# An initial investigation of the inverted trussed beam formed by wooden rectangular cross section enlaced with wire rope

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**Abstract.** This work presents a contribution to understand the inverted trussed beams behavior. The system has a main beam and struts with rectangular cross section associated to a wire rope enlaced to the main beam. It is an unpublished system with the advantage of easy positioning of the wire rope, once it is a continuous and connected by turnbuckles. It is a system that can be used as support for concrete formworks or for rehabilitation wooden beams proposal. The enlacement of the cable demands a small notch at the top of the cross section and a cross pin at the bottom. Six inverted trussed beams were tested, with spans of 180 cm with cables diameter of 1/4". Additionally, four simple beams without any external steel cable were also tested with material from the same lot of wood, allowing a comparison in rupture. The results showed capacity gain of around 60% compared to a simple beam. Once the wire rope characteristics and anchoring are very important for structure response, some improvement suggestions for the efficiency of the cables are also presented.

Keywords: inverted trussed beam; wire rope; rectangular section; notch

# 1. Introduction

Inverted trussed beam system is a type of structure which consists of a main beam with one or two vertical struts associated to steel cables. This structure can be used in roofs, bridges, runways, concrete formwork support, structural rehabilitation etc. The most classical and former use is in girders of railway vehicles as can be seen in Fig. 1 Girder of railway vehicle as an inverted truss. It is possible to note that the system is lightweight and stable. The vertical struts are compressed and no lateral contentions exist at its bottom end, but the system has stability due to the tensioned cable.

The main goal of this work is to create an original system able to be used as beams for concrete formwork using two materials wood and steel to produce a lightweight and efficient system. Although steel has a high density their pieces have small cross section and therefore its weight is convenient. As it is well known wood is a material with an excellent relationship between resistance and weight. So the system has appropriate features for the mentioned application by being reliable and to allow easy transport anywhere.

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Fig. 1 Girder of railway vehicle as an inverted truss

Although Feio *et al.* (2011) do not present details for inverted truss, they recommend the system as a way to recovery service life of old wooden beams. They emphasize that problem occurs when parts are aggregate in the already mounted structure. The proposal presented in this study can overcome this problem, once wire rope is flexible and adjustable to curve surface. This topic can be also included as a structural rehabilitation category for solving problems in the same range as presented by Augelli *et al.* (2005), Pantelides *et al.* (2010), Branco *et al.* (2010).

The interaction between wood and metal is very common as mentioned by Farrow (2009). It is estimated that approximately 75% of all new homes in the United States use metal plate connected wood trusses, showing that the conjunction of these materials is very interesting for structural purpose. Kim *et al.* (2009) investigated the introduction of Western style wooden roof trusses in early 20<sup>th</sup> century Korea and observed the new styles such king post truss, queen post truss, wood-steel composite roof truss as types of structures introduced. For the efficiency mixed structure wood-steel is mentioned as the mostly used in large scale construction.

For concrete formworks application it is necessary to understand appropriately pressures of flowable concrete that promotes displacements and forces in the timber sheathing. It also important to know appropriated systems that can be used to support formwork as described by Hurd (2005). This author lists several engineered wood product for this application as glued laminated timbers, structural composite lumber (SCL) and prefabricated wood I-joists. The intention is to include the inverted truss beam with special feature in the list of this product type. The system optimization is very important once it can contribute up to 40% of the overall cost for the concrete structure construction, as mentioned by Khayat and Assaad (2008).

Despite of the very common use in the railway segment, very little is known about the inverted trussed beam system, especially when the main beam is wood. Some authors as Boresi, Schmidt and Sidebottom (1993), Ritter and Faherty (1999), Herzog *et al.* (2004) deal with the subject superficially, considering the system in a conventional way as a reticulated system formed by one horizontal beam and one or two vertical struts in which the rods are supported, interconnecting the extremities of the beam. Details of the anchorages related to the vertical and horizontal positioning are not presented. These authors solve the structure in a conventional way, which means to consider vertical force acting on the main beam that comes from the strut. No detail is considered in the analysis as the location for the cable contact with the main beam, contact between cable and strut or friction due to the slip between strut and main beam and so on. It is known that any localized deformation is important for the system, as can be seen in Santana and Mascia (2009).

