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# Contact analysis of spherical ball and a deformable flat model with the effect of tangent modulus

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**Abstract.** The paper is on contact analysis of a spherical ball with a deformable flat, considering the effect of tangent modulus on the contact parameters of a non-adhesive frictionless elastic-plastic contact. The contact analysis of this model has been carried out using analysis software Ansys and Abaqus. The contact parameters such as area of contact between two consecutive steps, volume of bulged material are evaluated from the formulated equations. The effect of the tangent modulus is considered for determining these parameters. The tangent modulus are accounted between 0.1E and 0.5E of materials E/Y value greater than 500 and less than 1750. Result shows that upto an optimal tangent modulus values the elastic core push up to the free surface in the flat. The simulation is also carried out in Abaqus and result provide evidence for the volume of bulged material in the contact region move up and flow into the free surface of the flat from the contact edge between the ball and flat. The strain energy of the whole model is varied between 20 to 40 percentage of the stipulated time for analysis.

**Keywords:** tangent modulus; E/Y ratio; elastic-plastic; elastic core; strain energy

## 1. Introduction

Contact mechanics concerns with stress and deformation which arise when surfaces of two solid bodies are brought into contact. When two rough solids are brought to contact under a normal preload, contact junctions are formed at their contacting asperity tips, which may deform elastically, elastic-plastic or plastic. The so formed stress and deflections has practical application in hardness testing, wear and impact damage of engineering ceramics, the design of dental prostheses, gear teeth and ball and roller bearings. In a non-conforming body, the stresses are highly concentrated in the region close to the contact zone and are not greatly influenced by the shape of the bodies at a

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distance from the contact area. The existing contact analysis is carried out based on stress and strain in the contact bodies under loading and unloading conditions. The need of the present study is the influence of tangent modulus affection in the contact parameters such as contact area, volume of material squeezed and how the compressed material comes out from the contact region of deformed body under loading condition and an understanding on tribological phenomenon such as contact fatigue, wear and damage.

## 2. Theoretical background

The contact of a ball and a deformable flat is a fundamental problem in contact mechanics with important scientific and technological aspects. Classical work of Hertz (1881) derived an analytical solution for the frictionless (i.e., perfect slip) contact of two elastic spheres (Johnson 1985). It is important to analyse either a single asperity contact or contacting rough surfaces consisting multiple asperity contacts.

Chang *et al*. (1987) studied two fundamental approaches for modeling a single asperity contact either considering a deformable hemisphere in contact with a rigid flat (flatting approach). An intendation approach for solving the contact mechanics problem of a rigid spherical indenter penetrating in a deformable half space was given by (Polin and Lin 2006) (indentation approach). While in the elastic deformation regime, these two approaches are based on the Hertzian solution (Jackson and Green 2005) and hence produce identical results. Whenever beyond the elastic deformation, these two approaches yield different contact mechanics response. Bodies which have dissimilar profiles are said to be non-conforming contact.

## 3. Literature review

The theory developed by Hertz in 1880 remains the groundwork for most of the contact problems come across in engineering. Hertz theory deals, the small deformation with normal frictionless contact between elastic half-space. Abbott and Firestone (AF Model) (Abbott and Firestone 1933) introduced the basic plastic contact model, known as the surface micro-geometry model. In this model, it should be noted that Abbott and Firestone intended their model to be used to describe a wear process rather than a deformation process. Greenwood and Williamson (1966) used the Hertz theory and shows that rough surfaces can be modelled as a collection of individual asperities of various heights. In the connection of two models of GW (elastic model) and AF (plastic model), CEB model (Chang *et al*. 1987) has been developed on volume conservation of the plastically deformed asperities for an elastic-plastic contact model. Chang *et al.* (1988) pioneered the Poisson's ratio of the sphere which is related to the hardness coefficient. Davis (1999) (Metals Handbook) labeled that, hardness is a "Resistance of metal to plastic deformation, usually by indentation. Kogut and Etison (2002) (KE model) used the constitutive laws appropriate to any mode of deformation, be it elastic or plastic using Finite element method solution for the elastic-plastic contact of a deformable sphere and a rigid flat with out considered the material properties. Jackson and Green (2005) (JG model) presented some empirical relations of contact area and contact load with considered the material properties and the geometry of contacting body. Malayalamurthi and Marappan (2009) introduced the mode of material dependency of elastic-plastic contact behavior of

