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Strength of biaxially loaded high strength reinforced concrete columns

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Abstract. An experimental research was conducted to investigate the strength of biaxially loaded short and slender reinforced concrete columns with high strength concrete. In the study, square and L-shaped section reinforced concrete columns were constructed and tested to obtain the load-deformation behaviour and strength of columns. The test results of column specimens were analysed with a theoretical method based on the fiber element technique. The theoretical ultimate strength capacities and the test results of column specimens have been compared and discussed in the paper. Besides this, observed failure mode and experimental and theoretical load-lateral deflection behaviour of the column specimens are presented.

Keywords: high strength concrete column; ultimate strength; stress-strain relationship; slenderness effect

1. Introduction

High strength concrete has been commonly used in the structural members, such as columns, girders, piles etc. High strength concrete provides economy due to the reduced size of members and it offers performance advantages in multi storey or high rise buildings. However, it is widely believed that high strength concrete behaves less ductile owing to the brittleness of the material (Rangan 1990, Cusson and Paultre 1992, Hsu *et al.* 1995, Ibrahim and MacGregor 1996, ACI-ASCE Committee 441 1997, Saatcioglu and Razvi 1998). Therefore, concrete confinement provided by lateral ties becomes important issue for high strength concrete columns in seismically active regions (Saatcioglu and Razvi 1998).

High strength concrete columns located especially at the corner of the structures are commonly subjected to bending about both principal directions. It is very significant to describe the structural behaviour of such members for rational analysis and design. Several experimental and theoretical studies were carried out on eccentrically loaded high strength concrete columns. Cusson and Paultre (1992), Saatcioglu and Razvi (1998) presented experimental study of the behaviour of high strength concrete columns confined by rectangular ties under concentric loading. Hsu *et al.* (1995) explored experimental and theoretical load-deflection behaviour of square section slender high strength

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concrete columns subjected to biaxial bending and axial load. Ibrahim and MacGregor (1996), Lloyd and Rangan (1996), Foster and Attard (1997) investigated the behaviour of slender high strength concrete columns subjected to combined axial load and uniaxial bending by means of experimental program. Lee and Son (2000), Canbay *et al.* (2006) tested high strength concrete columns to investigate the structural behaviour and strength of eccentrically loaded reinforced concrete tied columns. Wang *et al.* (2011) investigated the mechanical properties of reinforced high strength concrete square columns confined by aramid fiber reinforced polymer jackets. Ho (2011) tested high strength concrete columns under combined high axial load level and large reverse inelastic displacement.

ACI-ASCE Committee (1997) reported the effects of the parameters related to concrete and transverse reinforcement on the behaviour of high strength concrete columns under monotonically increasing concentric or eccentric compression. Ibrahim and MacGregor (1997) suggested an equation to describe the rectangular stress block and verified the equation with the experimental data from the literature. Stewart and Attard (1999), Rangan (1999) studied on analytical methods to describe the strength capacity of high strength concrete columns. Yalcin and Saatcioglu (2000) developed computer software for inelastic analysis of reinforced concrete columns under combined axial compression and monotonically increased lateral loads. Mendis *et al.* (2000) attempted to provide a theoretical basis for the design of high strength concrete columns in terms of the spacing of lateral reinforcement. Sarker and Rangan (2003) performed an experimental and analytical investigation of high strength concrete columns subjected to unequal load eccentricities. Diniz and Frangopol (2003) presented analysis of the reliability of slender high strength concrete columns including long term effects under eccentric loading conditions. Campione (2008) suggested a mechanical model to predict the compressive response of high strength short concrete columns with square cross section confined by transverse steel. Lou and Xiang (2008) proposed a numerical method based on the finite element procedure to describe the nonlinear analysis of reinforced and prestressed concrete slender columns with arbitrary section subjected to combined biaxial bending and axial load. Tokgoz (2009) studied the effects of the various amounts of steel fiber addition on the behaviour of high strength concrete columns.

The main purpose of this study is to investigate the experimental and theoretical behaviour of eccentrically loaded short and slender reinforced concrete columns. A total of 11 square and L-shape both short and slender reinforced concrete columns were constructed and tested under biaxially applied short-term axial load. The structural behaviour and failure mode of the column specimens were observed during the tests. In the study, the column specimens have been analyzed based on a theoretical method previously suggested by Tokgoz (2009). The analysis results have been compared with the test results. In addition, the typical experimental and theoretical load-lateral deflection curves of the columns are presented and discussed in the paper.

