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Interval finite element analysis of masonry-infilled walls

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Abstract. This paper strongly addresses to the problem of the mechanical systems in which parameters are uncertain and bounded. Interval calculation is used to find sharp bounds of the structural parameters for infilled frame system modeled with finite element method. Infill walls are generally treated as non-structural elements considerably to improve the lateral stiffness, strength and ductility of the structure together with the frame elements. Because of their complex nature, they are often neglected in the analytical model of building structures. However, in seismic design, ignoring the effect of infill wall in a numerical model does not accurately simulate the physical behavior. In this context, there are still some uncertainties in mechanical and also geometrical properties in the analysis and design procedure of infill walls. Structural uncertainties can be studied with a finite element formulation to determine sharp bounds of the structural parameters such as wall thickness and Young's modulus. In order to accomplish this sharp solution as much as possible, interval finite element approach can be considered, too. The structural parameters can be considered as interval variables by using the interval number, thus the structural stiffness matrix may be divided into the product of two parts which correspond to the interval values and the deterministic value.

Keywords: interval analysis; interval finite elements; masonry infilled wall; uncertainty

1. Introduction

Structural design process requires that the performance of the system to be guaranteed over its lifetime. Real life engineering practice deals with the values of the structural geometric and material parameters such as Young's modulus, Poisson's ratio, length, or thickness of plates. The structural parameters which will describe the physical and mechanical behavior of a structure are often uncertain and cause modeling inaccuracies. These uncertain parameters are generally identified by random variables, and introduced in a stochastic approach of the problems. Different approaches can be used to solve these stochastic problems. The most common approach is to model the structural geometric and material parameters as random variables. The mean value, standard deviation of individual structural parameters and the correlation between different structural parameters are provided by the probabilistic information of the structural parameters. Unfortunately, the probabilistic approaches cannot give reliable results unless sufficient experimental data or statistical

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information is available to validate the expectations about the probability densities of the random variables. In some cases, only lower bound and upper bound of the structural parameters can be obtained. Hence, the interval analysis method has been a very useful method (Gao 2007). In the interval analysis (IA) method, a bounded uncertain structural parameter can be described as an interval variable and only its lower and upper bounds are required. In this context, a realistic way of representing uncertainty in engineering problems might be to consider the values of unknown variables which are defined within intervals that possess known bounds and it is possible to apply interval arithmetic in connection with the finite element methods (Dessombz *et al.* 2001).

The first applications of IA date back to the twenties (Burkil 1924, Young 1931, Sunaga 1958). No doubt, Ramon E. Moore made the most significant contribution to today's applications (Moore 1966, 1979). Nuding and Wilhelm's study may be mentioned as the first IA application in structural engineering, in which loads were defined as intervals (Nuding and Wilhelm 1972). Qiu *et al.* (1995, 1996a, b, 1998) have focused on the bounds of the eigenvalues of such dynamic systems. Chen and Yang (2000), Chen *et al.* (2003) have investigated the static responses and eigenvalue problems of structures with uncertain parameters by using the interval perturbation method. Gao (2007) has investigated a new method called the interval factor method for the natural frequency and mode shape analysis of truss structures with interval parameters. Koyluoglu and Elishakoff (1998) have introduced a comparison of the stochastic and interval finite element methods (IFEM) applied to shear frames exhibiting an uncertainty for the Young's modulus. The work has been restricted to narrow intervals and approximate results. In the works of Muhanna and Mullen (1995) and Mullen and Muhanna (1996-1999), an interval-based fuzzy finite element has been developed for treating uncertain loads in static structural problems. Additionally, Muhanna and Mullen (2001) have developed an IFEM based on an element-by-element technique and Lagrange multiplier where the uncertainty for the Young's modulus has been considered. Most sources of overestimation have been eliminated, and a sharp enclosure for the system response has been obtained (Muhanna *et al.* 2007). Recently, in the works of Muhanna *et al.* (2007), a new finite element formulation of frame structures with interval loading, lateral stiffness and bending stiffness have been considered. An element-by-element technique has been used to reduce overestimation and compatibility conditions have been ensured by a penalty method.