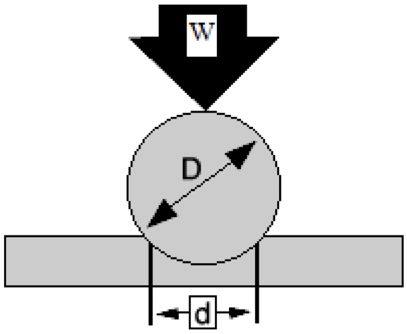


Fig. 1 Brinell hardness method

deformable sphere and a rigid flat by finite element analysis. The ratio of Young's modulus to yield strength (E/Y) value less than 300 show strikingly different contact phenomena. Shankar and Mayuram (2008) analyzed an axis-symmetrical hemispherical asperity in contact with a rigid flat is modeled for an elastic perfectly plastic material. This analysis shows the critical values in the dimensionless interference ratios $\omega/\omega_c$ for the evolution of the elastic core and the plastic region within the asperity for different Y/E ratios. The FE Analysis of single asperity model with the elastic perfectly plastic assumption depends on the Y/E ratio of the material. Tirupataiah and Sundararajan (1987) has been investigated the nature the nature of the indentation process in a low alloy steel heat treated to four different hardness levels with a tungsten carbide ball as the indenter. The effects of load and steel hardness on the various parameters have been studied. These results have been rationalized in terms of empirical equations suggested by others. Tabor (1951) knowable that hardness is not a unique material property. Tian (2010) model the dynamic contact stiffness has be inspected at the contact interface between a rigid sphere and a semi-infinite cubic solid. Spychalski *et al.* (2007) has been applied the finite element method for modelling of indentation process in polycrystalline metals. The indentation process has been quantitatively described by load-depth curves. These processes influenced the indent geometry and hardness measurements consequently. Hussein (2011) introduced an Adaptive Shifted Integration (ASI) technique for an elastic-plastic nonlinear analysis under dynamic load. That is capable of predicting with reasonable accuracy the behaviour of steel frame structures. Martin *et al.* (2012) an approached to case of forging by indentation by means of the application of the upper bound element technique in his development of triangular rigid blocks. This work developed the technique to determine a value of the energy that guarantees the essential deformation on the part.

From the literature review the contact analysis for both the models (Flattening and Indentation approach) using finite element analysis has been done by several researchers and some of these swotted the effect of material properties. The tangent modulus 0.1E is taken for analysis. Fig. 1 shows that the basic model (like as indentation approach). In the Brinell hardness test procedure a hard ball of diameter ‘D’ is pressed under a load ‘W’ into the plane surface under test.

## 4. Materials and methods

The present work aims to study the effect of tangent modulus for single asperity contact

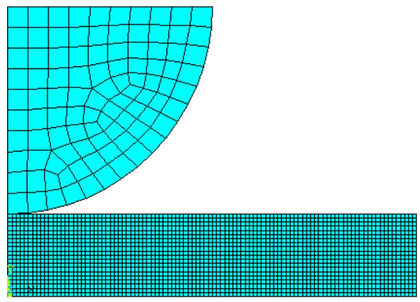

Fig. 2 (a) Meshed model - Ansys

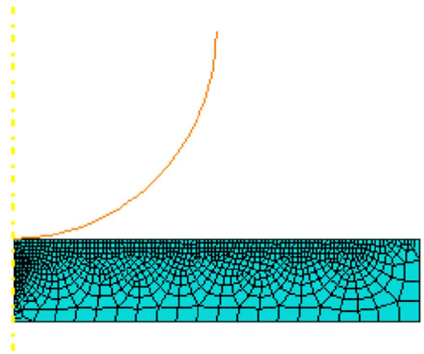

Fig. 2 (b) Meshed model - Abaqus

Table 1 Material properties

| S.No | $E \times 10^3$ (N/mm$^2$) | $Y$ (N/mm$^2$) | $E/Y$ |
|---|---|---|---|
| 1 | 210 | 380 | 552.63 |
| 2 | 70 | 95 | 736.84 |
| 3 | 100 | 130 | 769.23 |
| 4 | 120 | 121 | 991.74 |
| 5 | 120 | 69 | 1739.13 |

parameters for different materials under loading condition of rigid ball (sphere) and a deformable flat. In the indentation process like hardness testing, the indenter part is a rigid member and plate is a deformable member. In the present work the model is chosen for analysis is based on the similarity of an indentation process. Finite element analysis software 'Ansys' and 'Abaqus' are used to carry out the above analysis (Nakasone *et al.* 2006, Abaqus user's Manual 2006), with axisymmetric condition. Hence quarter portion of the sphere is considered for the analysis.

