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Time-history analysis based optimal design of space trusses: the CMA evolution strategy approach using GRNN and WA

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Abstract. In recent years, the need for optimal design of structures under time-history loading aroused great attention in researchers. The main problem in this field is the extremely high computational demand of time-history analyses, which may convert the solution algorithm to an illogical one. In this paper, a new framework is developed to solve the size optimization problem of steel truss structures subjected to ground motions. In order to solve this problem, the covariance matrix adaptation evolution strategy algorithm is employed for the optimization procedure, while a generalized regression neural network is utilized as a meta-model for fitness approximation. Moreover, the computational cost of time-history analysis is decreased through a wavelet analysis. Capability and efficiency of the proposed framework is investigated via two design examples, comprising of a tower truss and a footbridge truss.

Keywords: size optimization; space truss structure; time-history analysis; the covariance matrix adaptation evolution strategy (CMA-ES); generalized regression neural network (GRNN); wavelet analysis (WA)

1. Introduction

In the last two decades, advanced analysis procedures have been presented in order to reliably estimate the actual behavior of complex structures under strong earthquakes. One of the most comprehensive methods for seismic assessment of structures is achieved through a step-by-step time-dependent procedure that determines the history of response of structures subjected to earthquake loading. Such procedures are known as time-history analysis methods, which are becoming more popular among researchers, since in these procedures, nearly all sorts of complicated material and geometry can be directly included in the analysis. Although, the model complexity is not considered as an obstacle, however extensive computational demand has prohibited the widespread application of such analyses in practice. This problem will be resonated when these methods are applied to iterative procedures such as optimization. High analysis time prevents designers from thoroughly investigating the design search space that results in uneconomical designs (Kocer and Arora 1999, 2002).

During the recent years, extensive studies have been performed on finding methods to reduce the

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computational burden of time-history analyses, which are classified in two general categories. Firstly, studies on developing meta-models to quickly and precisely predict the time-history response of structures. Secondly, studies on producing surrogate records for original earthquake records that have larger time steps but have almost the same effects on the structures. Radial basis function (RBF) networks, emerged as a variant of artificial neural networks, have been successfully implemented as a reliable meta-model for predicting the response of structures under time-history loading. Fast training, reasonable accuracy and simplicity make RBF network a powerful tool for decreasing computational cost of time-history analysis in iterative procedures (Salajegheh *et al.* 2008, Gholizadeh *et al.* 2009). In the field of producing surrogate records, wavelet analysis has been shown to be much effective. Wavelet transform can divide an earthquake signal into two parts: Low frequency approximation and high frequency detail part. Low frequency part is the most influential part of the original signal on the response of structures. It can also be effectively used in dynamic analysis of structures to decrease the number of points of earthquake record involved in the time-history loading (Salajegheh *et al.* 2005, Gholizadeh *et al.* 2011).

The aim of this study is to propose a framework for size optimization of large-scale truss structures under time-history loading within an acceptable computational time. To achieve this goal, both introduced strategies for computational efficiency of time history analyses are employed in the process of the optimization.

The meta-heuristic developed here, belongs to a subclass of evolutionary algorithms. The CMA evolution strategy is a stochastic method for continuous optimization of non-linear, non-convex problems. In an ES, new candidate solutions are sampled according to a multivariate normal distribution. Pair-wise dependencies between the variables in this distribution are described by a covariance matrix. The CMA is a method to update the covariance matrix of this distribution (Talbi 2009). Self-adaptation is an important feature of the CMA-ES. Self-adaptation in its purest meaning is a method to adjust setting of the strategy parameters. It is called self-adaptive because the algorithm controls the setting itself. Self-Adaptation aims at biasing the distribution towards promising regions of the search space while maintaining sufficient diversity of the search (Kaveh *et al.* 2011b).

After this opening section, the paper is organized as follows: Section 2 explains the concept of time-history analysis. In section 3, the objective function and constraints of the optimization problem are formulated. Section 4 briefly introduces the CMA-ES. In section 5, the strategy employed for the fitness approximation is presented. In section 6, the main ideas behind using wavelet analysis are illuminated. The proposed framework is presented in section 7. Section 8 studies some test problems to verify the efficiency of the proposed algorithm and finally the paper is concluded with section 9.

2. Time-history analysis

Time-history analysis of a structure under a particular earthquake is a step-by-step procedure that determines the dynamic response of the structure over time during and after application of the ground acceleration. To obtain the time-history response, the equation of motion of the structure must be solved, given by

$$\mathbf{M}\ddot{X}(t)+\mathbf{C}\dot{X}(t)+\mathbf{K}X(t) = -\mathbf{M}l\ddot{x}_g(t) \tag{1}$$

where **M** is the mass matrix, **C** is the damping matrix and **K** is the stiffness matrix. $\ddot{X}(t), \dot{X}(t)$ and $X(t)$ are the acceleration, velocity and displacement vectors of the structure, respectively. These vectors are of size n, where n is the number of degrees of freedom of the structure. l is the unit vector and $\ddot{x}_g(t)$ is the ground acceleration (Chopra 1995).

There are two main ways to establish the mass matrix, namely consistent and lumped mass methods. The consistent mass matrix is based on the approximation for the kinetic energy. That is, while the approximation for the strain energy leads to the stiffness matrix, the use of the same shape functions in the approximation of the kinetic energy, leads to the consistent mass matrix. As a result, the consistent mass matrix provides an accurate representation of the inertial properties of the element. The simpler form of mass matrix is the lumped format that is obtained by placing concentrated masses at nodal points in the directions of the assumed degrees of freedom and hence the resulting element mass matrix is a diagonal matrix. The concentrated masses refer to translational and rotational inertia of the element and are calculated by assuming that the material within the mean locations on either side of the particular displacements behaves like a rigid body while the remainder of the element does not participate in the motion. Thus, this assumption excludes the dynamic coupling that exists between the element displacements and leads to natural frequencies that may be higher or lower than the exact ones, whereas by the consistent mass matrix, natural frequencies are bounded below by the exact values (Rao 2004).

In this study, structures are modeled using the consistent mass method. For constructing the damping matrix, Rayleigh damping model (Chopra 1995) is applied by assuming the damping ratio of 5% for the first two modes of free vibration of the structures.

The state-space representation of the equation of motion provides a convenient and compact way to model and analyze the corresponding dynamic system, which transforms the second-order differential equation to a first-order differential equation that has closed-form solution. Eq. (1) in the state space is reformulated as (Ogata 2010)

$$\dot{Z}(t) = \mathbf{A}Z(t) + h\ddot{x}_g(t) \tag{2}$$

where

$Z(t) = \begin{Bmatrix} X(t) \\ \dot{X}(t) \end{Bmatrix}$ is the $2n \times 1$ state vector,

$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$ is the $2n \times 2n$ system matrix,

$h = \begin{bmatrix} 0 \\ I \end{bmatrix}$ is the $2n \times 1$ location vector.

Solution of Eq. (2) is obtained as

$$Z(t) = \exp(\mathbf{A}t) \cdot Z(0) + \int_0^t \exp(\mathbf{A}(t-\tau)) \cdot h\ddot{x}_g(\tau)d\tau \tag{3}$$

where $Z(0)$ represents the initial conditions at time $t = 0$.

Since for the design procedure only the maximum response of the structure is needed, in order to have an optimal design, the effective duration of the ground motion can be used in the analysis instead of considering the entire earthquake record. The effective duration of a ground motion determines the start and end of the strong shaking phase that is the time interval between the accumulation of 5% and 95% of ground motion energy, where ground motion energy is defined by the Arias Intensity (Towhata 2008). The end of the duration indicates the time that the maximum response will occur definitely until then, therefore in order to achieve an optimal design, the record needs to be analyzed up to this time and further analysis is not necessary. The effective duration of an earthquake record can easily be computed by the available software like SeismoSignal®. In this paper, the concept of the effective duration is applied to optimize the design process with the difference that here the start of the duration is considered to be from the start of the corresponding earthquake record.

3. Optimization problem

Truss optimization is one of the most active fields in structural mechanics (Kaveh and Talatahari 2008, 2009, 2010). Size optimization of truss structures looks for optimum values of member cross-sectional areas that minimize the structural weight. This optimal solution should also satisfy the inequality constraints that limit design variable sizes and structural response. The objective function of size optimization can be expressed as

$$\text{Minimize} \quad W(A) = \sum_{i=1}^{m} \gamma_i \cdot L_i \cdot A_i \tag{4}$$

where $W(A)$ is the weight of the structure; m is number of members making up the structure; γ_i, L_i and A_i are material density, length and cross-sectional area of member i, respectively (A_i chosen between A_{min} and A_{max}).

