Optimum cost design of RC columns using artificial bee colony algorithm

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Abstract. Optimum cost design of columns subjected to axial force and uniaxial bending moment is presented in this paper. In the formulation of the optimum design problem, the height and width of the column, diameter and number of reinforcement bars are treated as design variables. The design constraints are implemented according to ACI 318-08 and studies in the literature. The objective function is taken as the cost of unit length of the column consisting the cost of concrete, steel, and shuttering. The solution of the design problem is obtained using the artificial bee colony algorithm which is one of the recent additions to metaheuristic techniques. The Artificial Bee Colony Algorithm is imitated the foraging behaviors of bee swarms. In application of this algorithm to the constraint problem, Deb's constraint handling method is used. Obtained results showed that the optimum value of numerical example is nearly same with the existing values in the literature.

Keywords: optimization; metaheuristic; artificial bee colony algorithm; reinforced concrete columns

1. Introduction

Since reinforced concrete structures are used extensively in the world, they are very important for civil engineers. In traditional design procedure shown in Fig. 1, the designer must verify problem requirements by mathematical analysis. If such requirements are not satisfied, then dimensions and/or reinforcement of RC elements are changed and a new solution is performed based on engineering perception. This repeated process consumes considerable time, until a suitable section is found. However optimal design procedure which consists of changing the design by minimizing an object function under some constraints is an alternative to the traditional design method (Fig. 2) (Coello Coello *et al.* 1997).

Haug and Kirmser (1967) used an iterative method based on generalized Newton's algorithm to solve statically determinate beams. The study was one of the first modern undertakings to use a digital computer as an optimal design tool. A method based on an energy criterion and a search algorithm based on constraint gradient values was developed by Venkayya (1971) for the design of structures under static loading. In the study the parameter to be minimized is the weight of the structural elements and his method is also applicable to the design of trusses, frames and beams.

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Karihaloo (1979) presented a solution of the problem of minimizing the maximum deflection of a simply supported beam under a transverse concentrated load. Sauma and Murad (1984) developed a method for minimizing the cost design of simply supported uniformly loaded partially prestressed concrete beams using the penalty functions method coupled with quasi-Newton unconstrained optimization techniques.

Zielinski (1995) used the internal penalty function for minimum cost design of reinforced concrete short tied rectangular columns based on Canadian Standard Specifications.Govindaraj and Ramasamy (2007) used Genetic Algorithm to optimize RC columns and compared the results with obtained in Zielinski's (1995) study. In these optimization problems strength, serviceability, ductility and side limitations are considered as the design constraints and they are implemented from Indian code of practice. Leroy Friel (1974) proposed an equation for RC beams to obtain optimum steel percentage and used moment strength constraints. Chou (1977) used Lagrange multiplier method to obtain minimum cost design of reinforced T beam sections based on ACI requirements. Kirsch (1983) presented three level iterative optimization procedure for multi span RC continuous beams. Prakash *et al.* (1988) were presented a model for optimal design reinforced concrete sections using Lagrangian and simplex methods. In their optimal design cost of steel, concrete and shuttering were taken into account. A similar but also detailed model developed by Chakrabarty (1992) and using geometric programming and Newton-Rapson method to minimize the cost. Al-Salloum and Siddiqi (1994) presented a closed form solution for steel area and depth of beam section to minimize the cost of RC beams. Coello Coello *et al.* (1997) presented a model using genetic algorithm for optimal design to minimize the cost of a rectangular reinforced concrete beam based on strength design procedures, but also considered the cost of concrete steel and shuttering. This work is one of the pioneering works on optimum design of RC structures using metaheuristic methods. Also in this paper we follow Coello Coello *et al.*'s and Chakrabarty's models to examine compatibility of our algorithm. Govindaraj and Ramasamy (2005) presented an
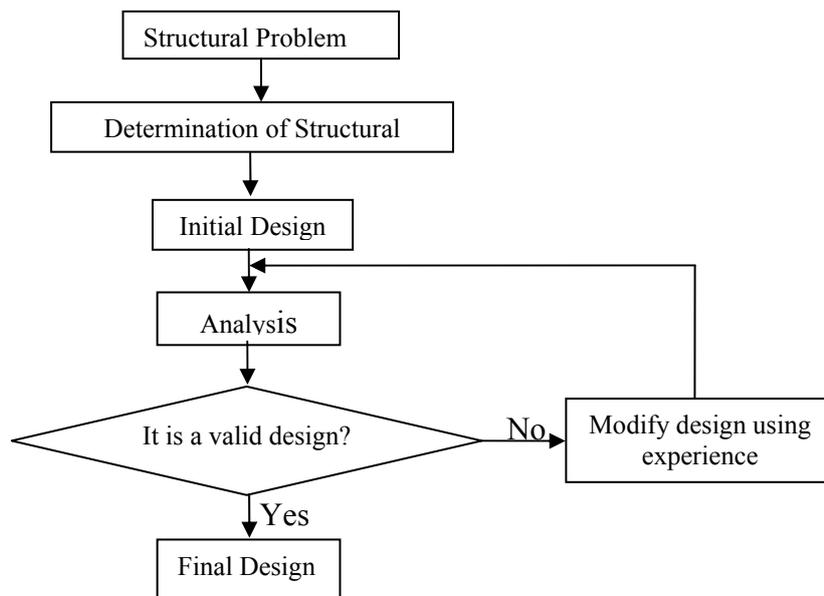

