Structural Engineering and Mechanics, Vol. 44, No. 1 (2012) 15-34
DOI: http://dx.doi.org/10.12989/sem.2012.44.1.015

Analytical solution for bending analysis of soft-core composite sandwich plates using improved high-order theory

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(Received September 15, 2011, Revised May 31, 2012, Accepted August 9, 2012)

Abstract. In the present paper, an improved high-order theory is used for bending analysis of soft-core sandwich plates. Third-order plate assumptions are used for face sheets and quadratic and cubic functions are assumed for transverse and in-plane displacements of the orthotropic soft core. Continuity conditions for transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and lower surfaces of the plate are satisfied. Also, transverse flexibility and transverse normal strain and stress of the orthotropic core are considered. The equations of motion and boundary conditions are derived by principle of minimum potential energy. Analytical solution for bending analysis of simply supported sandwich plates under various transverse loads are presented using Navier's solution. Comparison of the present results with those of the three-dimensional theory of elasticity and some plate theories in the literature confirms the accuracy of the proposed theory.

Keywords: bending analysis; sandwich plate; high-order theory; analytical solution; soft core

1. Introduction

Sandwich plates have been extensively used in many engineering applications such as automotive, aerospace, underwater, and building structures due to their highstrength to weight ratios, and ease of manufacturing. These plates are generally consisted of two thin high strength face sheets and a soft thick low strength core, whichare adhesively bonded together. In most cases, the core consisted of afoam polymer or honeycomb material, while composite laminates are commonly used as the face sheets. In these constructions, the core keeps the face sheets at sufficient distance and transmits the transverse normal and shear loads. Advantages of this construction method are to obtain the plates that have high-bending stiffness characteristics and an extremely low weight (Reddy 2004). To use these structures efficiently, an excellent understanding of their mechanical behavior is needed. There are some important points to be noted for more accuracy in the modeling and analysis of sandwich

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structures. The continuity conditions of the displacements and interlaminar transverse shear stresses should be satisfied to accurately model the mechanical behavior of these plates (Reddy 2004). But variation of material properties between the core and the face sheets, cause the slopes of displacements and transverse shear stresses change in the face sheet-core interfaces. In addition, the conditions of zero transverse shear stresses on the upper and lower surfaces must be enforced. Foam polymer cores are very flexible relative to the face sheets. As such, this behavior lead to un-identical displacement patterns through the depth of the panel and the displacements of the upper face sheet differ from those of the lower one (Reddy 2004).

To date, there are three approaches that are presented to analyze the mechanical behavior of sandwich plates and to predict their static and dynamic responses, that is: three-dimensional (3D) elasticity approaches, equivalent single layer (ESL) theories and layerwise (LW) theories. There are a few exact 3D elasticity solutions for static and dynamic analyses of the composite sandwich plates. Pagano (1969, 1970) derived the static bending response of rectangular composite sandwich plates based on the classical linear three-dimensional theory of elasticity.

Although, some authors analyzed the static and dynamic responses of sandwich plates using ESL theories. Kant and Swaminathan (2000, 2002), Swaminathan and Ragounadin (2004), Matsunaga (2002) presented the analytical solution for static analysis of sandwich plates using a high-order ESL theory. In all ESL theories, the plate is analyzed as two-dimensional (2D) equivalent single layer and the displacements are considered as continuous differentiable functions of the thickness coordinate. This in turn, results in continuous transverse strains, hence interlaminar transverse stresses are obtained as discontinuous functions at the interfaces between the layers with different stiffness properties. For thin laminated plates, this error can be neglected, but in thick laminated or sandwich plates, the ESL theories can give erroneous results. Also, two-dimensional (2D) treatment is not suitable for predicting the local behavior of sandwich plates such as delamination, matrix cracking or wrinkling.

To model thick laminated plates in a better manner and to evaluate their local behaviors, some layerwise plate theories were proposed by Reddy (1987), Robbins and Reddy (1993), Li and Liu (1995). To enforce the continuity conditions of the transverse shear stresses, Toledano and Murakami (1987), Carrera (1998) developed the mixed layerwise theories. Later, to account for the continuity of the transverse stresses at the layer interfaces, Rao and Desai (2004) developed a higher order mixed layerwise theory for the sandwich plates. Carrera and Demasi (2003) extended the mixed layerwise theory for sandwich plates. Extensive studies were presented by Carrera and Brischetto (2009), Demasi (2009) for the mixed layerwise plate theories. Plagianakos and Saravanos (2009) developed a theory for thick composite and sandwich plates by superposing two layerwise theories and imposed interlaminar shear stress continuity conditions. In these models, the number of unknowns depends on the number of the layers in composite and it becomes large as the number of the layers increases.

In order to overcome the high computational time in the layerwise theories, zigzag theories with linear or high-order local functions were proposed. Di Sciuva (1986) proposed a refined zigzag plate theory in which the unknowns for the in-plane displacements at each layer were assumed in terms of those at the reference plane and the transverse displacement was assumed constant along the plate thickness. Some researchers (Di Sciuva 1992, Bhaskar and Varadan 1989, Cho and Parmerter 1993) attributed further improvement in this model. Cho and Parmerter (1993) provided an efficient zigzag plate theory by combining the Reddy's higher order plate theory with the layer-wise zigzag plate theory of Di Sciuva (1986). This theory is sufficiently accurate for the analysis of sandwich plates with transversely incompressible core. But, for sandwich plates with soft core, the un-identical displacement patterns through the depth of the panel cannot be predicted. Some authors used high-order zigzag plate theories for analyzing the sandwich plates. A third-order zigzag plate theory was proposed by Kapuria and Achary (2004) for sandwich plates. Ganapathi *et al.* (2004) analyzed the nonlinear dynamic behavior of the sandwich plates using a third-order zigzag theory. Kim (2007) developed two enhanced plate theories for laminated and sandwich plates via the mixed variational formulation for the free vibration studies. In these models, the number of unknowns is independent of the number of the layers in the composite and the continuity conditions of the transverse shear stresses are enforced.

In addition to the continuity condition of the transverse shear stresses at the layer interfaces, the continuity conditions on the transverse normal stresses were also assumed. Also, the transverse flexibility of the soft cores significantly affects the overall behavior of the sandwich plate as well as the local effects. Pai and Palazotto (2001) presented a high-order layerwise theory to satisfy the continuity conditions of the interlaminar shear and normal stresses in the sandwich plates by introducing the concept of sub-laminates and accounting for non-uniform distribution of transverse shear stresses in each layer. A higher order layer-wise model was proposed by Dafedar *et al*. (2003) for buckling analysis of multi-core sandwich plates. They assumed that all displacement components in any layer are as cubic polynomial. As it involves a large number of unknowns, they proposed a simplified model which causes the plate treating as a single layer. Pandit *et al.* (2008) proposed an improved cubic zigzag theory for the static analysis of laminated sandwich plates with soft compressible core which the transverse displacement is assumed to vary quadratically within the core, while it remains constant through the faces. Kulkarni and Kapuria (2008) investigated the free vibration sandwich plates using an improved discrete Kirchhoff quadrilateral element based on third-order zigzag theory. Brischetto *et al.* (2009) developed a higher-order theory by adding the zigzag function to the known theories for bending analysis of sandwich plates. Most zigzag theories can satisfy the transverse shear continuity conditions, but in these theories, the transverse stresses continuity was not enforced in the governing equations. Post-processing method based on equilibrium consideration has to be adopted to calculate the transverse shear stresses.