The Figs. 2(a) and (b) shows the meshed model of Ansys and Abaqus respectively. For this investigation, element type plane 82, conta172 and target 169 and CAX4R are used for analysing the model in Ansys and Abaqus respectively. The nodes lying on the axis of the hemisphere are restricted to move in the radial direction for both Ansys and Abaqus.

Also the nodes in the bottom of the hemisphere are restricted in the axial direction. The average sphere size of radius 50 mm is used for this analyses. Here frictionless rigid deformable contact analysis is performed for different materials.

The material properties are selected based on the Young's modulus to yield strength ratio. The following are the materials shown in Table 1 used for the applications of contact problems such as cylinder over a flat plate, wheel and rail contact, roller bearings and meshing of gear teeth.

### *4.1 Finite element analysis using ansys*

The tangent modulus is the slope of the stress-strain curve. For strain less than the yield point it will be a constant and is the same as the elastic modulus. For higher strain it will vary with the strain, and depending on the material it could be any value from Zero, up to bigger than the elastic

Table 2 Tangent modulus and stress

| Tangent modulus $E_T$ | Stress in N/mm$^2$ E/Y 552.63 | Stress in N/mm$^2$ E/Y 991.74 | Stress in N/mm$^2$ E/Y 1739.13 | Linear Hardness $H = \dfrac{E_T}{(E - E_T)}$ E/Y = 991.74 |
|---|---|---|---|---|
| 0.1E | 43.226 | 6.66 | 8.081 | 0.11 |
| 0.2E | 43.066 | 26.067 | 7.815 | 0..25 |
| 0.3E | 44.301 | 14.596 | 49.827 | 0.43 |
| 0.4E | 56.812 | 23.289 | 21.976 | 0.67 |
| 0.5E | 60.132 | 45.974 | 20.891 | 1 |
| 0.6E | 59.932 | 21.172 | 22.477 | 1.5 |
| 0.7E | 22.374 | 8.317 | 21.268 | 2.33 |
| 0.8E | 15.679 | 9.319 | 8.264 | 4 |
| 0.9E | 18.196 | 13.926 | 6.649 | 9 |

modulus. The wide range of values of tangent modulus is taken to make a amend idea about the effect of this in different materials, hardening parameter and the area of contact. The tangent modulus is taken as %E for linear hardening material. The FE analysis using Ansys is carried out for different materials i.e., $500 \le E/Y \le 1750$. The stress values with respective to tangent modulus of different materials are presented in Table 2.

From the above table clearly known that the material E/Y = 991.74 and less than has maximum stress value at tangent modulus 0.5E and above that the maximum stress value at 0.3E.

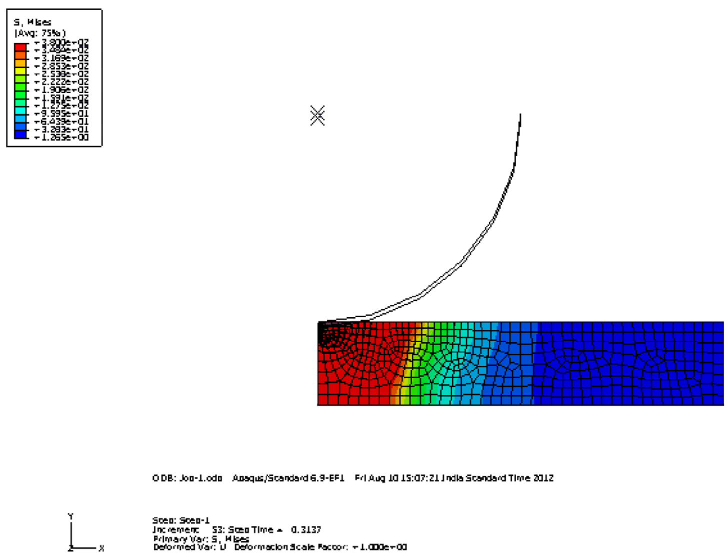


Fig. 3 Simulation model - Abaqus

### 4.2 Finite element analysis using abaqus

The axisymmetric model has been analyzed as shown in Fig. 3. The element type used for this analysis is CAX4R. For this analysis the mesh refinement are biased at near the top surface and left edge of the deformable plate where the large deformation is expected. The size of the deformable plate and the rigid spherical ball is $100 \times 20$ mm and radius 50 mm respectively. The boundary conditions are the bottom nodes of the plate are fixed, while the outer boundary is free to move.