The constraints are as follows

$$\begin{cases} \sigma_i(t) \le \sigma_{allowable} & i = 1, 2, \ldots, m \\ \delta_i(t) \le \delta_{allowable} & i = 1, 2, \ldots, n \end{cases} \tag{5}$$

where m and n are the number of members and nodes, respectively. $\sigma(t)$ and $\delta(t)$ represent the member stress and nodal displacement at time t determined by time-history analysis. *allowable* denotes the upper bounds.

According to (AISC 1995), the stress limitation for tension members is $\sigma_{allowable} = 0.6F_y$, in which F_y is the yield stress of steel. For compression members this limitation is given by

$$\sigma_{allowable} = \begin{cases} \left[\left(1 - \dfrac{\lambda_i^2}{2C_C}\right)F_y\right] \Big/ \left(\dfrac{5}{3} + \dfrac{3\lambda_i}{8C_C} - \dfrac{\lambda_i^3}{8C_C^3}\right) & \text{for} \quad \lambda_i < C_c \\ \dfrac{12\pi^2 E}{23\lambda_i^2} & \text{for} \quad \lambda_i \ge C_c \end{cases} \tag{6}$$

where

E is the modules of elasticity,

$\lambda_i = kl_i/r_i$, the slenderness ratio of member i where k and r_i are the effective length factor and the radius of gyration, respectively,

$C_c = \sqrt{2\pi^2 E/F_y}$ is the slenderness ratio dividing the elastic and inelastic buckling regions.

All structures before being exposed to ground motions have to resist the static loads imposed on them including their own self-weight. It means to achieve the ultimate response of a structure, the member stresses and nodal displacements calculated by static analysis must sum up to those of calculated by dynamic analysis. In this paper, the standard matrix stiffness method is used to perform the static analysis.

4. Optimization algorithm: the CMA-ES

In this section, a brief description of the CMA-ES is presented. For more information on the terminology and details, interested readers may refer to (Hansen 2011). A summary of the algorithm together with a table of the default strategy parameters and their values are provided in Appendix A.

4.1 Basic equation: sampling

In the CMA-ES, a population of new search points (individuals, offspring) is generated by sampling a multivariate normal distribution. At each generation, the basic equation for sampling is

$$x_k^{(g+1)} \sim m^{(g)} + \sigma^{(g)} N(0, \mathbf{C}^{(g)}) \quad \text{for} \quad k = 1, \ldots, \lambda \tag{7}$$

where

$N(0, \mathbf{C}^{(g)})$ is a multivariate normal distribution with zero mean and covariance matrix $\mathbf{C}^{(g)}$,

$x_k^{(g+1)} \in R^n$, k-th offspring from generation $g + 1$,

$m^{(g)} \in R^n$, mean value of the search distribution at generation g,

$\sigma^{(g)} \in R_+$, overall standard deviation, step-size, at generation g,

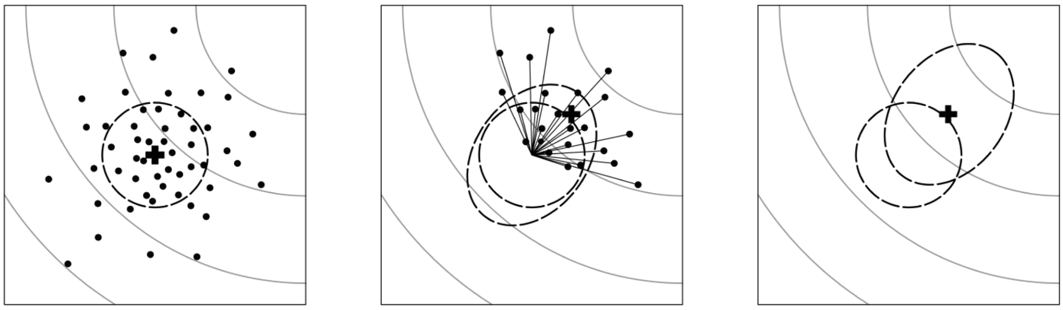
$\mathbf{C}^{(g)} \in R^{n \times n}$, covariance matrix at generation g,

$\lambda \geq 2$, population size, number of offspring.

To define the complete iteration step, the remaining question is how to define $m^{(g+1)}$, $\mathbf{C}^{(g+1)}$ and $\sigma^{(g+1)}$ for the next generation $g + 1$. The next three subsections deal with these questions. For sake of brevity, formulas of Appendix A are not repeated again and they are only addressed by their number.

4.2 Selection and recombination

The new mean $\mathbf{m}^{(g+1)}$ of the search distribution is a weighted average of μ selected points from the sample, Eq. (11), where $\mu \leq \lambda$ is the parent population size. The selected points are the best individuals out of $\mathbf{x}_1^{(g+1)}, \ldots, \mathbf{x}_\lambda^{(g+1)}$ from Eq. (7).



(a) (b) (c)

Fig. 1 Estimation of the search distribution for the second generation. (a) producing the first population by sampling of N (0, I), (b) selection of the new parents and updating the covariance matrix; solid lines determines the selected steps, (c) search distribution of the next generation (dashed ellipsoid). Contour lines (grayed) indicate that the strategy should move toward the upper right corner

4.3 Adapting the covariance matrix

The CMA-ES is based on two adaptation principles, which make it an efficient procedure for multimodal continuous problems. Firstly, a maximum-likelihood principle, based on the idea to increase the probability of successful candidate solutions and search steps. For this purpose, the algorithm updates the covariance matrix of the distribution such that the likelihood of already applied successful steps is increased. Rank-μ-update performs this principle.

Secondly, an evolution path principle, based on memorizing the time evolution path of the distribution mean. A sequence of successive steps, the strategy takes over a number of generations, is called an evolution path, Eq. (14). These paths contain substantial information about the correlation between consecutive steps. The evolution paths are exploited in two ways. One path is used for the covariance matrix adaptation procedure and facilitates a possibly much faster variance increase of favorable directions. Rank-one-update performs this. The other path is used to conduct an additional step-size control that effectively prevents premature convergence yet allowing a faster convergence (see Section 4.4). The final CMA update of the covariance matrix combines the advantages of the rank-μ-update and the rank-one-update, Eq. (15).

Fig. 1 demonstrates the concept behind the covariance matrix adaptation in the CMA-ES. As the generations progress, the algorithm approaches to the global optimum while simultaneously the distribution shape adapts to an ellipsoidal landscape and the search is directed along an evolution path.

4.4 Step-size control

The covariance matrix adaptation, introduced in the previous section, does not explicitly control the “overall scale” of the distribution. Step-size control defines how much the distribution ellipsoid should be elongated or shortened, to achieve an optimal scale. The evolution path is utilized to control the step-size.

The length of an evolution path is exploited, based on the following reasoning. Whenever the evolution path is short, single steps cancel each other out as is shown in Fig. 2(a). Hence, these are called anti-correlated. If steps annihilate each other, the step-size should be decreased. Whenever the

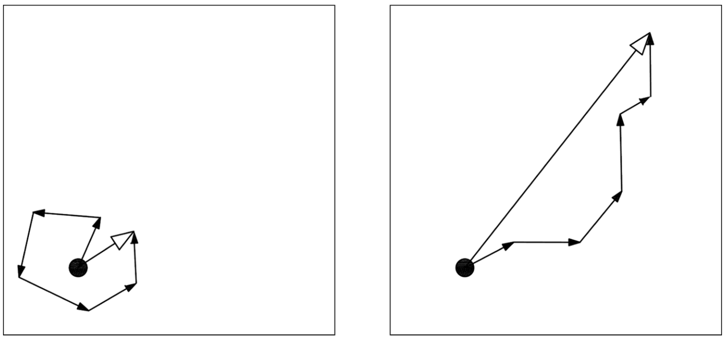


Fig. 2 Two evolution paths of six steps from various situations. The length of the evolution paths is remarkably different and is exploited for step-size control

evolution path is long, the single steps are pointing to similar directions and they are called correlated, Fig. 2(b). Since the steps are similar, the same distance can be covered by fewer but longer steps into the same direction. Consequently, the step-size should be increased. When the step-size approaches zero, the population converge to an optimum solution.