Fig. 1 Traditional design process (Coello Coello *et al.* 1997)

optimum design of RC continuous beams using Genetic Algorithm based on Indian standard specifications.

Optimum design of RC frames were investigated by many researchers and they used such methods as linear Programming (Krishnamoorthy and Munro 1973), optimality Criteria Method (Fadaee and Grierson 1996), Direct Search Method (Choi and Kwak 1990, Kwak and Kim 2008), Simulated Annealing (Balling and Yao 1997), Harmony Search (Akın 2010) and Genetic Algorithm (Rajaev and Krishnamoorthy 1998, Camp *et al.* 2003, Lee and Ahn 2003, Kwak and Kim 2009).

This proceeding focuses on the use of a metaheuristic artificial intelligence technique called Artificial Bee Colony (ABC) Algorithm (Karaboga and Basturk 2007, Degertekin 2012). The main purpose of this study is to present the optimal design with Artificial Bee Colony Algorithm that minimizes the cost of RC columns subjected to axial force and uniaxial bending moment, considering the cost of concrete, steel and shuttering.

2. Reinforced concrete design

The following basic assumptions are made when using strength design:
- Plane sections before bending remain plane after bending.
- At ultimate capacity, strain and stress are not proportional.
- Strains in the concrete is proportional to the distance from the neutral axis and the ultimate concrete strain is 0.003
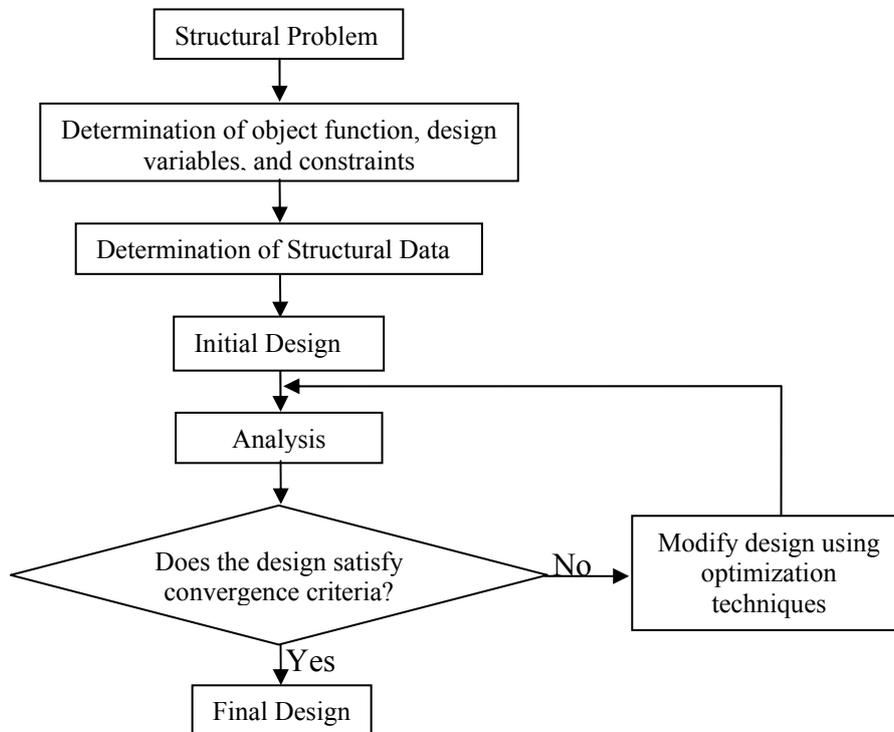