In this paper, structural uncertainties with intervals are investigated using finite element method in order to obtain the lower and upper bounds for the geometrical and mechanical parameters of an infilled frame system. In the IFEM used in this study, the uncertainties in wall thickness and Young's modulus are defined as interval values. Furthermore, the methodology is performed on a two story two-bay infilled frame system and results are given.

2. Numerical simulation of infill walls

Masonry-infilled walls as architectural elements of a typical frame structure can be found as interior and exterior walls in reinforced concrete and steel-framed structures. They are mostly considered as non-structural elements, and their presence is often ignored by structural engineers. These infill walls are often neglected in the analysis of building structures because of their complex nature and also because of the absence of a realistic analytical model. Ignoring the effect of the infill in stiffening and strengthening the surrounding frame is not always a safe approach, especially in seismic design. Such an approach may lead to significant inaccuracy in predicting the lateral stiffness, strength, and ductility of the structure. It will also lead to uneconomical design of the frame since the strength and stiffness demand on the frame could be largely reduced (Güney 2005). In addition, infill walls tend to interact with the surrounding frame when the structure is subjected to earthquake loads. If the wall failed from being overstressed under lateral loading, the high forces previously attracted and carried by the stiff infilled frame would be suddenly transferred to the more flexible frame after the infill is partially or fully damaged (El-Dakhakhni *et al.* 2003). Different failure modes of masonry-infilled frames occur during earthquake motion. In the last five decades, according to both the analytical and experimental studies available in the literature (Kakaletsis *et al.* 2011), five possible failure modes can be identified (Fig. 1). The most common earthquake damage is the diagonal cracking mode seen in the form of a crack connecting the two loaded corners. This mode is associated with a weak frame or a frame with weak joints and strong members with a rather strong infill (El-Dakhakhni *et al.* 2003). Different models have been used taking into account experimental results which simulate the behavior of infill walls. Experimental and theoretical

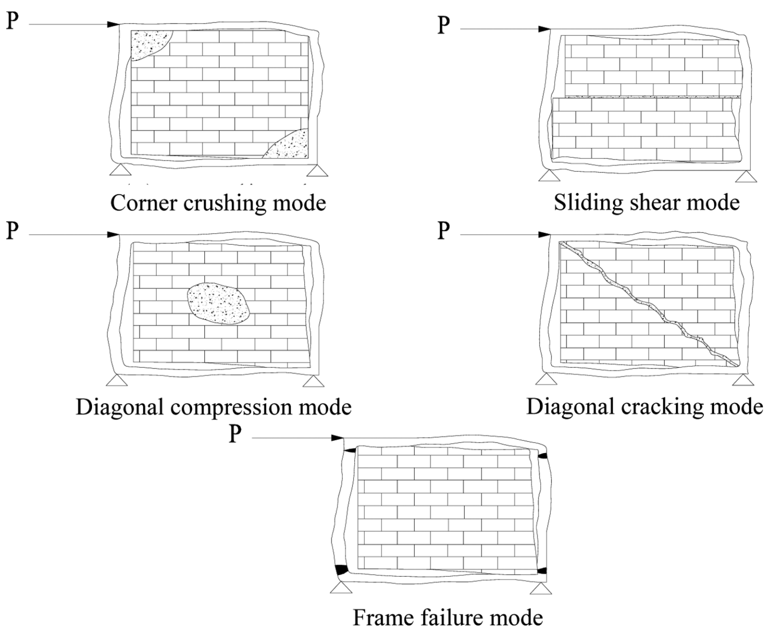

Fig. 1 Failure modes (El-Dakhakhni *et al.* 2003)