The surface to surface contact pair is created between the spherical ball and plate. The rigid body surface is selected as master surface and the plate top surface is selected as slave surface. The analytical body constraint has been created in the ball and the center of the ball. The model is analyzed in Abaqus/Standard and dynamic-Explicit. The displacement of the ball toward down is given in the step - 1 and upward movement is given in the step-2. The large displacement is given for this analysis says 2 mm for optimum visualization of output.

### 4.3 Ball indentation depth and width of contact

The rigid ball and a deformable flat contact model as shown in Fig. 4. The load is applied on the top of the ball. The sphere is pinched in to the flat.

In this study, the attempt has been made to modify the indentation depth in the new form by incorporating the tangent modulus in terms of %E. The loading relationship for the penetration depth is given by the relation

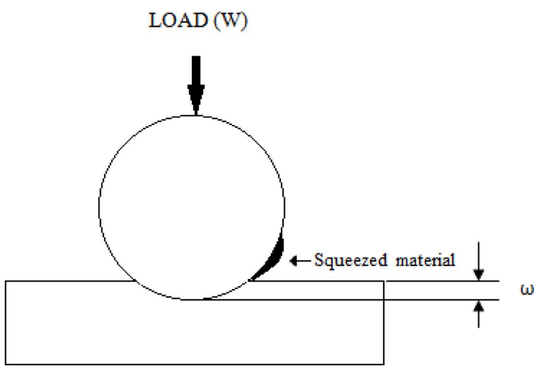


Fig. 4 Basic model

Table 3 Tangent modulus, Indentation depth, Projected area contact width

| $E_T$ (N/mm$^2$) | $\omega$ (mm) | $d/2$ (mm) |
|---|---|---|
| 0.1E | 0.095 | 3.081 |
| 0.2E | 0.089 | 2.982 |
| 0.3E | 0.084 | 2.897 |
| 0.4E | 0.080 | 2.827 |
| 0.5E | 0.077 | 2.774 |

$$\omega = \{9W^2/8D\}^{1/3}[2\{(1-\nu^2)/(E^* + E_T)\}]^{2/3} \tag{1}$$

$$1/E^* = \frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2} \tag{2}$$

$$d = 2[\omega(D-\omega)]^{1/2} \tag{3}$$

The significant material E/Y value of 991.74 is taken for observation of various parameters and it is related to the contact behaviour of the sphere with flat (indentation approach) with incorporating the tangent modulus is given in the Table 3.

From the above table, it is clearly known that with increase in tangent modulus, the indentation depth, contact width and contact area are reduced.

### 4.4 Evaluation of contact area and volume of squeezed material

The estimation of contact area between the two consecutive steps of penetration is very important for analysis the contact bodies in contact mechanics. The contact area between the two circles of the sphere and the flat is expressed by the double integral method in polar coordinates $\iint r dr d\theta$, with suitable limits. The limits are between $-\pi/2$ and $\pi/2$, $s\cos\theta$ and $2s\cos\theta$. The area lying inside the two circles is calculated from the following expression.

$$\text{Difference in area} = \iint r dr d\theta = \frac{3\pi s^2}{4} \tag{4}$$

The estimation of an area between two circles, the outer circle area is considered as zero. From this reference the remaining area between the two consecutive circles are calculated and shown in Table 4.

The evaluation of volume of material squeezed between the two contact bodies is very important for contact analysis. This parameter plays a vital role in the contact fatigue failure and wear analysis. The volume of material between the two circles of the sphere and the flat is expressed by the triple integral method in Cartesian form $8\iiint dx dy dz$ with suitable limits. The limits are between zero and $s$, zero and $\sqrt{s^2-x^2}$. 0 and $\sqrt{s^2-x^2-y}$. The volume lying inside the two circles is calculated from the following expression.

Table 4 Difference in area and volume of material between two circles

| $E_T$ | Projected area contact width ($d/2$) (mm) | Difference in area between two circles ($mm^2$) | Volume of material between two circles ($mm^3$) |
|---|---|---|---|
| 0.1E | 3.081 | 0 | 0 |
| 0.2E | 2.982 | 0.0231 | 0.0041 |
| 0.3E | 2.897 | 0.0170 | 0.0026 |
| 0.4E | 2.827 | 0.0115 | 0.0014 |
| 0.5E | 2.774 | 0.0066 | 0.0006 |

$$\text{Volume in between two circles} = 8\iiint dxdydz$$
$$= \frac{4\pi s^3}{3} \tag{5}$$

The estimation of volume between two circles, the outer circle volume is considered as zero. From this reference, the volume between the two consecutive circles are calculated and shown in Table 4.