Li and Liu (1995, 1997) developed the third-order global-local plate theory (GLPT) based on the double superposition hypothesis, in which the transverse shear stresses can be determined directly from the governing equations without post-processing technique. In this theory, the total number of unknowns is independent of the number of the layers. Based on this theory, the refined finite element method was proposed by Zhen and Wanji (2005) for composite laminated plates. Also, Zhen and Wanji (2006, 2007) studied the free vibration and thermo-mechanical buckling of laminated composites and incompressible sandwich plates using global-local higher-order theory. Based on the higher-order theory, they presented a refined three-node triangular element satisfying C^1 weak-continuity conditions. Cetkovic and Vuksanovic (2008) presented the analytical and the finite element solution for bending, buckling and free vibration of sandwich plates using the global-local higher-order theory of Zhen and Wanji (2006). Recently, Zhen and Wanji (2010) proposed a C^0-type higher-order theory for bending analysis of sandwich plates and presented its analytical solution. In this theory, the continuity conditions of the transverse shear stresses at the interfaces are a priori enforced as well as the conditions of zero transverse shear stresses are imposed on the upper and lower surfaces, but the transverse flexibility of the core is neglected. Zhen *et al.* (2010) also presented the analytical and finite element solution for the free vibration analysis of sandwich plates using C^0-type higher-order theory. Shariyat (2010) introduced a generalized global-local theory that guarantees the continuity conditions of all displacements and transverse stress components and considered the transverse flexibility of sandwich plates. Bending of sandwich plates subjected to thermo-mechanical loads was analyzed using the finite element method. Based on this theory, Shariyat (2010, 2010) also studied nonlinear bending and dynamic thermo-mechanical buckling of the imperfect sandwich plates. Most of researchers that used GLPT only presented the numerical or the semi-analytical solutions and omitted the closed-form analytical solutions.

Some investigators were assumed three-layer sandwich plates consisting of two laminated composite face sheets and a soft flexible core and postulated polynomial functions for in-plane and transverse displacements of each layer. Frostig (1998) developed a theory using the Kirchhoff-Love model (CLPT) for the face sheets and a postulated stress distribution in the core for overall and local buckling analysis of soft core sandwich plates. Frostig and Thomsen (2004) proposed a new high-order sandwich plate theory (HSAPT) for the free vibration analysis. In this theory, the CLPT model was used for the face sheets and cubic and quadratic polynomials were postulated for in-plane and transverse displacements of the core. Analytical solutions were presented for simply supported soft-core sandwich plates, but the transverse stress continuity conditions were neglected. Frostig and Thomsen (2009) improved their theory by using first-order shear deformable theory (FSDT) for the face sheets. Dawe and Yuan (2001) provided a model which uses a quadratic and linear expansion of the in-plane and transverse displacements of the core and represented the face sheets as either FDST or CLPT. A B-spline finite stripe method was given the buckling stresses of sandwich plates. Malekzadeh *et al.* (2004) improved Frostig's (1998) theory for free vibration analysis of sandwich plates with a viscoelastic soft flexible core. Also, Malekzadeh *et al*. (2005) predicted the low velocity impact response of composite sandwich panels using their improved theory. Analytical solution for static and dynamic behavior of sandwich plates and shells were presented by Hohe and Librescu (2004) and Hohe *et al*. (2006). They used the CLPT assumption for the face sheets and linear and quadratic function for transverse and in-plane displacements of the core. Transverse flexibility and shear effects are included in the soft-core layer. Recently, Kheirikhah *et al*. (2011) developed a new improved high-order sandwich plate theory and presented accurate analytical solution for biaxial overall buckling problem of sandwich plates. Also, Ahn and Lee (2011) studied transverse vibration of sandwich plates with asymmetrical faces. Rezaiee-Pajand *et al.* (2012) proposed a new higher-order triangular plate element for the bending analysis of laminated composite and sandwich plates. They studied different sandwich plates with different boundary conditions and fiber orientations.

Based on the above discussions, it can be concluded that in the ESL theories, interlaminar transverse stresses are obtained as discontinuous functions at the interface between the layers with different stiffness properties. Also, these theories are not suitable for predicting the local behavior of sandwich plates. In layerwise models, the number of unknowns depends on the number of the layers and large computational efforts needed. High order zigzag theories can accurately predict the behavior of sandwich plates, but transverse stresses continuity was not enforced in the governing equations. The high-order global-local theories are sufficient and accurate which their solutions usually were not presented analytically except in Ref. (Zhen and Wanji 2010). Analytical solution for static and dynamic behavior of sandwich plates which were presented by Refs. (Zhen and Wanji 2010, Frostig and Thomsen 2004, Hohe *et al*. 2006) use some simplifications, resulted in un-accurate results. But, Kheirikhah *et al.* (2011) presented a new improved high-order theory for biaxial buckling analysis of soft-core sandwich plates which the specifics of this theory can be defined as:

- Third-order shear deformable plate theory is used for the face sheets.
- Quadratic and cubic functions are assumed for the transverse and in-plane displacements of the soft orthotropic core.
- Continuity conditions of transverse shear stresses at the interfaces are satisfied.
- The conditions of zero transverse shear stresses on the upper and lower surfaces of the sandwich plate are satisfied.
- Transverse flexibility and transverse normal strain and stress of the core are considered.

In the present paper, the above improved high-order theory is used for bending analysis of soft-core sandwich plates. The equations of motion and boundary conditions are derived via principle of minimum potential energy. Analytical solution for bending analysis of simply supported sandwich plates under various transverse loads are presented using Navier's solution.

2. Mathematical formulation

A rectangular sandwich plate with the plane dimensions of $a \times b$ and total thickness of h is considered as shown in Fig. 1. The sandwich is composed of three layers: the top and the bottom face sheets and the core layer. All layers are assumed with uniform thickness and that the vertical coordinate of each layer is measured downward from its mid-plane. The face sheets are generally unequal in thickness i.e., h_t and h_b are the thicknesses of the top and bottom face sheets respectively, and they may be made of laminated composites. The core is assumed as soft orthotropic material with thickness h_c.

2.1 Kinematic relations

In the present structural model for sandwich plates, the third-order shear deformable theory is adopted for the face sheets. Hence, the displacement components of the top and bottom face sheets ($j = t$, b) are represented as (Reddy 2004)

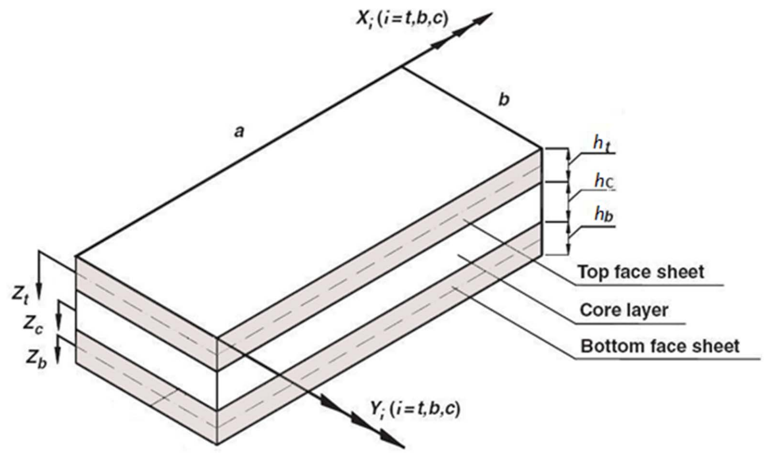

Fig. 1 A typical sandwich plate and its dimensions

$$u_j(x,y,z_t) = u_{0j}(x,y)+z_ju_{1j}(x,y)+z_j^2u_{2j}(x,y)+z_j^3u_{3j}(x,y)$$
$$v_j(x,y,z_t) = v_{0j}(x,y)+z_jv_{1j}(x,y)+z_j^2v_{2j}(x,y)+z_j^3v_{3j}(x,y)$$
$$w_j(x,y,z_t) = w_{0j}(x,y) \tag{1}$$

where u_{kj} and v_{kj} (k = 0, 1, 2, 3) are the unknowns of the in-plane displacements of each face sheet and w_{0j} are the unknowns of its vertical displacements, respectively.