To decide whether the evolution path is long or short, the length of the path is compared to its expected length under random selection, which is equal to the expectation of the Euclidean norm of a N (0, $\mathbf{I}$) distributed random vector. If selection biases the evolution path to be longer than expected, σ is increased, and vice versa, Eq. (13). To calculate the step-size, a conjugate evolution path is constructed, Eq. (12), because the expected length of the evolution path from Eq. (14) depends on its direction.

The CMA-ES does not require a tedious parameter tuning for its application. In fact, the choice of strategy internal parameters is not left to the user. Finding good strategy parameters is considered as a part of the algorithm design, and not part of its application. For the application of the CMA-ES, only an initial solution, an initial step-size, and the termination criteria need to be set by the user.

5. Fitness approximation strategy

In the present optimization problem, fitness function evaluation is the most time-consuming part of the solution algorithm. If all of the required fitness function evaluations are performed by time-history analysis, it may need many hours even for small structures. The solution of this problem is the use of computationally efficient approximations of the fitness function, a remedy utilized for solving optimization problems with extremely expensive objective functions.

In many real-world problems, due to the lack of data and the high dimensionality of the input space, it is very difficult to obtain a perfect global functional approximation (meta-model) of the original fitness function. To tackle this problem, two main tactics can be adopted. Firstly, the quality of the approximate model should be improved as much as possible, given a limited number of data. Several aspects are important to improve the model quality, such as selection of the model, use of the active data sampling and weighting, selection of training method, and selection of error measures. Secondly, the approximate model should be used together with the original fitness function. In the most cases, the original fitness function is available, although it is computationally very expensive. Therefore, it is very important to use the original fitness function efficiently. This is known as model management in conventional optimization or evolution control in evolutionary computation (Jin 2005). In the next subsections, these two concerns are reviewed in our specific optimization problem.

5.1 Meta-model selection and training

Neural networks are adaptive statistical models, which can be trained and utilized for predicting the response of a function. A neural network consists of an interconnected group of simple processing elements called artificial neurons, which exhibit complex global behavior determined by pattern of connections between them. Advanced neural networks have shown to be effective in modeling most complicated non-linear relationships between inputs and outputs (Galushkin 2010).

RBF neural networks have been successfully applied as a reliable meta-model for predicting the time-history response of structures. The obtained results demonstrate that with respect to the model precision and the required computational time, the RBF networks perform well (Salajegheh *et al.* 2008, Gholizadeh *et al.* 2009). In this study, the generalized regression neural networks are exploited for predicting the results of time-history analyses in the optimization process. GRNN is an advanced variant of RBF networks that is also known as normalized RBF network. GRNN has a radial basis layer and a special extra linear layer that performs normalization on the output set. Normalization yields accuracy improvement even more as input dimensionality increases (MATLAB 2011).

Selection of the input data should be done in a manner that firstly it can represent the considered structure properly, and secondly the trained network by these input data should be able to predict the time-history response of the structure with an acceptable precision. In this study, several alternatives for the input data are verified as well as the member's cross-sectional areas, the natural frequencies, the gyration radius of members and the slenderness ratio of members. Among these, the member's cross-sectional areas result in the most accurate estimates of the structural response.

Since the response of a structure includes nodal displacements and member stresses, two different output sets must be considered. Consequently, two distinct GRNNs with the same input data are constructed to predict each response. The first set covers the maximum displacement of nodes between three dimensions of the coordinate system. The second set contains the critical member stresses. To define the critical stress for a member, its maximum tensile and compressive stresses should be divided by the member's tensile and compressive capacities, respectively; the largest value determines the critical member stress.

Since in our problem the design search space is extremely large, the trained network with these widely ranged input data has low precision in estimating the response. When the input data of the GRNN are mostly similar, its estimate of the response to an arbitrary design is more accurate. In order to improve the quality of the meta-model, in this study, a concept of nearest data selection is applied (Kaveh *et al.* 2011a). In this approach, all the solutions, which are evaluated by the original fitness function, are stored in an archive. In the process of fitness approximation, when the optimization procedure generates a new solution, first, its k nearest neighbors in the archive (in the design search space) is determined and then a new GRNN is constructed and trained by these similar solutions. Finally, the trained network is used for estimating the fitness of the given solution. It should be noted that the value of k should be determined in a way that first, the trained network should be able to estimate the response precisely; second, the network should not be over trained. The distance between two different solutions is defined as follows

$$d = \left\| A^i - A^j \right\| \tag{8}$$

where A is the vector of member's cross-sectional areas and $\|\cdot\|$ is the norm of A. In this study, the value of k is set to 15.

5.2 Evolution control

Application of approximate models in the evolutionary optimization procedures is not as straightforward as one may expect. One important point is that it is very difficult to construct a meta-model that is globally accurate due to the high dimensionality, ill distribution, and limited number of training samples. There are three major concerns in using meta-models for the fitness approximation. Primarily, it should be ensured that the evolutionary algorithm converges to the global optimum or a near optimum of the original fitness function. Secondly, the computational cost should be reduced as much as possible. Thirdly, in the process of evolutionary optimization, the range of the solutions may change significantly and the model trained by the initial data may converge to a false optimum. Therefore, it is quite essential in most cases that the approximate model be used together with the original fitness function. The issue of incorporation of these two functions in the process of optimization is regarded as model management (Jin 2005).

Usually, model management in evolutionary computation is performed using two main approaches, one is individual-based and the other is generation-based evolution control. By individual-based control, it is meant that in each generation, some of the solutions use the meta-model and others employ the original function for fitness evaluation. While in generation-based control, in some specified generations all of the solutions are evaluated by the original fitness function (Hagan 1996).

The decision about the evolution control should be made based on the properties of the problem under consideration. In our specific problem, the following method is employed:

- In the first 15 generations of the optimization process, all solutions are evaluated by the original fitness function. All of these solutions are stored in an archive, regardless of whether they are acceptable due to the constraints or not. For each solution, the member's cross-sectional areas and the result of time-history analysis are recorded in the archive.
- In each generation, $\mu/2$ of the solutions are evaluated by the original fitness function and the others by the meta-models. In this way gradually some solutions of the new regions of the search space is added to the archive.
- After each 20 generations, in one generation all of the generated solutions are evaluated by the original fitness function. In this way, it is insured that the evolutionary computation converges to the main global optimum of the search space.

In the above mentioned method, all the solutions evaluated by the original fitness function are added to the archive and the meta-models utilize the provided data in this archive for training the GRNNs.

6. Wavelet analysis

Wavelet analysis is an advanced mathematical set of tools and techniques for signal-processing, which has aroused great attention in many fields of science and engineering. By WA we can denoise a signal from high-frequency components to understand the behavior of the primary signal better (see Fig. 3(a)). The theory and methods of WA are widely available in literature. In this paper, only the application of WA in our problem is explained. The interested reader may refer to (Strang and Nguyen 1996) for further information.

Wavelet transform is exploited for dividing data, functions and signals into different frequency components, where each of them is studied with a resolution matched to their scale. Wavelet transform can be simply attained by a tree of filter banks as shown in Fig. 3(b). In this figure, "downsampling" is an operation that keeps the even indexed elements of the input signal. The key scheme for filter banks is to cut up a signal into two parts; the first is the low-frequency and the other is the high-frequency part. This scheme is achieved by a set of filters (a low- and a high-pass filter), which separate a signal into different frequency bands. The low-pass filter removes the high-frequency bands of the signal and produces an approximate description of the primary signal. By

