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Stress-strain relationships for steel fiber reinforced self-compacting concrete

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Abstract. Steel fiber reinforced self-compacting concrete (SFRSCC) is a relatively new composite material which congregates the benefits of self-compacting concrete (SCC) technology with the profits derived from the fiber addition to a brittle cementitious matrix. Steel fibers improve many of the properties of SCC elements including tensile strength, toughness, energy absorption capacity and fracture toughness. Modification in the mix design of SCC may have a significant influence on the SFRSCC mechanical properties. Therefore, it is vital to investigate whether all of the assumed hypotheses for steel fiber reinforced concrete (SFRC) are also valid for SFRSCC structures. Although available research regarding the influence of steel fibers on the properties of SFRSCC is limited, this paper investigates material's mechanical properties. The present study includes: a) evaluation and comparison of the current analytical models used for estimating the mechanical properties of SFRSCC and SFRC, b) proposing new relationships for SFRSCC mixtures mechanical properties. The investigated mechanical properties are based on the available experimental results and include: compressive strength, modulus of elasticity, strain at peak compressive strength, tensile strength, and compressive and tensile stress-strain curves.

Keywords: steel fiber reinforced self-compacting concrete; compressive strength; modulus of elasticity; tensile strength; compressive and tensile stress-strain curve

1. Introduction

Self-compacting concrete (SCC) is considered as a concrete which can be placed and compacted under its self-weight with little or no vibration without segregation or bleeding. It is used to facilitate and ensure proper filling and good structural performance of restricted areas and heavily reinforced structural members. It has gained significant importance in recent years because of the advantages it offers. Recently, this concrete has gained wide use in many countries for different applications and structural configurations. SCC can also provide a better working environment by eliminating the vibration noise. Such concrete requires a high slump that can easily be achieved by superplasticizer addition to a concrete mix and special attention has to be paid to mix proportioning. The use of steel fiber reinforced self-compacting concrete (SFRSCC) probably, will promptly increase in the next years, since this composite material introduces several

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advantages on the concrete technology. In fact, the partial or total replacement of the conventional bar reinforcement by discrete fibers optimizes the construction process. The assembly of the reinforcement bars in the construction of concrete structures has a significant economic impact on the final cost of this type of constructions, due to the man-labor time consuming that it requires. In the modern societies, the cost of the man-labor is significant, from which diminishing the man-labor will decrease the overall cost of the construction. For this reason, SFRSCC is a very promising construction material with a high potential of application, mainly in the cases where fibers can replace the conventional reinforcement. At the present time, however, the SFRSCC technology is not yet fully developed and controlled, and, much less, the mechanical behavior of the SFRSCC material (Aslani and Nejadi 2012a-e).

In the fresh state, SFRSCC homogeneously spreads due to its own weight, without any additional compaction energy. To homogeneously fill a mold, SFRSCC has to fulfill high demands with regard to filling and passing ability, as well as segregation resistance. Driven by its own weight, the concrete has to fill a mold completely without leaving entrapped air even in the presence of dense steel bar reinforcement. All the concrete components have to be homogeneously distributed during the flow and at rest (Gräunewald 2004). The most benefited properties with the fiber addition to the concrete in the hardened state are the impact strength, the toughness and the energy absorption capacity. A detailed description of the benefits provided by the fiber addition to concrete can be found elsewhere (Balaguru and Shah 1992, Casanova 1996, ACI 544.1R 1997). The fiber addition might also improve the shear resistance (Rosenbusch and Teutsch 2003). Recently, Gräunewald (2004) compared the mechanical behavior of SFRSCC to the behavior of current fiber reinforced concrete (FRC). This author carried out bending and pull-out tests, and concluded that those properties were much better in the SFRSCC.

The field of possible application of SFRSCC include: highways, industrial and airfield pavements; hydraulic structures, tunnel segments, bridges components and concrete structures of complex geometry which present high difficulties of being reinforced by conventional steel bars, especially those who have high degree of support redundancy.

2. Research significance

The behavior of structural members can be rationally predicted by the given material properties, cross-sectional properties, and loading conditions when computerized non-linear structural analysis techniques are employed. For this purpose, materials properties can best described by their stress-strain relationships. Available material models are not able to accurately simulate the behavior of SFRSCC which requires more research should be done in this domain. The objectives of this study are: a) proposing new mechanical properties relationships for SFRSCC mixtures (i.e., compressive and tensile strengths, modulus of elasticity, and peak strain at maximum compressive strength), b) proposing new compressive and tensile stress-strain relationships for SFRSCC.

3. Database of SFRSCC experimental campaign

Using experimental results from various published investigations as a database is an effective tool for studying the applicability of the various SFRSCC mechanical properties. In order to apply the models to a particular concrete mixture accurately, it is necessary to use only the investigations that are adequately consistent with the applied testing methodology. The experimental results included in the database have been carried on mainly from the papers presented and published articles on SFRSCC. The database includes information regarding the composition of the mixtures, fresh properties of SFRSCC, testing methodology and conditions. However, it should be emphasized that the mechanical characteristics have not been investigated as much as the other aspects of SFRSCC, and the available published experimental data in the literature are still not very extensive. Using experimental results from different sources can frequently be problematic because of the following reasons: 1. there is often insufficient information regarding the exact composition of the concrete mixtures; 2. the size of the specimen, curing condition, and testing methodology vary between the different investigations and, in some cases, this information is not fully indicated; 3. in many cases it is difficult to extract the relevant experimental values because the published results are incomplete or are presented in graphical form and the data values still have to be extrapolated from the graphs.

Tables 1-3 are a general summary of the SFRSCC mechanical properties mixtures included in the database. The database includes 21 reference experimental results (i.e., Grünewald 2004, Corinaldesi and Moriconi 2004, Sahmaran *et al*. 2005, Cunha 2006, Liao *et al*. 2006, Schumacher 2006, Sengul *et al*. 2006, Dhonde *et al*. 2007, Ferrara *et al*. 2007, Aydin 2007, Torrijos *et al*. 2008, El-Dieb 2009, Buratti *et al*. 2010, Khaliq and Kodur 2011, Fantilli *et al*. 2011, Corinaldesi and Moriconi 2011, Ding *et al*. 2012a, b, Goel *et al*. 2012, Akcay and Tasdemir 2012, van Zijl and Zeranka 2012). Tables 1-3 also include additional information regarding the cement type, filler type, compressive strength test specimen type, aggregate type, fiber type, fiber shape, fiber aspect ratio, fiber length, mix label, fiber volume fraction (V_f), compressive strength at 28 days (f'_c), and fiber reinforcing index (R.I. = $V_f \times l_f / d_f$).

Table 1 SFRSCC experimental results database properties (including: cement type, filler type, compressive strength specimen type, and aggregate type)

Reference	Cement type	Filler type	f'_c Specimen type	Aggregate type
Grünewald (2004)	CEM III/A 52.5, CEM I 52.5 R	Silica fume	Cube (150 mm)	Natural crushed and round coarse aggregate and natural round sand
Corinaldesi and Moriconi (2004)	CEM II/ A-L 42.5 R	Limestone	Cube (100 mm)	Crushed limestone coarse aggregate and natural sand
Sahmaran *et al.* (2005)	OPC type I (ASTM C150-04)	Limestone	Cube (150 mm)	Crushed limestone and crushed sand
Cunha (2006)	CEM I 42.5R	Limestone	Cylinder (150 mm × 300 mm)	Crushed granite coarse aggregate and river sand
Liao *et al*. (2006)	ASTM Type III Portland	Fly ash	Cylinder (100 mm × 200 mm)	Crushed Limestone and Pea gravel, Silica Sand
Schumacher (2006)	CEM III/A 52.5, CEM I 52.5 R	Fly ash	Cube (150 mm)	Natural crushed and round coarse aggregate and natural round sand
Sengul *et al*. (2006)	OPC type I (ASTM C150-04)	Silica fume	-	-
Dhonde *et al*. (2007)	ASTM Type III Portland	Fly ash	Cylinder (150 mm × 300 mm)	Well-graded, rounded, river-bed, coarse aggregates and well-graded, river-bed sand

Table 1 Continued

Ferrara *et al*. (2007)	OPC type I (ASTM C150-04)	Fly ash	Cylinder (100 mm × 200 mm)	-
Aydin (2007)	OPC type I (ASTM C150-04)	Quartz powder	Cylinder (100 mm × 200 mm)	Natural gravel aggregate and natural sand
Torrijos *et al*. (2008)	CEM II 32.5 R	Limestone	Cylinder (150 mm × 300 mm)	Crushed limestone aggregates
El-Dieb (2009)	OPC type I (ASTM C150-04)	Silica fume	Cube	Natural crushed stone coarse aggregate and crushed natural stone sand
Buratti *et al*. (2010)	II/A-L 32.5R	II/A-L 32.5R	-	-
Khaliq and Kodur (2011)	OPC type I (ASTM C150-04)	Fly ash	Cylinder (100 mm × 200 mm)	Crushed Limestone coarse aggregate and natural sand
Fantilli *et al*. (2011)	A-LL 42.5 R	Carbonate	Cylinder (70 mm × 140 mm)	Natural gravel aggregate and natural sand
Corinaldesi and Moriconi (2011)	CEM II/A-L 42.5 R	Limestone	Cube (100 mm)	Gravel and quartz sand
Ding *et al*. (2012a)	OPC type I (ASTM C150-04)	Fly ash	Cube (150 mm)	Crushed gravel and natural sand
Ding *et al*. (2012b)	OPC type I (ASTM C150-04)	Fly ash	Cube (150 mm)	Crushed limestone aggregate and natural sand
Goel *et al*. (2012)	OPC type I (ASTM C150-04)	Fly ash	Cube (150 mm)	Crushed stone aggregate and natural sand
Akcay and Tasdemir (2012)	OPC type I (ASTM C150-04)	Silica fume	Cylinder (100 mm × 200 mm)	Crushed stone coarse aggregate and natural sand
van Zijl and Zeranka (2012)	OPC type I (ASTM C150-04) and CEM II 32.5	Fly ash	Cube (100 mm)	Greywacke stone and Malmesbury sand

