4506.45
2, 11	7901.77	9110.97	12757.68	2, 11	9284.27	9284.27	14121.47
3, 10	14729.51	13851.94	18761.39	3, 10	14084.96	14084.96	21105.80
4, 9	16819.63	17484.38	27726.28	4, 9	17690.77	17690.77	28265.14
5, 8	20057.52	19874.94	30767.88	5, 8	20114.46	20114.46	34586.75
6, 7	20965.40	20881.60	34992.85	6, 7	21308.15	21308.15	39781.27
13, 49	647.88	5440.65	5892.43	13, 36	7421.66	8641.44	10077.37
14, 48	8481.90	8310.15	7347.29	14, 35	7421.66	7421.66	10077.36
15, 47	926.63	7113.79	15265.85	15, 34	6220.51	7911.31	9726.29
16, 46	2194.37	100.00	100.00	16, 33	6220.51	6220.51	9271.95
17, 45	2054.12	7736.01	8601.43	17, 32	5019.36	7106.56	8389.55
18, 44	5879.23	4194.61	4186.40	18, 31	5019.36	5019.36	8389.55
19, 43	100.00	843.23	4173.00	19, 30	3818.20	6198.19	7402.74
20, 42	4206.93	6936.20	5475.50	20, 29	3818.20	3818.20	7402.73
21, 41	2178.69	3411.53	11541.60	21, 28	2617.05	5131.47	6253.65
22, 40	781.11	397.28	183.59	22, 27	2617.05	2740.74	6253.65
23, 39	1374.41	6054.62	7089.51	23, 26	1677.81	4108.73	4108.73
24, 38	3534.84	2564.51	2837.25	24, 25	1677.81	3426.82	3980.61
25, 37	100.00	903.94	1890.55	37, 47	6638.13	7309.77	10565.59
26, 36	1860.64	5058.66	4675.30	38, 46	12059.02	12059.02	18942.57
27, 35	1479.37	2373.69	3460.82	39, 45	16264.18	16264.18	24880.07
28, 34	713.94	737.36	359.27	40, 44	19297.69	19297.69	34057.33
29, 33	543.48	3916.68	3858.00	41, 43	21101.19	21101.19	37515.17
30, 32	1644.71	3163.77	4258.95	42	21674.68	21674.68	29706.30
31	100.00	559.96	677.81	Weight (kg)	10851.47	11625.68	18031.83
50, 61	647.88	5440.65	5892.43				
51, 60	10715.07	9533.92	11136.80				
52, 59	13538.65	14357.82	20444.61				
53, 58	18568.88	17980.65	24528.72				
54, 57	20171.39	20354.07	29310.55				
55, 56	21711.17	21734.79	35198.74				
Weight (kg)	10517.28	12559.90	17717.84				

In the following, the effect of the bulking in constraints is considered. Therefore, in tension members, the allowable stress is used. In the compression members, first, the critical buckling load is calculated and then its corresponding critical stress is computed. The minimum value of the allowable stress and the critical stress is used for the compression members.

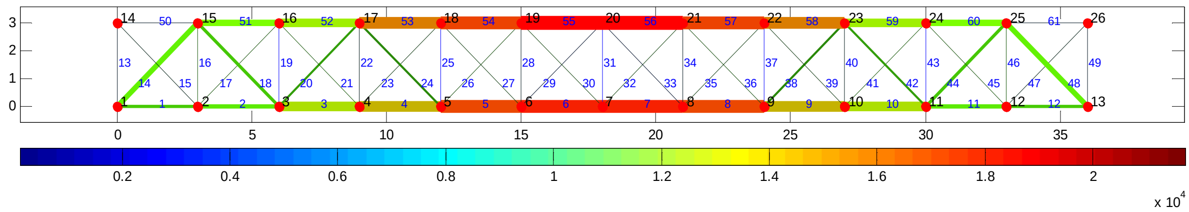
The effect of the buckling is demonstrated in Fig. 12. When, only the stress constraints are considered, (Fig. 12(a)) the indeterminate truss changes into the Warren truss. However, when the buckling constraints are entered in the optimization (Fig. 12(b)), the inclined members are similarly parallel. Obviously, this is due to the fact that when the members in one direction are under tension, the members on the opposite direction are under compression.