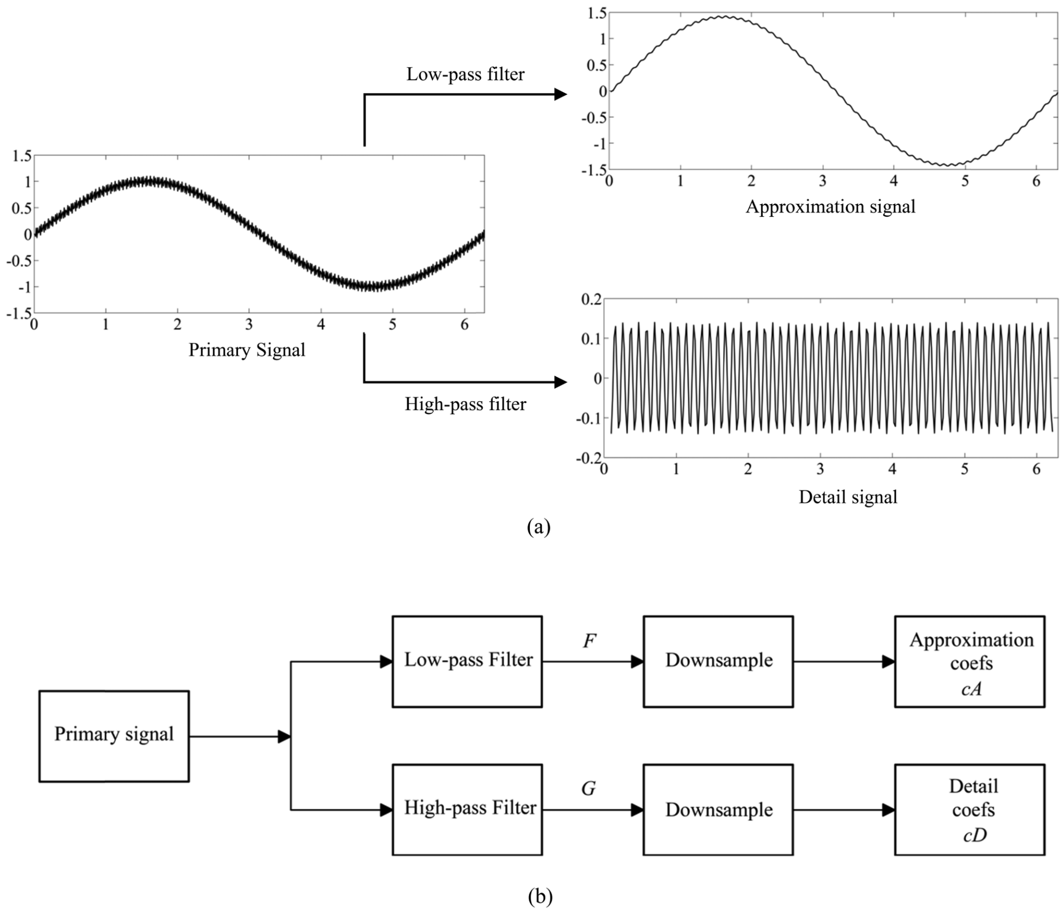


Fig. 3 (a) A signal processing example using WA, (b) general algorithm for discrete wavelet transforms

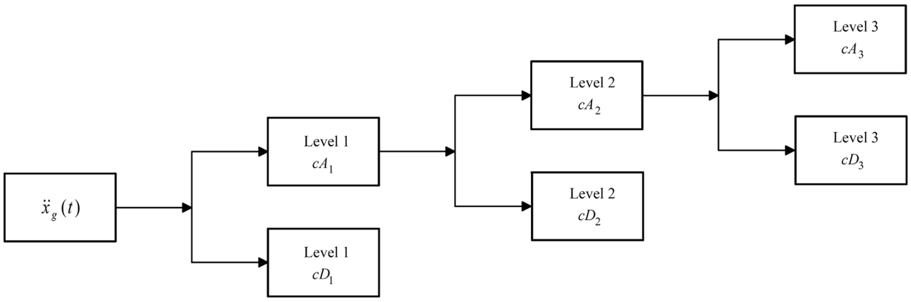


Fig. 4 A three-level wavelet decomposition of earthquake record $\ddot{x}_g(t)$

utilizing the high-pass filter, the low-frequency components of the signal are removed, and a signal including the details of the main signal is achieved. In other words, by constructing a filter bank with these two filters, the primary signal is separated into an approximation and a detail signal. Output of the filter bank is two sets of coefficients, (cA) and (cD) that include the low- and high-frequency contents of the main signal, respectively. In Fig. 3(a), the length of each filter is equal to $2N$ that N is the order of the wavelet function used as the filter. If n = length ($\ddot{x}_g(t)$), the signals F and G will be of length $n + 2N - 1$. And then the coefficients cA and cD are of length equals floor $(n-1/2)+N$, almost half of the primary signal length (MATLAB 2011).

Response of a structure under a given ground motion is mostly affected by the low-frequency content of the earthquake record. This content can be efficiently used in time-history analysis as a surrogate for the original record in order to decrease the number of points that are involved and then reduce the computational demand of the time-history analysis. The decomposition process can be repeated for the low-frequency content to achieve the desired scale of the earthquake record. This multilevel decomposition is called the wavelet decomposition tree (Gholizadeh *et al.* 2011). In this study, the decomposition process proceeds in three levels, as shown in Fig. 4, i.e., the approximate version of the earthquake record in the last step (cA_3) is utilized in the time-history analysis. Therefore the number of points involved is decreased to 0.125 of the primary record. It should be noted that when cA_3 is applied for the analysis, the time-step needs to be updated by

$$dt = dt \times \frac{length(\ddot{x}_g)}{length(cA_3)} \tag{9}$$

The decomposition process is invertible and the primary record can be reconstructed by convolving the obtained approximation and detail coefficients. This process is known as inverse wavelet transform that simply can be defined by reversing the trajectory of the wavelet transform algorithm. For computing the actual response of the structure under the original earthquake record, the reverse process is required. To achieve this, the dynamic response of the structure to the surrogate record is taken as the approximation coefficients of the last level and the detail coefficients of all levels are considered to be zero.

In our problem, WA is needed to be performed once before the start of the optimization procedure. In the phase of preparing the input data for optimization, WA produces a surrogate record for the given ground motion. This record is then used in all time-history analyses of the program instead of the original earthquake record. In the present study, Daubechies wavelet family (Db1 to Db6) is operated as the filter, i.e., the earthquake record is decomposed by each of these six wavelet functions and then the best approximation record is exploited in the optimization. The quality is tested with the use of the RRMSE (relative root mean squared error), and the R^2 (coefficient of determination) measures, as follows

$$RRMSE = \left(\left(\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \tilde{x}_i)^2\right) \Big/ \left(\frac{1}{n}\sum_{i=1}^{n}(x_i)^2\right)\right)^{\frac{1}{2}} \tag{10.a}$$

$$R^2 = 1 - \left(\sum_{i=1}^{n}(x_i - \tilde{x}_i)^2\right) \Big/ \left(\sum_{i=1}^{n}(x_i - \bar{x}_i)^2\right) \tag{10.b}$$

where x_i and $\tilde{x}_i$ are the ith component of the exact and approximate vectors, respectively. The mean value of exact vector and the dimension of vectors are expressed by $\bar{x}$ and n, respectively.

7. The proposed framework

Now, all of the introduced components in previous sections are incorporated in a simple framework, which makes the size optimization of space trusses subjected to time-history loading feasible. In this problem, all constraints are classified into two main groups:

- Initial constraints: Firstly, the cross-sectional areas of members of the newly generated design should be restricted to the permissible range. This can be done by selecting the upper or lower bounds for the violated areas. Secondly, the structural response of the new design under static loading must fulfill the requirements of the building code, before being subjected to the ground motion (see Sect. 3). If this constraint is violated, the design is omitted and a new one is generated.
- Final constraints: This group contains the checking of the nodal displacements and member stress ratios as mentioned in Section 3. In this step, ultimate response of the structure, a summation of the results of time-history and static analyses, is checked.

The main procedure, which is based on the CMA-ES algorithm, is as follows. The relevant sections to each step are noted in brackets:

Main procedure {

1. *Set parameters*.
2. Until termination criterion met
   - 2.1. Sample new population of search points [Sect 4.1].
     - 2.1.1. Sample a new individual.
     - 2.1.2. *Evaluate* the new individual.
   - 2.2. Step-size control [Sect 4.4].
   - 2.3. Covariance matrix adaptation [Sect 4.3].

}.

The first step is performed as follows:

Set parameters {

1. Set the CMA-ES user defined parameters.
2. Structural modeling.
3. Define the effective duration of the given earthquake record.
4. Perform WA on the effective duration [Sect 6].

}.

Evaluation of the generated population is expressed by:

Evaluate {

1. Prepare the input data for structural analysis.
2. Check the initial constraints.
3. If evolution control conditions are fulfilled then [Sect 5.2]

3.1. Perform time-history analysis for the given ground motion [Sect 2].

3.1.1. Store the obtained solution to the *archive* (member's cross-sectional areas, nodal displacements, critical member stresses).

3.1.2. Check the final constraints.

3.1.2.1. If the constraints are not violated then $\lambda = \lambda + 1$.

else

3.2. Select k nearest solutions in cross-sectional areas of the *archive* and train the required GRNNs by these input data [Sect 5.1].

3.3. Estimate the structural response by the trained GRNNs.

3.3.1. Check the final constraints.

3.3.1.1. If the constraints are not violated then $\lambda = \lambda + 1$.

}.