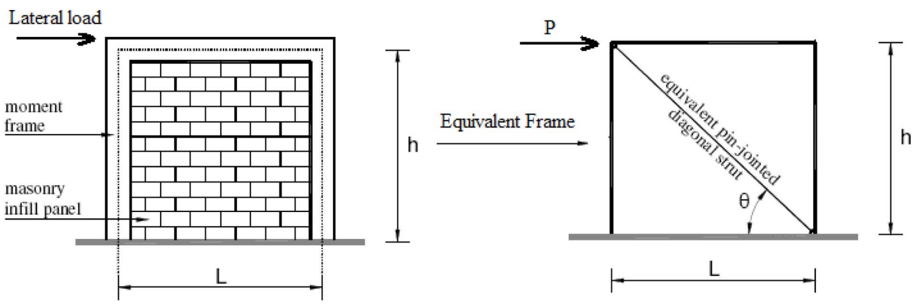

Fig. 2 Equivalent strut model for masonry infill walls

remarks have showed that a diagonal strut with appropriate geometrical and mechanical characteristics could simulate the behavior of infill walls. Polyakov (1960) has suggested the possibility of considering the effect of the infilling in each wall as equivalent to diagonal bracing. Holmes (1961) has replaced the infill by an equivalent pin-jointed diagonal strut made of the same material and having the same thickness as the infill wall and a width equal to one third of the infill diagonal length (Fig. 2).

Many structural parameters such as the Young's modulus and the thickness of the wall, which affect the strength and ductility under lateral loads, are the key parameters in establishing mathematical models of non-homogenous infill walls. Especially, the description of the Young's modulus causes some uncertainties. IA within defined bounds allows changing of all conditions in a numerical model. In this technique, it is sufficient to know only the lower and upper bounds with no need to know the probability distributions of the uncertain parameters. This reduces the need for statistical data. Thus, notwithstanding the cause of uncertainty, the accuracy of the results is assured.

3. Interval analysis

IA allows all conditions in a model within specified bounds to be changed. Therefore, it can offer a technique resulting in a parametric study which is almost perfect. A real interval x can be defined in the form [1, 2] if $1 \le x \le 2$. Here, 1 is lower bound and 2 is upper bound. The size of the uncertainty is measured by the width of the interval. An interval values group may be defined as follows

$$\mathbf{x} \equiv [a, b] = \{\tilde{x} \in \mathbf{R} | a \le \tilde{x} \le b\} \tag{1}$$

Here, a and b are members of R, the real numbers' group and the condition of $a \le b$ must be ensured under all circumstances. Interval values account for a sub-group of R, a real numbers' group at $a \equiv \inf(\mathbf{x})$ lower and $b \equiv \mathrm{sub}(\mathbf{x})$ upper bounds. From now on, the group of numbers to which the intervals belong will be symbolized as IR. The symbol "~" will be used to indicate each of the numbers, which takes place in the interval values' group ($\tilde{x} \in \mathbf{x}$). If the lower and upper bounds of an interval are equal to each other ($a = b$), then this interval is a thin interval and expresses "a", a single real number. Such an interval value is shown as $\mathbf{x} = [a, a]$. All interval quantities are denoted with boldface in this paper.

Fundamental principles

All operators used in mathematics: addition, subtraction, multiplication and division $\circ \in \{+, -, \times, /\}$ can also be used in analyzing interval values. If $\mathbf{x} = [a, a]$ and $\mathbf{y} = [c, d]$ are two interval values and also, $\tilde{x}$ and $\tilde{y}$ are two values in the said intervals, then the following definition will be valid

$$\mathbf{x} \circ \mathbf{y} = \{ \tilde{x} \circ \tilde{y} | a \le \tilde{x} \le b, c \le \tilde{y} \le d\} \tag{2}$$