Fig. 2 Optimum design process (Coello Coello *et al.* 1997)

- The modulus of elasticity of the reinforcing steel is 200000 MPa.
- The average compressive stress in the concrete is 0.85 fc'. fc' is indicates the specific compressive strength of the concrete.
- The average tensile stress in the reinforcement doesn't exceed fy. fy is indicates the specific yield strength of reinforcement.

Moment capacity of RC columns subjected to uniaxial combined bending is calculated by trial and error method using Fig. 3 (Ersoy and Özcebe 2001). The method is summarized below.

1) Choose depth of neutral axis (c).
2) Calculate strains (ε_{si}) for reinforcement at different levels.
3) Calculate stresses ($\sigma_{si}=\varepsilon_{si}E_s\leq f_y$) and their components ($F_{si}=A_{si}\sigma_{si}$) for these reinforcements.
4) Calculate resultant compression stress of section as $C=0.85f_c'A_{cc}$
5) Check the equilibrium condition as $N = C + \sum\limits_{i=1}^{n} A_{si}\sigma_{si}$
6) If equilibrium is provided, go to step 7. If equilibrium is not provided, select new values for c by return to step 1 until equilibrium is provided.
7) Calculate nominal moment capacity (M_n) when equilibrium is provided.

3. Optimum design problem

General three phases are considered in the optimum design of a structure or structural element. These are structural modelling, optimum design modelling and the optimization algorithm. For optimum design modelling, design variables, objective function, constraints and constraint handling method are decided in this section.

3.1 Design variables

The design variables chosen for the formulation are related to cross sectional dimensions of the column, number of steel reinforcement bar and steel reinforcement bar area (Fig.4). It is also seen in Fig. 4 that design variables are cross sectional dimensions (b,h), reinforcement bar area (ø) and number of middle reinforcement bar in x and y directions (n_x ve n_y). The upper and lower bounds of these variables are seen in Table 1.
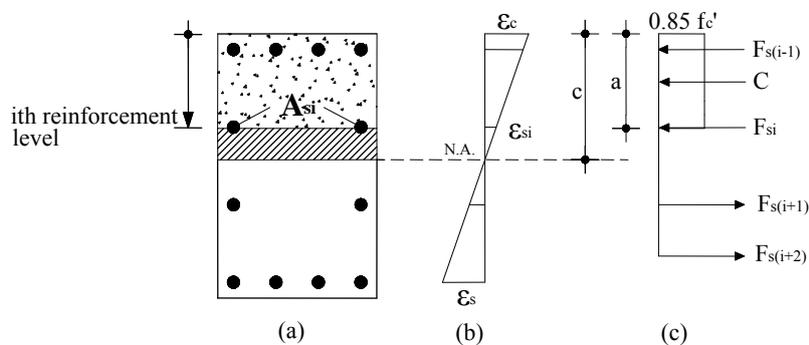

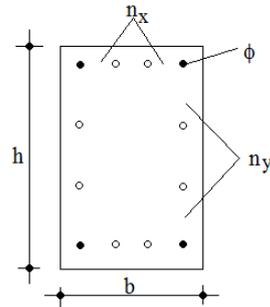Fig. 3 (a) Cross section, (b) strain diagram, (c) stresses and components

Fig. 4 Schematic cross section of column and design variables

Table 1 Bounds and increments of design variables

Design Variables	Lower Bound	Upper Bound	Increment	Number of possible values in the range
b (cm)	20	100	1 cm	81
h (cm)	20	100	1 cm	81
n_x and n_y	0	14	2	8
Ø (mm)	{14,16,18,20,22,25,28}		-	7

3.2 Objective function

Objective function is the total cost per unit length of the column consisting of cost components due to concrete, steel and shuttering. The cost per unit length of the column is calculated by the following expression

$$f_{cost}(x) = C_c V_c + C_s W_s + C_f A_f \tag{1}$$

Where C_c is the unit cost of concrete ($/m3), C_s is unit cost of steel ($/kg), C_r is the unit cost of shuttering ($/m2), V_c is the volume of concrete, W_s is the weight of steel reinforcement and A_f is the area of shuttering.