8. Design examples

In this section, the proposed framework is implemented in MATLAB® and some test problems are optimized. The structural analysis is completely executed by MATLAB®. As mentioned before, the direct stiffness method is used to perform the static analysis and for time-history analysis, all structures are modeled and solved in the state-space formulation. In order to validate the analysis code, all instances are verified by SAP®. For constructing a GRNN and wavelet decomposition of an earthquake record, neural network and wavelet toolboxes of MATLAB® are employed, respectively. The earthquake records have been selected from the PEER Strong Motion database (PEER 2012). The effective duration of each record is calculated by SeismoSignal®. In all the examples, the value of maximum allowable cross-sectional areas is taken to determine the start point in the search space for initializing the optimization procedure. The CPU-time consumption of the program is calculated for each case. All recorded times are obtained using an Intel® Core™ i7 @ 2.0 GHz processor equipped with 8 GBs of RAM.

8.1 A footbridge truss

The first design example is an actual-size footbridge truss consisting of 220 members and 68 nodes, shown in Fig. 5. As indicated in this figure, 44 member groups are considered. The effect of the superstructure dead loads on the truss, including the weight of the concrete deck, asphalt and sidewalks, is considered by a set of masses of weight 2500 lb (1134 kg) lumped at each top node.

The material and cross-sectional properties are as follows: the modulus of elasticity and the yield stress of the steel are taken as 10000 ksi (68943 MPa) and 35 ksi (241.3 MPa), respectively. The material density is 0.3 lb/in^3 (8304 kg/m^3). The radius of gyration of each member (r_i) is expressed in terms of its cross-sectional area as $r_i = aA_i^b$, where a and b are the constants depending on the types of sections adopted for the members. In this example, pipe sections of a = 0.799 and

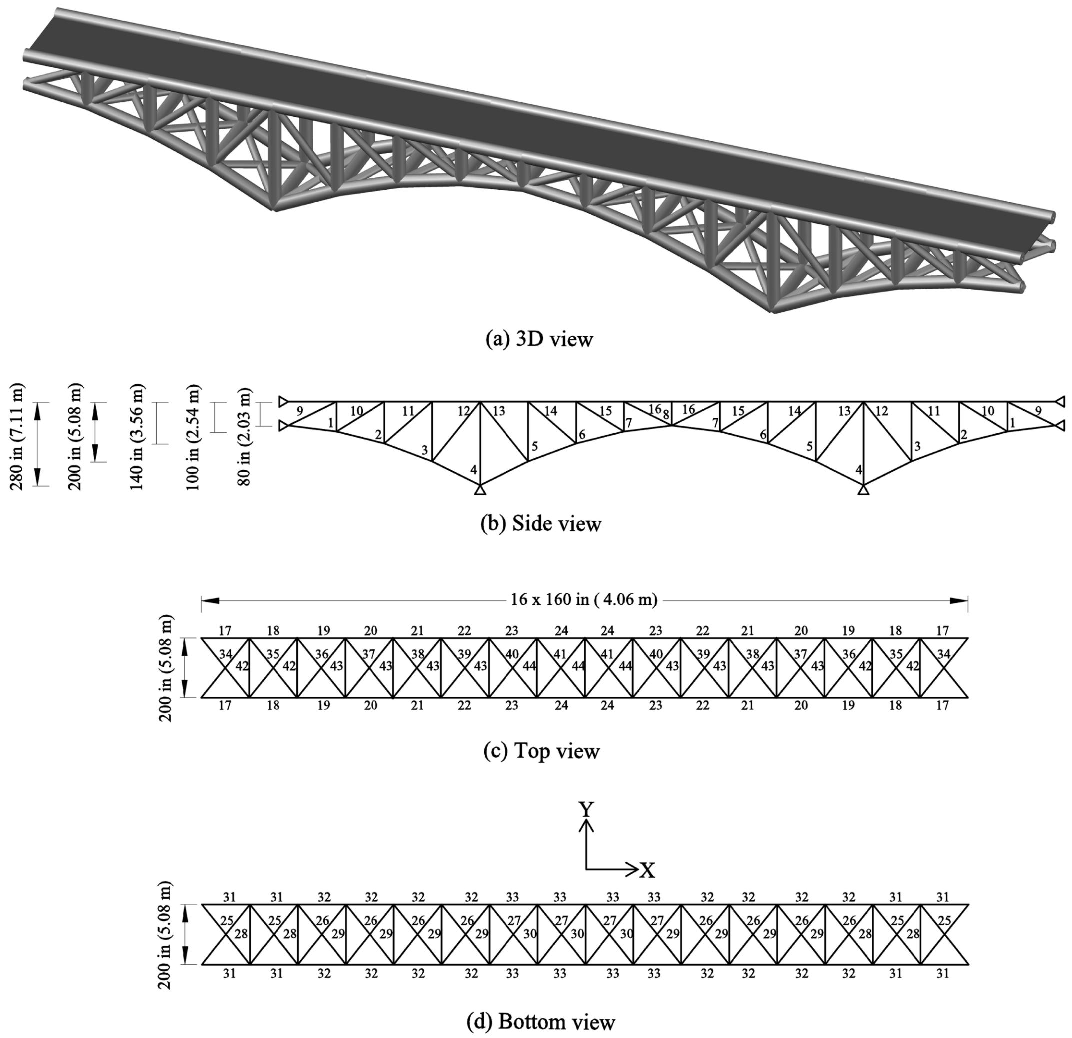


Fig. 5 A footbridge truss: topology and member grouping

$b = 0.669$ are utilized. Minimum cross-sectional area of all members is considered as 0.5 in^2 (3.226 cm^2) and the maximum as 10 in^2 (64.516 cm^2). Maximum displacement limitation of 2 in (5.08 cm) is imposed on every node in all directions.

This structure is subjected to the Loma Prieta ground motion with peak ground acceleration of 1.7 g in the *y*-direction, as shown in Fig. 6(a). The effective duration of this record stops at second 16.245, which leads to 3249 points with a time step of 0.005 sec, Fig. 6(b). In order to select a wavelet function for performing WA on the effective duration, 50 models of the structure are generated randomly and analyzed using the Daubechies wavelet family (Db1-Db6). Then the quality of each wavelet function for each model is evaluated by the RRMSE and R^2 measures. The mean values of the errors are reported in Table 1. As it can be seen, the Db3 outperforms other functions, i.e., produces the nearest results to the desired structural responses. Implementation of Db3 for wavelet decomposition reduces the number of points to 410 with a time step of 0.0396 sec,

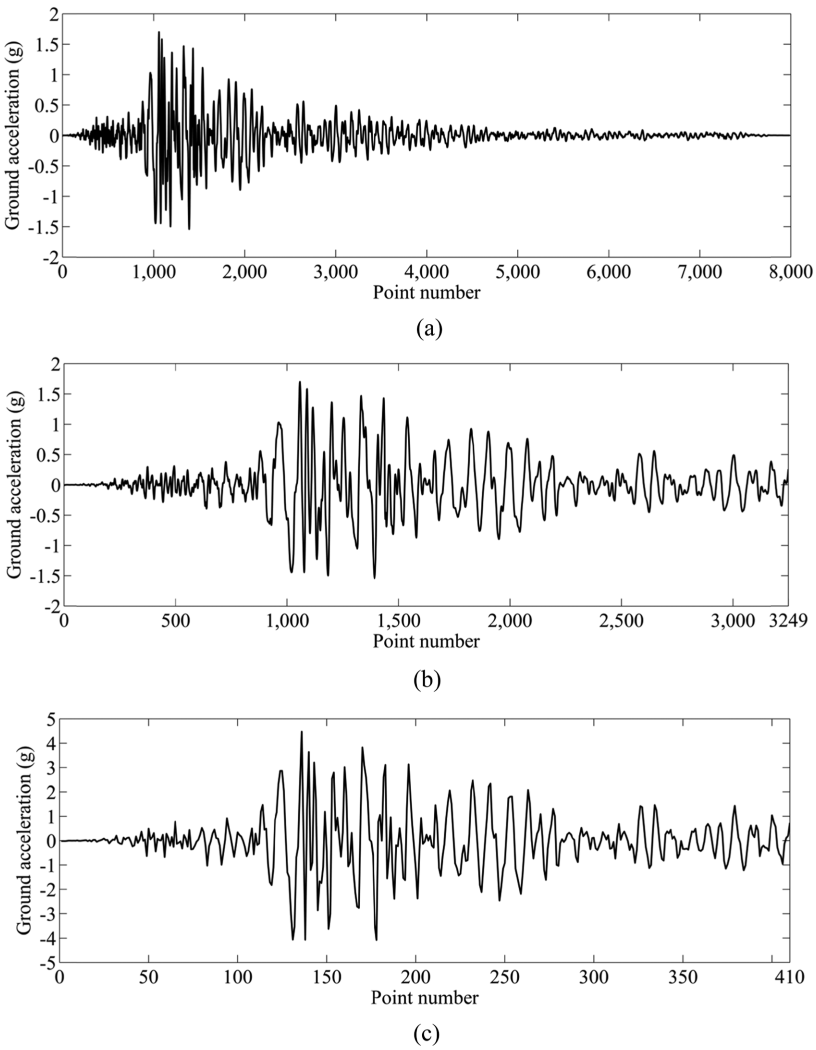


Fig. 6 Loma Prieta ground motion (station: Gilroy Array #7, 1989, PGA = 1.7 g). (a) original record, (b) original record in the effective duration, (c) filtered record