This topic is quite limited and it was not possible to detect studies carried out in last five years, unless that produced by the authors. It is a product with special properties especially due to the way the wire rope is placed in the system. All details are showed and considered in the analysis.

Several works have been carried out by the authors for this type of structure using finite element method to obtain the optimal struts positioning, struts length and the vertical anchoring point of the cables in the beam ends. Some details can be seen in Gesualdo and Cunha (2009) where is presented a numerical study analyzing details of inverted truss beam and its comparison with the regular evaluation as reticulated system.

An initial investigation of the inverted trussed beam formed by wooden rectangular cross section 241

# 2. The structural arrangement

The structural arrangement assessed in this study has the wire rope as the element used as rod. Fig. 2 presents a general view of the tested beam, composed by two rods. This metallic element is interesting due to its flexibility, easy handling, accommodation and fixation. It has a good resistance/density relation and, together with the wood, promotes a lightweight structural arrangement, good resistance and stiffness. A beam with rectangular cross section was devised, to which the mentioned element is bound. For this purpose, it was adopted a small vertically curved notch at the top of the main beam - Fig. 3 -, which allows the cable to be smoothly and harmonically accommodated, giving continuity to the cable that will have only one connection. At the bottom, a transversal metallic pin guarantees the support of the cable to the beam. The cable is



Fig. 2 General view: beam under test



(a) Anchoring detail of the cable and beam



(c) Detail of the notch at the top of the beam



(b) Detail of the bottom pin and the cable enlacement at the top



(d) Wire rope inserted in the notch – top view

Fig. 3 Notch details of the cable at the beam

F.A.R. Gesualdo and M.C.V. Lima



Fig. 4 Anchoring detail of the wire rope connection: stretcher, thimble and clips

connected in the central part, as shown in Fig. 2, for the perfect adjustment and cable stretch.

Fig. 4 illustrates the anchoring detail of the stretcher to the cable. The cable anchoring is made by means of a thimble to soften the cable curvature. Due to the diameter of the cable used in the test, it was employed two clips properly spaced to avoid relevant slipping between the parts.

# 3. Material and methods

The analyzed inverted trussed beams were numerically evaluated by the finite element method using ANSYS<sup>®</sup> (2010). It was also used the software GESTRUT (2010) produced by the author for framed three-dimensional structures. The first one was used to analyze the complete system and the second in order to develop the analysis of the reticular system. GESTRUT was used due to the facility of data generation, as it presents a module developed for this particular type of structure. It was also employed a software developed specifically to facilitate the data input into ANSYS<sup>®</sup>. It is an executable program, called AutoVV that generates a code in APDL language from the basic data of the inverted trussed beam. This software automatically generates the whole text in APDL language that will be read by ANSYS<sup>®</sup>. It allows the user the tranquility of not having to create procedures for the generation of the mesh, which is complex, neither for loading generation nor the analysis of the results. Every data will be inside in the generated code according to the user needs. Images and text files will be created in the specified directory. The computer program will choose the procedure for each type of beam chosen by user. It is a highly automated process. From the results files, of text type, data may be manipulated via worksheet or other procedure for graphs and images generation.

To better understand the effective performance of the system, seven beams were tested with different setups. The numerical and experimental results were individually compared, for each beam, and also for the representative lot. All beams were obtained from the same wood lot of *Angelim Red* specie.

The wood was mechanically and physically characterized through tests standardized by the Brazilian standard ABNT NBR7190:1997. The forms of cables anchoring were redefined according to the responses met during the tests and upon each stage. This was relevant, once it has been noted deformations located in the thimbles of cables connections, only observed in loco. These deformations significantly affect the elastic properties of the cable. Proposals for the stiffening of these thimbles will be presented forward. For better understanding wood theoretical behavior as stress concentration and Young's modulus definition it was used the Bodig and Jayne (1982) information, based on Carroll (1999) considerations. Poisson modulus of 0.3 was used for all numerical analysis.