Table 2 SFRSCC experimental results database properties (including: cement type, filler type, compressive strength specimen type, and aggregate type)

Reference	Fibre type	Shape	l_f/d_f	l_f(mm)
Grünewald (2004)	Dramix BP 80/60 C	Hooked	85.66	61.06
	Dramix BN 80/60 C		76.10	57.94
	Dramix BN 45/50 L		48.08	51.09
	Eurosteel 50/50		45.81	47.77
	Dramix BN 65/40 C		64.94	41.24
	Harex 01/32		32.82	32.40
	Dramix BP 80/30 C		78.50	30.48
	Dramix BN 45/30 L		46.34	28.80
	Harex 65/20		64.30	20.20
	Dramix OL 13/0.16		81.25	13.00
	Dramix OL 6/0.16		37.50	6.00
Corinaldesi and Moriconi (2004)	Straight steel fibers	Straight	27.50	11.00
Sahmaran *et al*. (2005)	Dramix ZP 305	Hooked	55.00	30.00
	Dramix OL 6/16	Straight	37.5	6.00
Cunha (2006)	Dramix RC-80/60-BN	Hooked	85.66	60.00
Liao *et al*. (2006)	Dramix RC-80/30-BP	Hooked	78.50	30.00
	Dramix ZP305		55.00	30.00

Table 2 Continued

Schumacher (2006)	Dramix BN 80/60 C	Hooked	85.66	61.06
	Dramix BP 80/30 C		78.50	30.48
	Dramix BN 45/30 L		46.34	28.80
Sengul *et al*. (2006)	-	Hooked	54.54	30.00
Dhonde *et al*. (2007)	Dramix RC-80/60-BN	Hooked	80.00	60.00
	Dramix ZP305		55.00	30.00
Ferrara *et al*. (2007)	Dramix 65/35	Hooked	65.00	35.00
Aydin (2007)	Dramix OL 6/16	Hooked	37.5	6.00
Torrijos *et al*. (2008)	-	Hooked	50.00	50.00
El-Dieb (2009)	HELIX 5-25	Twisted	50.00	25.00
Buratti *et al*. (2010)	Steel A		66.66	50.00
Khaliq and Kodur (2011)	NOVOCON XR	Corrugated	33.33	38.00
Fantilli *et al*. (2011)	Dramix RC 65/35 BN	Hooked	63.63	35.00
Corinaldesi and Moriconi (2011)	-	Hooked	43.00	30.00
Ding *et al*. (2012a)	Dramix BN 80/60 C	Hooked	80.00	60.00
Ding *et al*. (2012b)	-	Hooked	63.63	35.00
Goel *et al*. (2012)	-	Circular corrugated	30.00	30.00
Akcay and Tasdemir (2012)	HSS	Hooked	40.00	6.00
	NSH		55.00	30.00
	HSH		55.00	30.00
van Zijl and Zeranka (2012)	Dramix ZP305	Hooked	55.00	30.00

Table 3 SFRSCC compressive strength results database properties (including: fiber type, fiber volume fraction (V_f), 28 days compressive strength, and fiber reinforcing index R.I.)

Reference	Mix Label	Fibre type	V_f (%)	f'_c (28 days) (MPa)	$R.I. = (V_f \times l_f/d_f)$
Grünewald (2004)	L-R-60-60	80/60	2.5	54.00	2.14
	L-R-30-60	80/30	2.5	57.60	1.96
	L-R-40-100	65/40	4.23	51.90	2.74
	L-R-30-140	45/30	6.3	55.80	2.92
	M-R-30-40	80/30	1.7	70.30	1.33
	M-R-20-60	65/20	2.57	75.60	1.65
	M-R-60-60	80/60	2.56	75.10	2.19
	M-R-30-60	80/30	2.56	72.30	2.01
	M-R-40-100	65/40	4.2	73.50	2.73
	M-R-30-140	45/30	5.84	78.10	2.70
	M-F-60-60	80/60	2.5	75.30	2.14
	M-F-30-140	45/30	5.84	71.70	2.70
	H-R-60-60	80/60	2.5	116.70	2.14
	H-R-13-125	OL13/0.16	5.1	120.30	4.14
	P1	45/30	2.5	52.20	1.16
	P2	45/30	5	55.50	2.31
	P3	80/30	2.5	114.40	1.96
Corinaldesi and Moriconi (2004)	SCC-0.40	Straight	0.6	44.00	1.65
Sahmaran *et al*. (2005)	2	Dramix ZP 305	2.0	49.50	1.10
	6	Dramix OL 6/16	2.0	58.90	0.75

Table 3 Continued

Cunha (2006)	SFRSCC1	80/60-BN	0.55	69.70	0.47
	SFRSCC2		0.80	56.20	0.68
Liao *et al.* (2006)	SFRSCC1	ZP305	1.96	65.00	1.08
	SFRSCC2	80/30-BN	1.92	67.90	1.50
	SFRSCC3		1.47	65.00	1.15
	SFRSCC4		1.38	36.40	1.08
	SFRSCC5		1.50	43.60	1.18
	SFRSCC6		1.50	39.30	1.18
Schumacher (2006)	B45.45/30.60	45/30	2.50	55.70	1.16
	B45.45/30.120	45/30	5.00	56.40	2.32
	B45.80/30.60	80/30	2.50	56.10	1.96
	B45.80/60.60	80/60	2.50	60.70	2.14
	B105.80/30.60	80/30	2.50	116.70	1.96
	B105.80/60.60	80/60	2.50	116.70	2.14
Sengul *et al.* (2006)	V1350	-	1.50	86.0	0.81
	V1650		1.50	110.2	0.81
	V1900		1.50	124.2	0.81
	V2350		1.50	94.9	0.81
	V2650		1.50	123.7	0.81
	V2900		1.50	138.0	0.81
Dhonde *et al.* (2007)	SFRSCC1	80/60-BP	0.50	83.00	0.4
	SFRSCC2	ZP305	0.50	84.30	0.27
	SFRSCC3	ZP305	1.00	90.00	0.55
Ferrara *et al.* (2007)	1FRC	Dramix 65/35	2.75	58.26	1.78
	2FRC		2.91	81.60	1.89
	3FRC		3.10	61.40	2.01
	4FRC		2.75	76.70	1.78
	5FRC		2.91	79.44	1.89
	6FRC		3.10	66.50	2.01
	7FRC		2.75	73.95	1.78
	8FRC		2.91	82.78	1.89
	9FRC		3.10	68.07	2.01
Aydin (2007)	M2	Dramix OL 6/16	0.25	18.47	0.09
	M3		0.50	24.21	0.18
	M4		0.75	22.57	0.28
	M5		1.00	39.25	0.37
	M6		1.25	18.70	0.46
	M7		1.50	25.89	0.56
	M8		1.75	24.41	0.65
	M9		2.00	44.44	0.75
Torrijos *et al.* (2008)	SFR-SCC 25	-	1.00	54.00	0.50
	SFR-SCC 50		2.00	54.00	1.00
El-Dieb (2009)	A	HELIX 5-25	0.08	116.74	0.04
	B		0.12	99.48	0.06
	C		0.52	96.65	0.26
Buratti *et al.* (2010)	SF25a	Steel A	0.32	40.1	0.21
	SF25b		0.32	42.2	0.21
	SF35a		0.45	39.9	0.30
	SF35b		0.45	40.1	0.30

Table 3 Continued

Khaliq and Kodur (2011)	SCC-S	NOVOCON XR	1.75	57.00	0.58
Fantilli *et al*. (2011)	35SC0	Dramix RC 65/35 BN	0.45	34.50	0.28
	35SC1		0.45	37.30	0.28
	35SC3		0.45	42.50	0.28
	35SC10		0.45	67.80	0.28
	70SC0		0.90	21.80	0.57
	70SC1		0.90	29.50	0.57
	70SC3		0.90	38.30	0.57
	70SC10		0.90	64.90	0.57
Corinaldesi and Moriconi (2011)	S-RP	-	0.60	63.50	0.25
	S-LP		0.60	63.00	0.25
Ding *et al*. (2012a)	SF20	Dramix BN 80/60 C	0.44	36.00	0.35
	SF40		1.78	32.50	1.42
	SF60		2.60	41.20	2.08
Ding *et al*. (2012b)	SF40	-	0.51	64.00	0.32
	SF55		0.71	65.00	0.45
Goel *et al*. (2012)	SCFRC-1	-	0.50	40.40	0.15
	SCFRC-2		1.00	43.10	0.30
	SCFRC-3		1.50	45.70	0.45
Akcay and Tasdemir (2012)	C0.75N	0.5% HSS + 0.25% NSH	0.75	116.3	0.33
	C0.75H	0.5% HSS + 0.25% HSH	0.75	122.2	0.33
	C1.5N	1.0% HSS + 0.5% NSH	1.50	118.6	0.67
	C1.5H	1.0% HSS + 0.5% HSH	1.50	123.6	0.67
van Zijl and Zeranka (2012)	HPNFRC	Dramix ZP305	0.50	63.3	0.28
			1.00	71	0.55
			1.50	75.9	0.82
	HPSCFRC		0.50	85.5	0.28
			1.00	85.9	0.55
			1.50	91.8	0.82
	NFRC		0.50	34.9	0.28
			1.00	42.6	0.55
			1.50	42.2	0.82
	SCFRC		0.50	57	0.28
			1.00	56.7	0.55
			1.50	58.4	0.82

4. Database of SFRC and SFRSCC compressive stress-strain relationships

The stress-strain relationship of concrete essentially consists of two distinct branches−an ascending branch up to the peak stress followed by a descending branch until the concrete crushes. The key properties that are normally used to characterize the ascending branch of the curve are the initial tangent modulus, the compressive strength, and the strain at peak stress. In technical literature there are reported many analytical models developed to represent the stress-strain curves for plain concrete under compression. Among most important and known models, must be cited models Popovics (1973), Carreira and Chu (1985). Since the models of the compressive behavior of fiber-reinforced concrete were developed from models developed for plain concrete, it is necessary the inclusion of some parameters in these models to consider the influence of fibers on the properties of stress-strain curve (Ramadoss and Nagamani 2013, Bang *et al*. 2010). In Table 4 most of the SFRC compressive stress-strain relationships are summarized and include: Ezeldin *et al*. (1992), Hsu and Hsu (1994), Mansur *et al*. (1999), Nataraja *et al*. (1999), Neves and Almeida (2005), Bhargava *et al*. (2006), Oliveira Júnior *et al*. (2010). Also, Table 5 shows SFRSCC compressive stress-strain relationship as Cunha (2006).