The core layer is much thicker and softer than the face sheets. Thus, the displacements fields for the core are assumed a cubic pattern for the in-plane displacement components and a quadratic one for the vertical component (Reddy 2004)

$$u_c(x,y,z_c) = u_{0c}(x,y)+z_cu_{1c}(x,y)+z_c^2u_{2c}(x,y)+z_c^3u_{3c}(x,y)$$
$$v_c(x,y,z_c) = v_{0c}(x,y)+z_cv_{1c}(x,y)+z_c^2v_{2c}(x,y)+z_c^3v_{3c}(x,y)$$
$$w_c(x,y,z_c) = w_{0c}(x,y)+z_cw_{1c}(x,y)+z_c^2w_{2c}(x,y) \tag{2}$$

where u_{kc} and v_{kc} (k = 0, 1, 2, 3) are the unknowns of the in-plane displacement components of the core and w_{lc} (l = 0, 1, 2) are the unknowns of its vertical displacements, respectively. Finally, in this model there are twenty nine displacement unknowns.

In this study, the core is perfectly bonded to the face sheets. The interface displacement continuity requirements in each face sheet-core interface were defined in paper of Kheirikhah *et al.* (2011). Also, the transverse shear stresses on the upper and lower surfaces of the sandwich plate are zero (Reddy 2004). In addition, the continuity of transverse shear stresses at the top and bottom face sheet-core interfaces must be satisfied. These conditions were satisfied in the paper of Kheirikhah *et al.* (2011).

2.2 Strains

The linear strain-displacement relation for the face sheets (j = t, b) can be expressed as

$$\varepsilon_{xx}^j = u_{0j,x}+z_ju_{1j,x}+z_j^2u_{2j,x}+z_j^3u_{3j,x}$$
$$\varepsilon_{yy}^j = v_{0j,y}+z_tv_{1j,y}+z_j^2v_{2j,y}+z_j^3v_{3j,y}$$
$$\varepsilon_{zz}^j = 0$$
$$\gamma_{xy}^j = v_{0j,x}+z_jv_{1j,x}+z_j^2v_{2j,x}+z_j^3v_{3j,x}+u_{0j,y}+z_ju_{1j,y}+z_j^2u_{2j,y}+z_j^3u_{3j,y}$$
$$\gamma_{xz}^j = u_{1j}+2z_ju_{2j}+3z_j^2u_{3j}$$
$$\gamma_{yz}^j = v_{1j}+2z_jv_{2j}+3z_j^2v_{3j} \tag{3}$$

And the linear strain-displacement relation for the core can be expressed as

$$\varepsilon_{xx}^{c} = u_{0c,x} + z_c u_{1c,x} + z_c^2 u_{2c,x} + z_c^3 u_{3c,x}$$

$$\varepsilon_{yy}^{c} = v_{0c,y} + z_c v_{1c,y} + z_c^2 v_{2c,y} + z_c^3 v_{3c,y}$$

$$\varepsilon_{zz}^{c} = w_{1c} + 2z_c w_{2c}$$

$$\gamma_{xy}^{c} = v_{0c,x} + z_c v_{1c,x} + z_c^2 v_{2c,x} + z_c^3 v_{3c,x} + u_{0c,y} + z_c u_{1c,y} + z_c^2 u_{2c,y} + z_c^3 u_{3c,y}$$

$$\gamma_{xz}^{c} = u_{1c} + 2z_c u_{2c} + 3z_c^2 u_{3c} + w_{0c,x} + z_c w_{1c,x} + z_c^2 w_{2c,x}$$

$$\gamma_{yz}^{c} = v_{1c} + 2z_c v_{2c} + 3z_c^2 v_{3c} + w_{0c,y} + z_c w_{1c,y} + z_c^2 w_{2c,y} \tag{4}$$

2.3 Governing equations

The governing equations of motion for the face sheets and the core are derived through the principle of minimum potential energy

$$\delta\Pi = \delta U + \delta V = 0 \tag{5}$$

where U is the total strain energy, V is the potential of the external loads and δ denotes the variation operator. The variation of the external work equals to

$$\delta V = -\int_0^a\int_0^b [\bar{n}_{xt}\delta u_{0t} + \bar{n}_{yt}\delta v_{0t} + q_t \delta w_{0t} + \bar{n}_{xb}\delta u_{0b} + \bar{n}_{yb}\delta v_{0b} + q_b \delta w_{0b}]dxdy \tag{6}$$

where u_{0j}, v_{0j} and w_{0j} $(j = t, b)$ are the displacements of the mid-plane of the face sheets in the longitudinal, transverse and vertical directions, respectively, $\bar{n}_{xj}$ and $\bar{n}_{yj}$ $(j = t, b)$ are thein-plane external loads of the top and bottom face sheets and q_t and q_b are the vertical distributed loads applied on the top and bottom face sheets, respectively.

The first variation of the total strain energy can be expressed in terms of all stresses and strains of the face sheets and the core. In addition, six compatibility conditions at the interfaces, four conditions of zero transverse shear stresses on the upper and lower surfaces of the plate and four continuity conditions of transverse shear stresses at the interfaces are fulfilled by using fourteen Lagrange multipliers. Thus, the variation of the strain energy for sandwich plate with cross-ply laminated face sheets and orthotropic core can be read as

$$\begin{aligned}
\delta U = & \int_{v_t} (\sigma_{xx}^{t}\delta\varepsilon_{xx}^{t} + \sigma_{yy}^{t}\delta\varepsilon_{yy}^{t} + \tau_{xy}^{t}\delta\gamma_{xy}^{t} + \tau_{xz}^{t}\delta\gamma_{xz}^{t} + \tau_{yz}^{t}\delta\gamma_{yz}^{t})dv \\
& + \int_{v_b} (\sigma_{xx}^{b}\delta\varepsilon_{xx}^{b} + \sigma_{yy}^{b}\delta\varepsilon_{yy}^{b} + \tau_{xy}^{b}\delta\gamma_{xy}^{b} + \tau_{xz}^{b}\delta\gamma_{xz}^{b} + \tau_{yz}^{b}\delta\gamma_{yz}^{b})dv \\
& + \int_{v_c} (\sigma_{zz}^{c}\delta\varepsilon_{zz}^{c} + \tau_{xz}^{c}\delta\gamma_{xz}^{c} + \tau_{yz}^{c}\delta\gamma_{yz}^{c} + \sigma_{xx}^{c}\delta\varepsilon_{xx}^{c} + \sigma_{yy}^{c}\delta\varepsilon_{yy}^{c} + \tau_{xy}^{c}\delta\gamma_{xy}^{c})dv
\end{aligned}$$

$$+\delta\left\{\int_0^a\int_0^b\left[\lambda_x^t\left(u_t\left(z_t = \frac{h_t}{2}\right) - u_c\left(z_c = \frac{-h_c}{2}\right)\right) + \lambda_y^t\left(v_t\left(z_t = \frac{h_t}{2}\right) - v_c\left(z_c = \frac{-h_c}{2}\right)\right)\right.\right.$$

$$+\lambda_z^t\left(w_t\left(z_t=\frac{h_t}{2}\right)-w_c\left(z_c=\frac{-h_c}{2}\right)\right)+\lambda_x^b\left(u_b\left(z_b=\frac{-h_b}{2}\right)-u_c\left(z_c=\frac{h_c}{2}\right)\right)$$

$$\left.+\lambda_y^b\left(v_b\left(z_b=\frac{-h_b}{2}\right)-v_c\left(z_c=\frac{h_c}{2}\right)\right)+\lambda_z^b\left(w_b\left(z_b=\frac{-h_b}{2}\right)-w_c\left(z_c=\frac{h_c}{2}\right)\right)\Big]dxdy\right\}$$