If one of the bound values of the interval value of **y** is 0, then it is clear that the equation of **x/y** cannot be true. The following rules may be written for addition, subtraction, multiplication and division, which are basic operators, by considering interval values

$$[a, b] + [c, d] = [a + c, b + d] \tag{3}$$

$$[a, b] - [c, d] = [a - d, b - c] \tag{4}$$

The results depend on the sign of the interval bounds for multiplication and division. If $\mathbf{x} = [a, b]$ and $\mathbf{y} = [c, d]$ are two interval values, then the following rules may be written for the multiplication and division of the interval values of **x** and **y** ((**xy**) and (**x**/**y**)), respectively

$$[a, b] \times [c, d] = [\min(ac, ad, bc, bd),\ \max(ac, ad, bc, bd)] \tag{5}$$

$$[a, b]/[c, d] = [\min(a/c, a/d, b/c, b/d),\ \max(a/c, a/d, b/c, b/d)] \quad 0 \notin \mathbf{y} \tag{6}$$

The applications of operational features like commutation and association relating to the interval values on addition, subtraction, multiplication and division and the effects of the numbers of 1 and 0 are used as summarized below (x, y and z are interval values defined within two bound values)

$$\begin{array}{ll} x + y = y + x & x\,y = y\,x \\ (x + y) \pm z = x + (y \pm z) & (x\,y)z = x\,(y\,z) \\ x - (y \pm z) = (x - y) \pm z & (-x)(-y) = x\,y \\ x(y \pm z) \subseteq xy \pm xz & \\ x - y \subseteq (x + z) - (y + z) & x/y \subseteq (xz)/(yz) \\ 0 \in x - x & 1 \in x / x \\ -(x - y) = y - x & x\,(-y) = (-x)\,y = -x\,y \\ x + 0 = 0 + x = x & 1 \times x = x \times 1 = x \end{array}$$

A matrix with the size of $m \times n$ is introduced as the following

$$\mathbf{A} = (\mathbf{A}_{ik}) = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}, \quad A_{ik} \in \mathbf{IR} \tag{7}$$

In Eq. (7), all matrix components are interval numbers. **A** matrix can now be defined in a general expression to show the area in which the interval values' matrix of $\mathbf{IR}^{m \times n}$; $m \times n$ is valid, as the following

$$\mathbf{A} = \{\tilde{A} \in \mathbf{R}^{m \times n} | \tilde{A}_{ik} \in A_{ik} \quad i = 1, \ldots, m;\ k = 1, \ldots, n\} \tag{8}$$

4. Application of the interval analysis to masonry-infilled frames

As seen in Fig. 3, in a global coordinate system, the interval displacement vector of a frame at points i and j and nodal point load vector are expressed as the following

$$\mathbf{D}_e = \{\mathbf{u}_i\ \ \mathbf{v}_i\ \ \varphi_i\ \ \mathbf{u}_j\ \ \mathbf{v}_j\ \ \varphi_j\}^T \tag{9}$$

$$F_e = \{f_{i1}\ f_{i2}\ f_{i3}\ f_{j1}\ f_{j2}\ f_{j3}\}^T \tag{10}$$

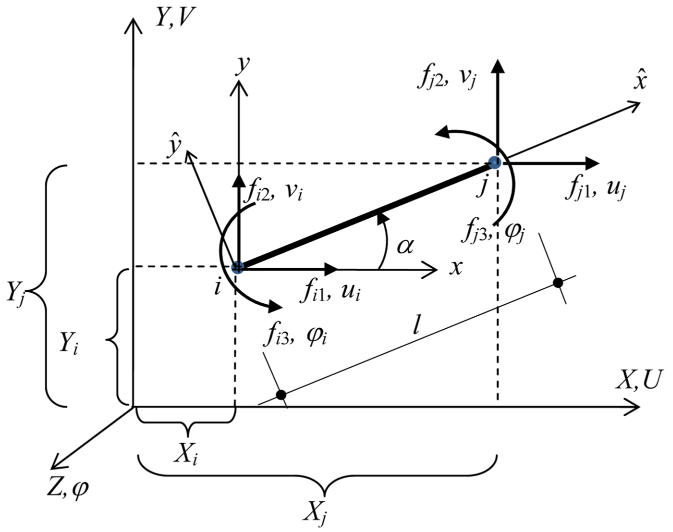

Fig. 3 Local $(\hat{x}, \hat{y})$ and Global (X, Y) Coordinate Systems for a Frame Element with Six Nodal Forces and DOF

