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Use of uncertain numbers for appraising tensile strength of concrete

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Abstract. Splitting tensile strength (STS) is a respectable mechanical property reflecting ability of the concrete. The STS of concrete is mainly related to compressive strength (CS), water/binder (W/B) ratio and concrete age. In this study, the assessment of STS is made by a novel uncertainty-oriented method which uses least square optimization and then predicts STS of concrete by uncertain (fuzzy) numbers. The approximation method addresses a novel integration of fuzzy set theory and multivariate statistics. The numerical examples showed that the method is applicable with relatively limited data. In addition, the prediction of uncertainty at various levels of possibility can be described. In conclusion, the uncertainty-oriented interval analysis can be suggested an effective tool for appraising the uncertainties in concrete technology.

Keywords: tensile strength; concrete; uncertainty; fuzzy number; interval analysis

1. Introduction

High performance concrete (HPC) is a specific product widely used in concrete industry that made with appropriate materials combined according to a selected mix design and properly mixed, transported, placed, consolidated, and cured so that the resulting concrete will give excellent performance in the structure in which it will be exposed, and with the loads to which it will be subjected for its design life (Forster 1994, Walraven 2009). There are some important quality characteristics of concrete materials include strength, volume stability, durability and permeability (Aliha *et al*. 2012) In particular, strength is often regarded as one of the most important properties and many factors can affect strength of concrete such as cement type, curing period and type, testing method, concrete age and water/binder ratio.

Prediction of splitting tensile strength (STS) in connection with compressive strength (CS) of concrete is still a novel problem and many empirical expressions were suggested to approximate the relationship between STS and CS. In general, the empirical expressions given in literature were developed based on some traditional statistical tools and codes (Arioglu *et al*. 2004, Parra *et al*. 2011). Recently, some new soft computing models have been developed to predict tensile strength

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from compressive strength of concrete (Zain *et al*. 2002, Choi and Yuan 2005, Tutmez 2009, Severcan 2012).

Soft computing has shown to be highly suitable for the modelling of complex and uncertain systems. The most attractive characteristics of soft computing models like fuzzy computing-based models compared with other conventional methods, such as statistical models, are transparency and flexibility (Ross 2004). In addition to these superiorities, some of soft computing tools, such as uncertain numbers, are very prevalent tools for system modelling and uncertainty quantification (Piegat 2001, Bardossy and Fodor 2004, Liu *et al*. 2013). A fuzzy number can be used as the main indicator of predictions and which can describe the prediction of uncertainty at various levels of possibility.

The method employed in this study uses regression analysis to express the model structure and then applies fuzzy arithmetic operations for assessment of splitting tensile strength (STS) of HPC. STS values are predicted using interval arithmetic and fuzzy numbers. One of the advantages of the method is its applicability by limited number of data (Tutmez 2007). Because the most of the strong interpolation methods in literature such as kriging computes a weighted average of the data and at least 20-25 observations are needed to obtain the reliable results (Davis 2002). Similarly, it is well-known that neural network algorithms produce reliable results only if a big number of data is used (Bishop 1996). However, fuzzy interval-based structure can be used in modelling works via smaller number of observations. In addition, by the uncertainty oriented approach, the prediction error (uncertainty) can be evaluated based on some intervals. Finally an approximate and fast solution is warranted.

The rest of the paper is organized as follows. Section 2 presents a description for STS of HPC and the conventional approaches. In this section theoretical frame of the fuzzy set based method is also stated. Section 3 expresses two case studies based on experimental data sets given in literature. Results and discussion are presented in section 4. Section 5 concludes the paper.

2. Material and methods

2.1 Statement of problem

The properties and performance of concrete, including strength development are difficult to predict because of variability in environmental conditions and the quality of constituent materials (Newman and Choo 2003). In order to evaluate mechanical properties of concrete, tensile strength is used one of the important indicators (Rocco 2001, Song and Hwang 2004). Providing STS from some experimental procedures is an ideal way but this process is very difficult in practice. The STS test involves compressing a cylinder or core on its side until a crack forms down the middle, causing failure of the specimen (Fig. 1).