2. Experimental program

2.1 Test specimens

The experimental study includes a total of 11 square and L-shaped section reinforced concrete columns with high strength concrete. The specimens were constructed with different length, cross section and different arrangement of reinforcements. Specimens HC0-HC2 were short square tied

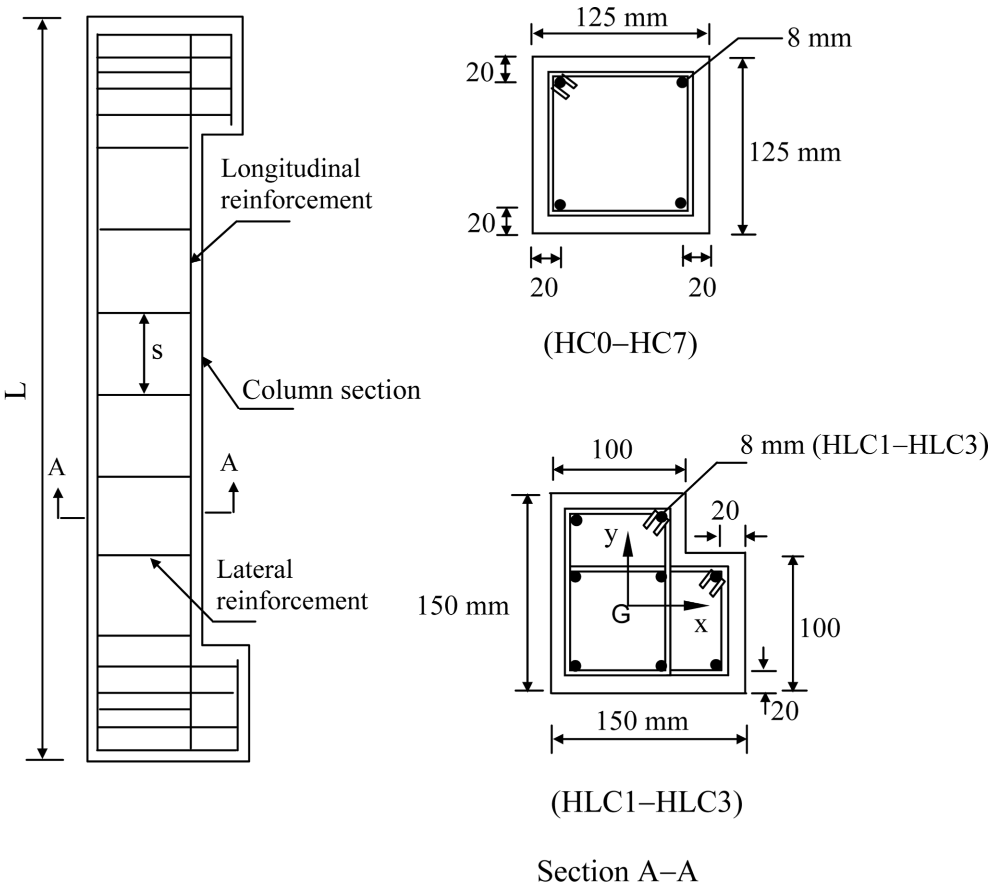


Fig. 1 Section features of the column specimens

columns, specimens HC3-HC7 were slender square tied columns and specimens HLC1-HLC3 were L-shaped section slender tied columns. The column specimens were designed in reduced scale. The cross section details and features of the column specimens are shown in Fig. 1.

The column specimens were reinforced with both longitudinal and lateral reinforcements. The longitudinal reinforcements were designed with 8 mm in diameter deformed bars had the yield strength of 550 MPa. The lateral reinforcements consisted of 6 mm in diameter deformed bars had the yield strength of 630 MPa. The lateral reinforcements were bent into 135 degree hooks at the ends.

The materials consisted of maximum size of 20 mm well dry and clean local aggregates, low water-cement ratio, Portland CEM I 42.5 R type cement content and plasticizer to maintain good workability. The reinforced concrete column specimens were cast horizontally inside a steel formwork in the Structural Laboratory at Cukurova University. Three standard cylinder specimens (150 mm in diameter and 300 mm in length) were cast from each column specimen concrete mixture. The cylinder specimens were compacted on a vibrating table and cured under the same condition as the column specimens in the Structural Laboratory. The cylinder specimens were axially tested at the same day of that column specimen to determine the main concrete compressive strength.