$$+\delta\left\{\int_0^a\int_0^b\Big[\lambda_{xz}^t\left(\gamma_{xz}^t\left(z_t=\frac{-h_t}{2}\right)\right)+\lambda_{yz}^t\left(\gamma_{yz}^t\left(z_t=\frac{-h_t}{2}\right)\right)+\lambda_{xz}^b\left(\gamma_{xz}^b\left(z_b=\frac{h_b}{2}\right)\right)\right.$$

$$+\lambda_{yz}^b\left(\gamma_{yz}^b\left(z_b=\frac{h_b}{2}\right)\right)+\lambda_{yz}^{tc}\left(Q_{44}^t\gamma_{yz}^t\left(z_t=\frac{h_t}{2}\right)\right)-G_{yz}^c\gamma_{yz}^c\left(z_c=\frac{-h_c}{2}\right)$$

$$+\lambda_{xz}^{tc}Q_{55}^t\gamma_{xz}^t\left(\left(z_t=\frac{h_t}{2}\right)=G_{xz}^c\gamma_{xz}^c\left(z_c=\frac{-h_c}{2}\right)\right)$$

$$+\lambda_{yz}^{bc}Q_{44}^b\gamma_{yz}^b\left(\left(z_b=\frac{-h_b}{2}\right)=G_{yz}^c\gamma_{yz}^c\left(z_c=\frac{h_c}{2}\right)\right)$$

$$\left.+\lambda_{xz}^{bc}Q_{55}^b\gamma_{xz}^b\left(z_t=\frac{-h_b}{2}\right)=G_{xz}^c\gamma_{xz}^c\left(z_c=\frac{h_c}{2}\right)\Big]dxdy\right\} \tag{7}$$

where $\lambda_i^j\ (i=x,y,z),\ (j=t,b)$ arethe six Lagrange multipliers for compatibility conditions at the top and bottom interfaces, $\lambda_{iz}^j\ (i=x,y),\ (j=t,b)$ are the four Lagrange multipliers for conditions of zero transverse shear stresses on the upper and lower surfaces of the plate and $\lambda_{iz}^{jc}\ (i=x,y)$, $(j=t,b)$ are the four Lagrange multipliers for continuity conditions of transverse shear stresses at the interfaces. The stress resultants for the two face sheets and the core $(j=t,b,c)$ can be defined as

$$\begin{Bmatrix} N_{xx}^j & N_{yy}^j & N_{xy}^j \\ M_{xx}^j & M_{yy}^j & M_{xy}^j \\ P_{xx}^j & P_{yy}^j & P_{xy}^j \\ R_{xx}^j & R_{yy}^j & R_{xy}^j \end{Bmatrix}=\int_{-h_j/2}^{h_j/2}[\sigma_{xx}^j\ \ \sigma_{yy}^j\ \ \tau_{xy}^j]\begin{Bmatrix}1\\ z_j\\ z_j^2\\ z_j^3\end{Bmatrix}dz_j$$

$$\left\{\begin{bmatrix} Q_{xz}^j & Q_{yz}^j \\ S_{xz}^j & S_{yz}^j \\ T_{xz}^j & T_{yz}^j \end{bmatrix}\right\}=\int_{-h_j/2}^{h_j/2}[\tau_{xz}^j\ \ \tau_{yz}^j]\begin{Bmatrix}1\\ z_j\\ z_j^2\end{Bmatrix}dz_j,\quad \begin{Bmatrix}N_{zz}^c\\ M_{zz}^c\end{Bmatrix}=\int_{-h_c/2}^{h_c/2}\sigma_{zz}^c\begin{Bmatrix}1\\ z_c\end{Bmatrix}dz_c \tag{8}$$

Integrating by part and doing some mathematical operations, the equations of motion for the top face sheet can be calculated as

$$
\begin{aligned}
&-N^t_{xx,x}-N^t_{xy,y}+\lambda^t_x-\bar{n}_{xt}=0\\
&-M^t_{xx,x}-M^t_{xy,y}+Q^t_{xz}+\frac{h_t}{2}\lambda^t_x+\lambda^t_{xz}+\frac{h_t}{2}\bar{n}_{xt}=0\\
&-P^t_{xx,x}-P^t_{xy,y}+2S^t_{xz}+\frac{h_t^2}{2}\lambda^t_x-h_t\lambda^t_{xz}+\lambda^{tc}_{xz}-\frac{h_t^2}{4}\bar{n}_{xt}=0\\
&-R^t_{xx,x}-R^t_{xy,y}+3T^t_{xz}+\frac{h_t^3}{8}\lambda^t_x+\frac{3h_t^2}{4}\lambda^t_{xz}+\frac{h_t^3}{8}\bar{n}_{xt}=0\\
&-N^t_{yy,y}-N^t_{xy,x}+\lambda^t_y-\bar{n}_{yt}=0\\
&-M^t_{yy,y}-M^t_{xy,x}+Q^t_{yz}+\frac{h_t}{2}\lambda^t_y+\lambda^t_{yz}+\frac{h_t}{2}\bar{n}_{yt}=0\\
&-P^t_{yy,y}-P^t_{xy,x}+2S^t_{yz}+\frac{h_t^2}{4}\lambda^t_y-h_t\lambda^t_{yz}+\lambda^{tc}_{yz}-\frac{h_t^2}{4}\bar{n}_{yt}=0\\
&-R^t_{yy,y}-R^t_{xy,x}+3T^t_{yz}+\frac{h_t^3}{8}\lambda^t_y+\frac{3h_t^2}{4}\lambda^t_{yz}+\frac{h_t^3}{8}\bar{n}_{yt}=0\\
&-Q^t_{xz,x}-Q^t_{yz,y}+\lambda^t_z-\lambda^t_{xz,x}-\lambda^t_{yz,y}-q_t=0
\end{aligned}
\tag{9}
$$

and for the bottom face sheet as

$$
\begin{aligned}
&-N^b_{xx,x}-N^b_{xy,y}+\lambda^b_x-\bar{n}_{xb}=0\\
&-M^b_{xx,x}-M^b_{xy,y}+Q^b_{xz}-\frac{h_b}{2}\lambda^b_x+\lambda^b_{xz}-\frac{h_b}{2}\bar{n}_{xb}=0\\
&-P^b_{xx,x}-P^b_{xy,y}+2S^b_{xz}+\frac{h_b^2}{2}\lambda^b_x+h_b\lambda^b_{xz}+\lambda^{bc}_{xz}-\frac{h_b^2}{4}\bar{n}_{xb}=0\\
&-R^b_{xx,x}-R^b_{xy,y}+3T^b_{xz}-\frac{h_b^3}{8}\lambda^b_x+\frac{3h_b^2}{4}\lambda^b_{xz}-\frac{h_b^3}{8}\bar{n}_{xb}=0\\
&-N^b_{yy,y}-N^b_{xy,x}+\lambda^b_y-\bar{n}_{yb}=0\\
&-M^b_{yy,y}-M^b_{xy,x}+Q^b_{yz}-\frac{h_b}{2}\lambda^b_y+\lambda^b_{yz}-\frac{h_b}{2}\bar{n}_{yb}=0\\
&-P^b_{yy,y}-P^b_{xy,x}+2S^b_{yz}+\frac{h_b^2}{4}\lambda^b_y+h_b\lambda^b_{yz}+\lambda^{bc}_{yz}-\frac{h_b^2}{4}\bar{n}_{yb}=0\\
&-R^b_{yy,y}-R^b_{xy,x}+3T^b_{yz}-\frac{h_b^3}{8}\lambda^b_y+\frac{3h_b^2}{4}\lambda^b_{yz}-\frac{h_b^3}{8}\bar{n}_{yb}=0\\
&-Q^b_{xz,x}-Q^b_{yz,y}+\lambda^b_z-\lambda^b_{xz,x}-\lambda^b_{yz,y}-q_b=0
\end{aligned}
\tag{10}
$$