Table 4 SFRC compressive stress-strain relationships database

Reference	Compressive stress-strain relationships for SFRC
Ezeldin *et al*. (1992)	$\sigma_{cf} = f'_{cf}\left(\dfrac{\beta\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)}{\beta-1+\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)^{\beta}}\right)$ $\beta = 1.093 + 0.7132(3R.I.)^{-0.926}$
Hsu and Hsu (1994)	$\sigma_{cf} = \dfrac{n\beta\varepsilon_c}{n\beta-1+\varepsilon_c^{n\beta}}$ $0 \le x < x_d$ $\sigma_{cf} = \eta_d \exp\left[-k_d(\varepsilon_c - x_d)^a\right]$ $x_d \le x$, $\eta_d = 0.6$, $k_d = 0.7$, $a = 0.8$ $\beta = \left[\dfrac{f'_c}{A}\right] + C$, $A = 1.717(V_f)^3 + 8.501$, $C = -0.26V_f + 2.742$, x_d : strain corresponding to $0.6f'_c$, $n = \begin{cases} 1.0 \text{ if } f'_c < 79.3, 1.5 \text{ if } 79.3 \le f'_c < 82.73, 2.0 \text{ if } f'_c \ge 82.73 & V_f = 0.5 \\ 1.0 \text{ if } f'_c < 79.3, 1.5 \text{ if } 79.3 \le f'_c < 86.18, 2.0 \text{ if } f'_c \ge 86.18 & V_f = 0.75 \\ 1.0 \text{ if } f'_c < 82.73, 1.5 \text{ if } f'_c \ge 82.73 & V_f = 1.0 \end{cases}$
Mansur *et al*. (1999)	$\sigma_{cf} = f'_{cf}\left[\dfrac{k_1\beta\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)}{k_1\beta-1+\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)^{k_2\beta}}\right]$ $k_2 = \left(\dfrac{50}{f'_{cf}}\right)^{1.3}\left[1-0.11\left(\dfrac{V_f l}{\phi}\right)^{-1.1}\right]$, $E_{cf} = (10300 - 400V_f)f_o^{1/3}$, $\varepsilon'_{cf} = \left(0.0005 + 0.00000072\left(\dfrac{V_f l}{\phi}\right)\right)(f'_{cf})^{0.35}$ For cylindrical specimens: $k_1 = \left(\dfrac{50}{f'_{cf}}\right)^{3.0}\left[1+2.5\left(\dfrac{V_f l}{\phi}\right)^{2.5}\right]$ For horizontally cast prisms: $k_1 = A\left\{\dfrac{40}{f'_{cf}}\right\}^{2.0}$, $k_2 = B\left\{\dfrac{40}{f'_{cf}}\right\}^{1.3}$ $A = 0.96$ and $B = 0.80$ for fibre concretes while $A = 1.00$ and $B = 1.00$ for plain concrete
Nataraja *et al*. (1999)	$\sigma_{cf} = f'_{cf}\left[\dfrac{\beta\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)}{\beta-1+\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)^{\beta}}\right]$, $\beta = 0.5811 + 1.93RI^{-0.7406}$

Table 4 Continued

Neves and Almeida (2005)	$\sigma_{cf} = f'_{cf}\dfrac{\left(\dfrac{\varepsilon_c}{\varepsilon'_{cf}}\right)}{(1-p-q)+q\left(\dfrac{\varepsilon_c}{\varepsilon'_{cf}}\right)+p\left(\dfrac{\varepsilon_c}{\varepsilon'_{cf}}\right)^{(1-q)/p}}$, $p+q=1-\dfrac{f'_c}{E_c\,\varepsilon'_c}$, $E_{cf}=\left(10.5-0.22\,V_f\right)f_c^{1/3}$ $\varepsilon'_{cf}=0.69\times f'^{\left[0.29+0.0002\,V_f\left(l_f/d_f^2\right)\right]}_{cf}$, $\dfrac{1-q}{p}>0$, Neves (2000) : $p=1-0.85\times f'^{\left[-0.0013\times V_f\left(l_f/d_f^2\right)\right]}_{cf}$
Bhargava *et al.* (2006)	$\sigma_{cf}=f'_{cf}\left(\dfrac{k_1\,\beta\left(\dfrac{\varepsilon_{cf}}{\varepsilon'_{cf}}\right)}{k_1\beta-1+\left(\dfrac{\varepsilon_{cf}}{\varepsilon'_{cf}}\right)^{k2\,\beta}}\right)$, $\beta=\dfrac{1}{1-\dfrac{f'_{cf}}{\varepsilon'_{cf}\,E_{if}}}$, Modifications: $\beta=\left[\dfrac{f'_{cf}}{A}\right]^3+C$ $A=50.35+22.31(R.I.)_s+19.13(R.I.)_l$, $C=2.04-0.313(R.I.)_s-0.155(R.I.)_l$ For the ascending portion of the curve: $k_1=1$, $k_2=1$ For the descending portion of the curve: $k_1=\left[\dfrac{D}{f'_{cf}}\right]^{3.79}$, $k_2=\left[\dfrac{G}{f'_{cf}}\right]^{1.46}$ $D=35.635+17.21(R.I.)_s+9.11(R.I.)_l$, $G=31.82+16.39(R.I.)_s+9.35(R.I.)_l$
Oliveira Júnior *et al.* (2010)	$\sigma_{cf}=f'_{cf}\left[\dfrac{\beta\left(\dfrac{\varepsilon_c}{\varepsilon'_{cf}}\right)}{\beta-1+\left(\dfrac{\varepsilon_c}{\varepsilon'_{cf}}\right)^{\beta}}\right]$, $\beta=\left(0.0536-0.5754\,V_f\right)f'_{cf}$, $\varepsilon'_{cf}=\left(0.00048+0.01886\,V_f\right)\ln f'_{cf}$

Ezeldin *et al*. (1992) proposed a model for complete stress-strain curve for non-silica-fume fiber reinforced concrete. Ezeldin *et al*.'s model (1992) is valid for the experimental stress-strain behavior of fiber reinforced concrete with compressive strength ranging from 35 MPa to 85 MPa. Three fiber volume fractions 30 kg/m^3, 45 kg/m^3, and 60 kg/m^3 and three aspect ratios of 60, 75, and 100 were investigated. The influence of the fiber reinforcing parameters on the peak stress, corresponding strain, the secant modulus of elasticity, the toughness of concrete, and the curve shape were reported. Empirical equations are proposed by Hsu and Hsu (1994) to represent the complete stress-strain relationships of high strength steel fiber concrete with compressive strength exceeding 69 MPa. Hsu and Hsu's model (1994) is based on a series of compression tests that were conducted on 75×150 mm cylindrical specimens using a modified test method that gave the complete stress-strain behavior for high-strength steel-fiber concrete with or without tie confinements. The volume fractions of steel fiber in the concrete were 0%, 0.5%, 0.75% and 1%, respectively. Various parameters were studied and their relationships were experimentally determined.

Mansur *et al*. (1999), based on their test data, proposed an analytical model to generate the complete stress-strain curves of high-strength fiber reinforced concrete derived from cylinders and horizontally cast prisms. The concrete strength investigated ranges from 70 to 120 MPa. Other