CS is one of the important mechanical properties considered in the mixture design of concrete. Similarly, STS, which is also a very respectable mechanical property, employed in the design of structural lightweight concrete members to evaluate the shear resistance obtained from concrete and to determine the development length of reinforcement. The STS of concrete is relatively much lower than its CS because it can be established more promptly with crack propagation (Saridemir 2011). In addition to CS, some other parameters such as volume, shape, aggregate type, concrete age, surface properties, degree of compaction and maximum size (Mindes and Young 2011, Eligehausen *et al*. 2006) are also influence on the STS value.

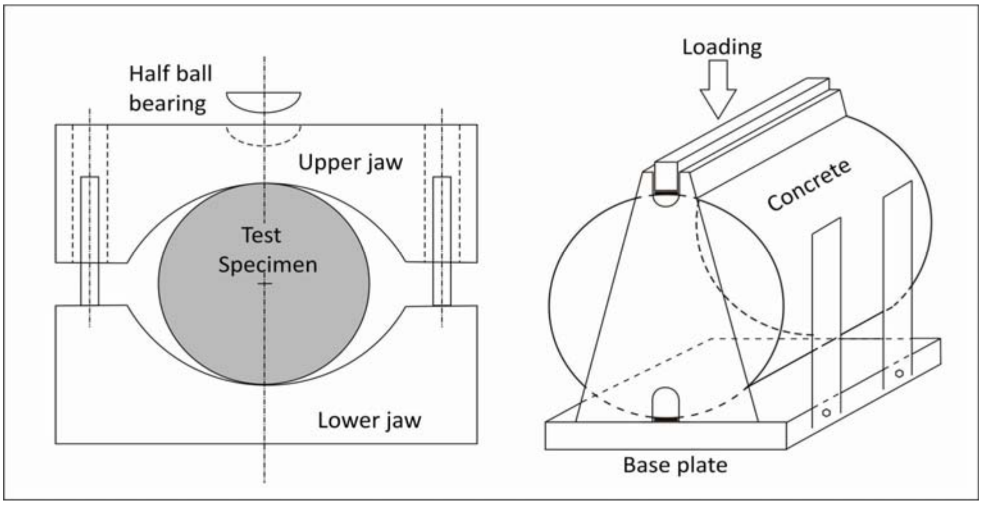

Fig. 1 Splitting tensile strength test design

2.2 Conventional approaches

Many empirical expressions were suggested in literature to model the STS relating to the compressive strength. One of the empirical formulae presented by Larrard and Malier (1992) in connection with the French regulations

$$f_{tj} = 0.6 + 0.06 f_{cj} \tag{1}$$

where f_{tj} and f_{cj} are average values of STS and CS, at *j* days (MPa). Similarly, another empirical formula has been given by CEB-PIP (1993) as follows

$$f_t = 0.301\left(f_c^{'}\right)^{0.67} \tag{2}$$

where f_t and $f_c^{'}$ denote STS and compressive strength, respectively. Recently, Parra *et al*. (2011) has proposed an alternative method to estimate STS from CS and W/B ratio as follows

$$f_{ts} = 0.54\sqrt{f_c^{'}}\left(W / B\right)^{-0.07} \tag{3}$$

2.3 Fuzzy set based methodology

Fuzzy set methodology addresses a novel soft computing approach that facilitates uncertainty analysis of systems where uncertainty arises due to vagueness or fuzziness. Because the assessment of tensile strength of concrete includes an uncertainty, the methodology can be considered as an evaluation tool for modelling STS of concrete. In the present model, combination of multiple regression analysis and fuzzy interval arithmetic is made and this combination is employed for appraising the STS values. First model structure is determined by least square estimation and then STS values of concrete are predicted using fuzzy numbers.