and also for the core as

$$
\begin{aligned}
&-N^{c}_{xx,x}-N^{c}_{xy,y}-\lambda^{t}_{x}-\lambda^{b}_{x}=0\\
&-M^{c}_{xx,x}-M^{c}_{xy,y}+Q^{c}_{xz}+\frac{h_c}{2}\lambda^{t}_{x}-\frac{h_c}{2}\lambda^{b}_{x}-\frac{G^{c}_{xz}}{2h_tQ^{t}_{55}}\lambda^{tc}_{xz}+\frac{G^{c}_{xz}}{2h_bQ^{b}_{55}}\lambda^{bc}_{xz}=0\\
&-P^{c}_{xx,x}-P^{c}_{xy,y}+2S^{c}_{xz}-\frac{h_c^2}{4}\lambda^{t}_{x}-\frac{h_c^2}{4}\lambda^{b}_{x}+\frac{G^{c}_{xz}h_c}{2h_tQ^{t}_{55}}\lambda^{tc}_{xz}+\frac{G^{c}_{xz}h_c}{2h_bQ^{b}_{55}}\lambda^{bc}_{xz}=0\\
&-R^{c}_{xx,x}-R^{c}_{xy,y}+3T^{c}_{xz}+\frac{h_c^3}{8}\lambda^{t}_{x}-\frac{h_c^3}{8}\lambda^{b}_{x}-\frac{3G^{c}_{xz}h_c^2}{8h_tQ^{t}_{55}}\lambda^{tc}_{xz}+\frac{3G^{c}_{xz}h_c^2}{8h_bQ^{b}_{55}}\lambda^{bc}_{xz}=0\\
&-N^{c}_{yy,y}-N^{c}_{xy,x}-\lambda^{t}_{y}-\lambda^{b}_{y}=0\\
&-M^{c}_{yy,y}-M^{c}_{xy,x}+Q^{c}_{yz}+\frac{h_c}{2}\lambda^{t}_{y}-\frac{h_c}{2}\lambda^{b}_{y}-\frac{G^{c}_{yz}}{2h_tQ^{t}_{44}}\lambda^{tc}_{yz}+\frac{G^{c}_{yz}}{2h_bQ^{b}_{44}}\lambda^{bc}_{yz}=0\\
&-P^{c}_{yy,y}-P^{c}_{xy,x}+2S^{c}_{yz}-\frac{h_c^2}{4}\lambda^{t}_{y}-\frac{h_c^2}{4}\lambda^{b}_{y}+\frac{G^{c}_{yz}h_c}{2h_tQ^{t}_{44}}\lambda^{tc}_{yz}+\frac{G^{c}_{yz}h_c}{2h_bQ^{b}_{44}}\lambda^{bc}_{yz}=0\\
&-R^{c}_{yy,y}-R^{c}_{xy,x}+3T^{c}_{yz}+\frac{h_c^3}{8}\lambda^{t}_{y}-\frac{h_c^3}{8}\lambda^{b}_{y}-\frac{3G^{c}_{yz}h_c^2}{8h_tQ^{t}_{44}}\lambda^{tc}_{yz}+\frac{3G^{c}_{yz}h_c^2}{8h_bQ^{b}_{44}}\lambda^{bc}_{yz}=0\\
&-Q^{c}_{xz,x}-Q^{c}_{yz,y}-\lambda^{t}_{z}-\lambda^{b}_{z}+\frac{G^{c}_{xz}}{2h_tQ^{t}_{55}}\lambda^{tc}_{xz,x}+\frac{G^{c}_{yz}}{2h_tQ^{t}_{44}}\lambda^{tc}_{yz,y}-\frac{G^{c}_{xz}}{2h_bQ^{b}_{55}}\lambda^{bc}_{xz,x}-\frac{G^{c}_{yz}}{2h_bQ^{b}_{44}}\lambda^{bc}_{yz,y}=0\\
&-S^{c}_{xz,x}-S^{c}_{yz,y}+N^{c}_{zz}+\frac{h_c}{2}\lambda^{t}_{z}-\frac{h_c}{2}\lambda^{b}_{z}-\frac{G^{c}_{xz}h_c}{4h_cQ^{t}_{55}}\lambda^{tc}_{xz,x}-\frac{G^{c}_{yz}h_c}{4h_tQ^{t}_{44}}\lambda^{tc}_{yz,y}-\frac{G^{c}_{xz}h_c}{4h_bQ^{b}_{55}}\lambda^{bc}_{xz,x}-\frac{G^{c}_{yz}h_c}{4h_bQ^{b}_{44}}\lambda^{bc}_{yz,y}=0\\
&-T^{c}_{xz,x}-T^{c}_{yz,y}+2M^{c}_{zz}-\frac{h_c^2}{4}\lambda^{t}_{z}-\frac{h_c^2}{4}\lambda^{b}_{z}+\frac{G^{c}_{xz}h_c^2}{8h_tQ^{t}_{55}}\lambda^{tc}_{xz,x}+\frac{G^{c}_{yz}h_c^2}{8h_tQ^{t}_{44}}\lambda^{tc}_{yz,y}-\frac{G^{c}_{xz}h_c^2}{8h_bQ^{b}_{55}}\lambda^{bc}_{xz,x}-\frac{G^{c}_{yz}h_c^2}{8h_bQ^{b}_{44}}\lambda^{bc}_{yz,y}=0
\end{aligned}
\tag{11}
$$

The resultants of Eqs. (9)-(11) can be related to the total strains which the corresponding equations were defined in the paper of Kheirikhah *et al*. (2011).

3. Analytical solution

The exact analytical solutions of Eqs. (9)-(11) exist for the simply-supported rectangular sandwich plate with cross-ply face sheets. Both face sheets are considered as a cross-ply laminated composite.

For simply-supported plates, the normal in-plane displacements on the boundary are admissible, but the tangential displacements are not as such. Therefore, the boundary conditions of simply-supported plates can be expressed as (Kant and Swaminathan 2000)

At edges $x = 0$ and $x = a$

$$v_{0j} = 0,\ v_{1j} = 0,\ v_{2j} = 0,\ v_{3j} = 0,\ j = t, b, c$$
$$w_{0t} = 0,\ w_{0b} = 0,\ w_{0c} = 0,\ w_{1c} = 0,\ w_{2c} = 0$$
$$N^j_{xx} = 0,\ M^j_{xx} = 0,\ P^j_{xx} = 0,\ R^j_{xx} = 0 \tag{12}$$

At edges $y = 0$ and $y = b$

$$u_{0j} = 0,\ u_{1j} = 0,\ u_{2j} = 0,\ u_{3j} = 0,\ j = t, b, c$$
$$w_{0t} = 0,\ w_{0b} = 0,\ w_{0c} = 0,\ w_{1c} = 0,\ w_{2c} = 0$$
$$N^j_{yy} = 0,\ M^j_{yy} = 0,\ P^j_{yy} = 0,\ R^j_{yy} = 0 \tag{13}$$

Using Navier's procedure, the solution of the displacement variables satisfying the above boundary conditions can be expressed in the following forms

$$u_{ij}(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} u^{mn}_{ij}\cos(\alpha_m x)\cdot\sin(\beta_n y)$$

$$v_{ij}(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} V^{mn}_{ij}\sin(\alpha_m x)\cdot\cos(\beta_n y)$$

$$w_{0j}(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} W^{mn}_{0j}\sin(\alpha_m x)\cdot\sin(\beta_n y)$$

$$u_{ic}(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} U^{mn}_{ic}\cos(\alpha_m x)\cdot\sin(\beta_n y)$$

$$v_{ic}(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} V^{mn}_{ic}\sin(\alpha_m x)\cdot\cos(\beta_n y)$$

$$w_{kc}(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} W^{mn}_{kc}\sin(\alpha_m x)\cdot\sin(\beta_n y) \tag{14}$$