Fig. 5 Cross section of wire rope type  $6 \times 19$  with fiber core – 6 strands with 19 wires

Wire ropes are offered in the market with a nominal cross section referent to its largest diameter. However, the effective section of steel is much smaller, once there are internal voids to accommodate the wires and strands that are coiled. In Fig. 5 the internal part corresponds to fiber core. Most of the external part is also empty. The nominal diameter is measured in the position as showed.

Catalog from manufactures informs properties for the effective steel area (A) named metallic area. This area related to a nominal diameter "d" is determined by the expression  $A = k \times d^2$ , where k is a correction factor given in specific table. The k value for wire rope  $6 \times 7$  is k = 0.395 and for wire rope  $6 \times 19$  is k = 0.416. Applying these coefficients to the conventional calculus of a circle  $(\pi \cdot d^2/4)$ , it may be concluded that the effective area corresponds to 31.0% for  $6 \times 7$  and 32.7% for  $6 \times 19$ . This significant reduction is important and represents an additional caution to be considered on the analyses.

Suppliers report that the value of the modulus of elasticity will range from 9000 kN/cm<sup>2</sup> to 10000 kN/cm<sup>2</sup> for wire ropes  $6 \times 7$  and, between 8500 kN/cm<sup>2</sup> and 9500 kN/cm<sup>2</sup> for the wire ropes  $6 \times 19$ . Cables with steel core will have higher values with an increase of around 16%. The wire ropes in accordance with manufacturer present additional deformations in the initial phase of loading due to the accommodations of wires and strands, which are interlaced. The deformations produce additional displacements that range from 0.50% to 0.75% of the initial length of the wire rope. This initial residual deformation can be eliminated by applying a pre-stretching tension force lower than the elastic limit of the cable.

## 4. Experimental program

## 4.1 Details of the beam specimens

The beams were tested under the action of two concentrated forces applied in the positions that coincide with the struts, as illustrated in Fig. 2. The vertical displacement in the central point was measured using two inductive transducers positioned over the beam allowing the data reading up to the rupture. The approximate dimensions of the beams are indicated in Fig. 6.

The dimensions were fixed for all the beams, adopting the relation between the strut position and span equal to 1/3, and strut length/span equal to 2/15.

The beams were designated by V1, V2, V3, V4, V5, V6 and V7. The experimental tests were performed for each beam model as presented at Table 1 Table 1 Test scheme for beams V1 to V7. For the beam V1 to V6 the tests were carried out taking into account a beam without wire rope in



Fig. 6 Forces application scheme

Table 1 Test scheme for beams V1 to V7

|                                  | Structural System |           | Beam                   |  |
|----------------------------------|-------------------|-----------|------------------------|--|
| Simple beam<br>Case A            | Δ                 | <u></u> Δ | V3, V4, V5, V6         |  |
| Beam without wire rope<br>Case B | Δ                 |           | V1, V2, V3, V4, V5, V6 |  |
| Inverted trussed beam<br>Case C  |                   |           | V1, V2, V3, V4, V5. V6 |  |

order to evaluate the elasticity modulus. After that, the cables were attached forming the structural system called inverted trussed beam. The beams V3, V4 e V5 were also tested as a simple beam – Case A –, without struts and cables, using wood of the same piece and carried to rupture.

For a better comprehension all curves showed further had the loading-unloading removed, once the final curve does not depend on these stages.

Although the beams V1 to V7 have been obtained from the same wood lot, there is a small variation on dimensions due adjustment during the beam finishings.

# 4.2 Beam V1

The beam V1 was tested with a span of 180 cm, 40.5 mm  $\times$  92.0 cm cross section, struts spaced by 60 cm and with strut length equal to 24 cm. The beam V1 performance results are presented in Fig. 7.