A, E and l denote the cross-section area, Young's modulus and length of a frame element, respectively. The cross-sectional areas and Young's modulus of the diagonal strut used in the modeling of infilled frame may take any value between two bound values. If A_o is the cross-section area of the bar and δA is a given tolerance value, then cross-section area of strut may be defined between lower bound A_o - δA and upper bound $A_o + \delta A$. In this study, the cross-sectional area of diagonal struts considered as the geometrical uncertainty, are defined as A, $\mathbf{A} \equiv [\underline{A}, \overline{A}]$ = $\{A \in R | \underline{A} \leq A \leq \overline{A}\}$, A = $[A_o - \delta A, A_o + dA]$, where $\underline{A}$ is lower bound and $\overline{A}$ is upper bound. A similar definition is valid for Young's modulus (E). If the degree of freedom of the entire system is n, the following equation can be written between the global interval stiffness matrix ($\boldsymbol{K}$) of size $n \times n$, the displacement vector (D) of size $n \times 1$ and nodal point load vector (F) of size $n \times 1$

$$\boldsymbol{K}D = F \tag{11}$$

The interval stiffness matrix of an element "e", $\mathbf{k}_e$, in the global coordinate system is expressed as

$$\mathbf{k}_e = T_e^T \hat{\mathbf{k}}_e T_e, \quad \hat{\mathbf{k}}_e = \tilde{k}_e \mathbf{d}_e \tag{12}$$

where, T_e is considered as transformation matrix of an element "e" and can be defined in the following form

$$T_e = \begin{bmatrix} C_e & S_e & 0 & 0 & 0 & 0 \\ -S_e & C_e & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_e & S_e & 0 \\ 0 & 0 & 0 & -S_e & C_e & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Here, C_e and S_e in the transformation matrix are defined as

$$C_e = \cos\alpha = \frac{X_j - X_i}{l_e}, \quad S_e = \sin\alpha = \frac{Y_j - Y_i}{l_e}, \quad l_e = \sqrt{(X_j - X_i)^2 - (Y_j - Y_i)^2} \tag{14}$$

$\hat{\mathbf{k}}_e$ and $\mathbf{d}_e$ in Eq. (12) are in the matrix form

$$\tilde{\mathbf{k}}_e = \begin{bmatrix} 1/l_e & 0 & 0 & -1/l_e & 0 & 0 \\ 0 & 12/l_e^3 & 6/l_e^2 & 0 & -12/l_e^3 & 6/l_e^2 \\ 0 & 6/l_e^2 & 4/l_e & 0 & -6/l_e^2 & 2/l_e \\ -1/l_e & 0 & 0 & 1/l_e & 0 & 0 \\ 0 & -12/l_e^3 & -6/l_e^2 & 0 & 12/l_e^3 & -6/l_e^2 \\ 0 & 6/l_e^2 & 2/l_e & 0 & -6/l_e^2 & 4/l_e \end{bmatrix} \tag{15}$$

$$\mathbf{d}_e = \begin{bmatrix} E_e\mathbf{A}_e & 0 & 0 & 0 & 0 & 0 \\ 0 & E_e\mathbf{I}_e & 0 & 0 & 0 & 0 \\ 0 & 0 & E_e\mathbf{I}_e & 0 & 0 & 0 \\ 0 & 0 & 0 & E_e\mathbf{A}_e & 0 & 0 \\ 0 & 0 & 0 & 0 & E_e\mathbf{I}_e & 0 \\ 0 & 0 & 0 & 0 & 0 & E_e\mathbf{I}_e \end{bmatrix} \tag{16}$$

