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Analysis of trusses by total potential optimization method coupled with harmony search

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Abstract. Current methods of analysis of trusses depend on matrix formulations based on equilibrium equations which are in fact derived from energy principles, and compatibility conditions. Recently it has been shown that the minimum energy principle, by itself, in its pure and unmodified form, can well be exploited to analyze structures when coupled with an optimization algorithm, specifically with a meta-heuristic algorithm. The resulting technique that can be called Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA) has already been applied to analyses of linear and nonlinear plane trusses successfully as coupled with simulated annealing and local search algorithms. In this study the technique is applied to both 2-dimensional and 3-dimensional trusses emphasizing robustness, reliability and accuracy. The trials have shown that the technique is robust in two senses: all runs result in answers, and all answers are acceptable as to the reliability and accuracy within the prescribed limits. It has also been shown that Harmony Search presents itself as an appropriate algorithm for the purpose.

Keywords: meta-heuristics; harmony search; total potential optimization method; truss; nonlinearity; robustness

1. Introduction

Recent decades have witnessed the introduction of many meta-heuristic methods for solving optimization problems. Harmony search (HS), which has been developed recently (Geem *et al.* 2001), has proved to be very efficient in solving many kinds of engineering optimization problems. It is a memory based random search method simulating musical improvisation. Similar to other meta-heuristic methods, it does not involve complex mathematical operations, the functions to be optimized need not be differentiable, and the constraints can be in the forms of equalities and/or inequalities. Since the roots of the method are in finding a harmony among its variables which are musical notes, HS was first designed for discrete problems, but it has also been successfully applied to problems with continuous variables (Lee *et al.* 2005, Lee and Geem 2005).

Revisions have been made to improve the method by Mahdavi *et al.* (2007) and Omran and Mahdavi (2008) under the names “improved harmony search” and “global – best harmony search”.

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Other improvements have taken place on adaptive determination of the parameters of the method (Hasancebi *et al.* 2010, Geem and Sim 2010) thus eliminating excessive a-priori knowledge about the problems and experience about the algorithm. Very recently, the explorative power of HS is analyzed and a modified form of HS, called Explorative HS (EHS), is introduced with the aim of outperforming preceding versions in solution quality and CPU time (Das *et al.* 2011).

For structural problems, HS has been applied successfully to optimum design of truss structures (Lee and Geem 2004), frames (Degertekin 2008, Saka 2009) grillage systems (Erdal and Saka 2009), and tuned mass dampers (Bekdaş and Nigdeli 2011). All these applications concern the design aspect of structures, and in all of them, whenever necessary, structural analyses were carried out using the very popular Finite Element Method (FEM).

It has been shown in the present study that structures can be analyzed through the HS method without having recourse to FEM or to any other classical matrix method. The technique is based on a very fundamental principle in mechanics which states that the statical equilibrium configuration of a structure corresponds to a minimum potential energy state. This principle is very well known and is applied in different ways for analyzing some simple structures under statical loadings (see for example Timoshenko and Gere 1961 and Oden 1967). Other applications include works on cable structures (see for example Buchholt (1985), Sufian and Templeman (1992, 1993)), geometrically nonlinear truss analyses (Rezaiee-Pajand and Naghavi 2011, Greco *et al.* 2012). But its combination with meta-heuristic methods gave rise to a method called the Total Potential Optimization with Meta-heuristic Methods (TPO/MA) which enabled problems involving profound geometric and material nonlinearities to be solved (Toklu 2004, Saffari *et al.* 2008).

2. Harmony Search

The HS method can be explained briefly as in the pseudo code given below:

- Choose a range R of notes
- Compose p number of partitions using notes in R
- While not satisfied
    - Compose $p+1^{st}$ partition applying special rules of HS
    - Determine the worst of the partitions among the $p+1$ ones and eliminate it
    - Decide if satisfied or not
- If satisfied, output the best partition and end

A partition in HS corresponds to a candidate solution vector. Thus, choosing p partitions actually means forwarding p candidate solutions for the given problem. The $p+1^{st}$ partition is in fact a candidate which is expected to be better than the p previous ones, or at least not worse than all of them. HS then imposes that the worst of these $p+1$ vectors will be wiped out to leave a new set of p vectors which is better than or the same as the previous set. The procedure continues until it can be said that no better sets can be obtained or some other stopping criterion, related to the number of iterations or the accuracy reached, becomes satisfied.

The determination of better and worse is done by calculating the objective function corresponding to each of these vectors. This evaluation will include the penalties calculated and added to the objective function in accordance with the constraints unsatisfied (Statnikov *et al.*

2009). The counterpart of this mathematical evaluation in music is the appreciation of the audience.

The special rules of HS mentioned above are important when defining a new partition, i.e. the $p+1^{st}$ vector. Any element of this vector will be chosen either from the set of corresponding elements of the existing p vectors, or independent of them, from the whole domain defined. If the first choice prevails, the value of the element may or may not be changed a small amount. This last choice corresponds to the musical interpretation that either the note will be played as is, or will be used with small variations, half a note above or half a note below. The above described procedure is accomplished through the following two parameters (Geem *et al.* 2001):

- *HMCR*: Harmony memory considering rate, probability with which the already chosen "notes" will be used in the new "partition" (with *1-HMCR* probability a note will be chosen from the whole range *R*).
- *par*: Pitch adjusting rate, probability that a component will be used with minor changes in case it is preferred to notes in the whole range (with *1-par* probability, value will be used as it is, without any change).

3. Total Potential Optimization Method for Structural Analysis of Trusses

As mentioned in the above sections, meta-heuristic methods have been recently applied to structural analysis problems defining them as optimization problems in contrast to the usual trend of formulating them as equilibrium problems (Toklu 2004). With this consideration, the equilibrium problem is defined as an optimization problem following the very basic principle of mechanics that of all the possible deformed shapes of a structure, the one corresponding to the state of equilibrium is the one corresponding to the minimum potential energy of the system. Thus the method is called Total Potential Optimization with Meta-heuristic Methods (TPO/MA).