The convergence procedure and percent variation of the optimum truss weight are presented in Figs. 13-14, respectively. In Fig. 14, in determinate trusses the convergence reached after 50 iterations. However, in indeterminate trusses the iteration number is 65. It can be concluded that the shape of the trusses is not the influential factor in the optimization convergence.

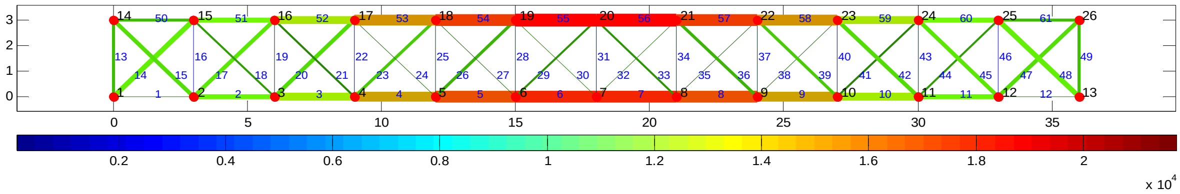
In determinate trusses, when the displacement constraint is considered, (Warren and Pratt) all cross sectional areas are increased and also the weight has increased about 56 percent. However, in indeterminate truss, some cross sectional areas have increased and some have decreased. Totally, there is a 41% increase in the optimum weight of the truss. In Parker truss, the displacement constraint has no effect on the optimum weight of the truss.

Table 9 The optimization results for 36-meter span trusses (Pratt and Parker trusses)

Parker truss				Pratt truss			
Group	Stress constraints A (mm^2)	Stress and Buckling constraints A (mm^2)	Stress, Buckling and deformation constraints A (mm^2)	Group	Stress constraints A (mm^2)	Stress and Buckling constraints A (mm^2)	Stress, Buckling and deformation constraints A (mm^2)
1, 12	6638.13	6638.13	6638.13	1, 12	6638.13	6638.13	9032.59
2, 11	6638.13	6638.13	6638.13	2, 11	6638.13	6638.13	9032.60
3, 10	8039.34	10666.44	10666.44	3, 10	12059.02	12059.02	17358.80
4, 9	9293.82	9293.82	9293.82	4, 9	16264.18	16264.18	24933.05
5, 8	10068.36	10068.36	10068.36	5, 8	19297.69	19297.69	31701.90
6, 7	10550.60	10550.60	10550.60	6, 7	21101.19	21101.19	37595.06
13, 35	9387.74	12293.52	12293.52	13, 35	9387.74	12293.52	12774.22
14, 34	1868.49	1868.49	1868.49	14, 34	1868.49	1868.49	1868.49
15, 33	2188.77	2417.86	2417.86	15, 33	7868.39	7868.39	11753.28
16, 32	1060.45	4382.45	4382.45	16, 32	4489.45	6011.42	7519.76
17, 31	2605.83	3686.78	3686.78	17, 31	6349.04	6349.04	10634.77
18, 30	1467.94	6015.52	6015.52	18, 30	3415.10	5243.03	6635.25
19, 29	2009.35	5060.71	5060.71	19, 29	4829.69	4829.69	9383.86
20, 28	1291.74	6180.38	6180.38	20, 28	2340.76	4340.69	5605.30
21, 27	1797.46	6132.86	6132.86	21, 27	3310.34	3899.04	7946.37
22, 26	1375.01	6130.31	6130.31	22, 26	1500.68	3475.56	3567.88
23, 25	1677.81	6853.63	6853.63	23, 25	2122.28	4875.07	5045.98
24	100.00	100.00	100.00	24	100.00	100.00	100.00
36, 45	8988.26	9509.85	9509.85	36, 45	12059.02	12059.02	17358.80
37, 44	9579.85	9579.85	9579.85	37, 44	16264.18	16264.18	24933.03
38, 43	10207.24	10207.24	10207.24	38, 43	19297.69	19297.69	31701.87
39, 42	10587.17	10587.17	10587.17	39, 42	21101.19	21101.19	37541.27
40, 41	10837.34	10837.34	10837.34	40, 41	21674.68	21674.68	42100.76
Weight (kg)	6882.98	10089.27	10089.27	Weight (kg)	10949.11	11705.39	18247.70