Table 1 Specimen features of the columns

Specimen no.	Specimen details				
	L (mm)	f_c (MPa)	e_x (mm)	e_y (mm)	s (mm)
HC0	850	43.94	45	45	100
HC1	850	45.25	50	50	80
HC2	850	58.64	55	55	100
HC3	1300	75.39	40	40	100
HC4	1300	58.49	45	45	100
HC5	1300	62.59	50	50	80
HC6	1300	58.54	35	35	120
HC7	1300	58.35	50	50	100
HLC1	1300	57.37	41.25	41.25	100
HLC2	1300	74.75	46.25	46.25	110
HLC3	1300	77.59	51.25	51.25	120

The column specimen features of the overall length (L), the average concrete compressive strength (f_c), the eccentricities of the applied axial load (e_x, e_y) and spacing of lateral reinforcements (s) are presented in Table 1.

2.2 Test procedure and observed failure mode

The column specimens were tested in the vertical position with pinned conditions at both ends under short-term axial load and biaxial bending in single curvature using a loading frame. The eccentrically applied axial load was measured by using a load cell and four linear variable displacement transducers were used on each mid-height of column face to obtain lateral deflections for both principal directions of the specimens. The typical test setup and test instruments are presented in Fig. 2. The load cell and the transducers were calibrated before they were used in the tests. Heavily reinforced brackets were designed at both ends of the column specimens to ensure the eccentric loading condition and to prevent local failures of the end zones. The digital readings of axial load and corresponding lateral displacements were recorded by using an electronic data acquisition system during testing of each specimen. The monotonically increased axial load was applied to the column specimens at a rate of 1 kN/s in the tests.

It was observed that the flexural cracks started to appear in a direction perpendicular to the column axis at a load level of about 50% maximum load on the tensile side of the specimens. The existing cracks propagated and new cracks were observed along the tensile side of the column specimens with increase of load level. Beyond the maximum load, major cracks appeared on the tensile side and the concrete crushed on the compression side at or close to mid-height of the column specimens. In the mean time, significant drop measured in load resistance. After that, buckling of longitudinal reinforcement was observed, and then test was terminated. It was seen that longitudinal reinforcement in the compressive region of the concrete buckled after the concrete crush had occurred for the specimens constructed with large stirrup spacing (HC6, HLC3). It was concluded that buckling of longitudinal reinforcement results in a brittle failure of the eccentrically loaded reinforced concrete columns. Decreasing the lateral tie spacing could delay the buckling of

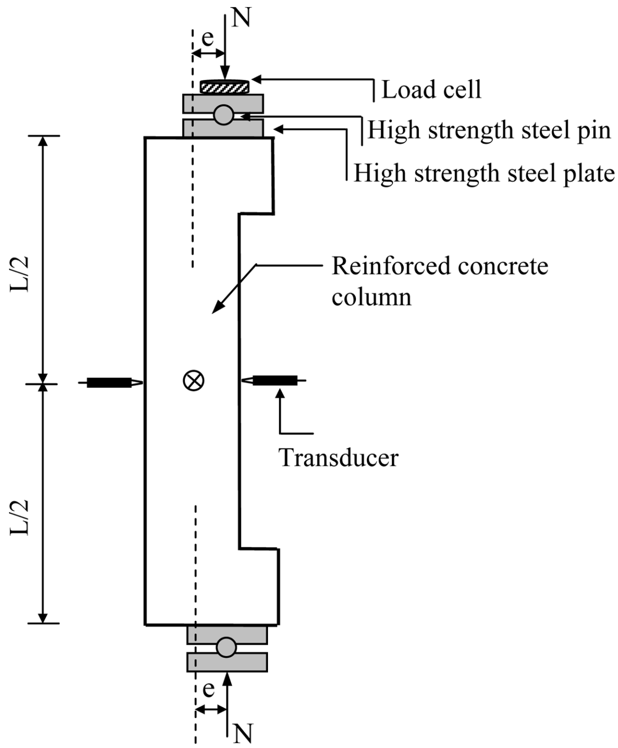
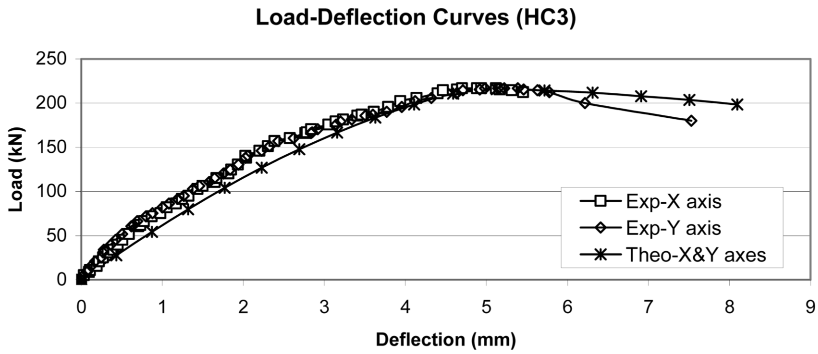