By using Navier's solution, the Lagrange multipliers can be expressed in the following forms

$$\lambda^j_x(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} X^{mn}_{j}\cos(\alpha_m x)\cdot\sin(\beta_n y)$$

$$\lambda^j_y(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} Y^{mn}_{j}\sin(\alpha_m x)\cdot\cos(\beta_n y)$$

$$\lambda^j_z(x, y) = \sum_{n=1}^{N}\sum_{m=1}^{M} Z^{mn}_{j}\sin(\alpha_m x)\cdot\sin(\beta_n y)$$

$$\lambda_{xz}^{j}(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} L_{xj}^{mn}\cos(\alpha_m x)\cdot\sin(\beta_n y)$$

$$\lambda_{yz}^{j}(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} L_{yj}^{mn}\sin(\alpha_m x)\cdot\cos(\beta_n y)$$

$$\lambda_{xz}^{jc}(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} L_{xjc}^{mn}\cos(\alpha_m x)\cdot\sin(\beta_n y)$$

$$\lambda_{yz}^{jc}(x,y) = \sum_{n=1}^{N}\sum_{m=1}^{M} L_{yjc}^{mn}\sin(\alpha_m x)\cdot\cos(\beta_n y) \tag{15}$$

which $j = t, b$, $i = 0, 1, 2, 3$, $k = 0, 1, 2$ and $\alpha_m = m\pi/a$, $\beta_n = n\pi/b$. By substituting Eqs. (14) and (15) into Eqs. (9, 10, 11) and collecting the coefficients, the generalized displacement components can be determined.

4. Numerical results and discussion

In this section, several examples of the bending problems of the sandwich plates are studied to verify the accuracy and applicability of the present higher order theory. The results obtained by present theory are compared with the published results. The material constants used in all results are assumed as:

For the face sheets (Pagano 1970)

$$E_1 = 25E,\ E_2 = E_3 = E,\ G_{12} = G_{13} = 0.5E,\ G_{23} = 0.2E,\ \nu_{12} = \nu_{13} = \nu_{23} = 0.25 \tag{16}$$

For the orthotropic core (Pagano 1970)

$$E_x = E_y = 0.04E,\ E_z = 0.5E,\ G_{xz} = G_{yz} = 0.06E,\ G_{xy} = 0.16E,\ \nu_{xy} = \nu_{xz} = \nu_{yz} = 0.25 \tag{17}$$

The following non-dimensional quantities used in the present analysis are defined as

$$(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\sigma}_z, \bar{\tau}_{xy}) = \frac{h^2}{a^2 q_0}(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}), \quad (\bar{\tau}_{xz}, \bar{\tau}_{yz}) = \frac{h}{a q_0}(\tau_{xz}, \tau_{yz})$$

$$\bar{w} = \frac{100 E_2 h^3}{a^4 q_0} w, \quad \bar{u} = \frac{E_2 h^2}{a^3 q_0} u \tag{18}$$

A square symmetric sandwich plate [0°/Core/0°] is subjected to a doubly sinusoidal transverse loading $q(x,y) = q_0\cdot\sin(\alpha_m x)\cdot\cos(\beta_n y)$ and all other loads are considered negligible. It has the total thickness of h in which the thickness of each face sheet is 0.1 h and the thickness of the core is 0.8 h. The non-dimensional stress components and deflections at the specific locations are

Table 1 Non-dimensional deflection and stresses at the important points of a simply supported square sandwich plate [0/Core/0] under sinusoidal load

a/h		$\bar{w}\left(\frac{a}{2},\frac{b}{2},0\right)$	$\bar{\tau}_{xz}\left(0,\frac{b}{2},0\right)$	$\bar{\tau}_{yz}\left(\frac{a}{2},0,0\right)$	$\bar{\sigma}_{z}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)$	$\bar{\sigma}_{y}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)$	$\bar{\tau}_{xy}\left(0,0,\frac{h}{2}\right)$
100	Present	0.8924	0.3240	0.0297	1.0975	0.0549	0.0437
	Pagano (1970)	0.8923	0.324	0.0297	1.098	0.0550	0.0433
	Pandit (2008)	0.8917	0.3323	0.0315	1.0643	0.0532	0.0423
	Kant (2002)	0.8910	-	-	1.0975	0.0549	0.0436
50	Present	0.9349	0.3231	0.0306	1.0988	0.0568	0.0446
	Pagano (1970)	0.9348	0.323	0.0306	1.099	0.0569	0.0446
	Pandit (2008)	0.9341	0.3314	0.0324	1.0658	0.0551	0.0432
	Kant (2002)	0.9294	-	-	1.0989	0.0566	0.0445
20	Present	1.2269	0.3174	0.0361	1.1089	0.0691	0.0508
	Pagano (1970)	1.2264	0.317	0.0361	1.110	0.0700	0.0511
	Pandit (2008)	1.2254	0.3255	0.0381	1.0763	0.0676	0.0496
	Kant (2002)	1.1939	-	-	1.1091	0.0682	0.0504
	Zhen (2010)	-	0.3174	0.0360	1.1101	0.0697	0.0511
10	Present	2.2034	0.3000	0.0528	1.1493	0.1067	0.0701
	Pagano (1970)	2.2004	0.300	0.0527	1.153	0.1104	0.0707
	Pandit (2008)	2.2002	0.3076	0.0555	1.1179	0.1058	0.0691
	Kant (2002)	2.0848	-	-	1.1495	0.1042	0.0688
	Zhen (2010)	-	0.3001	0.0523	1.1538	0.1091	0.0713
	Matsunaga (2002)	2.1435	0.3009	-	-	-	-
4	Present	7.6459	0.2401	0.1082	1.5177	0.2458	0.1395
	Pagano (1970)	7.5962	0.239	0.1072	1.512	0.2533	0.1437
	Pandit (2008)	7.6552	0.2453	0.1126	1.4820	0.2440	0.1430
	Kant (2002)	7.1539	-	-	1.5030	0.2391	0.1409
	Zhen (2010)	-	0.2395	0.1060	1.5370	0.2395	0.1476
	Matsunaga (2002)	7.3116	0.2409	-	-	-	-

obtained from the present model and compared with the results obtained by Pagano (1970), which used the elasticity solutions and some other published results in Table 1. In the following examples, the through-thickness normal stress σ_x and the transverse displacement w are calculated at $(x=a/2, y=b/2)$. The transverse shear stresses τ_{xz} and the in-plane displacement are calculated at $(x=0, y=b/2)$, whereas the transverse shear stress τ_{yx} is calculated at $(x=a/2, y=0)$. Also, the in-plane shear stress τ_{xy} is determined at $(x=0, y=0)$. The analysis is performed for different thickness ratios (*a/h* = 100, 50, 20, 10 and 4). It can be seen that the present results are in excellent agreement with elasticity solutions (Pagano 1970). The maximum error for all the results is less than 3 percent.