The first test was carried out without cables – V1B. The modulus of elasticity calculated corresponds to 22429 MPa. Later, the cables were connected and the load steps applied. Three loading and unloading stages were applied at the inverted trussed beam as showed in Fig. 7. It is possible to confirm that the cable efficiency increases after its adjustment, generating significant increase of stiffness. This beam V1C was carried to rupture and reached the maximum force of 28.13 kN.

#### 4.3 Beam V2

This beam was tested with a span of 180 cm, cross section of 41.2 mm× 90.6 cm, struts spaced





60 cm and with strut length equal to 24 cm. The beam V2 results are shown in Fig. 8.

The modulus of elasticity obtained for the beam without wire rope (V2B) was equal to 27381 MPa. The cables were attached and the inverted trussed beam (V2C) reached rupture at force equal to 43.61 kN.

# 4.4 Beam V3

Two tests were performed up to rupture for beam V3 with the same wood piece. The results are indicated in Fig. 9.

The dimension of the cross section of the beam without struts and cables (V3A) was of 39.1 mm  $\times$  88.2 mm and, with the struts and no cable and after attach the cables (V3B and V3C), 39.3 mm  $\times$ 91.3 mm.

The beam V3A reached rupture at 17.77 kN and the inverted trussed beam V3C at 28.60 kN. The inertia ratio between the two beams is 1.11, which means, the inverted trussed beam had a cross section 11% stiffer than the simple one, which should be considered in the analysis.

The modulus of elasticity of the V3A was 18476 MPa and for the V3B the value reached 20544 MPa. Taking into account an equivalent modulus of elasticity for the inverted trussed beam



Fig. 9 Results for beam V3

as an equivalent simple beam obtains 31792 MPa. This would represent a gain of 55% of stiffness. So the inverted trussed beam would be 61% more resistant than the simple beam, once the relation between rupture forces is 28.60/17.77.

#### 4.5 Beam V4

Beam V4 was tested for case A, B and C and taken to rupture (Fig. 10). The cross section dimensions were beam 39.1 mm  $\times$  90.6 mm for V4A and 39.9 mm  $\times$  90.6 mm for V4B and V4C.

The beam V4A reached the maximum force of 20.56 kN and the beam V4C of 33,30 kN. The inertia ratio between the two beams is of 1.075. The inverted trussed beam had a cross section 7.5% stiffer than the simple one.

The modulus of elasticity of the beam V4A was 17556 MPa and V4B was 17749 MPa. In this case, the beams presented almost the same mechanical property as expected. Considering the inverted trussed beam as an equivalent simple beam, the modulus of elasticity should be 25902 MPa which represents a stiffness gain about 46%. The inverted trussed beam would be 62% more resistant than the simple beam considering the relation between rupture forces of 33.30/20.56.

## 4.6 Beam V5

Beam V5 was tested for case A, B and C and carried to rupture as illustrated at Fig. 12. The V5A cross section dimensions are 40.7 mm  $\times$  92.5 mm and of 39.9 mm  $\times$  91.1 mm for V5B and V5C.

The beam V5A reached the rupture force of 19.48 kN and the V5C of 30.77 kN. The inertia ration between the two beams is 0.94. The inverted trussed beam had a cross section 6.0% less stiffer than the simple one.

The modulus of elasticity of the beam V5A was 18343 MPa and for beam V5B of 18859 MPa. In this case, the beams presented almost the same mechanical property. Taking into account an equivalent modulus of elasticity for the inverted trussed beam as an equivalent simple beam obtains 28204 MPa. The stiffness increase is about 50%. The inverted trussed beam would be 58% more resistant than the simple beam and relation between rupture forces is 30.77/19.48.



Fig. 10 Results for beam V4





Fig. 12 Results for beam V6

# 4.7 Beam V6

This beam was tested only for case B e C. A span of 180 cm, cross section of 40.4 mm $\times$  90.9 cm, struts spaced 60 cm and with strut length equal to 24 cm. The beam V6 results are shown in Fig. 12.

The modulus of elasticity obtained for the beam without wire rope (V6B) was equal to 17553.8 MPa. The cables were attached and the inverted trussed beam (V6C) reached rupture at force equal to 39.24 kN.