Fig. 2 The typical reinforced concrete column test setup and instrumentation

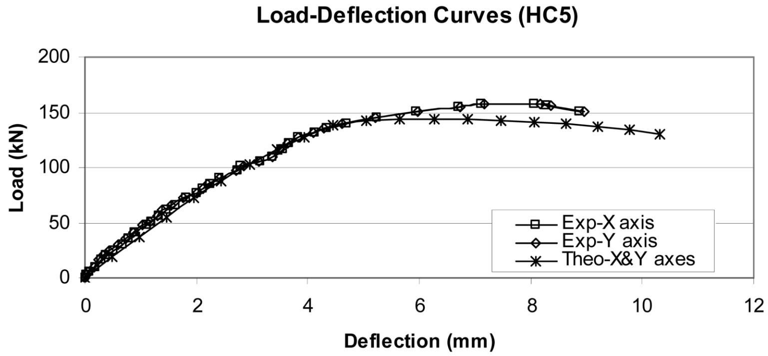
Fig. 3 Deflected shape of the column specimen HLC2

the longitudinal reinforcements and improve the ductility of the columns. As expected, the tested slender columns were more buckled than short columns. The typical deflected shape and failure mode of the column specimen HLC2 after test is presented in Fig. 3.

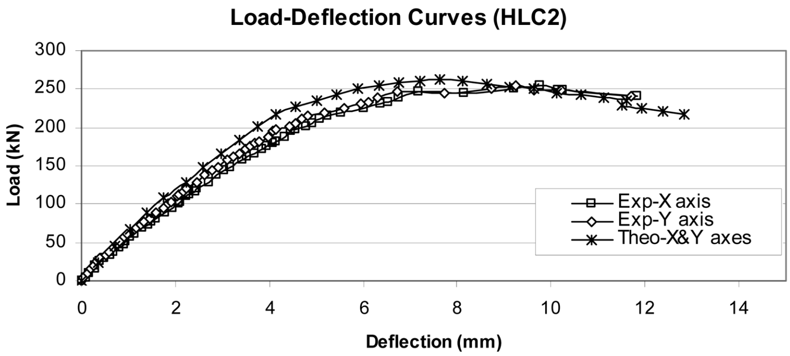
Failure was occurred in the most heavily compressed region for all the column specimens. It was observed that the columns had compressive strength more than 70 MPa failed explosively manner and a sudden loss of strength was measured in load resistance after the peak load. This type of



(a)



(b)



(c)

Fig. 4(a-c) Comparative load-lateral deflection diagrams of the column specimens

Fig. 5 Photo for all the column specimens after testing

failure was due to the brittle behaviour of high strength concrete, indicating unfavorable ductility capacity feature for the column specimens. The ductility capacity was strongly influenced by the concrete strength and load eccentricity. The ductility of tested columns considerably decreased with the increasing of concrete compressive strength and the load eccentricity. The typical experimental and theoretical load-lateral deflection diagrams of the column specimens HC3, HC5 and HLC2 are presented in Fig. 4(a-c) for each principal direction.

It is shown in the diagrams that the complete experimental load-lateral deflection curves compare well with the theoretical curves obtained based on the simplified theoretical method (Tokgoz 2009). A set of photographs of the reinforced concrete column specimens after test are shown in Fig. 5.