3.3 Constraints

Considered constraints are expressed in a normalized form as given below

$$
\left.
\begin{aligned}
& g_1(x) = \frac{N_u}{\Phi N_n} - 1 \le 0 \quad && g_2(x) = \frac{M_u}{\Phi M_n} - 1 \le 0 \quad && g_3(x) = \frac{\rho_{min}}{\rho} - 1 \le 0 \\
& g_4(x) = \frac{\rho}{\rho_{max}} - 1 \le 0 \quad && g_5(x) = \frac{(h/b)}{(h/b)_{max}} - 1 \le 0 \quad && g_6(x) = \frac{b_{min}}{b} - 1 \le 0 \\
& g_7(x) = \frac{h_{min}}{h} - 1 \le 0 \quad && g_8(x) = \frac{s_{min}}{s_x} - 1 \le 0 \quad && g_9(x) = \frac{s_x}{s_{max}} - 1 \le 0 \\
& g_{10}(x) = \frac{s_{min}}{s_y} - 1 \le 0 \quad && g_{11}(x) = \frac{s_y}{s_{max}} - 1 \le 0
\end{aligned}
\right\} \tag{2}
$$

Where Nu is the ultimate axial load normal to cross section, N_n is the nominal axial load capacity, Mu is the ultimate applied bending moment at cross section, M_n is the nominal moment strength and Φ is the strength reduction factor. ρ_{min} and ρ_{max} are the lower and upper limits on the reinforcement ratios, s_{min} and s_{max} are the minimum and maximum bar spacing respectively.

3.4 Constraint handling method

Most of the optimization problem; in science and engineering involve a number of constraints by which the optimal solution must be satisfied. A constraint optimization is usually written as

$$\begin{aligned} & \text{optimize} \, f(x) \\ & \text{Subject to} \quad g_j(x) \le 0 \quad i = 1,....,J \\ & \qquad\qquad\quad h_k(x) = 0 \quad k = 1,....,K \\ & \qquad\qquad\quad x_i^{min} \le x_i \le x_i^{max} \quad i = 1,....,n \end{aligned} \tag{3}$$

In these expressions there are n variables, J is the number of greater than and equal to type inequality constraints and K the number of equality constraints. If a solution in the algorithm is not satisfy these constraints, is required that using a constraint handling method to make a decision about choosing a new evaluated solution instead of old solution or not. Constraint handling methods used with evolutionary algorithm can be classified into some categories such as: (1) Methods based on preserving feasibility of solution, (2) Methods based on penalty functions, (3) Methods making distinction between feasible and infeasible solutions, (4) Methods based on decoders, and (5) Hybrid methods (Deb 2000, Coello Coello 2002).

In this paper Deb's constraint handling method is used. This method proposes to use a tournament selection operator, where two solutions are compared at a time and three criteria are always enforced such as; (1) Any feasible solution is preferred to any infeasible solution, (2) Among two feasible solutions, the one having better objective function value is preferred, and (3) among two infeasible solutions, the one having smaller constraint violation is preferred (Deb 2000).

3.5 Artificial bee colony algorithm

Behavior of real bees was modeled by Tereshko (2000). This model consists of three essential components such as; food sources, employed and unemployed bees. Bees select a food source according to its closeness, richness and taste of the nectar, ease of extracting this nectar. Employed bees employed at a specific food source which is discovered before. They carry information about distance, the direction and profitability of the source and share it with the other bees in the hive. Unemployed bees are divided into two groups. One of the groups is called scout bees who search the environment randomly and the other group called onlookers who try to find a food source by means of the information given by the employed bees.