In this study, the interval stiffness matrix $\mathbf{k}_e$ for the diagonal struts in the global coordinate system can be introduced by

$$\mathbf{k}_e = \frac{\mathbf{A}_e E_e}{l_e} \begin{bmatrix} C_e^2 & C_e S_e & -C_e^2 & -C_e S_e \\ C_e S_e & S_e^2 & -C_e S_e & -S_e^2 \\ -C_e^2 & -C_e S_e & C_e^2 & C_e S_e \\ -C_e S_e & -S_e^2 & C_e S_e & S_e^2 \end{bmatrix} \tag{18}$$

For the structural system consisting of "*m*" elements, the global interval stiffness matrix **K**, can now be defined as the sum of the element stiffness matrixes in the following form

$$\mathbf{K} = \sum_{e=1}^{m} \mathbf{k}_e \tag{19}$$

Proposed algorithm can also be seen in Fig. 4, for further information.

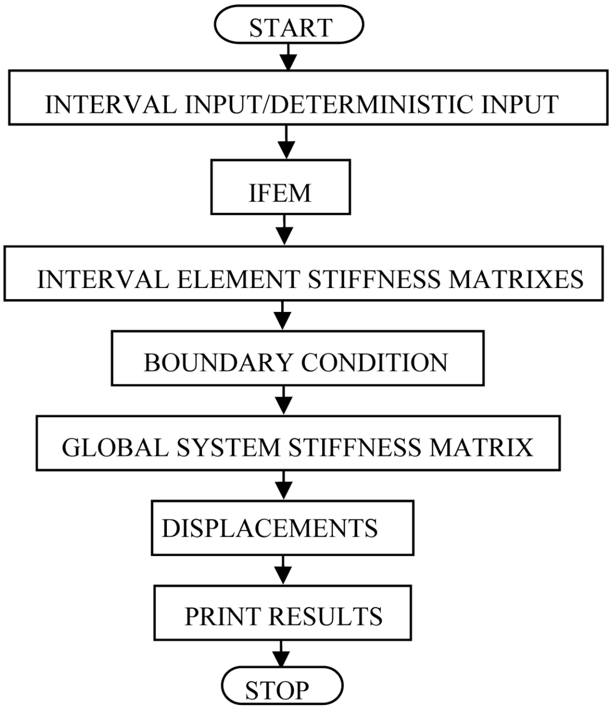

Fig. 4 Proposed algorithm

5. Assessment of the proposed method

Two story two-bay infilled frame system (Fig. 5) with its loading condition is modeled with walls (infilled frame)-without walls (bare frame) and analyzed using IFEM. In numerical analyses, the geometrical and/or material uncertainties of the frame elements have been considered as interval values and compared in various combinations. The values for the deterministic and non-deterministic area (A), moment of inertia (I) and Young's modulus (E) have been assumed to be constant for column and diagonal struts in the examples are given in Table 1. In the present study, the geometrical and/or material uncertainties of the diagonal struts have been considered as interval values. Variation has been assumed as ±2% in the cross-section area and as ±4% in the Young's modulus. Loads have been assumed as g_1 = 8.04 kN/m, g_2 = 7.83 kN/m, g_3 = 2.60 kN/m, F_1 = 6 kN and F_2 = 12 kN. Vertical-lateral displacements and rotations for nodal points of infilled frame with different conditions have been denoted in Tables 2-5.