From the above table, it is clearly known that at 0.3E the material push upto the free surface rapidly due to straining hardening effect of the material and after that the material is very slow in the free surface. And also the volume of material squeezed and comes out from the contact zone is most due to straining hardening effect of the material after that the volume of material is low down in the free surface.

## 5. Results and discussion

In this finite element analysis using Ansys, the effect of tangent modulus is studied. The result shows that increase in tangent modulus value the stress in the material (E/Y < 1000 close to 991.74) increases up to 0.5E after which stress decreases. With the increases in the tangent modulus value the stress in the material (E/Y > 1000) increases up to 0.3E after which stress decreases as shown in Fig. 5. This is due to the effect of strain hardening of the material.

It shows the non-linear behaviour in between stress and tangent modulus. It is observed that the higher stress is developed in the material E/Y < 1000 of hardness $H = 1$ and $H = 0.43$ for the material having E/Y > 1000. It is confirmed that the tangent modulus increases the hardness of the material. The material behaviour is dependent on the tangent modulus.

Fig. 6 shows the relation between difference in area, volume of material between two consecutive circles and tangent modulus. The tangent modulus of the material increases linearly upto 0.3E with increase in the difference in area and volume of material between two circles. Subsequently, the modulus of the material decreases gradually after 0.3E. When it is very close to the contact surface

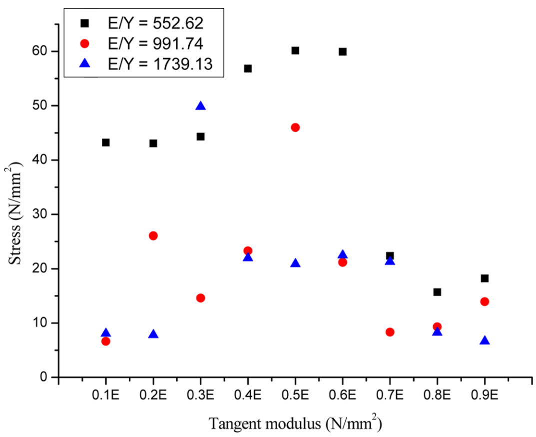


Fig. 5 Stress Vs. Tangent modulus for different materials

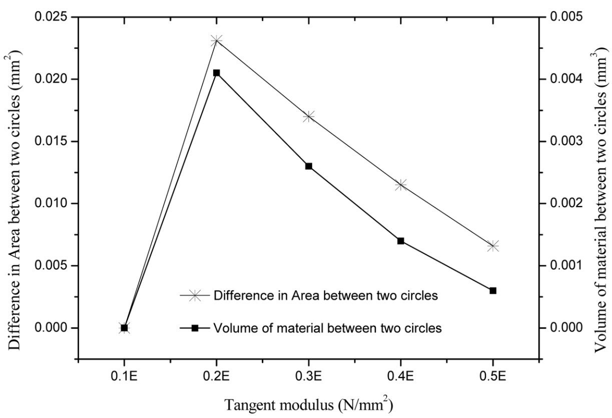


Fig. 6 Difference in area and volume of material between two circles Vs. Tangent modulus