Table 5 SFRSCC compressive stress-strain relationship database

Reference	Compressive stress-strain relationships for SFRSCC
Cunha (2006)	Fitted for SCC: $f_{cm}(t)=f_{cm,28}\exp\left\{0.052\left[1-\left(\frac{28}{t}\right)^{0.89}\right]\right\}$ Fitted for SCC: $E_c(t)=E_{c,28}\left\{\exp\left[0.1\left(1-\left(\frac{28}{t}\right)^{0.97}\right)\right]\right\}^{0.17}$ Fitted for SCC: $E_{ci}=13\left(\frac{f_{cm}}{10}\right)^{0.6}$ Strain at peak stress: $\varepsilon_{c1}=\frac{\varepsilon_{c1,28}\frac{t}{28}}{\frac{t}{28}-0.008}$ Stress–strain model: $\sigma(\varepsilon_c)=f_{cm}\frac{\frac{E_{ci}}{E_{c1}}\frac{\varepsilon_c}{\varepsilon_{c1}}-\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2}{1+\left(\frac{E_{ci}}{E_c}-2\right)\frac{\varepsilon_c}{\varepsilon_{c1}}}$ $\varepsilon_c\le\varepsilon_{c,\lim}$ $\varepsilon_{c,\lim}=\varepsilon_{c1}\left[\frac{1}{2}\left[(1-\alpha)\frac{E_{ci}}{E_{c1}}+2\alpha\right]+\left[\frac{1}{4}\left[(1-\alpha)\frac{E_{ci}}{E_{c1}}+2\alpha\right]^2-\alpha\right]^{0.5}\right]$ $\sigma(\varepsilon_c)=f_{cm}\left[\left[\frac{1}{\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}}}\xi\left(\frac{1}{2\alpha}\right)^2-\frac{1}{\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2}\frac{1}{\alpha}\right]\left(\frac{\varepsilon_c}{\varepsilon_{c1}}\right)^2+\left[\frac{1}{\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}}}\frac{2}{\alpha}-\xi\left(\frac{1}{2\alpha}\right)^2\right]\frac{\varepsilon_c}{\varepsilon_{c1}}\right]^{-1}$ $\xi=\frac{4\left[\left(\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}}\right)^2\left(\frac{E_{ci}}{E_{c1}}-2\right)+2\frac{\varepsilon_c}{\varepsilon_{c1}}-\frac{E_{ci}}{E_{c1}}\right]}{\left[\frac{\varepsilon_{c,\lim}}{\varepsilon_{c1}}\left(\frac{E_{ci}}{E_{c1}}-2\right)+1\right]^2}$; $\alpha=0.9\exp\left\{0.005\left[1-\left(\frac{28}{t}\right)^{1.16}\right]\right\}$

parameters include volume fraction of steel fibers and direction of casting in relation to the loading axis. Mansur *et al.* (1999) test results indicate that inclusion of fibers improves the strength and enhances the strain at peak stress but results in a smaller initial tangent modulus for specimens cast in an upright (vertical) position. Analytical models are proposed by Nataraja *et al.* (1999) to quantify the effect of fiber on compressive strength, strain at peak stress and the toughness of concrete in terms of fiber reinforcing parameter. These models are based on the experimental investigations to generate the complete stress-strain curve experimentally for steel fiber reinforced concrete for compressive strength ranging from 30 to 50 MPa. Round crimped fiber with three volume fractions of 0.5%, 0.75% and 1.0% (39, 59, and 78 kg/m^3) and for two aspect ratios of 55 and 82 were considered. The effect of fiber addition to concrete on some of the major parameters namely peak stress, strain at peak stress, the toughness of concrete and the nature of the stress-strain curve is studied.

Neves and Almeida (2005) proposed expressions to estimate the Young's modulus and the strain at peak stress, from the compressive strength results, knowing fiber volume, length and diameter. Also, an analytical model to predict the stress–strain relationship for steel fiber concrete in compression is also proposed by Neves and Almeida (2005). These relationships are achieved by using an experimental study to investigate the influence of matrix strength, fiber content and diameter on the compressive behavior of steel fiber reinforced concrete. Concrete compressive strengths of 35 and 60 MPa, 0.38 and 0.55 mm fiber diameter, and 30 mm fiber length, were considered. The volume of fiber in the concrete was varied up to 1.5 %. Bhargava *et al*. (2006) based on their test data, proposed a model to generate the complete stress-strain relationship for steel fiber reinforced high strength concrete. The experimental program consisted of testing 100×200 mm concrete cylinders. The experimental variables of this study were concrete strength levels (58.03 MPa and 76.80 MPa), volume fractions (0.5% to 2.0%) and aspect ratios (20 and 40) of flat crimped steel fibers. The effect of the mixed aspect ratio of fibers on the stress-strain behavior of steel fiber high strength concrete was also studied by blending short and long fibers.

Oliveira Júnior *et al*. (2010) presented a study on the compressive behavior of steel fiber reinforced concrete. In this study, an analytical model for stress-strain curve for steel fiber reinforced concrete is derived for concretes with strengths of 40 MPa and 60 MPa at the age of 28 days. Those concretes were reinforced with steel fibers with hooked ends 35 mm long and with aspect ratio of 65. Cunha (2006) has proposed stress-strain laws to model the behavior of the SFRSCC since the early ages. Additionally empirical expressions to predict the principal mechanical properties were presented. The requirements established for this SFRSCC were the following: average compression strength at 24 hours greater than 20 MPa, equivalent flexural tensile strength greater than 2 MPa at this age, content of cement not exceeding 400 kg/m^3. In this work, the compressive softening behavior of SFRSCC was investigated, within a structural point of view.

5. Understanding and interpreting regression analysis

In this study DataFit program is used to nonlinear regression analyses. The nonlinear regression understating and interpreting are described as follow. Similar to linear regression, the goal of nonlinear regression is to determine the best-fit parameters for a model by minimizing a chosen merit function. Where nonlinear regression differs is that the model has a nonlinear dependence on the unknown parameters, and the process of merit function minimization is an iterative approach. The process is to start with some initial estimates and incorporates algorithms to improve the estimates iteratively. The new estimates then become a starting point for the next iteration. These iterations continue until the merit function effectively stops decreasing. The nonlinear model to be fitted can be represented by $y = y(x;\ a)$. The merit function minimized in performing nonlinear regression the following $\chi^2(a) = \sum\{y_i - y(x_i;\ a)/\sigma_i\}^2$. where σ_i is the measurement error, or standard deviation of the *i*th data point. As with linear regression, we are minimizing is the sum of the squares of the distances between the actual data points and the regression line.

Nonlinear regression iterations proceed as follows: 1. Obtain initial estimates for all of the variables being fitted for in the model. These initial estimates can be obtained from linear regression, rules, or by examining the curve generated by the data points. For models pre-defined in DataFit, linear regression is used to obtain the initial estimates. For user defined models, either rules need to be created or the user must specify the initial estimates; 2. Using the initial estimates, compute the merit function; 3. Use an algorithm to adjust the variables in order to improve the fit of the model to the data points. DataFit utilizes the Levenberg-Marquardt method. Models pre-defined in DataFit use analytical derivatives during the optimization process, user defined models use numerical derivatives unless the user specifies the analytical derivatives; 4. Again, compute the merit function and compare it to the previous iteration; 5. Repeat steps 3 and 4 until there is essentially no change in the merit function, then cease the iterations; 6. Calculate the goodness of fit statistics.

The final results of nonlinear regression for parameters that are used in this study will be present as “Regression Variable Results” and “Variance Analysis”. About “Regression Variable Results” following parameters should be define; *Value* is the data in the value column are estimated or fitted parameter values; *Standard Error* is the data in the Standard Error column are the estimates of the standard deviations of the fitted regression parameters; *t-ratio* is the ratio of the estimated parameter value to the estimated parameter standard deviation. The larger the ratio is, the more significant the parameter is in the regression model. This is a test statistic to determine if the actual parameter value is zero; *Prob(t)* or *p* value is used to test the null hypothesis for each parameter. The smaller the value of Prob(t), the less likely the parameter is actually zero. For example, if Prob(t) = 0.01, there is a 1% chance that the actual parameter is zero. If Prob(t) = 0.95, there is a 95% chance that the actual parameter value is zero. In cases like the latter, the parameter in question can usually be removed from the model without affecting the regression accuracy.

Moreover, about “Variance Analysis” following parameters should be define; *n* is the number of data points, or observations; *p* is the number of parameters or variables in the regression model; *Predicted Value*, the *i*th predicted, or fitted value of the dependent variable *Y*, is denoted by $\hat{Y}_i$. This value is obtained by evaluating the regression model $\hat{Y} = f\left(X, \hat{\beta}_j\right)$, where $\hat{\beta}_j$ are the regression parameters, or variables; SSR = regression sum of squares = $\sum_{i-1}^{n}(Y_i - \hat{Y}_i)^2$; SSE = Residual or Error Sum of Squares (Absolute) = $\sum_{i-1}^{n}(Y_i - \hat{Y}_i)^2$; SST = total sum of squares = $\sum_{i=1}^{n}\left(\hat{Y}_i - \overline{Y}\right)^2$. Also, other variance analysis parameters are defined in Table 6.

Table 6 Variance analysis definition

Variance Analysis				
Source	DF	Sum of Squares	Mean Square	F Ratio
Regression	p-1	SSR	SSR/(p-1)	(SSR/(p-1))/(SSE/(n-p))
Error	n-p	SSE	SSE/(n-p)	
Total	n-1	SST		

6. Proposed relationships for SFRSCC mechanical properties

In this study, relationships for the mechanical properties of the SFRSCC (e.g., compressive strength (f'_{cf}), tensile strength (f_{ctf}), modulus of elasticity (E_{cf}), and strain at peak stress (ε'_{cf})) are proposed. These relationships are based on nonlinear regression analyses on the existing experimental database. Eqs.(1)-(8) express these relationships. The experimental results raw SFRSCC compressive strength/ normal compressive strength (f'_{cf} / f'_c) versus R.I., SFRSCC tensile strength / normal tensile strength (f_{ctf} / f_{ct}) versus R.I., SFRSCC modulus of elasticity / normal modulus of elasticity (E_{cf} / E_c) versus R.I., and strain at peak stress (ε'_{cf}) versus fiber compressive strength are presented in Table A.1. to A.4. in Appendix, respectively.