When analyzing a structure, the total potential energy of the structure can be calculated as

$$U(\varepsilon) = \int_V e(\varepsilon)dV - \sum_{i=1}^{N_P} P_i u_i \tag{1}$$

where strains in the body are characterized by ε, creating the generalized deflections u_i coupled with the generalized loads P_i, and $e(\varepsilon)$ is the strain energy density

$$e(\varepsilon) = \int_0^{\varepsilon} \sigma(\varepsilon)d\varepsilon \tag{2}$$

The relationship between σ and ε can be found in the constitutive equation $\sigma = \sigma(\varepsilon)$, which can be linear or not, and is assumed to be known and integrable. N_P is the number of loads; V is the volume of the body.

Consider a truss with N_m prismatic members, N_j joints and N_P loads. Consider the element ij (see Fig. 1) with original end coordinates (x_i, y_i, z_i) and (x_j, y_j, z_j) and with original length $L(0)$ as given in Eq. (3). After end displacements (u_i, v_i, w_i) and (u_j, v_j, w_j) corresponding to a configuration c, the final length will be $L(c)$ as given in Eq. (4).

$$L(0) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2} \tag{3}$$

$$L(c)=\sqrt{\left(x_j-x_i+u_j-u_i\right)^2+\left(y_j-y_i+v_j-v_i\right)^2+\left(z_j-z_i+w_j-w_i\right)^2} \tag{4}$$

The elongation of the member then is $\Delta L(c) = L(c)-L(0)$ and the uniform strain in a member is

$$\varepsilon(c)=\Delta L(c)/L(0) \tag{5}$$

With ε at hand for a given deformed shape, σ can be calculated using the constitutive equation, which enables the calculation of the strain energy density *e* from Eq. (2), for a given member. For example, for a truss member made of linear elastic material with the constitutive equation $\sigma = E\varepsilon$, *E* being the modulus of elasticity, the strain energy becomes $e = E\varepsilon(c)^2/2$. For nonlinear materials, this calculation will necessitate the integration Eq. (2). Thus, it becomes possible to determine the potential energy of a structure with Eq. (6) if the end displacements are known

$$U(\varepsilon)=\sum_{j=1}^{N_m} e_j A_j L_j-\sum_{i=1}^{N_P} P_i u_i \tag{6}$$

where A_jL_j is the volume of the truss element *j*, A_j being the cross sectional area. The method of analysis is based on the minimization of the total potential energy U, which forms the objective function of the problem, calculated using Eq. (6). The constraints are the restrictions on nodal displacements. In the nodes that correspond to supports of a truss, some displacements may be limited from above, from below, or from both sides. The last case corresponds to a fixed nodal displacement; zero in most of the cases, or a finite value in cases where there exist imposed support displacements.

According to the formulation described above, the unknowns of the problem are the displacement components at the joints of the truss which form the unknown vector $\mathbf{d} = [u_1\ v_1\ w_1 \ldots u_{Nj}\ v_{Nj}\ w_{Nj}]^T$, with $2N_j$ components for a plane truss, and with $3N_j$ components for a space truss. As stated above, there may be constraints on the components of **d**, forcing some of them to be zero at the supports, or limiting them from above and/or below, depending on the geometrical conditions of the truss. There may even be linear relations between some components, corresponding for instance to inclined supports.

TPO/MA then involves determination of **d** that minimizes U, the total potential of the system, satisfying the constraints of the problem, using a meta-heuristic optimization technique. In the