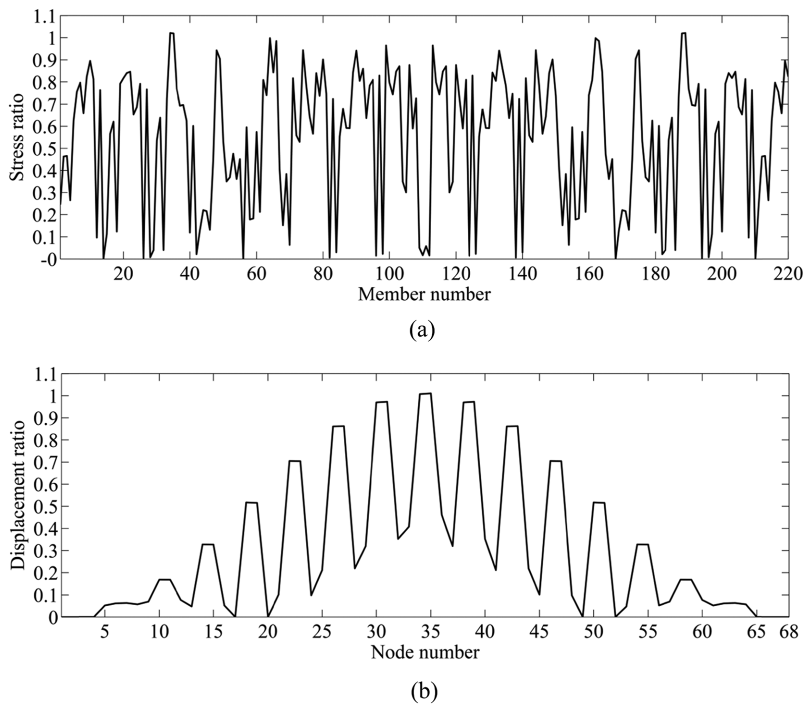
Table 1 Performance evaluation of the Daubechies wavelet family for the footbridge truss

	Displacement estimation error		Member stress estimation error	
	RRMSE	R^2	RRMSE	R^2
Db1	0.0320	0.9975	0.0852	0.9872
Db2	0.0267	0.9981	0.0564	0.9945
Db3	0.0231	0.9985	0.0517	0.9944
Db4	0.0320	0.9972	0.0595	0.9936
Db5	0.0393	0.9958	0.0696	0.9915
Db6	0.0426	0.9950	0.0707	0.9915

Fig. 6(c). This filtered record is a surrogate for the original earthquake record that is used instead of it throughout the optimization. 410 is obtained as follows: 1627 = floor $\left(\frac{3249-1}{2}\right)+3$ at level 1, 816 = floor $\left(\frac{1627-1}{2}\right)+3$ at level 2 and 410 = floor $\left(\frac{816-1}{2}\right)+3$ at level 3 (see Sect. 6).

Table 2 The optimal design achieved for the footbridge truss

Optimal cross-sectional areas (in^2)									
Group No.	Area	Group No.	Area	Group No.	Area	Group No.	Area	Group No.	Area
1	1.4140	10	2.8925	19	6.1497	28	1.8105	37	3.2134
2	1.9246	11	2.3478	20	4.4274	29	0.8222	38	2.8068
3	2.8063	12	2.0725	21	4.0988	30	0.5606	39	2.4567
4	4.2512	13	1.1088	22	3.0404	31	3.3227	40	2.0864
5	3.0668	14	1.3568	23	3.3237	32	2.9906	41	1.3771
6	1.7862	15	1.5787	24	2.4963	33	3.4147	42	0.9927
7	1.1837	16	1.4362	25	3.6890	34	1.9357	43	0.6369
8	3.9350	17	3.7705	26	3.3017	35	1.9771	44	0.5000
9	3.8811	18	2.7488	27	2.3377	36	1.9272		
Structural weight = 33987 lb (15416 kg)									



(a)

(b)

Fig. 7 Obtained (a) stress ratios, (b) displacement ratios for the footbridge truss

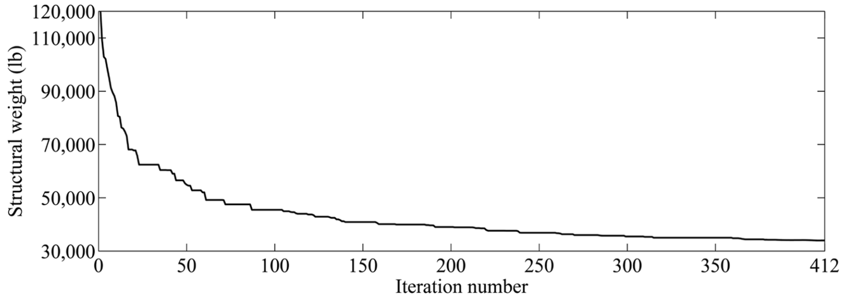


Fig. 8 Convergence history of the CMA-ES for the footbridge truss

Table 3 Performance evaluation of the proposed fitness approximation strategy for the footbridge truss

	Optimization method	
	Design A: With use of meta-model	Design B: Without use of meta-model
Structural weight	33987 lb (15416 kg)	32154 lb (14494 kg)
Computation time	438 mins (7.30 hours)	1295 mins (21.58 hours)
Number of original fitness function evaluation	2842	7393
Number of fitness function approximation	4734	0
Total number of fitness function evaluation	7576	7393
Quality improvement	0%	5.39%
Time improvement	66.18%	0%

Due to the stochastic nature of the solution algorithm, this problem is solved 5 times. The value of initial step-size in all runs is set to 1.75 and the termination criterion is met when the step-size approaches 0.05.The best found design is reported in Table 2. Fig. 7 shows the obtained stress ratios, calculated by dividing the critical member stress by the allowable capacity, and the obtained displacement ratios, computed by dividing the nodal displacement by the allowable value. The convergence history of the algorithm is displayed in Fig. 8.

In order to provide a measure to assess the performance of the proposed fitness approximation strategy, this example is solved again by evaluating the original fitness function instead of fitness approximation during the entire optimization procedure. The obtained results are compared as illustrated in Table 3.

As can be seen from Table 3, by implementing the developed fitness approximation strategy a significant improvement in computational effort is achieved, at the expense of only a small loss of accuracy. It should be noted that without use of the employed WA, the solution process would have required roughly eight times more time, e.g., for design B 173 hours will be needed.

8.1 A tower truss

The second design example, shown in Fig. 9, is a tower truss consisting of 314 members and 84 nodes, where all members are categorized into 45 groups employing the symmetry of the structure.