(a) Stress constraints



(b) Stress and buckling constraints

Fig. 12 The visual results of the indeterminate truss optimization

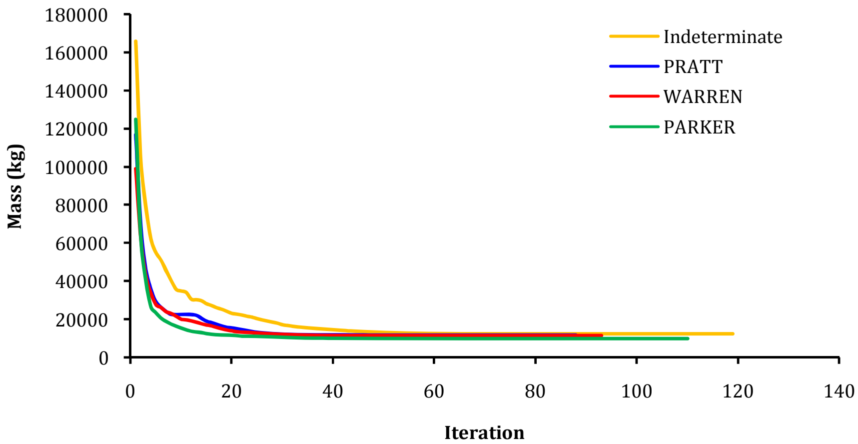


Fig. 13 The convergence procedure of the optimization algorithm for different trusses

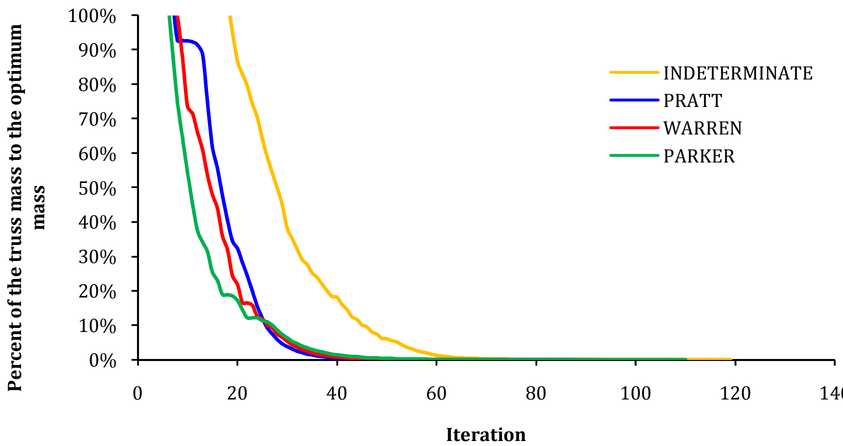
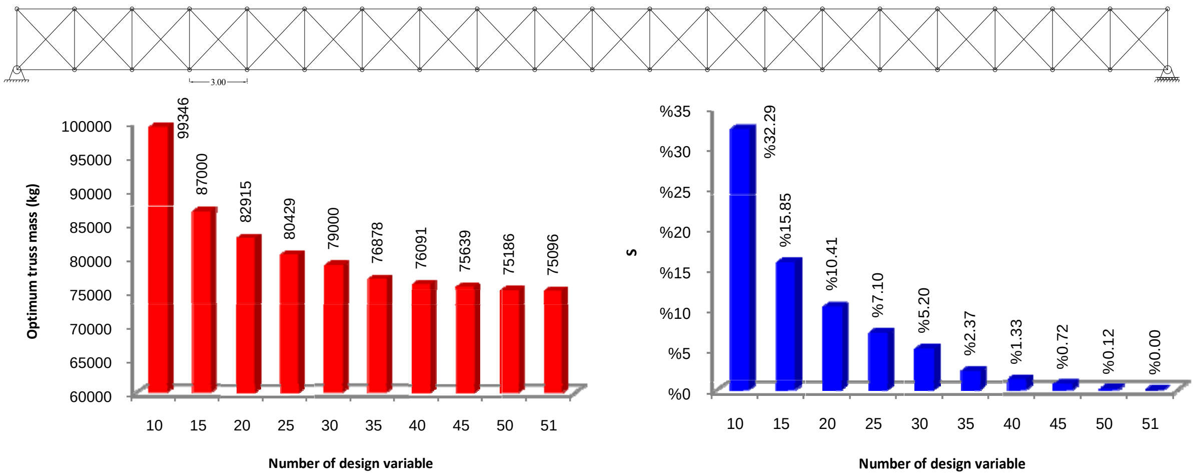