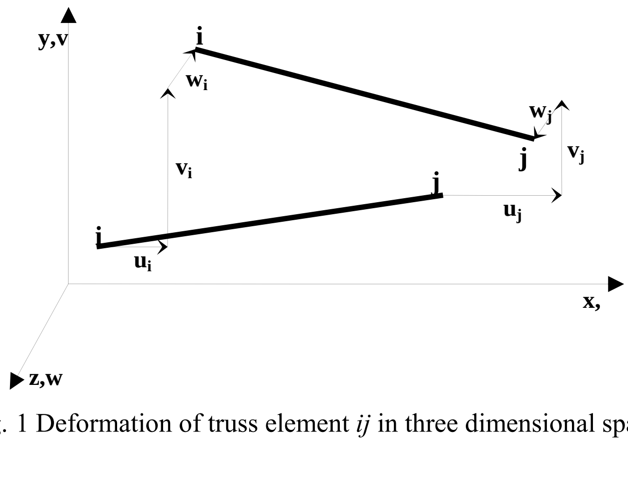


Fig. 1 Deformation of truss element *ij* in three dimensional space

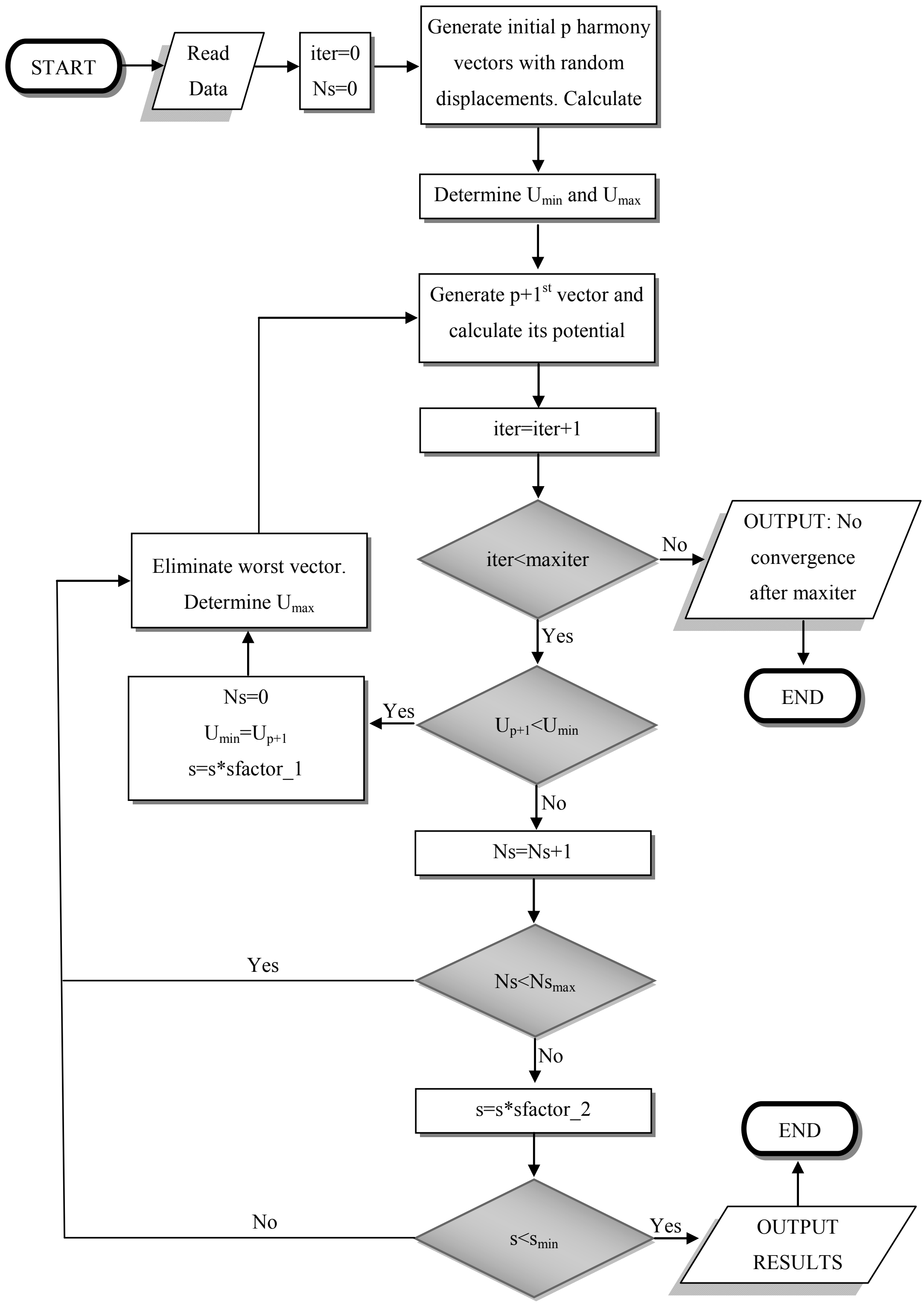


Fig. 2 Flow chart of the program

program developed in the scope this study, random displacements are given to the nodes of the truss, forming several vectors **d**, and total potential energy of the structure is calculated depending on the formulation above. By using the HS technique the iterations are continued until no smaller potential can be found by changing the configuration of the system within the range imposed by the accuracy desired or when a prescribed number of cycles have been performed.

4. Harmony Search Applied to Truss Analysis

In this study, in order to perform structural analysis of trusses using the HS technique, special purpose software was prepared based on the Visual Basic 2008 programming language.

The flow chart of the program is given in Fig. 2. Data read into the program consists of three blocks, one being related to the problem being solved, the second being related to the parameters of the HS method, and the third being related to convergence and stopping of the cycles.

Structural data includes the number of joints, number of members, original joint coordinates, constraints on joint displacements, i.e., support conditions, member connectivity information, member areas and material properties, and finally loads acting on the structure. As can be seen, these are the normal data which would be necessary for analyzing the performance of trusses by any other method.

Data inherent to the HS method are p, population number of vectors to be kept in memory, *HMCR*, harmony memory considering rate, and, p*ar*, pitch adjusting rate, which are already described above.

Third group of data, which are related to stopping criteria are *s, s_min, Ns, s_factor1, s_factor 2,* and *maxiter*, which are described below. Their role in the calculations can be followed from Fig. 2.

The neighborhood for fine adjustment is characterized by the parameter *s*. Its original value is increased by factor *s_factor1* each time a better solution is obtained and decreased by factor *s_factor2* whenever a better solution is not obtained after *Ns* cycles. Obviously *s_factor1* ≥ 1.0 and *s_factor2* < 1.0. The stopping operations are triggered when s goes down below a prescribed value *s_min*.

The limiting number of iterations is *maxiter*. When number of iterations reach this level, another type of stopping operations are triggered, meaning that no convergence is obtained with the accuracy defined by *s_min*.

As shown in Fig. 2, the cycles begin by the creation of p harmony vectors and calculation of potentials corresponding to these vectors $U_1, \ldots, U_p$. Then the best and worst vectors among them are chosen corresponding to the minimum and maximum of potentials, U_{min} *and* U_{max}, respectively. The rest of the procedure consists of cycles involving the creation of a new vector following the well-known rules of HS, increasing the population from p vectors to $p+1$ vectors, and then eliminating the vector with the largest potential again reducing the population number down to p.

Originally, HS was defined in such a way that the fine adjustment was to be done to an upper half tone or to a lower half tone, i.e., in a discrete way. In this application of truss analysis, since the deflections of joints are continuous, their fine adjustments are done by picking randomly a number within that variable's neighborhood defined by *s*. It is to be noted that *s* diminishes as iterations gets closer to the final solution.

For a given problem, the initial choices of the deflected shape yield in general very different values for the total potential, resulting in a great discrepancy between the best and worst choices.

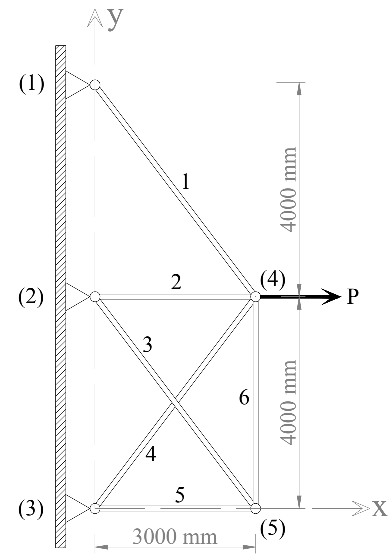


Fig. 3 6-bar plane truss

As iterations progress, both the best and worst choices in the *p* vectors in the memory gets better and better, the discrepancy between them diminishing with the iterations. After a sufficient number of iterations, the difference in energy between the best and worst vectors becomes negligible. In this case, a great number of additional iterations become necessary for obtaining even slightly better solutions. The stopping operations are then triggered when it becomes clear that new iterations do not result in obtaining better solutions. At this stage, the required joint displacements become determined as the components of the vector corresponding to the minimum total potential energy U_{min}. Determination of joint displacements in this way enables calculation of both member elongations and member forces. This completes the analysis of a truss.