An interval of confidence is one way of reducing the uncertainty of using lower and upper bounds. It is a practical and logical process for treating uncertainty with whatever information is available (Ayyub and Klir 2006). Therefore, fuzzy numbers are very convenient tools for

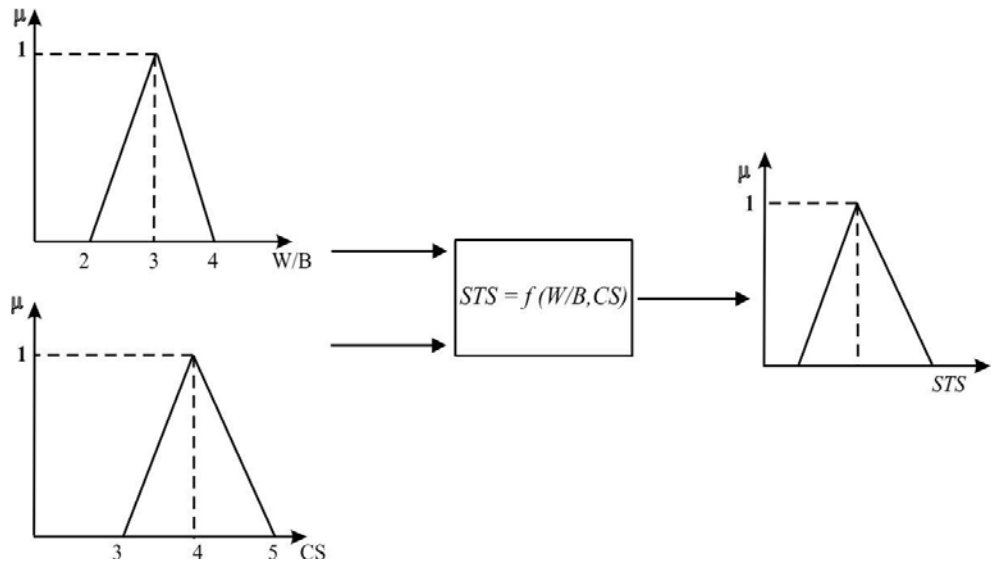

Fig. 2 The mechanism of proposed modelling schema

evaluating the uncertainty. A fuzzy number is defined as a fuzzy quantity with an upper semicontinuous membership function,

- a bounded support,
- a unique modal value *f*

Such a fuzzy number is a description of the imprecisely described quantity "approximately *x*". A fuzzy number has optimistic and pessimistic bounds which are obtained from the endpoints of the support. In addition, core of the fuzzy number describes the most plausible values only.

Suppose a known input/output mapping given in terms of a mathematical model $y = f(X)$, where the input parameter values cannot be measured precisely but only approximately, e.g.:

W/S = "approximately 3"
CS = "approximately 4",
STS = 4(W/S) + 7(CS)

In a basic form, the STS value may be estimated in a triangular fuzzy number form as seen in Fig. 2.

In this problem, fuzzy intervals are applied in STS estimation. A general representation of fuzzy intervals (numbers) has been suggested by Carlsson and Fuller (2002). Some of the operations have been summarized in Appendix A. In the simplest case of a Left-Right (*LR*) fuzzy number shape functions are linear (Kaufmann and Gupta 1991).

Trapezoidal fuzzy numbers are a special case of the L-R numbers (Fig. 2). A trapezoidal fuzzy number may be seen as a fuzzy quantity

"*x* is approximately in the interval $[a_L, a_R]$"

The form of a trapezoidal fuzzy number can be written as $A = (a_L, a_R, \alpha, \beta)$. Fig. 2 shows a fuzzy number (interval) which is represented by a few parameter values, and the operations on fuzzy numbers are applied by appropriate operations on these parameters. The family of fuzzy numbers denoted by *F* and any fuzzy number $A \in F$ can be given as follows (Carlsson and Fuller 2002)

$$A(t) = \begin{cases} L\left(\frac{a_L - t}{\alpha}\right) & if\ t \in [a_L - \alpha, a_L] \\ 1 & if\ t \in [a_L, a_R] \\ R\left(\frac{t - a_R}{\beta}\right) & if\ t \in [a_R, a_R + \beta] \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $[a_L, a_R]$ is the core of A, and $(a_L - \alpha, a_R + \beta)$ is the support of A. Arithmetic operations on trapezoidal fuzzy numbers were presented in detail by Kaufmann and Gupta (1991).