Fig. 14 Percent variation of the optimum truss weight in different iterations



S = (Optimum truss mass with “N” design variable - Optimum truss mass with “51” design variable)/ Optimum truss mass with “51” design variable $\times 100$

Fig. 15 The effect of grouping of the members on the optimum weight of the indeterminate truss

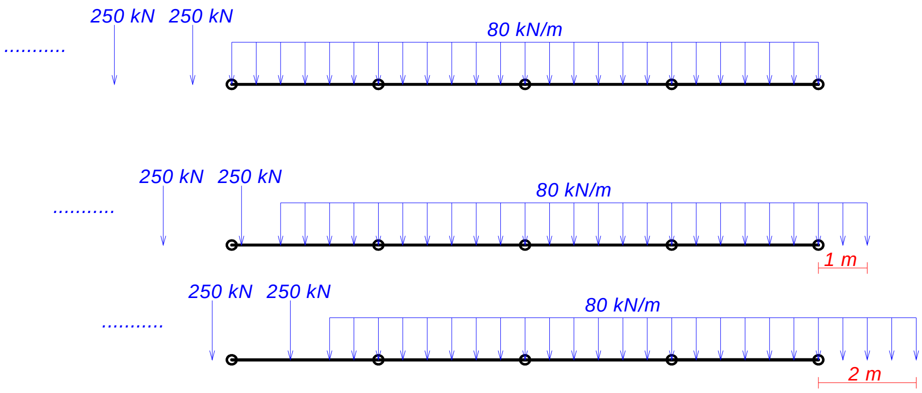


Fig. 16 The step length of the moving load

5.3 The effect of member grouping and optimal grouping of cross section

The important issue is that, considering all cross sectional areas in design, leads to large number of design parameters. Reduction in the number of the parameters is possible by considering symmetry in geometry. However, member grouping is an alternative method that can be used to reduce the number of parameters. Practically, this issue leads to considerable deduction in the final cost of construction projects. If a certain number of grouping increases, the response does not change significantly. Therefore, to reduce the solution time, a proper number of member groups should be utilized. Such a procedure for the current example is depicted in Fig. 15.