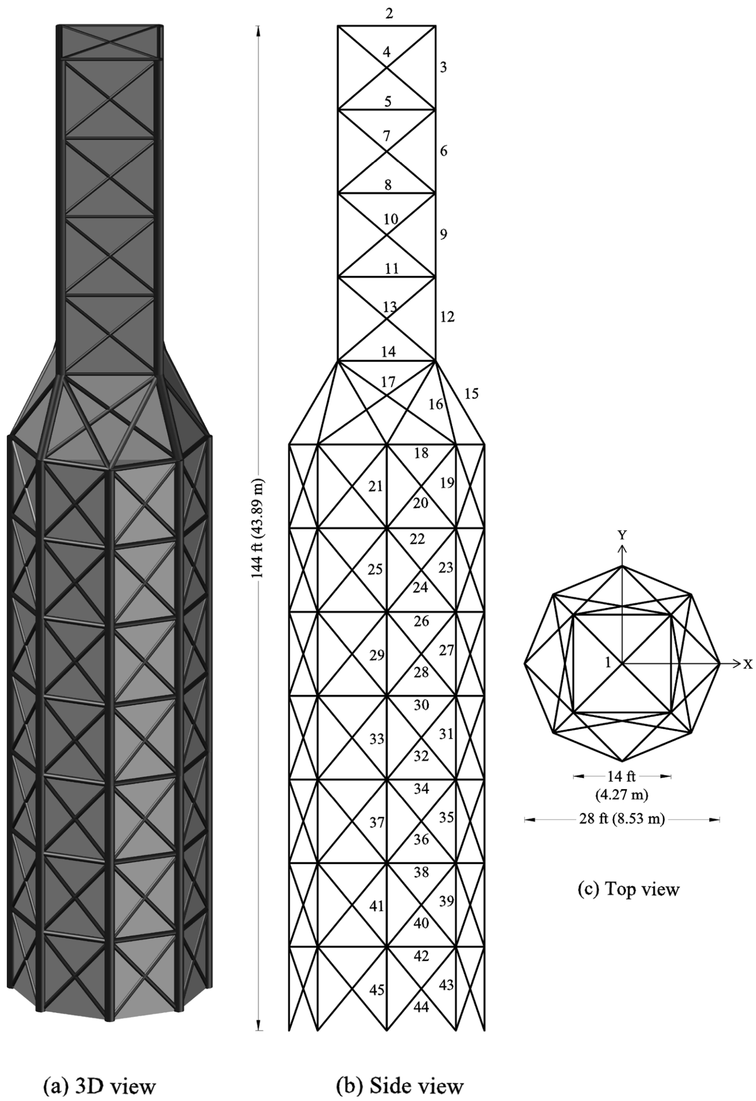


Fig. 9 A tower truss: topology and member grouping

The material properties and the geometry of sections are considered as in the previous example. The range of cross-sectional areas varies from 0.5 to 15 in^2 (3.226 to 96.774 cm^2) and the nodal displacements are bounded to 8 in (20.32 cm).

This structure is subjected to the Coalinga ground motion with peak ground acceleration of 2.0 g in both the x- and y-directions, displayed in Fig. 10(a). The effective duration of this record stops at second 9.51, which leads to 1902 points with a time step of 0.005 sec, Fig. 10(b). Performance evaluation of the Daubechies wavelet family, which is obtained by analyzing 50 randomly generated models of the structure, is presented in Table 4. As it can be seen, the Db2 outperforms other functions. Implementation of Db2 for wavelet decomposition reduces the number of points to 240 with a time step of 0.0396 sec, Fig. 10(c).

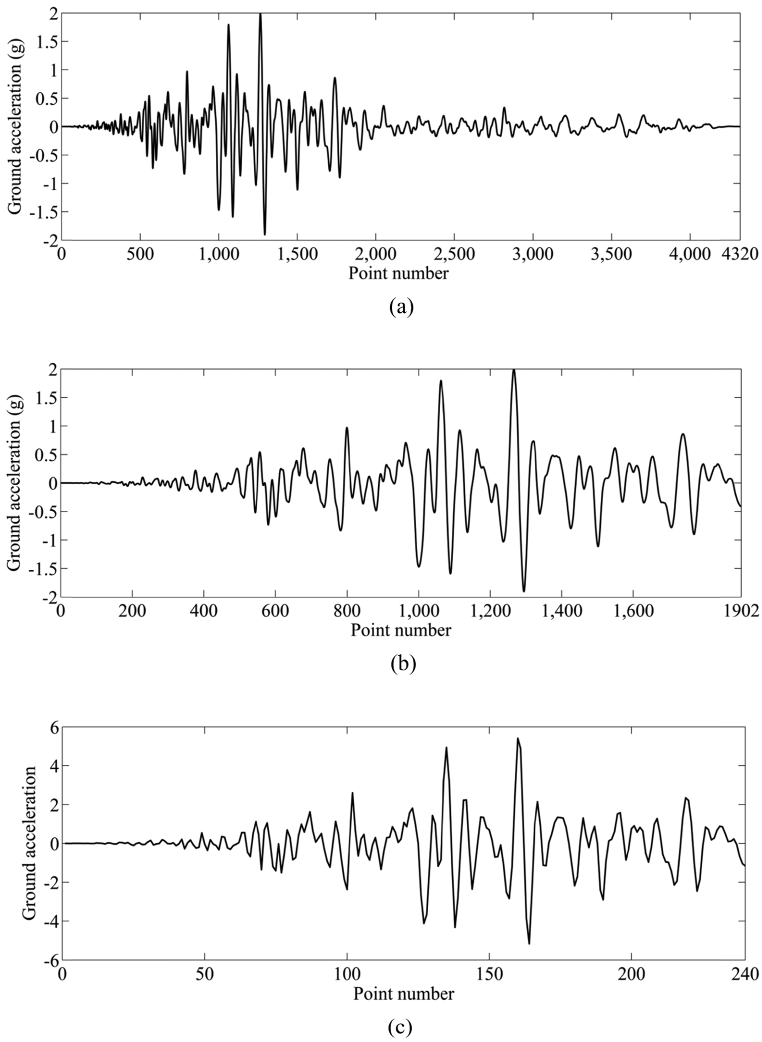


Fig. 10 Coalinga-05 ground motion (station: Oil city, 1983, PGA = 2.0 g), (a) original record, (b) original record in the effective duration, (c) filtered record

Table 4 Performance evaluation of the Daubechies wavelet family for the tower truss

	Displacement estimation error		Member stress estimation error	
	RRMSE	R^2	RRMSE	R^2
Db1	0.0391	0.9966	0.0480	0.9950
Db2	0.0232	0.9988	0.0279	0.9983
Db3	0.0342	0.9974	0.0475	0.9951
Db4	0.0356	0.9972	0.0522	0.9941
Db5	0.0548	0.9933	0.0637	0.9913
Db6	0.0722	0.9883	0.0757	0.9877

Table 5 The optimal design achieved for the tower truss

Optimal cross-sectional areas (in^2)									
Group No.	Area	Group No.	Area	Group No.	Area	Group No.	Area	Group No.	Area
1	2.0912	10	3.3100	19	6.3445	28	3.6660	37	5.24849
2	1.2039	11	2.2384	20	3.3805	29	6.3279	38	2.66310
3	8.9772	12	5.1180	21	5.3277	30	4.9212	39	11.1094
4	3.2545	13	4.4017	22	2.2399	31	6.7704	40	4.03505
5	8.3155	14	6.8504	23	6.4469	32	3.3951	41	5.27015
6	6.0712	15	6.1707	24	3.6142	33	11.7020	42	3.22022
7	2.8874	16	9.1762	25	3.4791	34	2.37664	43	7.61367
8	4.7540	17	1.7435	26	2.2465	35	7.77521	44	4.43820
9	6.8169	18	8.8065	27	7.0078	36	3.83677	45	7.29304
Structural weight = 72006 lb (32661 kg)									

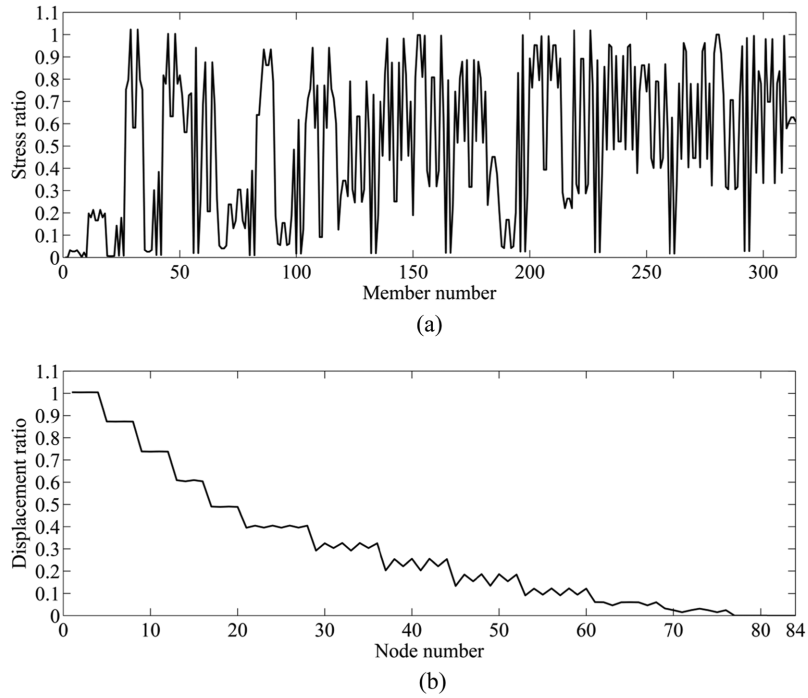


Fig. 11 Obtained (a) stress ratios, (b) displacement ratios for the tower truss

This example was solved 5 times and the value of initial step-size was set to 2 in all the runs. The best found solution is reported in Table 5. Fig. 11 demonstrates the obtained stress and displacement ratios. The convergence history is shown in Fig. 12.