3. Case studies

To show relationships between CS and STS of concrete by the uncertainty oriented approach, two case studies were established. The data in the studies were obtained from the papers that were focused on these relationships. To illustrate the results based on some confidence intervals, trapezoidal membership functions were preferred for the applications. As special types of triangular functions trapezoidal functions have flexibility and smoothing capacity and which provide a line (interval) instead of a point to represent the maximum of the global output function (Piegat 2001).

3.1 Case study I

The data set derived from Zain *et al*. (2002) was employed in the first application (Table 1). Before starting of the modelling work, sample values were standardized by using a linear transformation between minimum and maximum STS values [3.6, 6.5].

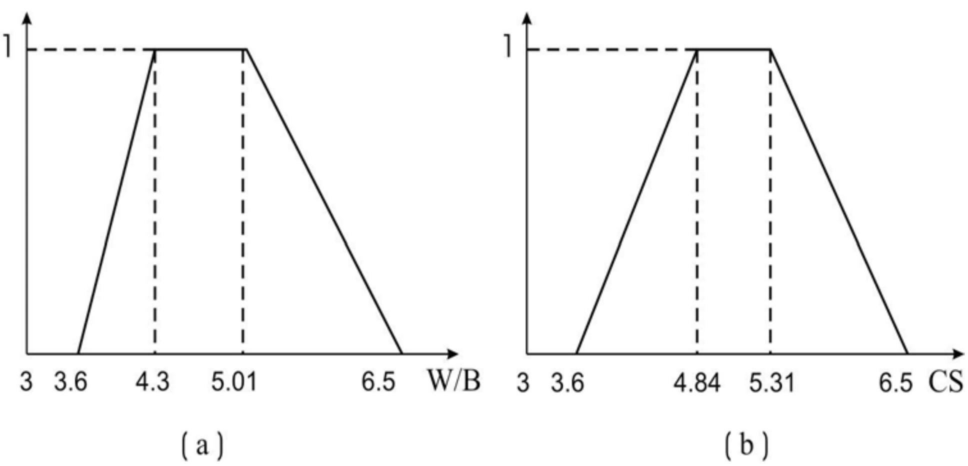

Fig. 3 Input memberships derived from projections for first application (a) W/B (b) CS

Table 1 Data set for case study 1

Age of Concrete (days)	W/B	CS (MPa)	STS (MPa)
28	0.55	41.00	3.90
28	0.44	42.00	4.00
28	0.40	44.80	3.60
28	0.40	56.20	4.40
28	0.38	56.70	4.60
28	0.38	64.50	4.80
28	0.36	66.50	4.70
28	0.36	65.40	4.30
28	0.35	73.80	4.90
28	0.35	74.00	5.30
28	0.34	72.50	5.00
28	0.32	74.60	4.90
28	0.30	77.90	5.50
28	0.30	79.10	5.20
28	0.30	84.20	5.70
28	0.30	83.30	5.90
28	0.29	86.50	5.50
28	0.28	102.00	5.50
28	0.27	101.00	6.50
28	0.25	111.00	6.20
28	0.25	94.50	5.80
28	0.22	118.00	6.20

In order to define the model structure and parameters, multiple regression analysis was applied. By using the global least square optimization, a regression equation was determined. The resulting model equation is given as follows

$$STS = 1.427 - 0.103(W/B) + 0.846(CS) \tag{5}$$

The input membership functions which represent W/B ratio and CS, were determined by one dimensional data projections as given in Fig. 3. Next, these functions were described as trapezoidal fuzzy numbers and the STS predictions were conducted by the following operations

$$STS = (1.427, 1.427, 0, 0) + 0.103(-5.01, 4.30, 1.49, 0.7) + 0.846(4.84, 5.31, 1.24, 1.19) \tag{6}$$

$$STS = (1.427, 1.427, 0, 0) + (-0.516, -0.443, 0.154, 0.072) + (4.095, 4.492, 1.049, 1.007) \tag{7}$$

The results obtained from the arithmetic operations described in Appendix A, provide a basis for developing global membership function, which characterized the prediction capacity. The final global membership structure was calculated as follows

$$STS = (5.006, 5.476, 1.203, 1.079) \tag{8}$$

Fig. 4 indicates the smooth structure of the global model given in Eq. (8) via fuzzy trapezoidal membership function.