#### 4.8 Beam V7

Beam V7 was exclusively tested as simple beam - V7A. The wood used for beam V7A was from a random piece of the same lot. It was carried out to rupture and reached a maximum force of 22.25 kN. The modulus of elasticity resulted in 19910 MPa. This testing purpose was to have an additional sample for the wood lot characterization.

## 5. Results analysis

## 5.1 Overall results

Table 2 shows the experimental results for beams tested in the elastic range. The modulus of elasticity value was evaluated by the classical theory of bi-supported beam with two concentrated forces, i.e.,  $E = F \cdot a \cdot (3\ell^2 - 4a^2)/(48 \cdot I \cdot v)$ , where F is the total applied force (Fig. 6), a is the distance between the support and the force F/2,  $\ell$  is the span, I is the moment of inertia of the cross section and v is the experimental value for the corresponding F. In Table 2 the column  $I_{caseC}/I_{caseA}$  indicates the ratio of the effective inertia of the main beam, between the simple beam and the inverted truss, which had pairs tested up to rupture.

|      | 2       |         |                                     |          |                   |
|------|---------|---------|-------------------------------------|----------|-------------------|
| Beam | b<br>mm | h<br>mm | $\frac{I_{case \ C}}{I_{case \ A}}$ | E<br>MPa | $F_{rup} \  m kN$ |
| V1C  | 40.5    | 92.0    |                                     | 28290.6  | 28.13             |
| V2C  | 41.2    | 90.6    |                                     | 31737.2  | 43.61             |
| V3C  | 39.2    | 91.3    | 1 117                               | 31792.2  | 28.60             |
| V3A  | 39.1    | 88.2    | 1.110                               | 18475.8  | 17.77             |
| V4C  | 40.5    | 92.3    | 1 075                               | 25902.3  | 33.30             |
| V4A  | 39.9    | 90.6    | 1.075                               | 17556.4  | 20.56             |
| V5C  | 39.9    | 91.1    | 0.026                               | 28203.9  | 30.77             |
| V5A  | 40.7    | 92.5    | 0.936                               | 18342.6  | 19.48             |
| V6C  | 40.4    | 90,9    |                                     | 29230,5  | 39,24             |
| V7A  | 41.4    | 93.8    |                                     | 19910.3  | 22.25             |
|      |         |         |                                     |          |                   |

Table 2 Results summary obtained from the tests

# 5.2 Experimental results comparison: case A and C

# 5.2.1 Beams V3, V4 and V6

The pairs of force resulted from the tests for beams V3, V4 and V5 is presented in Table 2. Considering the direct comparison, i.e., neglecting the variation of inertia moments, the resistance gain is about 60%. The modulus of elasticity indicated for inverted trussed beams in Table 4 were determined based on the real displacements (tests), assuming the inverted trussed beam equivalent to a simple beam. It can be inferred that the three inverted truss beams (V3, V4 and V5) are about 58% stiffer than a simple beam.

Results from Table 3 and Table 4 permit to establish that the inverted truss beam has a gain about 60%, even in resistance as in stiffness.

Table 3 Analysis in rupture for beams V3, V4 and V5

|        | F <sub>inverted</sub> trussed<br>kN | F <sub>simple</sub><br>kN | $rac{F_{\mathit{inverted trussed}}}{F_{\mathit{simple beam}}}$ |
|--------|-------------------------------------|---------------------------|---|
| <br>V3 | 28.60                               | 17.77                     | 1.61  |
| V4     | 33.30                               | 20.56                     | 1.62  |
| V5     | 30.77                               | 19.48                     | 1.58  |
|        |                                     |                           |   |