5. Numerical examples

To verify the program developed, two problems relevant to the subject have been solved, one with 3 loadings. A computer having a Core 2 Duo T7700 processor has been used for the analyses. In the algorithm of the program developed, random numbers have been created by using the Visual Basic 2008 Rnd function. CPU time for the first example is between 4-52 seconds and for the second one between 25-591 seconds depending on the vector populations and loadings. These problems, which involve nonlinearity due to large deflections, were also analyzed by using FEM software capable of solving linear and geometrically nonlinear problems. The comparisons are done with respect to joint displacements, member forces, and the total potential energy calculated using Eq. (6).

5.1 Example 1. 6-bar truss

The first example is a 6-bar plane truss system, which has previously been used in structural analysis through TPO/MA (Toklu 2004). The system is shown in Fig. 3. The point load P at the node 4 equals 150 kN, the modulus of elasticity E is 200 GPa for all the members, the cross-sectional area of the members marked 1, 5, 6 is 200 mm^2 and of the members marked 2, 3, 4 is 100 mm^2. This example is used for checking results with solutions obtained using other methods and also for making trials on the parameters of the technique.

Fig. 4 shows the general behavior in minimization of the total potential of the system as a function of the number of iterations performed. It is to be noted that the best and worst solutions in the population approach each other as the number of iterations increases. It is also to be noted that improvements take place in the worst solution much more frequently than improvements in the best solution.

It is evident that the total potential of the system is null in the undeformed configuration if one adopts an appropriate datum for the applied loads. The fact that the total potential may be of the order of 3×10^7 MNm in the first iterations, as can be seen in Fig. 4, indicates that iterations may start from very unrealistic configurations and still approach the best possible solution that corresponds to a total potential which is around -1.06 kNm.

While the total potential approaches a minimal value, the displacements and member forces approach their final values. This behavior is demonstrated in Fig. 5 for two displacements of the 6-bar truss corresponding to best values in the successive populations. It is to be noted that in the relevant run the trials for the y displacement of joint 5, v_5, start from around 1572 mm and end at the value 2.33 mm. The corresponding values for the x displacement of joint 4, u_4, are -1241 mm and 14.12 mm.

The roles of the parameters of the technique applied are investigated through this example. First trials have shown that the initial diameter can be taken as the half of the bigger exterior dimension of the system, which is 4000 mm for the current example. This dimension is then modified through the computations using a value of the parameter *s_factor1* of 1.005, *s_factor2* of 0.995 and *Ns* as 200. This means that the radius is taken as 4000 mm at the start, it is increased

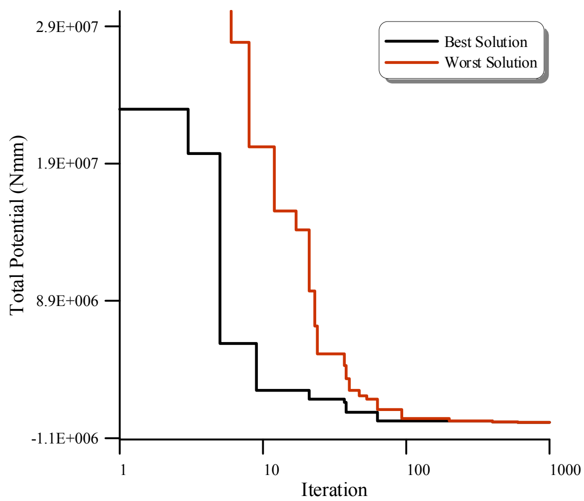


Fig. 4 Change of the total potential corresponding to best and worst solution vectors in the population as a function of the iterations for the 6-bar plane truss

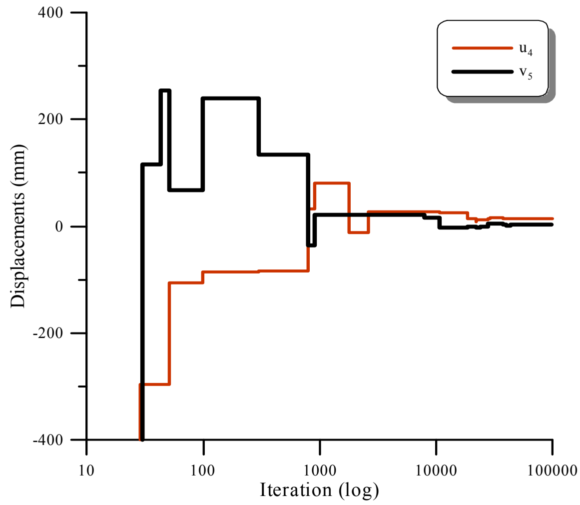


Fig. 5 Variation of displacements u_4 and v_5 in the 6-bar truss with respect to the number of iterations

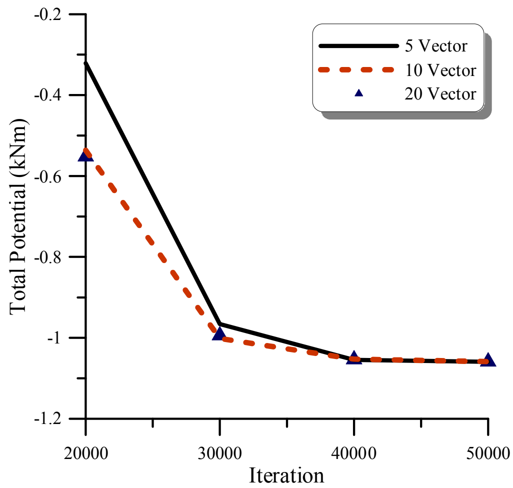


Fig. 6 Variation of total potential energy with respect to iterations and the number of vectors in the population for the 6-bar truss. The energy values are the averages of three independent runs

whenever a better solution is found by multiplying it by 1.005, and whenever a better solution is not found in 200 successive iterations, and it is reduced by the factor 0.995. This continues until the radius falls below a prescribed value which is of the order of $s\_min = 10^{-3}$ mm depending on the accuracy desired.

Fig. 6 demonstrates the effect of the number of vectors in the population. It is evident that number of iterations multiplied by the number of vectors is a measure of the number of total potential evaluations. With this view in mind, one would expect that for a small number of iterations, solutions corresponding to larger populations should give better results. This expectation is clearly validated in Fig. 6 where the potentials are averages of three independent runs starting with different seeds of random numbers. It is concluded in this respect that if a small population of vectors is used in the calculations, the number of iterations needed to find a solution with sufficient accuracy will be increased. Based on these considerations the iterations are run with 20 vectors in the applications that follow.