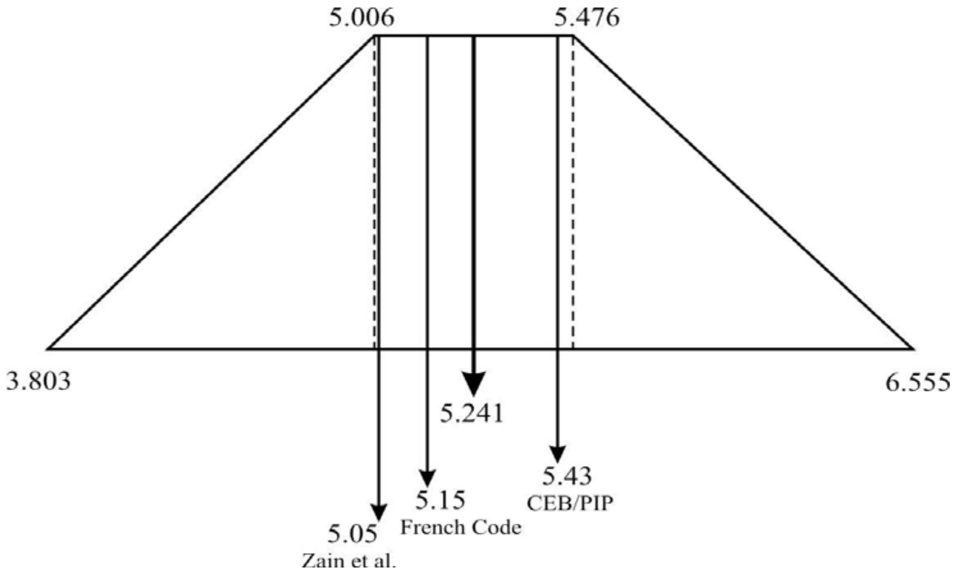

Fig. 4 Global output membership function and results with former studies

Table 2 Data set for case study 2

Age of Concrete (days)	W/B	CS (MPa)	STS (MPa)
28	0.46	44.05	3.66
28	0.46	50.29	3.69
28	0.36	55.58	4.39
28	0.36	58.24	4.68
28	0.36	66.16	5.09
28	0.28	33.25	2.53
28	0.30	66.55	4.20
28	0.43	36.60	2.93
28	0.34	62.66	3.61
28	0.29	34.10	2.82
28	0.43	19.34	1.82
28	0.39	49.60	3.69
28	0.40	61.10	5.30
28	0.40	64.70	5.50
28	0.40	63.40	4.90
28	0.40	56.10	4.40
28	0.45	44.80	4.19
28	0.45	27.00	2.60
28	0.45	35.00	3.50
28	0.45	24.00	2.20
28	0.45	40.00	3.40
28	0.45	40.00	3.80
28	0.60	30.00	3.20
28	0.45	40.00	4.50
28	0.60	31.00	3.90