5.4 Selection of the appropriate step length

In theory, the load should move continuously along the length of the bridge. However, in practice, it is not continuous and the load moves in specific intervals. The moving of the load pattern is

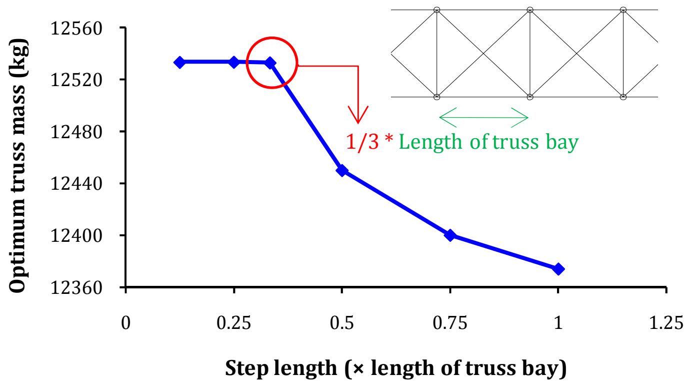


Fig. 17 The effect of the step length on the truss optimal weight

indicated in Fig. 16. Large moving intervals lead to inaccurate solutions. This comes from the fact that some constraints may have been ignored. On the other hand, small intervals decrease efficiency of the solution procedure. Due to some considerations, the appropriate step length is chosen as 1/3 of the bay of the truss (Fig. 17).

5.5 Approximate concepts for the optimum shape and geometry selection

It is already stated that, the structure shape parameters such as the truss height and the step length are the prominent factors. In several cases when one unique type of the truss was used in the railway network, the optimization of the shape and geometry highly affected the cost of the projects. Therefore, it makes sense to study the effect of the structure height on the optimum solution. It is highlighted that the accurate solutions are easily reachable, if the vital parameters of the optimization are considered.

To estimate the optimum height for a 60-meter truss, initially, the optimum solutions for the different heights of 3, 5 and 9 meters are determined. Then by fitting a second order curve, the minimum value is calculated. Afterward, in increasing the truss height from 5 to 6 meters, the step length of 10 centimeters was used. Finally the optimum height is assessed. A comparison shows that the optimum height is obtained with an accuracy of 0.7%, to fit a second order curve (Fig. 18).

5.6 The effect of the moving load on the optimum solution

The noticeable point is that the structure response envelop to the moving load cannot be used for the optimal analysis. It does not lead to optimal solutions. Besides, the solution is highly dependent on the structure shape and this may lead to inaccurate results.

In determinate trusses, using the force envelope (when only stress constraint is considered) may lead to accurate and real solutions. However, when the displacement constraints are entered in the optimization the envelope is not useful any more. This is due to the dependency of the displacement to the cross sectional area of the members. On the other hand, for indeterminate structures, with the internal forces of members dependent on the cross sectional area, using the force envelope does not lead to accurate solutions.