The through thickness distributions of some parameters for symmetric three-layer sandwich plate [0/Core/0] subjected to doubly sinusoidal loads are plotted in Figs. 2-6 and are compared with

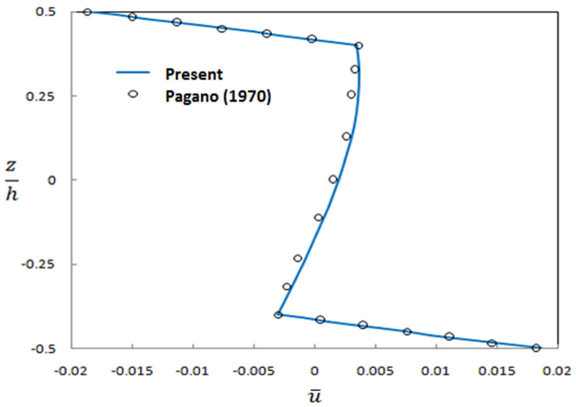

Fig. 2 Distributions of in-plane displacements through the thickness of three-layer sandwich plate [0/Core/0] subjected to doubly sinusoidal loads (a/h = 4)

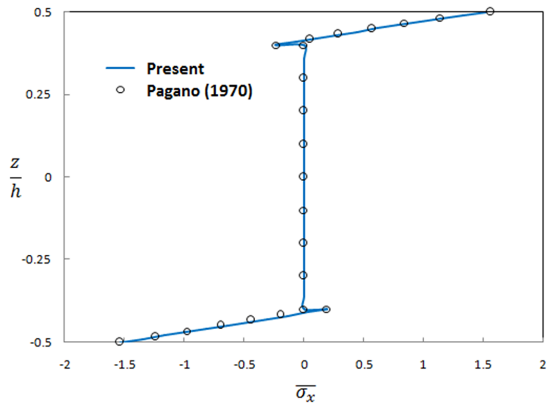

Fig. 3 Distributions of in-plane stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to doubly sinusoidal loads (a/h = 4)

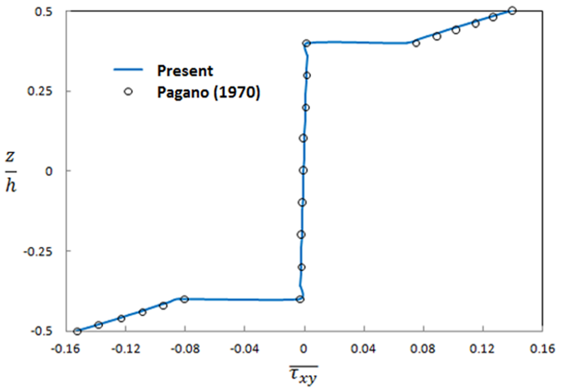

Fig. 4 Distributions of in-plane shear stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to doubly sinusoidal loads (a/h = 4)

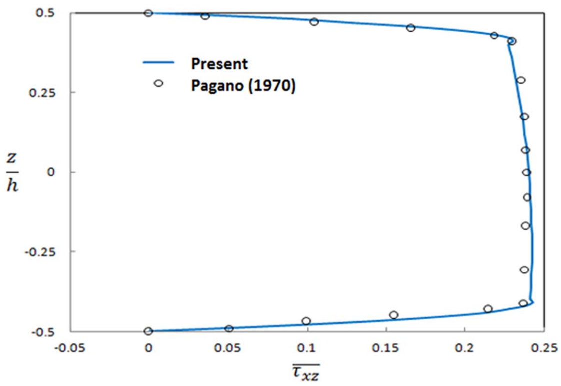

Fig. 5 Distributions of transverse shear stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to doubly sinusoidal loads (a/h = 4)

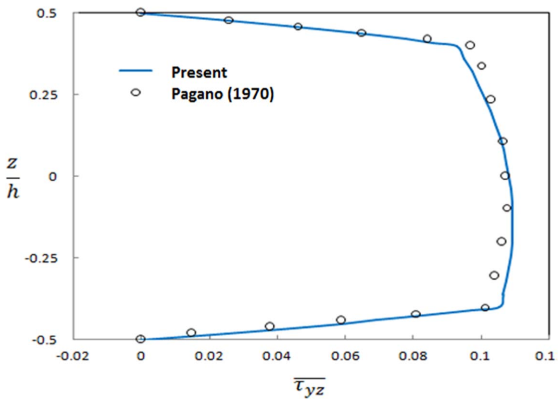

Fig. 6 Distributions of transverse shear stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to doubly sinusoidal loads (a/h = 4)

three-dimensional elasticity solutions (Pagano 1970). In these figures, the thickness ratio of sandwich plate is considered a/h = 4. Fig. 2 shows the distributions of in-plane displacements through the thickness of the point (x = 0, y = b/2) of sandwich plate. As shown in this figure, the displacement continuity between each face sheet-core interface is satisfied and the results are compatible with elasticity solutions (Pagano 1970). The distributions of in-plane stresses through the thickness of the mid-point (x = a/2, y = b/2) of sandwich plate is shown in Fig. 3. This figure shows that the in-plane stresses are discontinues at the face sheet-core interfaces. Also, it can be observed that the in-plane stresses in the core are very small in comparison with those obtained in the face sheets. The results shown in this figure are in good agreement with the results obtained by elasticity solutions (Pagano 1970).

The through thickness distribution of in-plane shear stresses of the corner-point (x = 0, y = 0) for the sandwich plate is shown in Fig. 4 and compared with the elasticity solution results(Pagano, 1970). Similar to the in-plane stresses, the in-plane shear stresses distribution is also discontinue at the face sheet-core interfaces and are negligible in the core. Figs. 5 and 6 show the through thickness distribution of transverse shear stresses τ_{xz} and τ_{yz} of the edge-points (x = 0, y = b/2) and (x = a/2, y = 0) for the sandwich plate, respectively. As shown in these figures, the transverse shear stresses distribution is continues in the face sheet-core interfaces and are zero in the upper and lower surfaces of the sandwich plate. Also, comparison of these results with the elasticity solutions results (Pagano 1970) shows that the results obtained by present analysis are very accurate.

The non-dimensional central deflections at the mid-plane of un-symmetric rectangular sandwich plate [0º/90º/Core/0º/90º] subjected to a doubly sinusoidal transverse loading are presented and compared with the published results Table 2. In this example, the thickness of each face sheet is considered as 0.1 h, whereas the thickness of the core is considered as 0.8 h. The maximum error of all the results is less than 0.5 percent.

Figs. 7 and 8 show the distribution of non-dimensional deflections along the length of symmetricthree-layer and un-symmetric five-layer square sandwich plates, respectively. The plates are subjected to a doubly sinusoidal transverse loading on the upper surface of the sandwich plate. Distributions of vertical deflectionsare obtained at the lines $(y = b/2,\ Z_i = 0,\ i = t, b, c)$ for each face sheet and the core separately. These figures show that the transverse flexibility of the core causes to create different pattern for deflection of the face sheets and the core. Also, it can be seen that the deflection of the top face sheet is larger than the others and the deflection of the core and the bottom face sheet are close together.

In addition, the throughthickness distributions of non-dimensional displacement and stress components for square sandwich platesubjected to a uniform transverse loading $q(x, y) = q_0$ are shown in Figs. 9-13. The plate has three layers [0º/*Core*/0º] and the thickness of each face sheet is 0.1 h, whereas the thickness of the core is 0.8 h. In these figures,various thickness ratios (a/h = 50, 20, 10 and 4) are analyzed. Fig. 9, shows the throughthickness distributions of in-plane

Table 2 Non-dimensional central deflection at the mid-plane of a simply supported un-symmetric sandwich plate [0/90/Core/0/90] under sinusoidal load

	b/a = 1, a/h = 10	b/a = 1, a/h = 5	b/a = 2, a/h = 10	b/a = 2, a/h = 5
Present	1.7282	4.2564	3.1954	7.3850
Pagano (1970)	1.7272	4.2447	3.1944	7.3727
Pandit (2008)	1.7270	4.2577	3.1923	7.3795

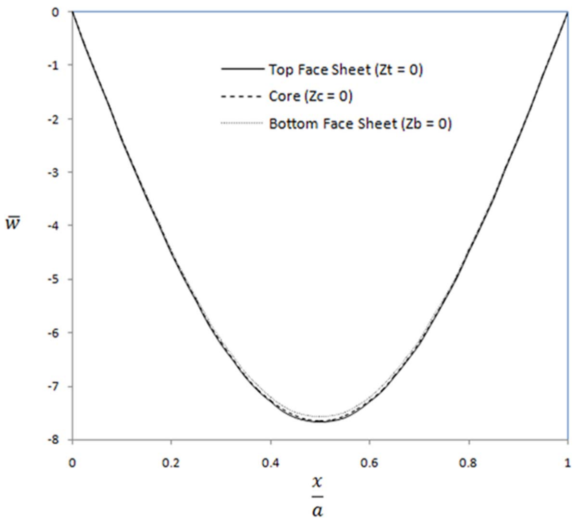

Fig. 7 Distributions of non-dimensional deflections along the length of three-layer symmetric square sandwich plate [0/Core/0] subjected to doubly sinusoidal loads (*a/h* = 4)