3. Analysis method

The suggested analysis method has been previously reported by Tokgoz (2009) to describe the strength and load-deformation behaviour of eccentrically loaded high strength reinforced concrete columns. In the study, the method has been used for the analysis of columns by application of constitutive models for high strength concrete available in the literature (Hsu and Hsu 1992, Cusson and Paultre 1995). The fundamental algorithm of the method has been presented in the study.

3.1 Assumptions

1. Plane sections remain plane before and after bending (Bernoulli's assumption). Therefore, the strain distribution is linear across the concrete section.
2. Nonlinear stress-strain relations are used for the concrete compression zone.
3. The reinforcing steel bars are assumed to be elastic-perfectly plastic in both tension and compression.

4. Perfect bond exists between steel and concrete.
5. The effect of creep, shrinkage and the tensile strength of concrete are ignored.
6. Shear deformation is neglected.

3.2 Material modelling for concrete

In the presented study, the reinforced concrete columns have been analyzed by using the nonlinear stress-strain relationships recommended for high strength concrete (Hsu and Hsu 1992, Cusson and Paultre 1995). These empirical models were described to represent the complete mechanical behaviour of high strength concrete.

Hsu and Hsu (1992) suggested the complete stress-strain relationship for high strength concrete by the following empirical equations

$$\sigma_c = \left[\frac{n_1\beta\left(\frac{\varepsilon_c}{\varepsilon_o}\right)}{n_1\beta - 1 + \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^{n_1\beta}}\right] f_c\,, \quad \left(0 \le \frac{\varepsilon_c}{\varepsilon_o} \le x_d\right) \tag{1}$$

and

$$\sigma_c = 0.3\exp\left[-0.8\left(\frac{\varepsilon_c}{\varepsilon_o} - x_d\right)^{0.5}\right] f_c \quad \left(x_d \le \frac{\varepsilon_c}{\varepsilon_o}\right) \tag{2}$$

where

$$\beta = 1/\{1 - [f_c/(\varepsilon_o E_c)]\} \text{ for } \beta \ge 1.0 \tag{3}$$

$$E_c = 4730\sqrt{f_c}\,, \text{ (ACI Committee 363 1984, ACI 318 2005)} \tag{4}$$

In which, f_c is expressed in MPa. Here, n_1 and β are the material parameters (n_1 depends on the strength of materials and β depends on the shape of the stress-strain curve (Hsu and Hsu 1992); σ_c is the concrete stress in general; ε_c is the concrete strain in general; f_c is the peak stress of concrete; ε_o is the strain corresponding to the peak stress; and x_d is the strain at $0.3f_c$ in the descending portion of the stress-strain curve. Eq. (1) and Eq. (2) represent both ascending and descending portions of the stress-strain relationship.

The complete stress-strain curve proposed for high strength concrete by Cusson and Paultre (1995) is described by the following equations

The ascending part of the model is described as

$$\sigma_c = \frac{f_c\left(\frac{\varepsilon_c}{\varepsilon_o}\right)r}{r - 1 + \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^r}, \quad \varepsilon_c \le \varepsilon_o \tag{5}$$

where

$$\varepsilon_o = 0.0028 - 0.0008k_1 \tag{6}$$

$$k_1 = \frac{40}{f_c} \leq 1.0 \tag{7}$$

$$r = \frac{E_c}{E_c - (f_c/\varepsilon_o)} \tag{8}$$

The descending part of the model is expressed as

$$\sigma_c = f_c \cdot \exp[k_2(\varepsilon_c - \varepsilon_o)^{0.58}], \quad \varepsilon_c > \varepsilon_o \tag{9}$$

where

$$k_2 = \frac{\ln 0.5}{(0.004 - \varepsilon_o)^{0.58}} \tag{10}$$

3.3 Fundamental equations for the analysis

An arbitrarily shaped reinforced concrete cross section subjected to compressive axial load N with the eccentricities e_x and e_y is shown in Fig. 6. According to Bernoulli's assumption, the strain (ε_i) at any point (x_i, y_i) in the cross section can be expressed as

$$\varepsilon_i = \varepsilon_{cu}\left[\left(\frac{y_i}{c} + \frac{x_i}{a}\right) - 1\right] \tag{11}$$

where ε_{cu} is the ultimate concrete compressive fiber strain; a and c are the horizontal and the