According to numerical results, displacement and rotation values of the bare frame and infilled frame as seen in Tables 2 and 3, the nodal point displacement tends to decrease as expected. The nodal interval displacement values presented in Table 4 and Table 5, embrace the deterministic displacement values of all nodes in Table 3. In other words, the intervals in which the uncertainties are denoted by interval numbers include deterministic solutions in which the uncertainties are ignored. Table 5 shows nodal displacement values obtained by taking both the geometrical and material uncertainties into account (A, E) while Table 4 shows that the nodal displacement values obtained by only the geometrical uncertainty considered (A). According to the numerical results

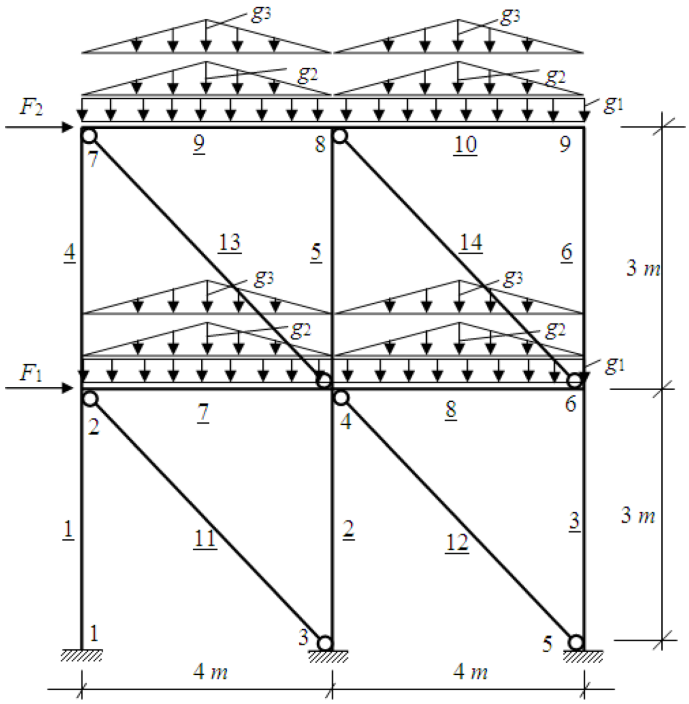

Fig. 5 Two story two-bays frame and loading conditions

Table 1 Interval and deterministic values of frame elements

Element	A (m^2)	A (m^2) ×10^{-3}	I (m^4) ×10^{-4}	I (m^4) ×10^{-4}	E (kN/m^2) ×10^7	E (kN/m^2) ×10^6
1-10	0.24	-	72	-	3.2	-
11-14	0.10	[98, 102]	20.858	[19.608, 22.108]	4.5	[4.32, 4.68]

Table 2 Vertical and lateral displacement and rotation values of a two-bay bare frame (1-10 elements)*

Nodal point	U (mm)	V (mm)	φ (rad)
1	0.0000	0.0000	0.0000
2	0.1000	-0.0232	-0.0518
3	0.0000	0.0000	0.0000
4	0.1008	-0.0551	-0.0215
5	0.0000	0.0000	0.0000
6	0.1023	-0.0287	-0.0116
7	0.2163	-0.0350	-0.0598
8	0.2073	-0.0831	-0.0119
9	0.2014	-0.0425	0.0203

*All values are deterministic.

Table 3 Vertical and lateral displacement and rotation values of a two-bay infilled frame (1-14 elements)*

Nodal point	*U* (mm)	*V* (mm)	φ (rad)
1	0.0000	0.0000	0.0000
2	0.0472	-0.0229	-0.0349
3	0.0000	0.0000	0.0000
4	0.0494	-0.0531	-0.0102
5	0.0000	0.0000	0.0000
6	0.0535	-0.0285	0.0045
7	0.1052	-0.0345	-0.0488
8	0.0966	-0.0798	-0.0055
9	0.0917	-0.0417	0.0313

*All values are deterministic.