These foraging behaviors of bee swarms are imitated by several algorithms such as; Bee Colony Optimization (Teodorovic 2003, Teodorovic and Orco 2005), Virtual Bee (Yang 2005), Bee (Pham *et al.* 2006), and Artificial Bee Colony (Karaboga and Basturk 2007, Singh 2009, Sonmez 2011) algorithms. Although all the algorithms are basically similar, there are some differences between them (Karaboga and Basturk 2008).

In Artificial Bee Colony (ABC) algorithm, each food source corresponds to a possible solution of a given optimization problem. Quality and the location of the food source represent fitness of solution and design variables respectively. First half of all bees consist of the employed bees and second half includes onlooker bees. At the beginning, algorithm generates random solutions for all bees. This operation can be defined as

$$x_{ij} = x_j^{min} + rand(0,1)\left(x_j^{max} - x_j^{min}\right) \tag{4}$$

where, i=1,2…SN and j=1,2…D, D is the number of design variables and SN is the number of employed or onlooker bees.

Then employed bees determine candidate food sources in the neighbourhood of the food sources in their memory and evaluate its nectar amount. When they produce a candidate food source, algorithm uses following expression

$$v_{ij} = \begin{cases} x_{ij} + \theta_{ij}\left(x_{ij} - x_{kj}\right) & R_j < MR \\ x_{ij} & \text{otherwise} \end{cases} \tag{5}$$

where, $k \in \{1,2,…,SN\}$ and $j \in \{1,2,…,D\}$ are randomly chosen indexes, but k must be different from i. θ_{ij} is a random number between [-1,1]. This parameter controls the production of neighbor food sources around x_{ij}. R_j is a random number between [0,1] and MR is a control parameter between [0,1].

Employed bees share their information with onlooker bees in the hive and onlooker bees select one of the food sources depending on the information given by the employed bees. After this step onlooker bees produce candidate food sources, according to probability value of the old sources, in the neighbourhood of the food source chosen by them. In other words onlooker bees select a food source according to a probability proportional to the amount of nectar and constraint violations. Probability value calculated by

$$p_i = \begin{cases} \left(0.5 + \left(\dfrac{fitness_i}{\sum\limits_{j=1}^{SN} fitness_j}\right)\right) \cdot 0.5 & \text{if constraints are satisfied} \\[3em] \left(\left(1 - \dfrac{violation_i}{\sum\limits_{j=1}^{SN} violation_j}\right)\right) \cdot 0.5 & \text{if constraints aren't satisfied} \end{cases} \tag{6}$$

Where, violation$_i$ is the constraint violation of ith food source (solution). Fitness of a food source is determined as

$$fitness_i = 1/\left(1 + f_i\right) \tag{7}$$

Where, f_i is the value of objective function for ith solution. In these equations, probability values of infeasible solutions are between 0 and 0.5 while those of feasible ones are between 0.5 and 1. By a selection mechanism like roulette wheel, solutions are selected probabilistically proportional to their fitness values in case of feasible solutions and inversely proportional to their violation values in case of infeasible solutions. Thus, solutions in feasible region have a threshold

value of 0.5. (Karaboga and Akay 2011).

Produced new solutions performance is compared with that of its old one. If the new food source has an equal or better nectar than the old source, it is replaced with the old one in the memory. If there is no improvement in the amount of nectar from a source after predefined iteration number (LIMIT), this source is discarded and its employed bee becomes scout bee. A new food source is randomly generated by the scout bee. This process is repeated until the iteration is reached a predefined maximum cycle number (MCN) or a termination criterion is satisfied (Karaboga and Basturk 2009). Another optimization parameter called Scout Production Period (SPP) which is the period of scout bee production used in the algorithm.

4. Numerical example

We selected three column examples from the literature to demonstrate the effectiveness and efficiency of optimum design algorithm using artificial bee colony algorithm developed for RC columns subjected to uniaxial combined bending. These problems were solved by Zelinski *et al*. (1997) using two part Penalty Function optimization technique according to Canadian Standard Specifications. Govindaraj and Ramasamy (2007) used Genetic Algorithm Indian provisions for RC design. And also these problems were designed according to ACI 318-05 by Akın and Saka (2010) using Harmony Search Algorithm.