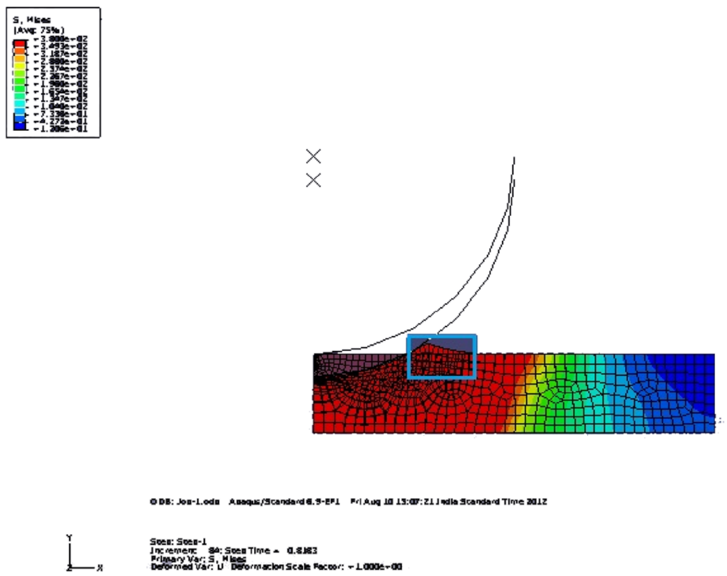

Fig. 7 Simulation of pile-up condition - Abaqus

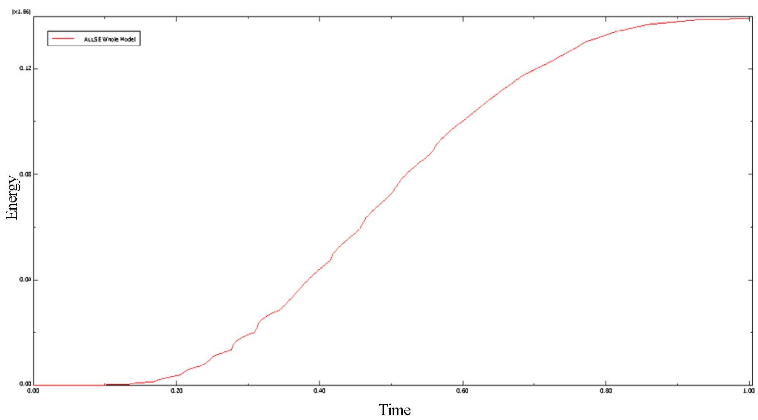


Fig. 8 Simulation of strain energy Vs. Time

this area reduces due to the strain hardening effect of the material for any further increase in the tangent modulus. The effect of tangent modulus has greater influence in the contact parameter.

Fig. 7 shows the analysis of ball and a deformable flat contact model is simulated in the Abaqus software. It shows, the ball penetrated in to a deformable flat with the maximum displacement of 2 mm. Initially the maximum stress induced underneath of the contact surface, when the ball slowly penetrated the stress propagated throughout the flat. And also the bulged material in the contact region is pile-up slowly in an upward direction near the edge of contact and move up into the top surface of the flat. The pile-up of material as shown in the box provided in the plot (Both deformed and undeformed shape).

Fig. 8 shows the strain energy of the model analyzed in the Abaqus software. This plot is taken an output from the simulation. The plot between the simulated time and the strain energy of the whole model.

The output shows that the strain energy variation in between 20 and 40 percentage of the simulated time. This variation is due to the effect of strain hardening of the material.

## 6. Conclusions

The tangent modulus of the material is considered for the study of rigid ball and a flat contact model. In the contact phenomena, the effect of tangent modulus in the contact parameters is very important. The detailed study has been done for the effect of tangent modulus and strain hardness is carried out by finite element analysis using Ansys and analytical solutions. The increasing in tangent modulus the straining action of the material is also increased in considerable rate. For analysing the contact parameter such as area between two consecutive steps, volume of squeezed material the effect of straining action of the material is well thought-out. This effect makes the decrease in area between the two consecutive circles near the top surface of the flat and the volume of material squeezed also decreases. The simulation results from the Abaqus is an substantiation for bulged material escape from the contact region through the edge of the contact between the ball and the flat. And also the output graph shows that due to the effect of strain hardening the strain energy is varying upto 40 percentage of the stipulated time for the process.

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## Notations

| | | | |
|---|---|---|---|
| $W$ | : | Load applied | N |
| $D$ | : | Diameter of spherical ball | mm |
| $d$ | : | Projected area diameter | mm |
| $E$ | : | Young's modulus | $N/mm^2$ |
| $Y$ | : | Yield strength | $N/mm^2$ |
| $E/Y$ | : | Young's modulus to Yield strength ratio | - |
| $E_T$ | : | Tangent modulus | $N/mm^2$ |
| $H$ | : | Linear Hardness | - |
| $\omega$ | : | Penetration depth | mm |
| $\nu$ | : | Poisson's ratio | - |
| $E^*$ | : | Equivalent Young's modulus | $N/mm^2$ |
| $d/2$ | : | Projected area contact width | mm |
| $A$ | : | Area of contact circle | mm |
| $r$ | : | Position of circle from the centre of contact between bodies | mm |
| $\theta$ | : | Angle at which the circle propagated | Degree |
| $dr$ | : | Radius between two circles | mm |
| $s$ | : | Difference in projected area contact width | mm |

## Subscripts

| | | |
|---|---|---|
| 1 | : | Ball (sphere) material |
| 2 | : | Flat (Plate) material |