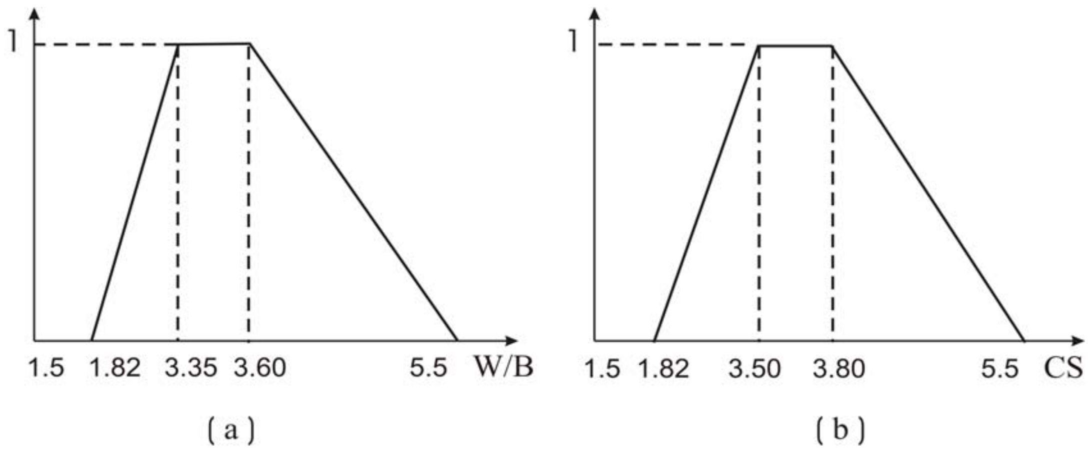

Fig. 5 Input memberships derived from projections for second application (a) W/B (b) CS

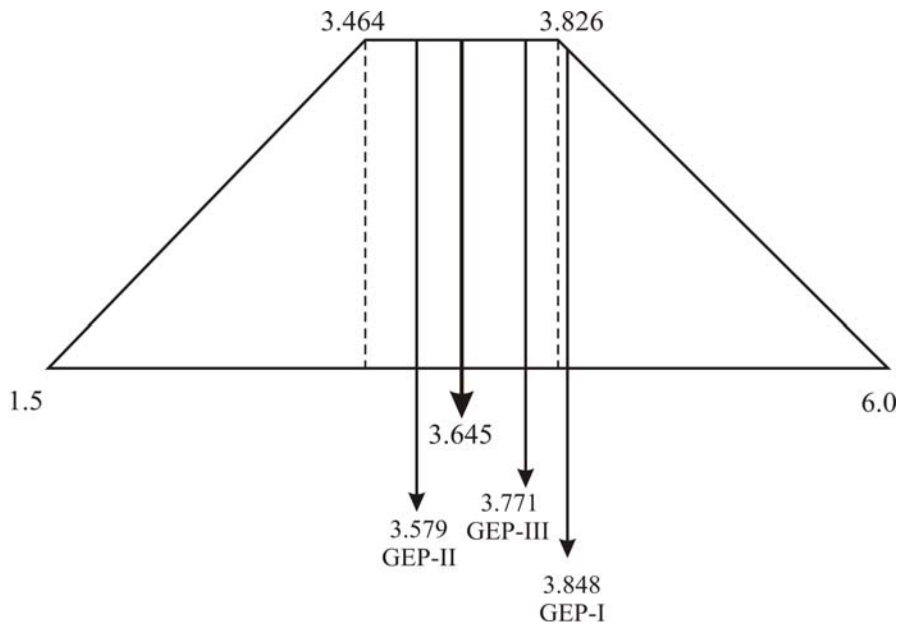

Fig. 6 Model output function and results with previous studies

3.2 Case Study II

In the second case study, the data collected by Severcan (2012) from different resources were employed (Table 2). First the data set was standardized between 1.82 and 5.50 then regression procedure was applied. The following linear equation has been obtained as result of the application

$$STS = -0.837 + 0.384(W/B) + 0.862(CS) \tag{9}$$

The trapezoidal membership functions of (W/C) and CS were defined by one dimensional data projections (Fig. 5). Finally, the STS estimations were provided as follows

$$STS = (-0.837, -0.837, 0, 0) + 0.384(3.35, 3.60, 1.53, 1.90) + 0.862(3.50, 3.80, 1.68, 1.70) \quad (10)$$

$$STS = (-0.837, -0.837, 0, 0) + 0.384(3.35, 3.60, 1.53, 1.90) + 0.862(3.50, 3.80, 1.68, 1.70) \quad (11)$$

As a combination of least squares regression and fuzzy arithmetic, the global membership function which states the frame of the final estimations was provided. Fig. 6 illustrates this final membership all together with the former works given in Severcan (2012).