The trials on the parameters *HMCR* and *par* defined above have shown that the combination *HMCR* = 0.9, *par* = 0.4 gives better results for a given number of iterations meaning that, in creating a new vector, in 90% of the cases previously used components will be used, and in 10 % of the cases the components will be chosen from the whole range. If the first case is valid, that is if already used components are to be used again, in 60% of cases these components will be used as they are, and small adjustments will be made with about 40% probability. In other words when creating a new vector, 54% of its components will be taken from existing ones and used without any change, 36% will be taken from existing ones and used with minor changes, and the remaining 10% will be chosen randomly from the whole range defined.

The solutions obtained for the 6-bar truss using the HS technique as proposed in this study are compared to three other solutions for this truss, one corresponding to a previous study obtained again by TPO/MA but with the local search technique (Toklu 2004), and the two others obtained from linear and geometrically nonlinear FEM applications. Though the truss in question demonstrates nonlinear behavior at the considered level of loading, the linear FEM analysis is included to provide another basis for comparison. Non-zero joint displacements, member forces,

Table 1 Solutions for the 6-bar plane truss

		Present study – TPO/MA			Toklu (2004)	FEM	
		5 vectors	10 vectors	20 vectors		Linear	Nonlinear
Joint displace-ments (mm)	u4	14.119	14.119	14.118	14.12	14.150	14.120
	v4	2.825	2.828	2.830	2.83	2.843	2.828
	u5	0.301	0.302	0.304	0.30	0.300	0.302
	v5	2.311	2.316	2.319	2.32	2.309	2.317
Member forces (N)	1	49825.84	49808.69	49789.18	49811	49725.13	49810.25
	2	94134.8	94134.82	94131.58	94142	94331.33	94140.09
	3	-6672.743	-6686.72	-6687.501	-6688	-6669.14	-6688.4
	4	42961.98	42970.56	42977.97	42974	43055.99	42973.59
	5	4025.542	4032.102	4074.446	4035	4001.49	4034.16
	6	5374.823	5354.307	5354.774	5352	5335.31	5351.45
Total potential (kNm)		-1.059734	-1.059734	-1.059734	-1.059735	-1.059727	-1.059735

and total potentials are given in Table 1 corresponding to these solutions.

A good way of comparing these results is by looking at the total potential energies. It can be seen in this way that FEM-linear is the worst solution and the solutions obtained by the present application are the ones with the lowest potential energies thus they correspond to a more feasible deformed configuration. One interesting point in these results is that although the difference between the highest and lowest total potential is of the order of 0.005% (between the linear FEM solution and the results from the present study), the difference between the lowest and highest member forces for the most significant member force may go up to 0.2% (between the linear and nonlinear FEM solutions for member 2, and even larger for other members which have less force). If the linear FEM solution is excluded in this comparison, the corresponding values become 0.002% difference in the total potentials and 0.01% difference in the most significant member force.

Another observation relates to the three independent runs made using the present method: although the energy level reached is the same, the member forces are not the same, corresponding to different solutions. This can be explained by the fact that, around the optimal point which is characterized by a pit in the multi-dimensional energy surface, small changes in the position result in very small changes in the energy.

5.2. Example 2. 25-bar space truss

To verify the program in three dimensions, a 25-bar space truss system (Venkaya 1971) shown in Fig. 7 is analyzed. All the members are characterized with the same material having a modulus of elasticity of 200 GPa and the same cross sectional area of 10 mm^2. The problem is solved with three loadings:

- Loading 1: at joint 1, P_y = 80 kN and P_z = -20 kN
  at joint 2, P_y = -80 kN and P_z = -20 kN
- Loading 2: at joint 1, P_x = 800 kN and P_z = -200 kN
  at joint 2, P_x = -800 kN and P_z = -200 kN
- Loading 3: at joint 1, P_x = 800 kN, P_y = 800 kN and P_z = -200 kN.

The solutions obtained for these loadings using the presented method and the nonlinear FEM method are presented in Table 2.