Table 4 Vertical and lateral displacement and rotation values of a two-bays infilled frame (1-14 elements)*

Nodal point	*U* (mm)	*V* (mm)	φ (rad)
1	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]
2	[0.0431, 0.0513]	[-0.0232, -0.0226]	[-0.0361, -0.0338]
3	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]
4	[0.0451, 0.0536]	[-0.0533, -0.0528]	[-0.0109, -0.0094]
5	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]
6	[0.0491, 0.0579]	[-0.0287, -0.0283]	[0.0034, 0.0057]
7	[0.0979, 0.1125]	[-0.0350, -0.0340]	[-0.0494, -0.0482]
8	[0.0893, 0.1038]	[-0.0802, -0.0795]	[-0.0059, -0.0052]
9	[0.0844, 0.09896]	[-0.0419, -0.0414]	[0.0307, 0.0318]

*Diagonal strut area values are interval-other values are deterministic.

Table 5 Vertical and lateral displacement and rotation values of a two-bays bare frame (1-10 elements)*

Nodal point	*U* (mm)	*V* (mm)	φ (rad)
1	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]
2	[0.0333, 0.0611]	[-0.0239, -0.0218]	[-0.0388, -0.0311]
3	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]
4	[0.0349, 0.0638]	[-0.0539, -0.0523]	[-0.0128, -0.0075]
5	[0.0000, 0.0000]	[0.0000, 0.0000]	[0.0000, 0.0000]
6	[0.0387, 0.0683]	[-0.0292, -0.0278]	[0.0006, 0.0085]
7	[0.0804, 0.1299]	[-0.0361, -0.0329]	[-0.0509, -0.0467]
8	[0.0719, 0.1212]	[-0.0811, -0.0786]	[-0.0068, -0.0043]
9	[0.0670, 0.1162]	[-0.0425, -0.0409]	[0.0294, 0.0331]

*Diagonal strut area and Young's modulus values are interval, other values are deterministic.

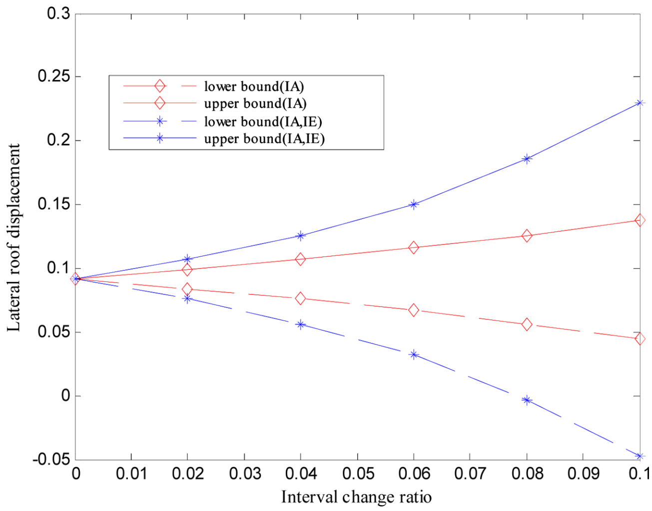

Fig. 6 Variations for lateral roof displacements

given in Tables 4 and 5, it can be seen that the first option offers a wider range of solution intervals. Besides, variations for 0%-±10% in the lateral roof displacements are depicted in Fig. 6.

6. Conclusions

In this study, the effect of uncertainty in the geometrical and material parameters of the structural static responses is investigated by using IFEM. The approach presented here is based on the use of the interval values within the scope of the finite element method. The geometrical and material uncertainties such as wall thickness and Young's modulus are defined as interval values in which the actual value is defined between the two bounds. The lower bound, upper bound, and interval change ratio on the displacement of infilled frame structures with interval parameters can be obtained practically. From Tables 2-5 and Fig. 6, the following conclusions are drawn:

1. Results of IA point out that the uncertainty of Young's modulus and wall thickness produce different lateral roof displacement in contrast to the structural responses for infilled frame under vertical and lateral loadings.
2. If the interval change ratio of the structural parameter is increased, the uncertainty of the structural displacement is increased, as well, remarkably.

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