6.1 Compressive strength for the SFRSCC

SFRSCC compressive strength relationships are separated in three different range of strength as: 35 to 60 MPa, 60 to 80 MPa, and 80 to 120 MPa. Regression analyses have been done on the raw SFRSCC compressive strength/ normal compressive strength (f'_{cf}/f'_c) versus R.I. data. Normal compressive strength (without fiber) has important impact to consist available raw data for doing reliable nonlinear regression.

f'_{cf}: 35 to 60 MPa

$$f'_{cf} = a \times R.I. + b \times f'_c \tag{1}$$

Regression Variable Results					
Variable	**Value**	**Standard Error**	**t-ratio**	**Prob(t)**	
a	0.10	1.39	2.01	0.050	
b	1.46	6.27	23.29	0.0	
Variance Analysis					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	0.23	0.23	4.014	0.050
Error	38	2.21	5.82E-02		
Total	39	2.44			

f'_{cf}: 60 to 80 MPa

$$f'_{cf} = a \times R.I. + b \times f'_c \tag{2}$$

Regression Variable Results					
Variable		**Standard Error**	**t-ratio**	**Prob(t)**	
a		6.19E-03	2.06	0.049	
b		1.03E-02	106.79	0.0	
Variance Analysis					
Source		**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	2.74E-03	2.74E-03	4.26	0.049
Error	24	1.54E-02	6.42E-04		
Total	25	1.81E-02			

f'_{cf}: 80 to 120 MPa

$$f'_{cf} = a \times R.I. + b \times f'_c \tag{3}$$

Regression Variable Results					
Variable	**Value**	**Standard Error**	**t-ratio**	**Prob(t)**	
a	1.56E-04	3.79E-03	0.04	0.048	
b	1.25	0.004	285.21	0.0	
Variance Analysis					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	2.36E-07	2.36E-07	0.0016	0.047
Error	21	2.94E-03	1.40E-04		
Total	22	2.94E-03			

Fig. 1 (a-c) shows proposed SFRSCC compressive strength relationship compared to the experimental results database for three different limitations of compressive strength 35 to 60 (MPa), 60 to 80 (MPa), and 80 to 120 (MPa).

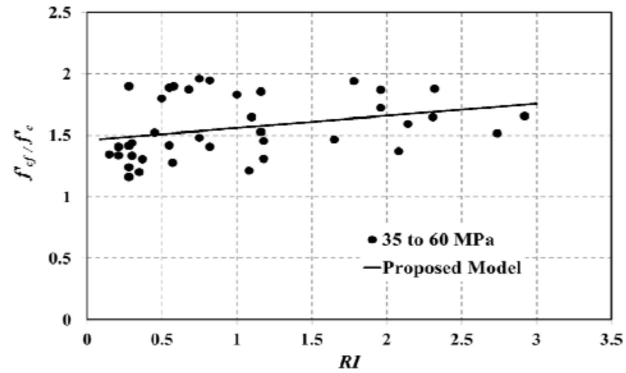

Fig. 1 (a) Proposed relationship for compressive strength of SFRSCC versus reinforcing index of fiber for 35-60 MPa

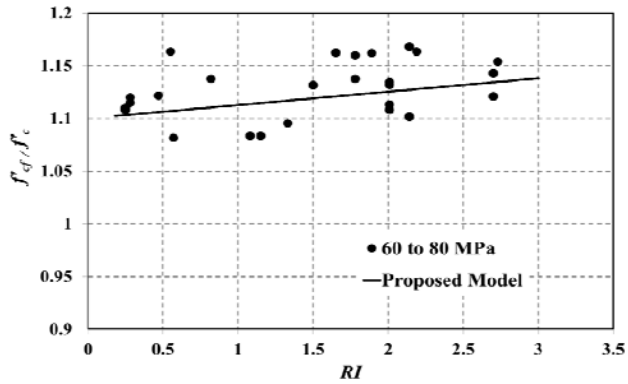

Fig. 1 (b) Proposed relationship for compressive strength of SFRSCC versus reinforcing index of fiber for 60-80 MPa

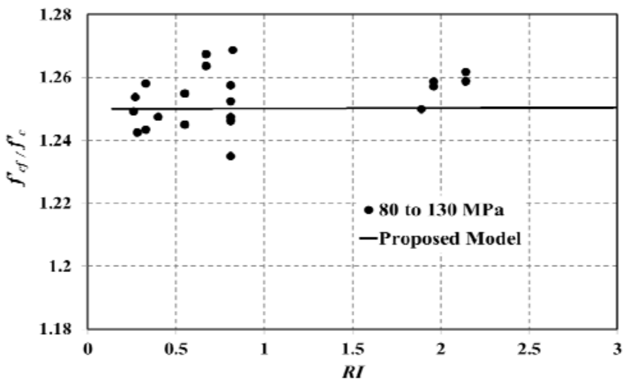

Fig. 1 (c) Proposed relationship for compressive strength of SFRSCC versus reinforcing index of fiber for 80-120 MPa

6.2 Tensile strength for the SFRSCC

SFRSCC tensile strength relationships are separated in three different range of SFRSCC compressive strength as: 35 to 60 MPa, 60 to 80 MPa, and 80 to 120 MPa. Regression analyses have been done on the raw SFRSCC tensile strength / normal tensile strength (f_{ctf} / f_{ct}) versus R.I. data. Normal tensile strength (without fiber) has important impact to consist available raw data for doing reliable nonlinear regression.

f'_{cf}: 35 to 60 MPa

$$f_{ctf} = a \times R.I. + b \times f_{ct} \tag{4}$$

Regression Variable Results					
Variable	**Value**	**Standard Error**	**t-ratio**	**Prob(t)**	
a	0.55	4.48E-02	12.29	0.049	
b	0.99	5.98E-02	16.65	0.0	
Variance Analysis					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	5.50	5.50	151.1	0.0485
Error	27	0.98	3.64E-02		
Total	28	6.49			

f'_{cf}: 60 to 80 MPa

$$f_{ctf} = a \times R.I. + b \times f_{ct} \tag{5}$$

Regression Variable Results					
Variable	**Value**	**Standard Error**	**t-ratio**	**Prob(t)**	
a	0.215	2.28E-02	9.43	0.050	
b	0.90	4.28E-02	20.98	0.0	
Variance Analysis					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	0.39	0.39	88.96	0.0485
Error	12	5.26E-02	4.38E-03		
Total	13	0.44			

f'_{cf}: 80 to 120 MPa

$$f_{ctf} = a \times R.I. + b \times f_{ct} \tag{6}$$

Regression Variable Results					
Variable	**Value**	**Standard Error**	**t-ratio**	**Prob(t)**	
a	0.407	3.23E-02	12.60	0.048	
b	1.14	3.27E-02	34.83	0.0	
Variance Analysis					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	1.49	1.49	158.91	0.0485
Error	21	0.195	9.42E-03		
Total	22	1.69			

Fig. 2 (a-c) shows proposed SFRSCC tensile strength relationship compared to the experimental results database for three different limitations of compressive strength 35 to 60 (MPa), 60 to 80 (MPa), and 80 to 120 (MPa).

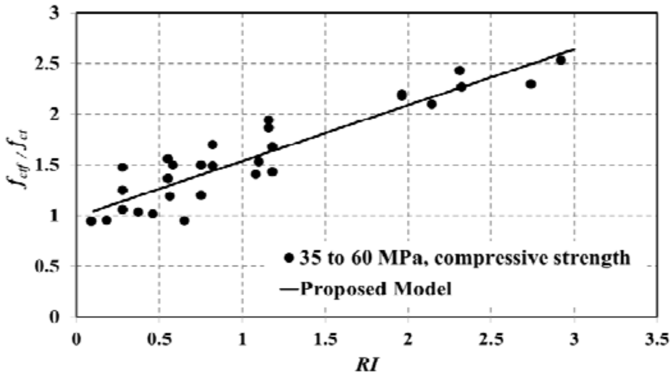

Fig. 2 (a) Proposed relationship for tensile strength of SFRSCC versus reinforcing index of fiber for 35-60 MPa

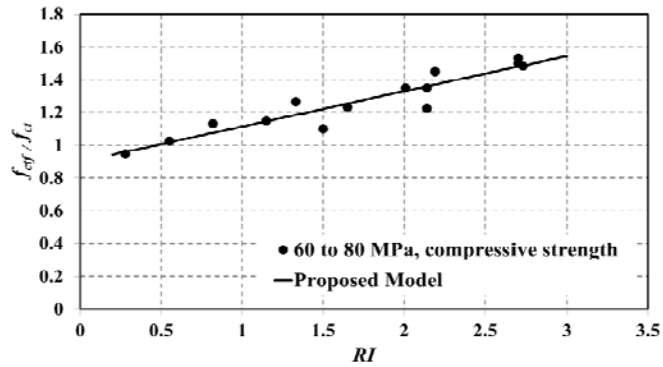

Fig. 2 (b) Proposed relationship for tensile strength of SFRSCC versus reinforcing index of fiber for 60-80 MPa

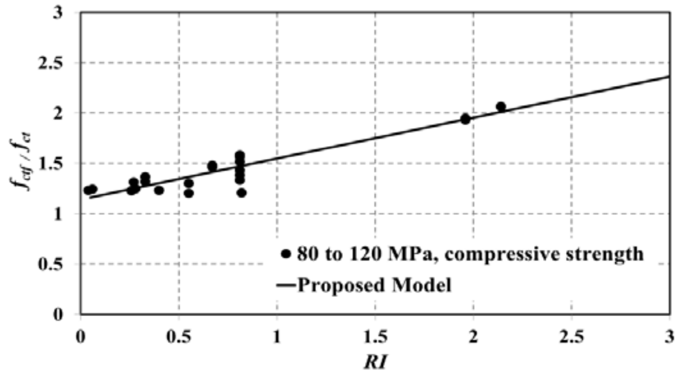

Fig. 2 (c) Proposed relationship for tensile strength of SFRSCC versus reinforcing index of fiber for 80-120 MPa

6.3 Modulus of elasticity for the SFRSCC

Regression analyses have been done on the raw SFRSCC modulus of elasticity / normal modulus of elasticity (E_{cf} / E_c) versus R.I. data. Normal modulus of elasticity (without fiber) has important impact to consist available raw data for doing reliable nonlinear regression.

$$E_{cf} = a \times R.I. + b \times E_c \tag{7}$$

Regression Variable Results					
Variable	**Value**	**Standard Error**	**t-ratio**	**Prob(t)**	
a	2.42E-02	8.92E-03	2.71	0.0475	
b	1.25	9.25E-03	135.86	0.0	
Variance Analysis					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	6.62E-03	6.62E-03	7.35	0.0473
Error	28	2.52E-02	9.00E-04		
Total	29	3.18E-02			

Fig. 3 shows proposed SFRSCC modulus of elasticity relationship compared to the experimental results database by considering corresponding compressive strength to the each modulus of elasticity.