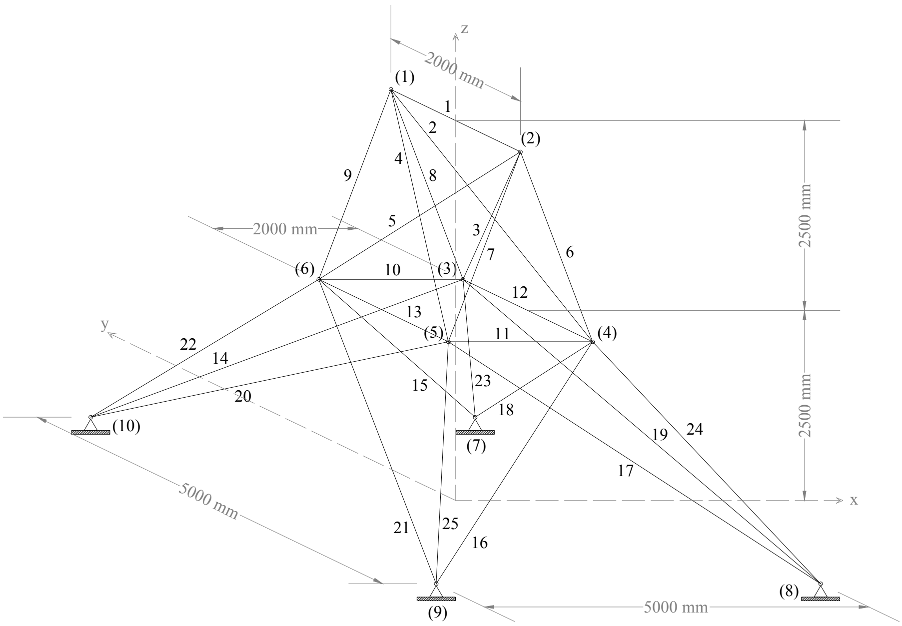


Fig. 7 25-bar space truss

Table 2 Solutions for the 25-bar space truss under three loadings

		Loading 1		Loading 2		Loading 3	
		Present study	FEM nonlinear	Present study	FEM nonlinear	Present study	FEM nonlinear
Joint displacements (mm)	u1	-0.076	0.000	1257.533	1261.101	2522.412	2522.974
	v1	37.910	37.847	-527.509	-528.823	1840.283	1840.856
	w1	-37.214	-37.199	-455.548	-456.667	-3078.900	-3083.146
	u2	-0.064	0.000	-1264.620	-1261.101	2437.300	2441.255
	v2	-37.766	-37.847	529.739	528.823	2522.971	2523.148
	w2	-37.153	-37.199	-457.905	-456.667	-1435.857	-1441.611
	u3	0.892	0.867	-48.858	-48.610	-393.940	-394.610
	v3	-1.731	-1.744	-288.344	-288.313	-371.211	-371.344
	w3	-16.342	-16.392	-362.120	-362.743	-904.675	-905.218
	u4	0.924	0.867	-203.107	-202.750	579.074	579.039
	v4	1.750	1.744	-296.619	-296.792	198.927	199.088
	w4	-16.355	-16.392	-67.535	-68.466	44.972	44.423
	u5	-0.822	-0.867	48.774	48.610	1013.812	1015.303
	v5	1.751	1.744	288.714	288.313	1037.329	1038.580
	w5	-16.402	-16.392	-363.583	-362.743	-353.069	-356.179
	u6	-0.844	-0.867	202.232	202.750	407.347	408.348
	v6	-1.746	-1.744	297.259	296.792	802.637	803.160
	w6	-16.430	-16.392	-69.186	-68.466	-36.622	-36.688

Table 2 Continued

Member forces (N)	1	75675.54	75693.19	692590.19	692496.23	107639.85	106577.59
	2	3914.70	3893.38	-334407.98	-334606.7	273603.10	274287.68
	3	3882.78	3893.38	73046.40	72764.01	-310371.76	-310685.58
	4	3890.05	3893.38	72342.98	72764.01	247022.79	246394.01
	5	3885.09	3893.38	-334711.31	-334606.7	-70531.59	-70014.67
	6	-13844.06	-13883	275293.86	274675.19	-10172.10	-10676.67
	7	-13875.23	-13883	-189136.75	-189232.9	359300.79	359448.95
	8	-13899.23	-13883	-189278.62	-189232.9	187013.94	187700.44
	9	-13893.96	-13883	273926.72	274675.19	471844.41	472058.79
	10	1735.60	1733.72	-132534.71	-132732.2	-110993.33	-111425.75
	11	1745.12	1733.72	-132868.16	-132732.2	-180274.51	-180522.63
	12	-3481.69	-3488.52	35618.38	35767.34	-26908.42	-26811.14
	13	-3497.44	-3488.52	35797.63	35767.34	-106788.78	-106798.40
	14	-3376.39	-3394.96	-52294.17	-52346.49	-260641.29	-260940.19
	15	-3411.65	-3394.96	-125338.75	-125276.5	-237250.26	-237646.69
	16	-3366.09	-3394.96	-125143.20	-125276.5	238850.10	238740.82
	17	-3411.79	-3394.96	-52509.14	-52346.49	-177596.69	-178358.89
	18	-4657.28	-4655.87	116226.47	116024.86	-125783.32	-125978.40
	19	-4643.16	-4655.87	-174650.86	-174821.5	-248091.29	-248092.67
	20	-4654.23	-4655.87	-175120.58	-174821.5	-190077.70	-190716.96
	21	-4662.30	-4655.87	115946.41	116024.86	332334.87	332665.42
	22	-7378.16	-7367.45	-46776.43	-46167.64	-52548.72	-52284.93
	23	-7354.86	-7367.45	-54924.78	-55263.71	-106706.62	-106579.02
	24	-7364.58	-7367.45	-45581.17	-46167.64	-50751.51	-50960.27
	25	-7357.67	-7367.45	-55421.23	-55263.71	542044.64	542081.79
TP (kNm)		-3.7645	-3.7645	-1444.6	-1444.6	-2860.5	-2860.5

Table 3 Comparison FEM and TPO/MA analysis results for the 25-bar space truss

	Parameter	Difference	FEM	TPO/MA
Loading 1	v_2	0.21%	37.847 mm	37.766 mm
	f_1	0.02%	75693.19 N	75675.54 N
Loading 2	u_1	0.28%	1261.101 mm	1257.533 mm
	f_1	0.01%	692496.2 N	692590.2 N
Loading 3	w_1	0.14%	3083.146 mm	3078.90 mm
	f_{25}	0.007%	542081.8 N	542044.6 N

It can be observed from the given results that for all three cases investigated, the solutions are very close to each other as to the energy, deflected shape and member forces. In fact, the energy levels are the same. The most significant displacements and member forces for the three cases and the discrepancies can be seen in Table 3.

Thus the maximum discrepancy in the most significant displacements is smaller than 0.3%. For member forces this limiting value is much smaller, 0.02%. These values are somewhat greater than those found in the 1st example.

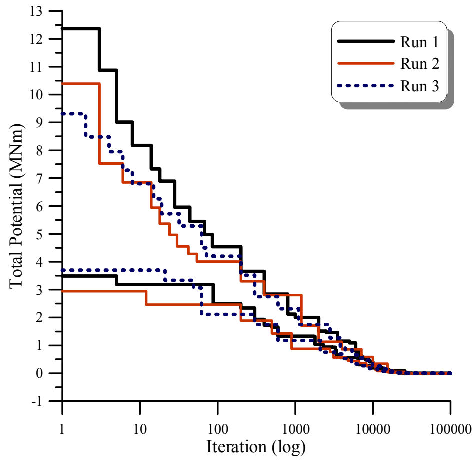
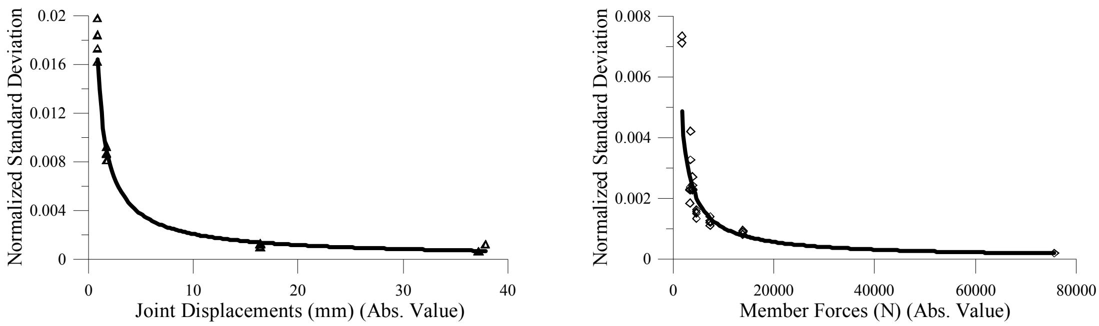