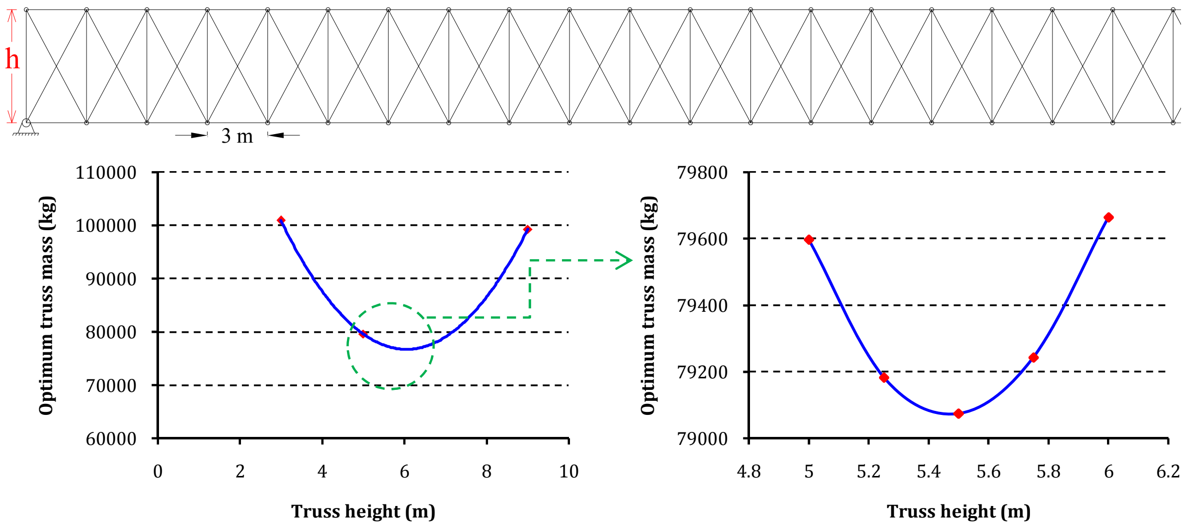


Fig. 18 The optimum height estimation

6. Conclusions

This research aimed at the modification of the hyper-sphere algorithm in order to optimize the truss structures. A major advantage of the proposed algorithm is the speed of convergence. While exercising a sample optimization, the Eurocode moving load pattern was applied to the structure. This led to the increased calculation time and convergence iterations. To speed up the convergence, the moving load was applied in three-stages and the starting point of the optimization was accordingly estimated.

The proposed methodology of optimization is comprehensive, very accurate and highly reliable. It can easily take into account the various constraints of the problem. The optimization indicates reasonable convergence and the algorithm stability is independent of the number of constraints. The algorithm is highly capable of finding the optimum solutions while allowing for various constraints, (such as the fatigue constraints (Nouri 2011)).

The algorithm is capable of minimizing the problem and omitting unnecessary constraints. This in turn indicates the efficiency in solving problems of higher scale. Due to such reasons, the algorithm was used for the optimization of a structure subjected to moving load. Such a structure is considered as a multi-constraint problem.

Furthermore, the algorithm can take in new variables such as the radius and thickness for a pipe section. Such new set of variables can easily be introduced without altering the convergence speed of the solution processes.

The versatility of the proposed algorithm examined for the railway truss optimization for some known examples. The effect of buckling constraint on the indeterminate truss evaluated. In addition, the minimum weight observed in Parker truss.

While some trusses seem to be sensitive to a series of constraints some of the others seem to be indifferent. While surveying the Parker truss, it is concluded that the displacement constraint is not effective in optimum weight. However, it is highly effective in Pratt, Warren and indeterminate trusses (up to 56 percent).

The calculation time is directly dependent on the number of the design parameters. It is observed that after specific increase in the number of the parameters, the grouping has negligible effect on the optimum weight. This on the other hand increases the solution efficiency.

The truss geometry is amongst the significant parameters. This article also presented an innovative method for calculating the optimized height of an arbitrary truss.

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Nomenclature

$\ell_j^{(p)}$	Distance between the design point and the *j*th constraint
$\vec{S}_p$	Vector variations of the design parameters
M	Mass of truss
W	Weight of truss
E	Young's modulus of elasticity
ρ	Mass density
σ_{yield}	Yield stress
σ_{all}	Allowable stress
σ^t_{all}	Allowable tension stress
σ^c_{all}	Allowable compression stress
σ_i	Stress in the *i*th member
σ_{ik}	Stress in the *i*th member under the *k*th loading condition
δ_{mid}	Deformation in mid-span

δ_{jk}	Displacement in the *j*th degree of freedom under the *k*th loading condition
Δ_{all}	Allowable deflection
L	Span length
L_i	Member length
r_i	Radius of gyration
A^L	area section lower bound
A_i	Cross sectional area of the *i*th member
k*	Ratio of effective length in buckling
NM	Number of members
NL	Number of loading conditions
eps	Convergence accuracy
I	Impact factor for highway bridge
Φ_3	Impact factor for railway bridge with standard maintenance
cs_{ik}	Stress constraint in the *i*th member under the *k*th loading condition
cd_{jk}	Displacement constraint in the *j*th degree of freedom under the *k*th loading condition
AASHTO	American Association of State Highway and Transportation Officials
AISC	American Institute of Steel Construction
MHSM	Modified hyper sphere method
SLP	Sequential linear programming
SCP	Sequential convex programming
SQP	Sequential quadratic programming
EVOL	Evolution strategy