6.4 Strain at peak stress for the SFRSCC

Regression analyses have been done on the raw SFRSCC compressive strength versus strain at peak stress data. Strain at peak stress for the SFRSCC model is developed based on this analyses as Eq. (8).

$$\varepsilon'_{cf} = a \times f'_{cf} + b \times \varepsilon_c \tag{8}$$

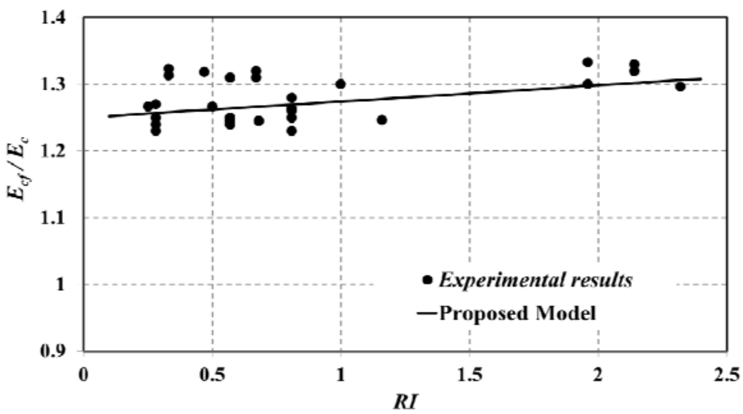

Fig. 3 Proposed relationship for modulus of elasticity of SFRSCC versus reinforcing index of fiber

Regression Variable Results					
Variable	**Value**	**Standard Error**	**t-ratio**	**Prob(t)**	
a	-8805.66	2558.91	-3.44	0.051	
b	84.74	7.45	11.36	0.0	
Variance Analysis					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**	**Prob(F)**
Regression	1	949.42	949.42	11.84	0.052
Error	28	2244.94	80.176		
Total	29	3194.365139748			

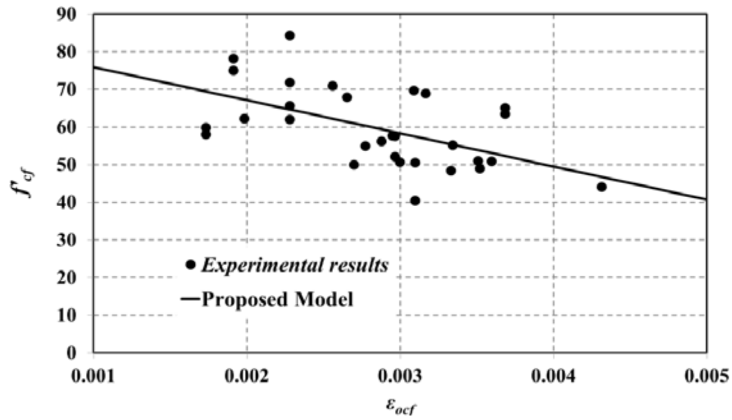

Fig. 4 Proposed relationship for strain at peak stress of SFRSCC versus compressive strength

Fig. 4 shows proposed SFRSCC strain at peak stress relationship compared to the experimental results database by considering corresponding compressive strength to the each strain at peak stress.

7. Proposed stress-strain relationship for SFRSCC

7.1 Compressive stress-strain relationship

The proposed compressive envelope curve is based on Aslani and Jowkarmeimandi (2012), as given by Eqs. (9-16). Also, proposed compressive stress-strain relationship does not allow performing a cracking analysis. In this normal and high strength SFRSCC compressive stress-strain relationship, the SFRSCC compressive strength (f'_{cf}) as Eqs. (1-3), SFRSCC modulus of elasticity (E_{cf}) as Eq. (7) and strain at peak stress (ε'_{cf}) as Eq. (8) are used

$$\frac{\sigma_{cf}}{f'_{cf}} = \frac{n\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)}{n-1+\left(\frac{\varepsilon_c}{\varepsilon'_{cf}}\right)^n} \tag{9}$$

$$n = n_1 = \left[1.02 - 1.17\left(E_{sec} / E_{cf}\right)\right]^{-0.74} \quad if\ \varepsilon_c \le \varepsilon_{cf}^{'} \tag{10}$$

$$n = n_2 = n_1 + \left(\gamma + 28 \times \mu\right) \quad if\ \varepsilon_c \ge \varepsilon_{cf}^{'} \tag{11}$$

$$\gamma = 3 \times \left(12.40 - 0.0166 f_{cf}^{'}\right)^{-0.46} \tag{12}$$

$$\mu = 0.83\, exp\left(-911 / f_{cf}^{'}\right) \tag{13}$$

$$E_{sec} = f'_{cf} / \varepsilon'_{cf} \tag{14}$$

$$\varepsilon'_c = \left(\frac{f'_c}{E_c}\right)\left(\frac{\psi}{\psi - 1}\right) \tag{15}$$

$$\psi = \frac{f'_c}{17} + 0.8 \tag{16}$$

Figs. 5-11 show comparisons between Liao *et al*. (2006) (SFRSCC3 to SFRSCC6 mixtures), Cunha (2006) (SFRSCC1 and SFRSCC2 mixtures), and Dhonde *et al*. (2007) (SFRSCC2 and SFRSCC3 mixtures) experimental results and available compressive fiber reinforced stress-strain relationships database (Tables 4-5).

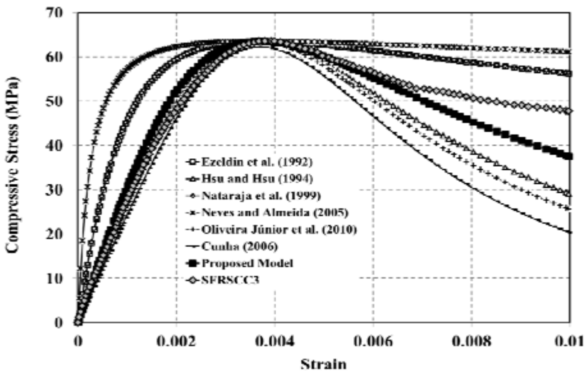

Fig. 5 Comparison between Liao *et al*. (2006), SFRSCC3 mixture, experimental test with compressive stress-strain relationships

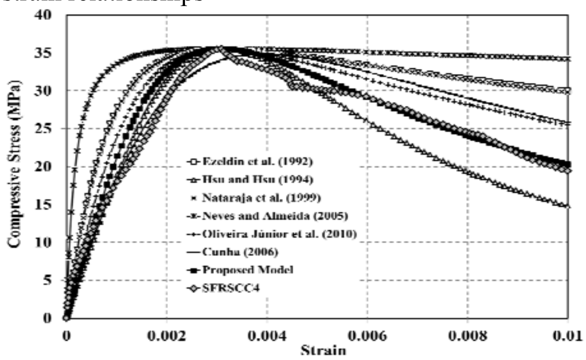

Fig. 6 Comparison between Liao *et al*. (2006), SFRSCC4 mixture, experimental test with compressive stress-strain relationships

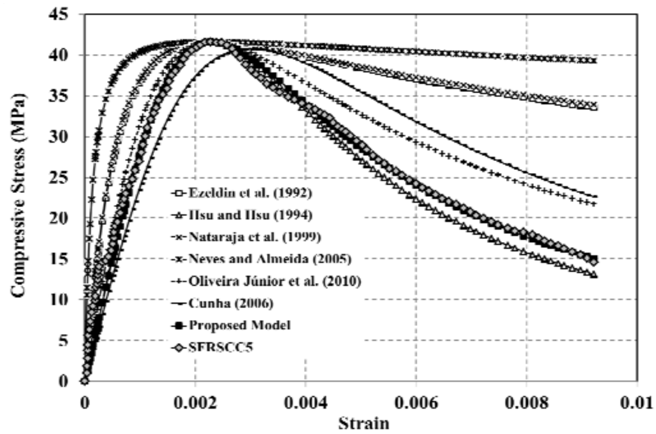

Fig. 7 Comparison between Liao *et al*. (2006), SFRSCC5 mixture, experimental test with compressive stress-strain relationships

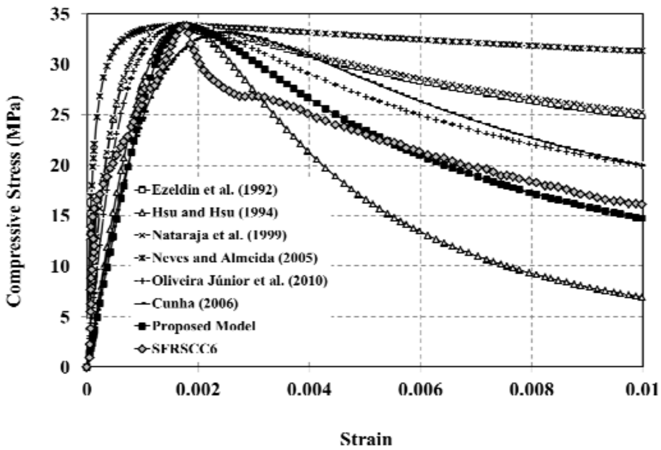

Fig. 8 Comparison between Liao *et al*. (2006), SFRSCC6 mixture, experimental test with compressive stress-strain relationships