Fig. 8 Convergence of the best and worst vectors in three independent runs for the 25-bar space truss under Loading 1



(a) (b)

Fig. 9 Normalized standard deviations in 100 independent runs for the 25-bar space truss under Loading 1, solutions with p = 20 vectors (a) Joint displacements (b) Members Forces

In Fig. 8, the convergence behavior of the best and worst vectors in the HS population are shown for the 25-bar space truss problem for three independent runs under Loading 1 obtained with 20 vectors in the HS population.

Table 4 gives the results corresponding to multiple runs carried out on 25-bar space truss. In the table, complete results are presented for 100 independent runs with three different choices of population vectors, 5, 10 and 20 namely. For each choice of population number and for each variable of the truss, i.e., nodal displacements, member forces and total potential energy, minimal and maximal values, average, and standard deviation calculated for these 100 runs are tabulated.

The first observation that can be made here is the one made for the 6-bar plane truss: The minimum energy is not sensitive to small changes in the configuration near the optimal point. Indeed, the total potential is the same up to 6 significant digits for all the runs considered. The standard deviation for total potentials is of the order of 0.5×10^{-6} kNm for all three cases p = 5, 10 and 20. The results show that the choice p = 5 is as acceptable as the choices p = 10 and p = 20.

As to the joint displacements and member forces, the same behavior can be observed with the additional and reasonable advertence that for more significant values, the deviations from the

Table 4 Evaluation of 100 independent solutions of the 25-bar space truss under Loading 1 using TPO/MA

		p = 5 vectors				p = 10 vectors				p = 20 vectors			
		Min	Max	Aver.	St. Dev.	Min	Max	Aver.	St. Dev.	Min	Max	Aver.	St. Dev.
Joint displacements (mm)	u1	-0.099	0.099	0.004	0.035	-0.080	0.063	-0.002	0.027	-0.076	0.062	-0.001	0.029
	v1	37.730	37.941	37.834	0.043	37.756	37.930	37.843	0.041	37.756	37.980	37.845	0.043
	w1	-37.245	-37.143	-37.195	0.021	-37.239	-37.150	-37.197	0.022	-37.261	-37.151	-37.199	0.020
	u2	-0.074	0.090	0.003	0.036	-0.079	0.079	-0.001	0.037	-0.070	0.068	0.001	0.029
	v2	-37.960	-37.768	-37.857	0.044	-37.941	-37.745	-37.849	0.041	-37.933	-37.719	-37.851	0.043
	w2	-37.242	-37.169	-37.204	0.017	-37.238	-37.157	-37.199	0.018	-37.247	-37.132	-37.200	0.021
	u3	0.828	0.898	0.866	0.015	0.838	0.898	0.867	0.013	0.831	0.912	0.872	0.016
	v3	-1.783	-1.709	-1.745	0.016	-1.782	-1.711	-1.744	0.013	-1.782	-1.715	-1.744	0.015
	w3	-16.431	-16.348	-16.390	0.017	-16.434	-16.350	-16.391	0.018	-16.467	-16.342	-16.387	0.018
	u4	0.823	0.905	0.868	0.015	0.834	0.907	0.867	0.015	0.837	0.924	0.869	0.015
	v4	1.713	1.782	1.744	0.016	1.711	1.776	1.745	0.014	1.697	1.789	1.745	0.016
	w4	-16.436	-16.350	-16.396	0.018	-16.430	-16.348	-16.390	0.017	-16.437	-16.330	-16.393	0.020
	u5	-0.899	-0.832	-0.868	0.015	-0.903	-0.823	-0.866	0.016	-0.896	-0.822	-0.867	0.014
	v5	1.705	1.779	1.743	0.016	1.707	1.773	1.743	0.015	1.708	1.775	1.745	0.014
	w5	-16.437	-16.359	-16.394	0.017	-16.431	-16.353	-16.392	0.017	-16.425	-16.358	-16.390	0.015
	u6	-0.905	-0.831	-0.869	0.015	-0.902	-0.831	-0.869	0.015	-0.906	-0.831	-0.863	0.017
	v6	-1.779	-1.711	-1.746	0.015	-1.776	-1.713	-1.745	0.013	-1.794	-1.714	-1.746	0.015
	w6	-16.436	-16.341	-16.388	0.021	-16.431	-16.351	-16.390	0.019	-16.460	-16.338	-16.391	0.019
Member Forces (N)	1	75650.45	75721.20	75690.92	12.73	75656.04	75730.11	75692.38	14.10	75659.18	75730.36	75696.01	13.92
	2	3871.38	3917.71	3892.29	9.74	3875.27	3913.21	3892.85	8.60	3867.07	3918.56	3893.39	9.46
	3	3868.04	3914.54	3893.44	9.11	3876.71	3914.24	3894.07	7.73	3866.81	3916.95	3893.72	8.96
	4	3872.51	3915.60	3892.91	8.64	3873.19	3917.09	3893.26	8.36	3875.26	3915.78	3891.98	8.82
	5	3869.55	3919.12	3893.76	10.38	3875.43	3914.98	3893.67	8.73	3872.23	3923.64	3893.46	10.54
	6	-13909.65	-13859.26	-13883.46	10.76	-13913.77	-13851.61	-13883.32	12.35	-13906.80	-13844.06	-13882.37	12.24
	7	-13906.94	-13861.84	-13883.60	9.29	-13912.01	-13857.79	-13882.85	12.02	-13923.35	-13840.75	-13883.95	12.86
	8	-13905.49	-13849.83	-13882.38	11.37	-13908.04	-13860.39	-13881.64	11.98	-13919.32	-13853.82	-13884.21	11.16
	9	-13910.92	-13858.18	-13881.03	11.58	-13914.45	-13851.49	-13882.35	11.90	-13918.23	-13848.32	-13884.40	13.02
	10	1705.80	1765.05	1734.62	12.60	1707.54	1764.53	1735.78	11.44	1705.93	1766.72	1734.79	12.75
	11	1705.51	1763.35	1736.01	12.52	1690.66	1770.07	1732.89	13.85	1697.75	1775.15	1735.52	12.37
	12	-3516.06	-3453.07	-3489.27	11.92	-3522.60	-3461.71	-3488.94	12.14	-3524.45	-3449.70	-3489.14	14.67
	13	-3521.55	-3459.36	-3488.81	12.28	-3514.75	-3460.53	-3488.54	10.90	-3518.21	-3459.22	-3490.59	11.40
	14	-3413.21	-3375.77	-3394.85	7.10	-3407.95	-3371.07	-3394.72	6.94	-3414.31	-3374.47	-3392.02	7.79
	15	-3413.06	-3377.86	-3393.14	7.98	-3410.32	-3376.20	-3393.63	6.80	-3422.24	-3370.15	-3396.07	8.04
	16	-3413.70	-3378.92	-3395.62	6.72	-3409.07	-3377.53	-3394.53	7.26	-3412.64	-3366.09	-3394.44	7.72
	17	-3410.18	-3378.61	-3395.33	7.00	-3411.58	-3375.91	-3395.57	6.71	-3411.79	-3378.43	-3394.58	6.23
	18	-4682.18	-4638.81	-4656.94	7.74	-4670.87	-4637.61	-4655.73	7.12	-4677.98	-4626.20	-4656.63	7.53
	19	-4673.59	-4639.01	-4655.78	6.90	-4677.70	-4639.61	-4655.54	6.81	-4677.72	-4640.55	-4655.44	6.98
	20	-4676.09	-4640.97	-4656.10	7.29	-4670.56	-4635.93	-4655.47	7.16	-4672.47	-4638.43	-4655.67	6.23
	21	-4673.19	-4637.34	-4655.80	6.72	-4671.07	-4637.99	-4656.14	6.39	-4675.76	-4638.74	-4655.69	7.21
	22	-7396.32	-7341.05	-7365.85	11.27	-7390.28	-7340.46	-7366.96	10.73	-7388.04	-7340.15	-7365.53	9.14
	23	-7383.33	-7345.04	-7366.15	7.76	-7387.93	-7348.53	-7366.81	8.04	-7402.04	-7347.83	-7366.83	8.86
	24	-7396.88	-7341.28	-7369.94	10.47	-7383.57	-7345.91	-7366.79	8.01	-7392.49	-7343.60	-7368.13	10.25
	25	-7387.32	-7350.26	-7369.08	7.41	-7387.65	-7344.12	-7367.67	9.47	-7390.95	-7345.83	-7366.72	8.07
TP (kNm)		-3.764502	-3.764499	-3.764501	0.58×10^{-6}	-3.764502	-3.764499	-3.764501	0.49×10^{-6}	-3.764502	-3.764499	-3.764501	0.54×10^{-6}