Table 4 Modulus of elasticity for beams V3, V4 and V5

| Beam | $E_{inverted\ trussed}$ MPa | E <sub>simple</sub><br>MPa | $rac{E_{\it inverted trussed}}{E_{\it simple beam}}$ |
|------|-----------------------------|----------------------------|---|
| V3   | 31792.2                     | 18475.8                    | 1.72  |
| V4   | 25902.3                     | 17556.4                    | 1.48  |
| V5   | 28203.9                     | 18342.6                    | 1.54  |

| Doom   | $E_{inverted\ trussed}$ | $E_{simple}$ | $E_{inverted\ trussed}$ | $F_{rup}$ |
|--------|-------------------------|--------------|-------------------------|-----------|
| Dealli | MPa                     | MPa          | $E_{simple \ beam}$     | kN        |
| V1     | 28290.6                 | 22429.2      | 1.26                    | 28.13     |
| V2     | 31737.2                 | 27381.1      | 1.16                    | 43.61     |
| V3     | 31792.2                 | 20544.3      | 1.55                    | 28.60     |
| V4     | 25902.3                 | 17748.8      | 1.46                    | 33.30     |
| V5     | 28203.9                 | 18858.9      | 1.50                    | 30.77     |
| V6     | 29230,5                 | 17553.8      | 1,67                    | 39,24     |

Table 5 Stiffness comparison between simple beam and inverted trussed beam

# 5.2.1 All beams

It may be noted in Table 5 that beams V1 and V2 present a small stiffness increase when the cables are attached to the system – the inverted trussed beam. In these two beams, thimbles were used in the conventional way, without reinforcement. This generates located deformation of the thimble with significant change of the modulus of elasticity of the cable. While the remaining beams have stiffness gains more than 50%, these two had about 20%.

Taking into account this behavior, reinforcements in the thimbles were used for all the other beams by introducing a metallic welded element, as shown in Fig. 16. Some thimbles characteristics will be discussed further in Section 6.2.3. If only beams V3 through V6 are considered in the analysis the inverted trussed beam is 54.5% stiffer than the simple beam, as can be obtained by the fourth column of Table 7.

# 6. Experimental and numerical results

#### 6.1 Discussion

The displacements obtained numerically for each tested beam, using the effective characteristics of each piece, and the modulus of elasticity of the main beams and the struts are indicated in Table 6. The *E* value was obtained in the previous tests of each inverted trussed beam before the cables placement. The cables modulus of elasticity considered in the analysis was equal to  $80 \times 10^3$  MPa. Two calculations were made. The first one, considers the model in standard form as a reticular system, where the nodes correspond to the meeting of the members, without eccentricities, Fig. 13(a). The second one considers the model by the finite elements method, where the model considers the accurate position of each element and the transfer of forces, in some points, is done by contact surface. In this case, the model was generated by the computer program AutoVV mentioned before. The chosen force *F* (total) applied was equal to 14 kN, as it encompasses all the experimental results. The modulus of elasticity used for the wood was that indicated in the 2nd column of Table 6 as they are effectively for all inverted trussed beam without cables.

From Table 6 it is possible to conclude that if the simplified model (reticulated) is used, the displacement will be underestimated in comparison with the real case (60%). In the same way, the displacement obtained via MEF adopted model also produce an error of 42%. The direct comparison between MEF and the reticulated model reveals that the MEF is a better way to

|    | <i>E<sub>simple</sub></i><br>MPa | u <sub>simple</sub><br>mm | u <sub>reticulated</sub><br>mm | u <sub>MEF</sub><br>mm | u <sub>real</sub><br>mm | $\frac{u_{MEF}}{u_{retivul.}}$ | $\frac{u_{real}}{u_{MEF}}$ | $\frac{u_{real}}{u_{retivul.}}$ | $\frac{u_{simple}}{u_{real}}$ |
|----|----------------------------------|---------------------------|--------------------------------|------------------------|-------------------------|--------------------------------|----------------------------|---------------------------------|-------------------------------|
|    | ۵                                |                           |                                |                        |                         |                                |                            |                                 |                               |
| V1 | 22429.2                          | 24.3                      | 11.5                           | 13.1                   | 19.6                    | 1.14                           | 1.50                       | 1.70                            | 1.24                          |
| V2 | 27381.1                          | 20.5                      | 10.5                           | 11.9                   | 17.8                    | 1.13                           | 1.50                       | 1.70                            | 1.15                          |
| V3 | 20544.3                          | 28.0                      | 12.2                           | 14.1                   | 19.1                    | 1.15                           | 1.35                       | 1.57                            | 1.47                          |
| V4 | 17748.8                          | 30.4                      | 12.7                           | 14.7                   | 20.1                    | 1.16                           | 1.37                       | 1.58                            | 1.51                          |
| V5 | 18858.9                          | 30.2                      | 12.6                           | 14.6                   | 17.9                    | 1.16                           | 1.23                       | 1.42                            | 1.68                          |
| V6 | 17553.8                          | 30.4                      | 12,6                           | 14.6                   | 20.7                    | 1.16                           | 1.42                       | 1.64                            | 1.47                          |
|    |                                  |                           |                                |                        | Mean:                   | 1.15                           | 1.40                       | 1.60                            | 1.42                          |