$$STS = (3.464, 3.826, 2.036, 2.196) \quad (12)$$

4. Results and discussion

The experimental studies conducted for apprising the STS of concrete have provided some reliable findings. According to Fig. 4, the full possibility is represented by a core interval [5.006 5.476]. The relative error of the estimation and the average STS can be calculated using the length of the core interval. It is possible to states the overall relative error of this STS estimation in percentages. The limits of the core interval [5.006 5.476] give the overall relative error. For this prediction, the middle point of the interval (average STS) is 5.241. The amount of the relative error is determined by the following expression

$$\varepsilon_R = \frac{C_R - C_L}{C_M} * 100 \quad (13)$$

where C_R and C_L represent the right and left limit points of the core interval, respectively. C_M is the midpoint of the core (average STS). By using the Eq. (13), relative error of the prediction is calculated as ±8.9%. The predicted values of the previous methods such as French Code, Zain *et al.* (2002) are illustrated in Fig. 4 together with the results of the proposed model.

As seen in Fig. 6, the full possibility of the second application is determined as the core interval [3.464 3.826]. By using the limits and the middle point of the core interval that is 3.645, relative error of the prediction is calculated as $\pm 9.9\%$. Fig. 6 illustrates the comparison of the uncertainty-oriented interval model and the previous models developed by Gene Expression Programming.

The amount of errors obtained from (< 10 %) can be accepted as appropriate results for uncertain system modelling works performed in engineering (Bardossy and Fodor 2002). Furthermore, it is important that the proposed modelling method uses limited numbers of data and provides some reliable results.

The performance of the proposed approach has been compared to scientific models (French Code, CEP-PIP), a conventional model (Zain *et al.* 2002) and a computational intelligence model (Severcan 2012). Therefore, there can be mentioned a capacity and generality for the proposed methodology. Moreover, differently from the previous works, the proposed method provides some opportunities such as transparency and flexibility. In addition to them, the proposed model can eliminate computational complexity and it also allows performing confidence level-based final evaluation. Finally, it should be presented that the proposed methodology also can be used for assessing different types of materials such as rock and asphalt concretes.

5. Conclusions

The methodology presented in this paper provides a suitable uncertainty oriented tool for the evaluation of the parameters in concrete industry. The method uses global regression modelling and fuzzy numbers for appraising the uncertainties and predicting the tensile strength of concrete based on interval arithmetic. The main superiority of the proposed uncertainty-oriented model is its flexibility. In addition, the method gives an opportunity to analyse the system based on confidence limits.

The case studies have indicated that the method produces reliable outcomes via limited numbers of data. In addition, the fuzzy numbers describe prediction of uncertainty at various levels of possibility. Finally, the results show that the method is convenient for system modelling and uncertainty evaluation works in concrete industry.

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Nomenclature

f_t: tensile strength (empirical formula)
f_c: compressive strength (empirical formula)
TS: tensile strength
CS: compressive strength
STS: splitting tensile strength
W/B: water/binder ratio
C: limit point
L-R: Left-Right fuzzy number
α: range for left site of fuzzy number
β: range for right site of fuzzy number
μ: membership value

Appendix A

The operations on two trapezoidal fuzzy numbers $A = (a_L, a_R, \alpha, \beta)$ and $B = (b_L, b_R, \gamma, \delta)$ are presented below.

Sum
The sum *A+B* can be calculated directly from the extension principle,

$$A + B = (a_L + b_L, a_R + b_R, \alpha + \gamma, \beta + \delta)$$

The core of new trapezoidal fuzzy number *(A+B)* is calculated the sum of the peaks of *A* and *B*. Similarly, its left and right expansions are calculated from the expansions of *A* and *B*.

Difference
Difference of trapezoidal fuzzy numbers *A* and *B* can be calculated from the following rule:

$$A - B = (b_L - a_L, b_R - a_R, \alpha + \gamma, \beta + \delta)$$

Product
Let $A = (a_L, a_R, \alpha, \beta)$ be a trapezoidal fuzzy number, and $k \in \Re$ a real number. Then, the product of *A* and *k* can be given as follows

$$k.A = \begin{cases} (k.a_L, k.a_R, k.\alpha, k.\beta) & \text{if } k \geq 0, \\ (k.a_R, k.a_L, |k|.\beta, |k|.\alpha) & \text{if } k < 0 \end{cases}$$

where, operation depends on the sign of real number *k*.