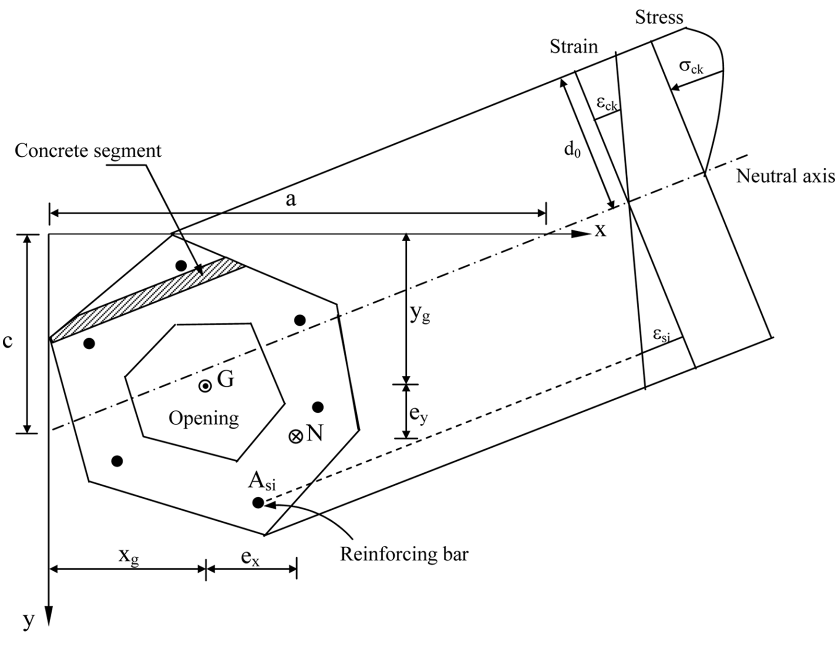


Fig. 6 An arbitrarily shaped reinforced concrete column cross section

vertical distances between the origin of the x-y axis system and the neutral axis. The curvature (ψ) can be determined using the strain distribution as follows (Fig. 6)

$$\psi = \frac{\varepsilon_c}{d_0} \tag{12}$$

where ε_c is the concrete strain; d_0 is the neutral axis depth. In the procedure, the nonlinear behaviour of the materials is assumed. Thus, the concrete compression zone of the column section is divided into parallel segments to the neutral axis (Fig. 6). The stress resultants of the materials are determined with using Eq. (11) and the stress-strain relationships.

In order to satisfy the force equilibrium, the following equations can be written

$$\sum_k^t \bar{A}_{ck}\sigma_{ck} - \sum_i^n A_{si}\sigma_{si} - N = 0 \tag{13}$$

$$\sum_i^n A_{si}\sigma_{si}(y_i - y_g) - \sum_k^t \bar{A}_{ck}\sigma_{ck}(\bar{y}_{ck} - y_g) - M_x = 0 \tag{14}$$

$$\sum_i^n A_{si}\sigma_{si}(x_i - x_g) - \sum_k^t \bar{A}_{ck}\sigma_{ck}(\bar{x}_{ck} - x_g) - M_y = 0 \tag{15}$$

where, M_x and M_y are the bending moments about x-axis and y-axis, respectively; $\bar{A}_{ck}$ and $(\bar{x}_{ck}, \bar{y}_{ck})$ indicate the area and the centroid coordinates of kth concrete segment respectively; σ_{ck} is the concrete compressive stress corresponding to the strain, ε_{ck}, computed at the centroid of the kth segment; A_{si} is the area of each reinforcing bar within the cross-section; t and n are the number of segment of the concrete in compression zone and the total number of reinforcing bars, respectively; σ_{si} is the stress of ith reinforcing bar; x_i and y_i are the coordinates of the ith reinforcing bar; (x_g, y_g)