The performance of the proposed fitness approximation strategy is evaluated as is provided in Table 6. As it can be observed, the achievement of the previous example can be accomplished in

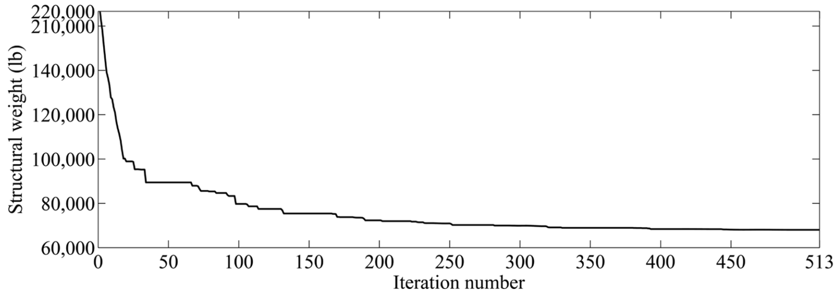


Fig. 12 Convergence history of the CMA-ES for the tower truss

Table 6 Performance evaluation of the proposed fitness approximation strategy for the tower truss

	Optimization method	
	Design A: With use of meta-model	Design B: Without use of meta-model
Structural weight	72006 lb (32661 kg)	68783 lb (31199 kg)
Computation time	719 mins (11.98 hours)	2205 mins (36.75 hours)
Number of original fitness function evaluation	2858	8352
Number of fitness function approximation	5733	0
Total number of fitness function evaluation	8591	8352
Quality improvement	0%	4.48%
Time improvement	67.39%	0%

solving the optimal design of more complex structures, where the calculation of the original fitness function is much more time consuming.

9. Conclusions

A new framework is proposed for optimal design of large-scale truss structures subjected to time-history loading. Minimizing the structural weight through a size optimization procedure is considered as the objective. The CMA-ES is employed as the optimization algorithm. Three methods are implemented for reducing the computational effort of time-history analyses during the optimization process. Firstly, the effective duration of the ground motion is utilized instead of considering the entire earthquake record, since for the design procedure only the maximum response of the structure is needed. The effective duration indicates the time that the maximum response will occur definitely until then. Therefore, the record needs to be analyzed up to this time and further analysis is not required.

Secondly, a WA is applied to decrease the number of involved points of earthquake record up to 0.125 of the effective duration through a three-level wavelet decomposition process. In order to select a wavelet function for performing WA, 50 models of the structure are generated randomly and analyzed using the Daubechies wavelet family (Db1-Db6). Then the quality of each wavelet function for each model is evaluated by the RRMSE and R^2 measures. The one that averagely outperforms others is selected as the filter. Based on our findings, there is no specific wavelet function that always results in minimum errors for all types of structures, contrary to what has been employed by some researchers in the past. We have found that performance of the wavelet functions depends on the topology and properties of the given structure, in addition to the time-frequency characteristics of the applied earthquake record.

Finally, a specific fitness approximation strategy is developed for predicting the time-history response of structures. For this purpose, a GRNN is used as the meta-model and a special evolution control scheme is developed. In order to improve the quality of the estimations, a concept of nearest data selection is applied. In this approach, all the solutions, which are evaluated by the original fitness function, are stored in an archive. In the process of fitness approximation, when the optimization procedure generates a new solution, first, its k nearest neighbors in the archive are determined and then a new GRNN is constructed and trained by these similar solutions.

This framework is operated for the design of two practical structures: a footbridge truss and a tower truss. It is demonstrated that by the use of the proposed framework a significant improvement in computational effort can be achieved, at the expense of only a small loss of accuracy. Also, the obtained results confirm that optimal design of similar structural problems can be performed for much larger structures within acceptable amount of computational time.

Acknowledgements

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Appendix A. Summary of the CMA-ES algorithm

Set parameters
Set parameters $\lambda, \mu, \omega_{i=1,\dots,\mu}, c_\mu, c_1, c_c, c_\sigma$ and d_σ to their default values according to Table 4.
Set evolution paths $\mathbf{p}_c^{(0)} = 0, \mathbf{p}_\sigma^{(0)} = 0$, and covariance matrix $\mathbf{C}^{(0)} = I$.
Choose distribution mean $\mathbf{m}^{(0)} \in R^n$ and step-size $\sigma^{(g)} \in R_+$, problem dependent.

Until termination criterion met
Sample new population of search points

$$\mathbf{x}_k^{(g+1)} \sim \mathbf{m}^{(g)} + \sigma^{(g)} N(0, \mathbf{C}^{(g)}) \quad \text{for} \quad k = 1, \dots, \lambda \tag{10}$$

Selection and recombination

$$\mathbf{m}^{(g+1)} = \sum_{i=1}^{\mu} \omega_i \mathbf{x}_{i:\lambda}^{(g+1)} \text{ where } \sum_{i=1}^{\mu} \omega_i = 1, \quad \omega_i \geq 0 \tag{11}$$

Step-size control

$$\mathbf{p}_\sigma^{(g+1)} = (1 - c_\sigma)\mathbf{p}_\sigma^{(g)} + \sqrt{\mathbf{c}_\sigma(2 - c_\sigma)\mu_{eff}}\, \mathbf{C}^{(g)-1/2} \frac{\mathbf{m}^{(g+1)} - \mathbf{m}^{(g)}}{\sigma^{(g)}} \tag{12}$$

$$\sigma^{(g+1)} = \sigma^{(g)} \exp\left(\frac{c_\sigma}{d_\sigma}\left(\frac{\|p_\sigma^{(g+1)}\|}{E\|N(0, \mathbf{I})\|} - 1\right)\right) \tag{13}$$

Covariance matrix adaptation

$$\mathbf{p}_c^{(g+1)} = (1 - c_c)\mathbf{p}_c^{(g)} + h_\sigma^{(g+1)} \sqrt{c_c(2 - c_c)\mu_{eff}}\, \frac{\mathbf{m}^{(g+1)} - \mathbf{m}^{(g)}}{\sigma^{(g)}} \tag{14}$$

$$\text{where} \qquad h_\sigma^{(g+1)} = \begin{cases} 1 \ \text{ if } \ \dfrac{\|p_\sigma^{(g+1)}\|}{\sqrt{1 - (1 - c_\sigma)^{2(g+1)}}} < \left(1.4 + \dfrac{2}{n+1}\right) E\|N(0, \mathbf{I})\| \\ 0 \ \text{ Otherwise} \end{cases}$$

$$\mathbf{C}^{(g+1)} = (1 - c_1 - c_\mu)\mathbf{C}^{(g)} + c_1 \mathbf{p}_c^{(g+1)} \mathbf{p}_c^{(g+1)T} + c_\mu \sum_{i=1}^{\mu} \omega_i \mathbf{y}_{i:\lambda}^{(g+1)} \mathbf{y}_{i:\lambda}^{(g+1)T} \tag{15}$$

Table 4 Default strategy parameters

Selection and recombination

$$\lambda = 4 + [3\ln n], \quad \mu = [\mu'], \quad \mu' = \frac{\lambda}{2} \tag{16}$$

$$\omega_i = \frac{\omega_i'}{\sum_{j=1}^{\mu} \omega_j'}, \quad \omega_i' = \ln(\mu' + 5) - \ln i, \quad \text{for } i = 1, \ldots, \mu \tag{17}$$

Step-size control

$$c_\sigma = \frac{\mu_{eff} + 2}{n + \mu_{eff} + 5}, \quad d_\sigma = 1 + c_\sigma + 2\max\left(0, \sqrt{\frac{\mu_{eff} - 1}{n + 1}} - 1\right) \tag{18}$$

Covariance matrix adaptation

$$c_c = \frac{4 + \mu_{eff}/n}{n + 4 + 2\mu_{eff}/n} \tag{19}$$

$$c_1 = \frac{2}{(n + 1.3)^2 + \mu_{eff}} \tag{20}$$

$$c_\mu = \min\left(1 - c_1, \frac{2(\mu_{eff} - 2 + 1/\mu_{eff})}{(n + 2)^2 + \mu_{eff}}\right) \tag{21}$$