Table 6 Comparison of stiffness for the same beam as simple and as inverted trussed beam at level F = 14 kN

calculate displacement, once the difference is around 15%. In this model it is also necessary to calibrate parameters related to cable, wood and accessories which influence the result.

# 6.2 Modeling techniques

The beams were modeled in order to represent the real case. It was considered a three dimensional model having solid elements that resulted in hexagonal elements. The quality of the mesh was a point of concern, in view of the accuracy problems generated by an efficient mesh, Sarrate and Huerta (2009). It was conducted some tests in terms of precision and mesh density for the consistence of results.

To perform well numerical analysis of the tested beams it was used the software ANSYS<sup>®</sup>. For generating more accurate results, it was taken into consideration the actual conditions by using the contact elements, solid elements, support conditions and details for fixing the cable to the beam. Fig. 13 illustrates some details for model as the mesh and the cable embedded in the wood beam.

From it is possible to conclude that the difference between the theoretical reticular model and the finite elements model corresponds to 15% on average, that means, the reticular model indicates more conservative displacements; it is when the model details are not considered in the calculus. On the other hand, the real displacements, obtained in the experiment are beyond the predicted by the numerical models. On average, they are 39% higher. It should be considered that beams V1 and V2 did not have their thimbles reinforced and, therefore, the comparison should remain with beams V3, V4, V5 and V6 where the stiffness reduction would be of 32%. In the same way, when comparing the experimental result with the simple beam, it should be considered 1.41 and 1.56, respectively.

Numerically, the contact between cable and beam in the notch was included in the analysis admitting a frictional coefficient between steel and wood equal to 0.2 and the possible penetration caused by cable in the wood. Experimentally, no penetration was observed in the notch, especially because the wood species used has a high hardness.



Fig. 13 Models used in numerical analysis

#### 6.2.1 Overview

The experimental results indicate that there is an error of, at least, 32% in the displacement prediction when the adopted model is used and the calculus is done by the finite elements method. From the numerical analysis, it may be concluded that the modulus of elasticity of the cable represents a very sensitive parameter for the model. Furthermore, it is the parameter with greater complexity in defining the elastic characteristics due to the difficulty in determining the modulus of elasticity, experimentally, and the anchorage deficiencies, described in 5.4. In the theoretical calculus it was considered the modulus of 80000 MPa for the wire rope. However, tests performed with cables by authors indicated that the modulus of elasticity is lower than the nominal value informed by the manufacture. The value experimentally obtained is equal to 6832 MPa.

#### 6.2.2 Nonlinearity effects

It was observed in several examined cases that the effect of geometric nonlinearity is negligible, because it is less than 1%. The physical nonlinearity of the material also introduces no effects for the analysis associated to the displacements because, as it can be seen by the diagrams shown for all beam tested, Fig. 7 through Fig. 12, the curves approximately fit to a straight line. On the other hand, for the rupture force analysis it is not possible to obtain an appropriate curvature for that, because it is not possible to find this relation for the reference beam, which requires a tension level within the elastic range.