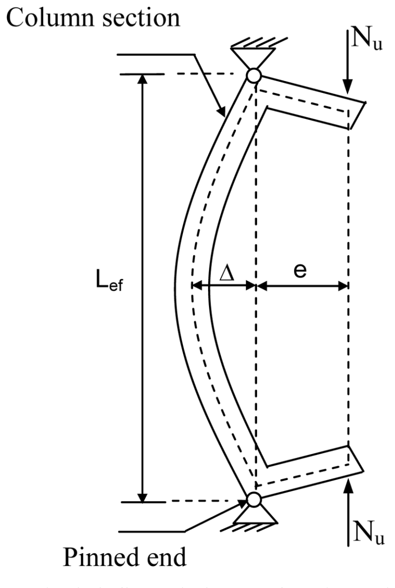


Fig. 7 The typical deflected shape of a pin ended column

Table 2 Test, analysis and comparative results of the column specimens

Column no.	Test results	Analysis results			Ratio	
	N_{test} (kN)	N_{u1} (kN)	N_{u2} (kN)	M_{ux2} & M_{uy2} (kN.cm)	N_{u1}/N_{test}	N_{u2}/N_{test}
HC0	179	160.36	166.85	750.87	0.896	0.932
HC1	150	143.03	146.82	734.13	0.954	0.979
HC2	146	140.41	146.24	804.12	0.962	1.002
HC3	217	206.65	210.27	977.89	0.952	0.969
HC4	182	163.18	159.75	825.45	0.897	0.878
HC5	158	143.63	143.18	817.04	0.909	0.906
HC6	248	222.03	223.54	914.35	0.895	0.901
HC7	152	140.48	138.71	790.28	0.924	0.913
HLC1	260	259.53	263.05	1271.37	0.998	1.012
HLC2	254	243.84	260.62	1406.46	0.960	1.026
HLC3	198	214.12	228.73	1357.37	1.081	1.155
		Mean ratio			0.948	0.970
		Standard deviation			0.0555	0.0789

indicate the centroid coordinates of the cross section.

The solution of algebraic equations (Eqs. (14), (15)) gives the neutral axis parameters (a, c). When substituting these parameters in Eq. (13), the biaxially eccentric ultimate load N_u can be obtained as the following equation

$$N_u = \sum_k^t \bar{A}_{ck}\sigma_{ck} - \sum_i^n A_{si}\sigma_{si} \tag{16}$$

Additional detailed procedure for the strength and load-deformation analyses of high strength reinforced concrete columns can be obtained from the study reported by Tokgoz (2009).

4. Analysis of column specimens

The column specimens were analysed for the ultimate strength capacities and load-lateral deflection curves by using the computer program (Tokgoz 2009). In the analysis, nonlinear stress-strain curves were used for the concrete compression zone of the columns (Hsu and Hsu 1992, Cusson and Paultre 1995). The test results, the predicted ultimate strength values of N_{u1} and N_{u2} computed using the concrete models suggested by Hsu and Hsu (1992) and Cusson and Paultre (1995), respectively, bending moments and comparative ratios of the analytical load to test load of the column specimens are presented in Table 2.

Good agreement has been achieved between the predicted ultimate strength capacities and the test results for the most of the column specimens (Table 2). By comparing the ultimate strength capacities, it is seen that both the concrete stress-strain models used in this study (Hsu and Hsu 1992, Cusson and Paultre 1995) have proven reasonable results. The ratios of N_{u1}/N_{test} and N_{u2}/N_{test}

have been obtained as 0.948 and 0.970 respectively, indicating a good degree of accuracy between the experimental and theoretical values. It is concluded from the analysis that the ultimate concrete compressive fiber strain is the significant parameter on the computation of the strength capacity of high strength reinforced concrete columns.

5. Conclusions

An experimental study of square and L-shaped both short and slender biaxially loaded reinforced concrete columns have been presented. The experimental and theoretical load-lateral deflection behaviour, strength capacities and failure mode of the specimens have been discussed in the study.

The experimental study exposed that buckling of longitudinal reinforcement and the brittle behaviour of failure are the main critical issues for high strength reinforced concrete columns. It is observed that loss of ductility has been appeared with increasing of concrete compressive strength and load eccentricity. Decreasing the lateral reinforcement spacing could improve the ductility of the eccentrically loaded reinforced concrete columns. The slenderness and the load eccentricity parameters have considerable effects on the strength capacity of reinforced concrete columns with high strength concrete.

The column specimens have been analysed and compared with the test results. The proposed analysis method yields a reasonable accuracy in predicting the load-lateral deflection and strength of the reinforced concrete columns. The results indicate that the use of the empirical stress-strain relationships for high strength concrete gives satisfactory aggreement in this study. In addition, the analysis has revealed that the ultimate concrete compressive fiber strain is the most effective parameter in predicting the ultimate strength capacities of high strength reinforced concrete columns.

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