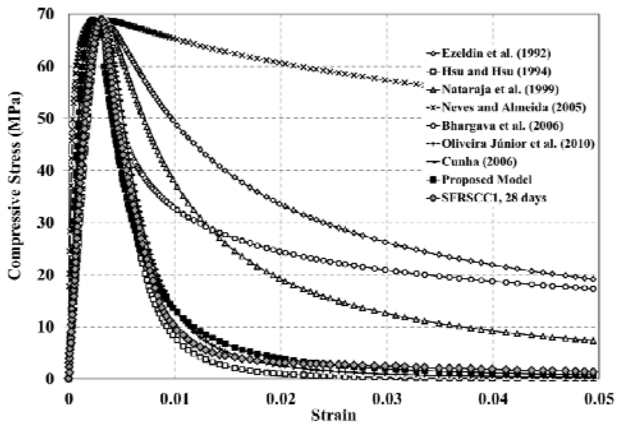

Fig. 9 Comparison between Cunha (2006), SFRSCC1 mixture, experimental test with compressive stress-strain relationships

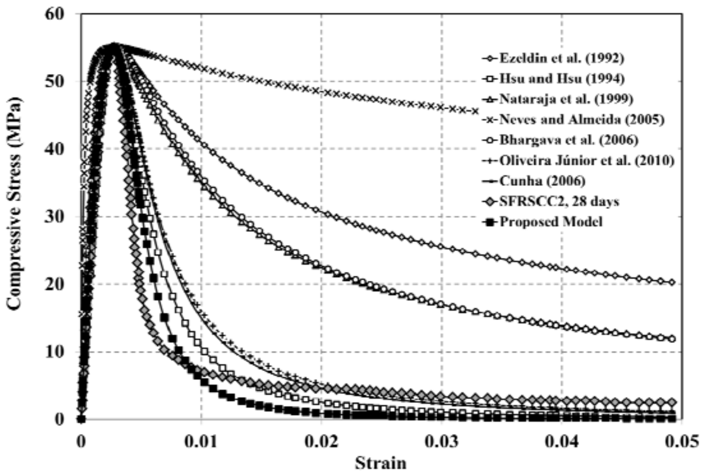

Fig. 10 Comparison between Cunha (2006), SFRSCC2 mixture, experimental test with compressive stress-strain relationships

7.2 Compressive stress-strain relationship

Much less attention has been directed towards the modeling of SFRSCC under tensile loading. Several expressions have been documented in the literature to represent the softening branch, including straight lines (Bažant and Oh 1983), polylinear curves (Gustafsson 1985, Gylltoft 1983, Hillerborg *et al.* 1976, Rots *et al.* 1985, Petersson 1981), exponential curves (Gopalaratman and Shah 1985), polynomial curves (Lin and Scordelis 1975), Yankelevsky and Reinhardt 1987, 1989), combinations of them (Cornelissen *et al.* 1985), a continuous damage-based formulation to represent post-peak stress-strain curves of concrete (Mazars 1981) and tension softening in terms of prescribed drops (Scanalon 1971). The proposed tensile envelope curve is a very simple model, as given by Eqs. (17)-(19). In this normal and high strength SFRSCC tensile stress-strain relationship, the SFRSCC tensile strength (f'_{ctf}) as Eqs. (4)-(6), SFRSCC modulus of elasticity (E_{cf}) as Eq. (7) are used

$$\sigma_{ctf} = E_{cf}\ \varepsilon_{ct} \qquad \varepsilon_{ct} < \varepsilon^* \tag{17}$$

$$\sigma_{ctf} = f_{ctf} \qquad \varepsilon_{ct} = \varepsilon^* \text{ to } 5.66\,\varepsilon^* \tag{18}$$

$$\sigma_{ctf} = f_{ctf}\left(\frac{5.66\,\varepsilon^*}{\varepsilon_{ct}}\right)^{0.78} \qquad \varepsilon_{ct} > 5.66\,\varepsilon^* \tag{19}$$

where ε^* is the corresponding strain to the 0.85 f_{ctf} and ε_{ct} is the tensile concrete strain in general, and constant factor "5.66" is the factor that shows tensile stress is reached to maximum tensile strength equal to "5.66 ε^*". Fig. 12 shows comparisons between Liao *et al.* (2006) (SFRSCC2 to SFRSCC6 mixtures) experimental results and proposed tensile fiber reinforced stress-strain relationship.

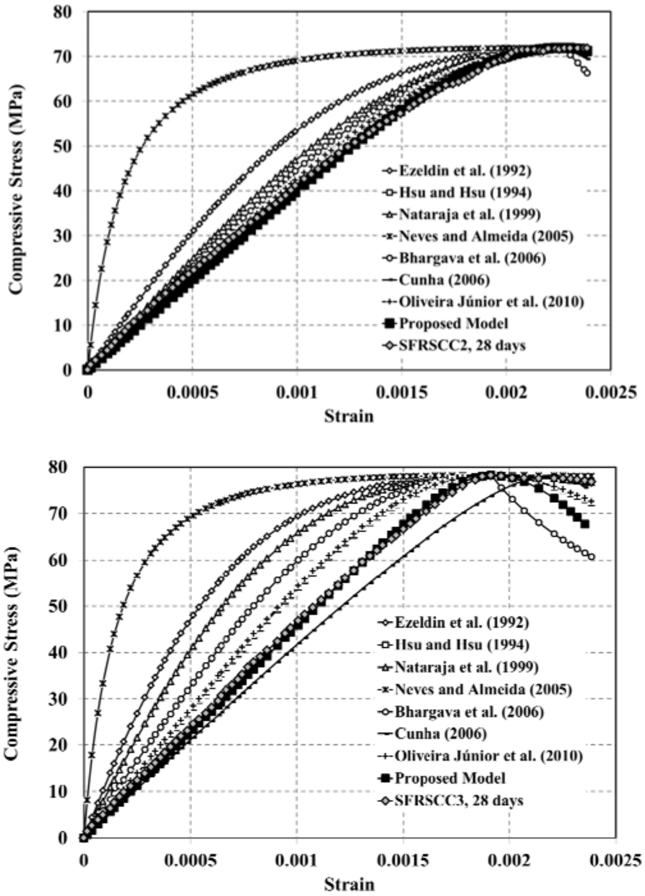

Fig. 11 Comparison between Dhonde *et al*. (2007), SFRSCC2 and SFRSCC3 mixtures, experimental test with compressive stress-strain relationships

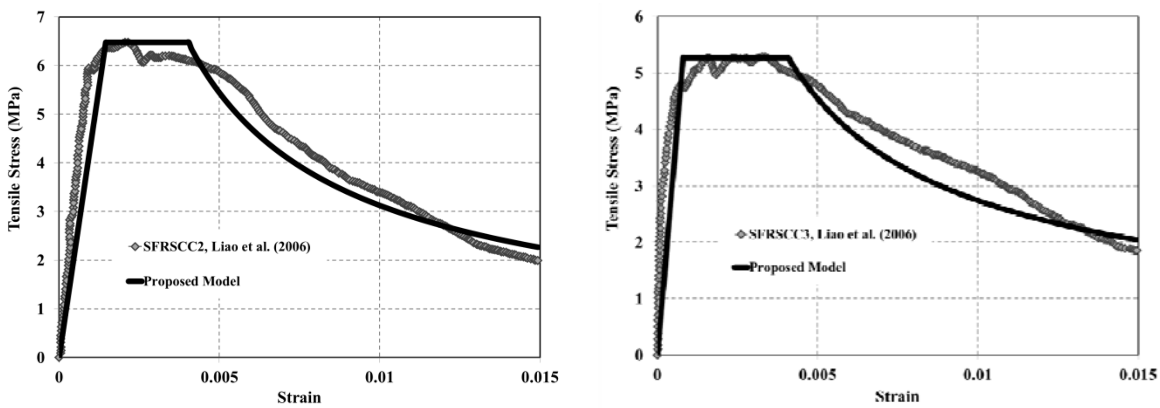

Fig. 12 Comparison between Liao *et al*. (2006), SFRSCC2- SFRSCC6 mixtures, experimental test with proposed tensile stress-strain relationship