average get smaller. This behavior is visualized in Fig. 9 where the ratios standard deviation/ average or the normalized standard deviation values which are also known as coefficients of variation are plotted against average values for the case p = 20 vectors. It can be seen on this figure that as the calculated values get bigger in absolute value, the discrepancies from the average values diminish. This behavior can be explained by the fact that the variables which affect more the total potential of the system are found more accurately as compared to variables which have negligible effect on the total potential energy. That is why the above mentioned ratio, which can be considered as a measure of relative accuracy, is smallest for the displacements v_1, w_1, v_2 and w_2, where the external loads are applied, and for the elements 22, 23, 24 and 25, where the member forces are more important. A closer look at the Fig. 9(a) which is drawn excluding the displacements u_1 and u_3 which are practically zero, shows that the relative accuracy is less than 0.01% in the example problem for more significant joint displacements, but may go up to 2% for insignificant variables which do not affect the total potential of the system. For member forces these figures are of the order of 0.02% and 0.7% for significant and insignificant member forces respectively, as seen from Fig. 9(b).

6. Conclusions

Total Potential Optimization Method in combination with meta-heuristic techniques has a very simple but sound principle behind it. In this paper two and three dimensional problems with geometric nonlinearity are treated to demonstrate the power of the named method. It has been shown that Harmony Search can be incorporated into this method to yield an efficient and accurate method for treating relevant problems. The results obtained by using this technique are compared with the results obtained from well-known nonlinear structural analysis programs which are based on FEM. As seen from the presented results, the solutions found using these two methods are close to each other within tolerable limits for the treated examples. Further study is necessary for checking the correctness of this observation for all types of trusses and for all levels of loads.

One observation about the application of HS in TPO/MA is that the values assigned to parameters inherent to the technique are important only to a certain degree: by increasing the number of iterations, sufficiently accurate results can be obtained for almost any combination of parameters. On the other hand, if one aims to minimize the number of iterations, then a comprehensive search on them will be necessary. One such effort in this study gave the best values for the main parameters as *HMCR* = 0.9 and *par* = 0.40 meaning that the components of the new vectors will be chosen among the already used options with 90% probability with a further search within a “small” neighborhood around them in 40% of the cases. It has been seen that such a choice results in a meaningful reduction in the number of function evaluations.

One further remark about the method presented is its robustness. Effectively, it has been seen that every run performed in this study has ended with acceptable results independent of the truss type, loading, choice of parameters, and starting point related to the seed of the random numbers. This is certainly very important for a meta-heuristic method considering that many optimization problems are very sensitive to problem types and starting options.

The study has also drawn attention to an interesting point on the accuracy of the results obtained by TPO/MA. It has been observed that the total potential energy of the system after the application of loads can be optimized and calculated with a very high accuracy. But the level of accuracy is not the same for all variables of the system, it is higher for the ones which contribute more to the total potential of the system and less for the ones with smaller order of magnitude thus with less significant effect on the total potential. This of course is due to the fact that the system is solved by minimizing a single scalar objective function which depends on a great number of variables.

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