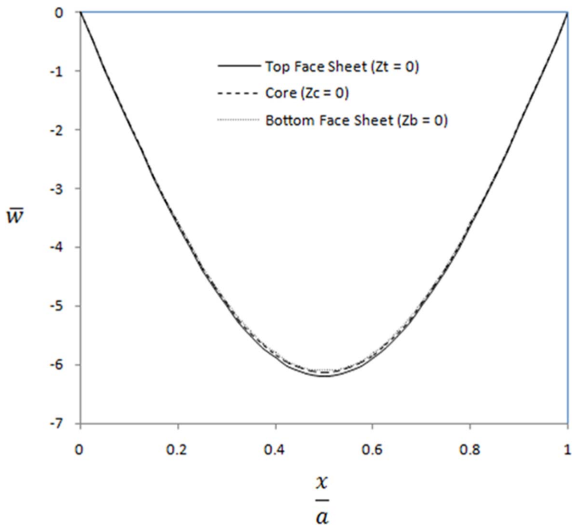

Fig. 8 Distributions of non-dimensional deflections along the length of five-layer un-symmetric square sandwich plate [0/90/Core/0/90] subjected to doubly sinusoidal loads (*a/h* = 4)

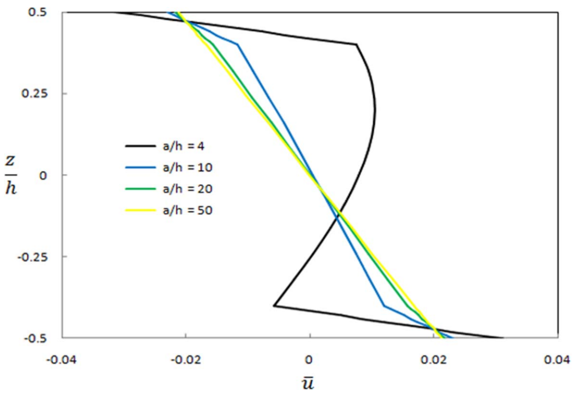

Fig. 9 Distributions of in-plane displacements through the thickness of three-layer sandwich plate [0/Core/0] subjected to uniform loads

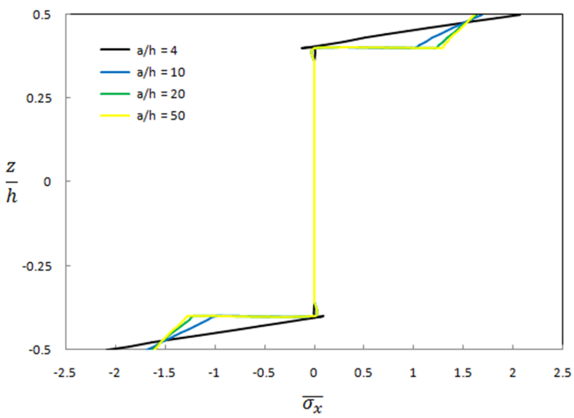

Fig. 10 Distributions of in-plane stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to uniform loads

displacement of sandwich plate. This figure shows that the in-plane displacement of the face sheet varies linearly through the thickness. As shown in Fig. 9, for high thickness ratios, the variation of in-plane displacement of the core can be assumed to be linear in which the slop of these lines decreases with increase in the thickness ratios. Also, for thick sandwich plate (*a/h* = 4), the in-plane displacement of the core varies cubically through the thickness of the plate. Figs. 10 and 11 show the in-plane stress and shear stress through the thickness of the sandwich plate, respectively. These figures show that the in-plane stress and shear stress of the face sheets vary linearly through the thickness of the plate. It can be seen that the slop of these lines increase with the increase in

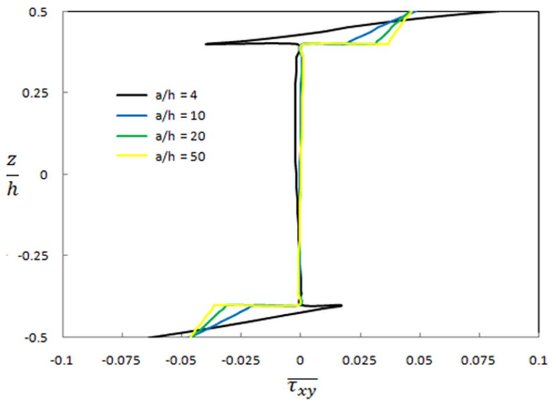

Fig. 11 Distributions of in-plane shear stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to uniform loads

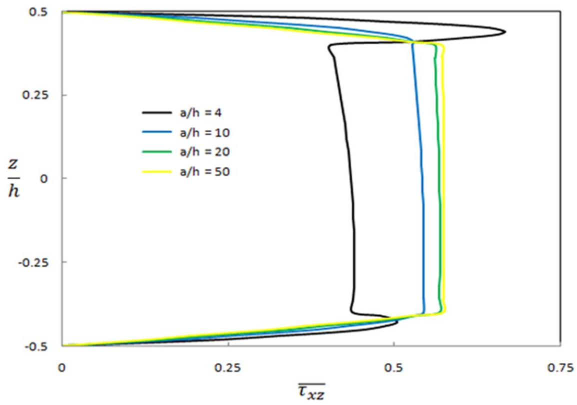

Fig. 12 Distributions of transverse shear stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to uniform loads

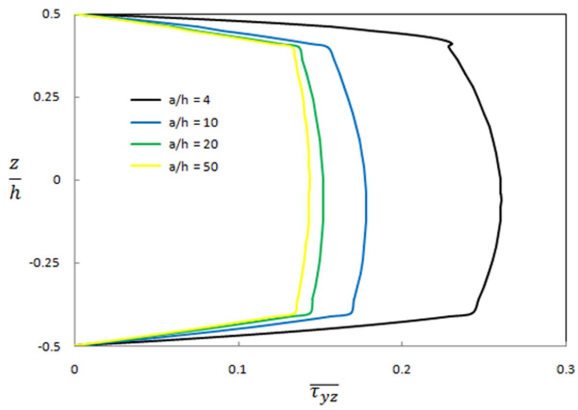

Fig. 13 Distributions of transverse shear stresses through the thickness of three-layer sandwich plate [0/Core/0] subjected to uniform loads

thethickness ratios. Also, these figures show that the in-plane stress and shear stress of the soft core are vey smaller than those ofthe face sheets. Figs. 12 and 13 show the transverse shear stress through the thickness of the sandwich plate, respectively.As shown in these figures, the transverse shear stresses distribution is continues in the face sheet-core interfaces and are zero in the upper and lower surfaces of the sandwich plate.

5. Conclusions

In this paper, an improved high-order theory was used for bending analysis of soft-core sandwich plates. Analytical solution for bending analysis of simply supported sandwich plates under various transverse loads are presented using Navier's solution. The used theory can be satisfied the continuity conditions of transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and lower surfaces of the plate. Also, transverse flexibility and transverse normal strain and stress of the core are considered. Some non-dimensional throughthickness displacements and stresses were obtained for symmetric and un-symmetric square sandwich plates subjected to sinusoidal transverse loadingand compared with the elasticity solutions. It can be drawn from the results that the presented theory is in excellent agreement with elasticity solution. Also, distributions of non-dimensional throughthickness displacements and stresses were presented for symmetric square sandwich plates subjected to uniform transverse loading.

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