Data used in three examples are presented in Table 2. Costs are taken from the literature as 2.1$/daN, 110$/m3 and 27 $/m2 for steel, concrete and shuttering respectively. Unit weight of steel and concrete are taken 78.5 kN/m3 and 25 kN/m3 respectively. Note that ACI 318-05 is used for reinforced concrete design. Minimum and maximum reinforcement ratios are taken 0.008 and 0.06 respectively. Minimum and maximum clear distances between reinforcement bars, minimum width of column section and maximum aspect ratio of column section dimentions are taken 50mm, 300mm, 200mm and 3 respectively as the studies in the literature (Zelinski *et al*. 1995, Govindaraj and Ramasamy 2007, Akın and Saka 2010)

The optimization software based on the Artificial Bee Colony Algorithm was developed using MATLAB (2008) version 7.6.0. It was run on a personal computer with a Pentium Dual Core 2.0 GHz processor and 3 GB memory under The Microsoft Windows Vista operating system. To investigate the effect of colony size on the convergence rate of the Artificial Bee Colony Algorithm, four different colonies consisting of 10, 20, 30, and 40 bees were used. Twenty independent runs were performed for the each colony sizes. Since the convergence rates for all sizes are very close to each other, the colony size may be set at any value between 10 and 40 for this problem.

Maximum cycle numbers (MCN) and colony size are selected as 3000 and 20 respectively. Karaboğa and Basturk (2007) purposed useful ranges for optimization parameters MR, SPP, and LIMIT such as between 0.3 to 0.8 for MR, $0.5 \cdot SN \cdot D$ and $SN \cdot D$ for SPP, and $0.1 \cdot SN \cdot D$ and $2 \cdot SN \cdot D$

Table 2 Some data about examples

	fc'	fy	Cover	Nu	Mu
	N/mm2	N/mm2	mm	kN	kNm
Example 1	25	400	65	2460	443
Example 2	30	400	70	1780	362
Example 3	30	400	75	2000	562

for LIMIT. In this study MR, SPP and LIMIT parameters were selected as 0.7, 30 and 60 respectively.

Comparison of nonlinear programming approach used by Zelinski *et al.* (1995), Genetic Algorithm used by Govindaraj and Ramasamy (2007), Harmony Search Algorithm used by Akın and Saka (2010) and Artificial Bee Colony Algorithm are presented in Table 3. Optimization history and axial force-moment interaction diagrams for optimum designs are presented in Fig.5 and Fig. 6 respectively.