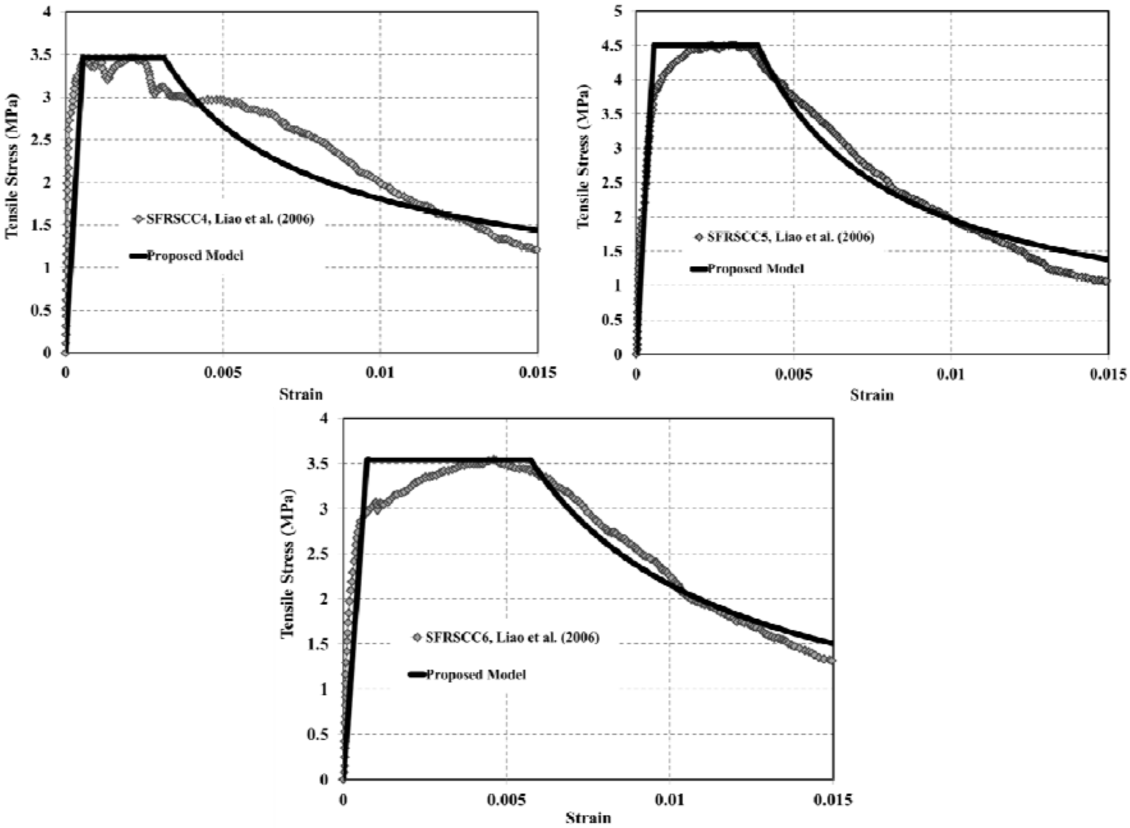

Fig. 12 Continued

8. Results and discussions

Experimental results database (Tables 1-3) shows that the major cement type that is used is ordinary Portland cement (ASTM C150-04), the major fillers that are used in the mix designs are fly ash and limestone, and compressive strength test specimen type is variable. Major aggregate types that used are crushed limestone, natural coarse aggregate and natural sand. Major type of fiber is Dramix and major shape is hooked end with different lengths and aspect ratios.

In this study, the compressive strength test specimen type of 100 mm × 200 mm cylindrical is considered as main compressive strength and other types of test results (i.e., 150 mm × 300 mm cylindrical, 100 mm cube, and 150 mm cube) must convert to it. Yi *et al*. (2006) reported that the relationship between 100 mm × 200 mm cylindrical with 150 mm cube was: $f'_{cy}(100 \times 200) = (f'_{cu}(150)-8.86)/0.85$ and the relationship between 100 mm × 200 mm cylindrical with 100 mm cube was: $f'_{cy}(100 \times 200) = (f'_{cu}(100)-7.07)/0.95$. Also, Carrasquillo *et al*. (1981) stated that the average ratio of compressive strength of 150 mm × 300 mm to 100 mm × 200 mm cylinders was 0.9, regardless of strength and test age.

As shown in Fig. 1(a), the proposed 35 to 60 (MPa) compressive SFRSCC strength relationship is developed based on the 40 values. In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.16, coefficient of multiple determination (R^2) is 0.17, proportion of variance explained is 9.65%, and Durbin-Watson statistic is 0.96. Also, presented regression variable results have shown that Prob(t) for *a* constant factor is 0.050. The proposed 60 to 80 (MPa) compressive SFRSCC strength relationship is developed based on the 26 values (see Fig. 1(b)). In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.11, coefficient of multiple determination (R^2) is 0.15, proportion of variance explained is 15.09%, and Durbin-Watson statistic is 1.55. Also, presented regression variable results shown that Prob(t) for *a* constant factor is 0.049. Furthermore, as shown in Fig. 1(c), the proposed 80 to 120 (MPa) compressive SFRSCC strength relationship is developed based on the 23 values. In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.11, coefficient of multiple determination (R^2) is 0.10, proportion of variance explained is 0.008%, and Durbin-Watson statistic is 1.80. Presented regression variable results have shown that Prob(t) for *a* constant factor is 0.048.

The 35 to 60 (MPa) tensile SFRSCC strength relationship is developed based on the 29 values (see Fig. 2(a)). In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.84, coefficient of multiple determination (R^2) is 0.84, proportion of variance explained is 84.83%, and Durbin-Watson statistic is 1.23. Also, presented regression variable results have shown that Prob(t) for *a* constant factor is 0.049. Fig. 2(b) shows that the proposed 60 to 80 (MPa) tensile SFRSCC strength relationship is developed based on the 14 values. In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.87, coefficient of multiple determination (R^2) is 0.88, proportion of variance explained is 88.11%, and Durbin-Watson statistic is 1.96. Also, presented regression variable results have shown that Prob(t) for *a* constant factor is 0.050. Moreover, as shown in Fig. 2(c), the proposed 80 to 120 (MPa) tensile SFRSCC strength relationship is developed based on the 23 values. In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.87, coefficient of multiple determination (R^2) is 0.88, proportion of variance explained is 88.32%, and Durbin-Watson statistic is 1.19. Also, presented regression variable results have shown that Prob(t) for *a* constant factor is 0.048.

The proposed normal and high strength SFRSCC modulus of elasticity relationship is established based on the 29 values (see Fig. 3). In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.17, coefficient of multiple determination (R^2) is 0.20, proportion of variance explained is 20.79%, and Durbin-Watson statistic is 1.08. Also, presented regression variable results have shown that Prob(t) for *a* constant factor is 0.0475. Furthermore, as shown in Fig. 4, the proposed SFRSCC peak strain at maximum compressive strength relationship is developed based on the 29 values. In the nonlinear regression analysis, 11 nonlinear iterations are performed, adjusted coefficient of multiple determination (Ra^2) is 0.27, coefficient of multiple determination (R^2) is 0.29, proportion of variance explained is 29.72%, and Durbin-Watson statistic is 2.32. Also, presented regression variable results have shown that Prob(t) for *a* constant factor is 0.051. Although, the available peak strain data is limited but application of its relationship for using in proposed compressive stress-strain relationship is appropriate.

Ezeldin *et al*. (1992), Nataraja *et al*. (1999), Mansur *et al*. (1999), Neves and Almeida (2005), and Bhargava *et al*. (2006) relationships have not good prediction in both ascending and descending branches of stress-strain curve compare with Liao *et al*. (2006) experimental results for all mixtures (see Figs. 5-8). In comparison with Cunha (2006) experimental results, these relationships have good prediction in ascending branch but in descending portion are overestimated (see Figs. 9-10). Also, these relationships compare to Dhonde *et al*. (2007) experimental tests are overestimated (see Fig. 11). Hsu and Hsu (1994) relationship has good prediction in ascending branch but for descending branch prediction compared with Liao *et al*. (2006) experimental results for all mixtures, it is underestimated (see Figs. 5-8). In comparison with Cunha (2006) experimental results, it has a good prediction in both ascending and descending portions (see Figs. 9-10). Also, this relationship compared to Dhonde *et al*. (2007) experimental tests (it is just for ascending portion) is overestimated (see Fig. 11).

Oliveira Júnior *et al*. (2010) and SFRSCC Cunha (2006) compressive stress-strain relationships have good prediction in ascending branch but for descending branch prediction compared with Liao *et al*. (2006) experimental results for all mixtures are underestimated except for SFRSCC3 mixture (as shown in Figs. 5-8). In comparison with Liao *et al*. (2006) experimental results, these relationships have good prediction in both ascending and descending portions. Also, these relationships compared to Dhonde *et al*. (2007) experimental tests are overestimated (as shown in Fig. 11). The compressive SFRSCC stress–strain relationship suggested in this study calculates the ascending branch of the stress–strain curve in comparison with Liao *et al*. (2006), Cunha (2006) and Dhonde *et al*. (2007) appropriately. Also, it calculates the descending branch within a minimum range of deviations with a reasonably accuracy. Besides, simple proposed tensile SFRSCC stress–strain relationship estimates Liao *et al*. (2006) experimental results for both ascending and descending portions (as shown in Fig. 12) in accurate manner.

9. Conclusions

Summarizing the obtained results following conclusions can be drawn from this study:

• Available SFRC and SFRSCC compressive stress-strain relationships prediction are not in good agreement with the SFRSCC experimental results especially descending portion of stress-strain curve.

• Nonlinear regression analyses have been conducted in order to develop SFRSCC mechanical properties (e.g., compressive strength, tensile strength, modulus of elasticity, and strain at peak stress). The nonlinear regression results are presented by “regression variable results” and “variance analysis”.

• Regression variable results are shown with calculated values of variables, standard error of variables, t-ratios of variables and probability of t-ratios. Variance analyses indicated number of data points, number of parameters or variables in the regression model, SSR of regression, SSE of error, SST of total, mean squares of regression and error, and F-ratio. Appropriate relationships are developed based on the nonlinear analysis for SFRSCC mechanical properties and all required information about these relationships is presented.

• Normalized mechanical SFRSCC properties data (e.g., fiber compressive strength/ normal compressive strength (f'_{cf} / f'_c) versus R.I.) are more suitable and compatible for nonlinear regression analyses rather than just mechanical SFRSCC properties data (e.g., fiber compressive strength (f'_{cf}) versus R.I.).

• Proposed SFRSCC relationships are more valid for SFRSCC mixtures that used ordinary Portland cement, fly ash and limestone fillers, limestone and natural coarse aggregate and natural sand, and hooked end fibers.

• The suggested SFRSCC stress-strain compressive relationship is based on author model with several modifications (i.e. changing the ascending and descending portions). In this compressive stress-strain relationship for normal and high strength SFRSCC the proposed compressive strength, modulus of elasticity and strain at peak stress models are used.

• The proposed SFRSCC tensile envelope curve is a simple relationship based on the author model. In this normal and high strength SFRSCC tensile stress-strain model, the proposed tensile strength and modulus of elasticity models are used.

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