F.A.R. Gesualdo and M.C.V. Lima



Fig. 14 Thimble anchoring failure: rotation due to clip fastening deficiency



Fig. 15 Thimbles conditions

## 6.2.3 Thimbles

A problem associated to the use of cable and thimble refers to the slipping possibility in the cable anchoring through the clips, as it was observed during the tests with cables, thimbles, clips and turnbuckles, Fig. 14. If the clips are inadequately fastened, it can slip, leaving the thimble loose, enabling it to spin round. This shows the importance of cable anchoring, through the clips, and the accommodation to be perfectly adjusted to the available space.

With the two previous conditions considered, yet there will be the problem of the located deformation of the thimble under the loading action – Fig. 15(a). It can be noted that the thimble deformation provokes an additional lengthening, which will affect the modulus of elasticity of the cable.

In order to stiffen this element, one metallic part in the internal side was included, welded in the thimble walls to reduce its non-closure, as illustrated in Fig. 15(b).

# 7. Complementary analysis

An additional numerical analysis was carried out in order to compare the efficiency of inverted truss beam in terms of span. It was considered an inverted truss beam having two struts L/10 long, located at L/3 from supports, with a constant cross section for main beam and cables, where is applied two forces coinciding with the struts location. The span was varied in a range between 100 cm through 400 cm. Three situations were considered: a simple beam and an inverted truss beam with two modulus of elasticity for the cable (E = 8000 MPa and E = 20500 MPa). A fourth line was included in diagram showed in Fig. 16(a), which represents a displacement limit, established as L/200 in accordance with the Brazilian standard ABNT NBR7190:1997 for wooden structures. From this representation it can be noted that the inverted truss beam produces a



Fig. 16 Comparison between simple beam and inverted truss beam with two different modulus of elasticity

considerable gain in stiffness if compared with the simple beam. The gain is proportional to the span. There is a point where displacement in the simple beam is greater than the limit (L/200) close to span equal to 370 cm. For cable having different modulus of elasticity the variation is represented in Fig. 16(b). From this figure can be noticed that the variation in displacement is not proportional to the modulus of elasticity, because the main beam produce an expressive contribution in the final behavior of the beam.

# 8. Conclusions

By comparing the numerical and experimental results it was verified that the wire ropes represent the most important element for the characterization of this type of beam. In spite of this, they have high deformation due to their structure. The modulus of elasticity is affected by adjustments and the anchorage mechanisms that should have a severe quality control. The use of thimbles stiffener is fundamental to avoid additional deformations.

It was also shown that the theoretical calculation made for an inverted trussed beam by employing the numerical model adopted in this work, and the classical model – reticular system – results in a difference of 15%, in other words, the model via finite elements method represents better the real condition of the beam than a simple reticulated system. All information found in literature does not consider this case.

The comparison made directly among three pairs of beams as simple beam and as inverted trussed beam, assembled with the same wood, showed that the inverted trussed beams have rupture and deflections around 60% more than the one of the simply supported beam. When all beams are considered in the results it been concluded that the inverted trussed this value is reduced to 55%. The geometric nonlinearity does not affect results for the model. Due to the variation of the parameters that influence the beam, it can be concluded that the numerical model appropriately represents the phenomenon.

It is an initial study that must be complemented, specially for spans longer than the case studied, when the cable produce more effective contribution. Numerically, it can be addressed that displacements in inverted truss beam is significantly lower than that of the simple beam. As the span increases, dramatically the inverted truss beam efficiency increases as showed in Fig. 16.

The wire ropes present great constructive ease, because they allow enlacements that promote appropriate continuity and positioning. On the other hand, the elements have a widely variation of properties which depends on the degree of pre-tension and the assembling accessories. Thus, more specific studies should be conducted for the assessment of cables properties, as well as for the verification of other types of tie rods, as for example, steel round bars. Both types can be combined.

Although this is not a conclusive investigation, it reveals and quantifies several important points associated to the problem in discussion and provides great perspective for the development of new works on this topic.

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254

An initial investigation of the inverted trussed beam formed by wooden rectangular cross section 255

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