Table 3 Comparison of optimum results

| | Studies | Design Variables | | | | | Cost $/m |
		b	h	ø	n_x	n_y	
Example 1	Zelinski *et al.* (1995)	400	700	3000 mm^2			139.65
	Govindaraj and Ramasamy (2007)	325	900	14	0	0	138.94
	Akın and Saka (2010)	375	750	16	4	4	131.46
	This study	290	860	16	0	6	122.68
Example 2	Zelinski *et al.* (1995)	350	600	3000 mm^2			123.85
	Govindaraj and Ramasamy (2007)	300	750	16	2	4	114.61
	Akın and Saka (2010)	275	725	16	2	2	102.45
	This study	250	730	18	0	2	98.16
Example 3	Zelinski *et al.* (1995)	500	500	7500 mm^2			205.14
	Govindaraj and Ramasamy (2007)	350	825	16	4	4	135.00
	Akın and Saka (2010)	325	825	14	4	6	127.12
	This study	300	830	16	2	4	121.56
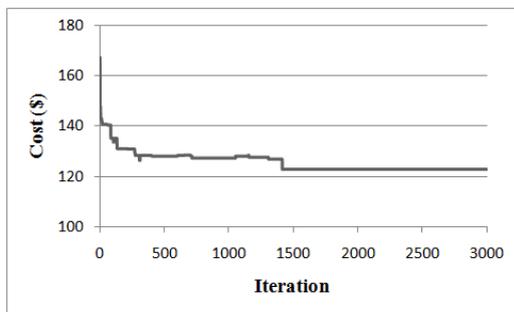
(a) Example 1
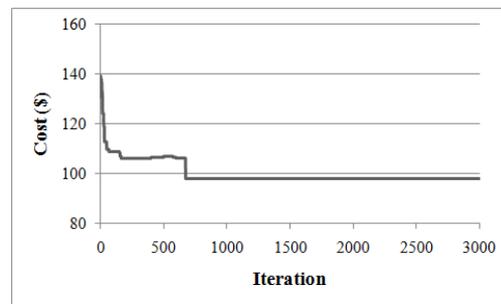
(b) Example 2
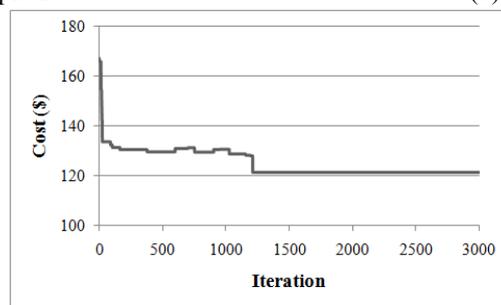
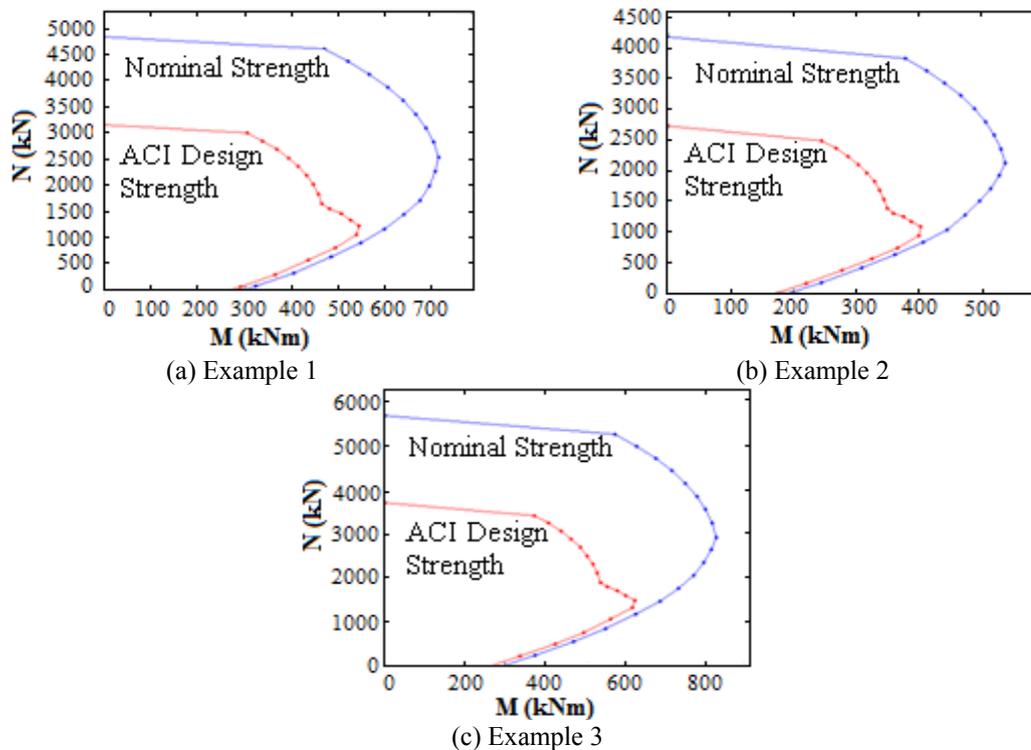(c) Example 3
Fig. 5 Convergence histories

(a) Example 1   (b) Example 2

(c) Example 3

Fig. 6 Axial force-moment interaction diagrams for optimum designs

5. Conclusions

The artificial bee colony (ABC) algorithm, based on mimicking the behavior of honeybee swarms, is proposed as a method of optimization of singly reinforced concrete rectangular beams. Deb's method, which makes distinction between feasible and infeasible solutions, is used as the constraint handling method. The objective function is the cost of the column for unit length. Constraints and material costs were taken from literature. Obtained results were compared with their results. Bee colony size effect is investigated and it is presented that the colony size may be set at any value between 10 and 40. The design algorithm performs effectively in finding the optimum values of design variables. Because the results are nearly same with compared studies, the ABC algorithm is an effective tool for optimization of RC columns subjected to uniaxial combined bending.

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